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LINEAR PROGRAMMING ASSIGNMENT ONE.

QUESTION

(a) If $X_1 = 1, X_2 = 2, X_3 = 1, X_4 = 1$ is a feasible soln of the system of linear equations

$$11X_1 + 2X_2 - 9X_3 + 4X_4 = 10$$

$$15X_1 + 3X_2 - 12X_3 + 6X_4 = 15$$

Reduce the above feasible solution to basic feasible solutions [10mks]

q. $11X_1 + 2X_2 - 9X_3 + 4X_4 = 10$
 $15X_1 + 3X_2 - 12X_3 + 6X_4 = 15$

$A = \begin{pmatrix} 11 & 2 & -9 & 4 \\ 15 & 3 & -12 & 6 \end{pmatrix} \cdot \underline{x} = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{pmatrix}$ and $b = \begin{pmatrix} 10 \\ 15 \end{pmatrix}$

$\underline{q}_1 = \begin{pmatrix} 11 \\ 15 \end{pmatrix}$ $\underline{q}_2 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ $\underline{q}_3 = \begin{pmatrix} -9 \\ -12 \end{pmatrix}$ $\underline{q}_4 = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$ $(X_1, X_2, X_3, X_4) = (1, 1, 1, 1)$

exist 4 scalars s_1, s_2, s_3, s_4 such that $s_1 \underline{q}_1 + s_2 \underline{q}_2 + s_3 \underline{q}_3 + s_4 \underline{q}_4 = 0$

$\Rightarrow s_1 \begin{pmatrix} 11 \\ 15 \end{pmatrix} + s_2 \begin{pmatrix} 2 \\ 3 \end{pmatrix} + s_3 \begin{pmatrix} -9 \\ -12 \end{pmatrix} + s_4 \begin{pmatrix} 4 \\ 6 \end{pmatrix} = 0$

$\Rightarrow \begin{pmatrix} 11s_1 + 2s_2 - 9s_3 + 4s_4 \\ 15s_1 + 3s_2 - 12s_3 + 6s_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \begin{cases} 11s_1 + 2s_2 - 9s_3 + 4s_4 = 0 \\ 15s_1 + 3s_2 - 12s_3 + 6s_4 = 0 \end{cases}$

(b) Find the optimum solution of the following programming problem:

$$\text{Maximize } Z = 2X_1 - 3X_2 + X_4$$

$$\text{Subject to } 3X_1 + 2X_2 + X_3 = 15$$

$$2X_1 + 4X_2 + X_4 = 8$$

$$X_1, X_2, X_3, X_4 \geq 0$$

[10mks]

TOTAL : [20mks]

b. Maximize $Z = 2X_1 - 3X_2 + X_4$
s.t $3X_1 + 2X_2 + X_3 = 15$
 $2X_1 + 4X_2 + X_4 = 8$
 $X_1, X_2, X_3, X_4 \geq 0$

$A = \begin{pmatrix} 3 & 2 & 1 & 0 \\ 2 & 4 & 0 & 1 \end{pmatrix}$, $X = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{pmatrix}$, $b = \begin{pmatrix} 15 \\ 8 \end{pmatrix}$

$a_1 = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$, $a_2 = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$, $a_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $a_4 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$B_1 = (a_1, a_2) = \begin{pmatrix} 3 & 2 \\ 2 & 4 \end{pmatrix}$, $B_2 = (a_1, a_3) = \begin{pmatrix} 3 & 1 \\ 2 & 0 \end{pmatrix}$, $B_3 = (a_1, a_4) = \begin{pmatrix} 3 & 0 \\ 2 & 1 \end{pmatrix}$

$B_4 = (a_2, a_3) = \begin{pmatrix} 2 & 1 \\ 4 & 0 \end{pmatrix}$, $B_5 = (a_2, a_4) = \begin{pmatrix} 2 & 0 \\ 4 & 1 \end{pmatrix}$, $B_6 = (a_3, a_4) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$|B_1| = 8$, $|B_2| = -2$, $|B_3| = 3$, $|B_4| = -4$, $|B_5| = 2$, $|B_6| = 1$

i. $3X_1 + 2X_2 = 15$
 $2X_1 + 4X_2 = 8$
 $\Rightarrow X_1 = 5.5$
 $X_2 = -0.75$

ii. $3X_1 + X_3 = 15$
 $2X_1 + 0X_3 = 8$
 $\Rightarrow X_1 = -3.5$
 $X_3 = 3$

iii. $3X_1 + 0X_4 = 15$
 $2X_1 + X_4 = 8$
 $X_1 = 5$
 $X_4 = -8$