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Introduction

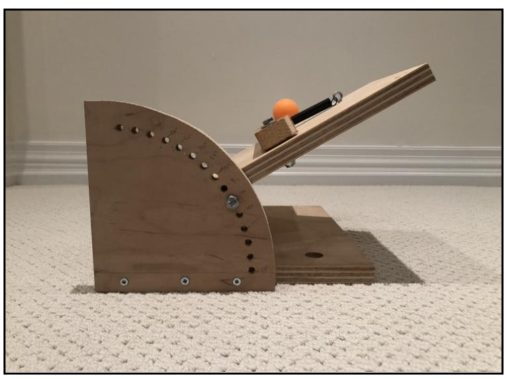
At what angle would you launch a non-ideal projectile to optimize for its range and would this result (angle) change using different launch speeds? An ideal projectile is one which is only affected by the force of gravity (appreciably) once launched. A non-ideal projectile is affected by the force of air resistance alongside the force of gravity and therefore has different flight characteristics.

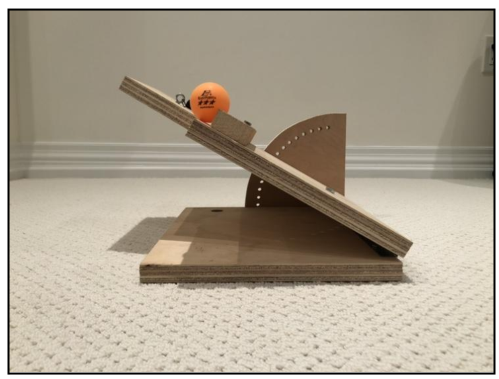
The range of an ideal projectile can be described by: $\Delta x = sin(2\Theta)v^2/g$, where v is the initial velocity (just after launch) and θ represents the launch angle above the horizontal. To optimize the range, we find the angle which would yield the maximum value of sin, meaning the ideal launch angle would be 45°.

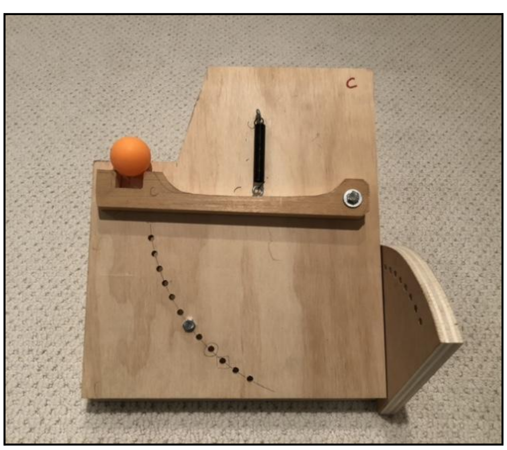
The ideal launch angle for a ping-pong ball (non-ideal projectile) was discovered experimentally, through a series of trials (launches) at a consistent launch speed (≈6.0 m/s). To determine if/how the ideal angle would change among different factors, the trials were repeated at an increased launch speed (≈8.2 m/s). Through the analysis of the gathered data, a mathematical model of the launch range was developed. This model factored in the force of air resistance (represented by the constant K) and the initial velocity (U) of the projectile to calculate the range (Δx).

Procedure

The launcher was set up at a fixed position on the ground and rotated to account for the overextension of the launcher arm. Rotation was necessary to ensure the ping-pong ball remained parallel to the camera during its flight. A 30cm (0.3m) ruler was taped to the wall to allow for later calibration in *Physics Tracker*. The ball was launched at angles between 23° and 68°. Two trials (launches) were recorded for each angle and the trial in which the ball had a more parallel flight path was used to collect the final data. Each video was uploaded into *Physics Tracker* to collect detailed x and y position data as the projectile flies through the air. The process was repeated with two different launch speeds (≈6.0 m/s, ≈8.2 m/s).



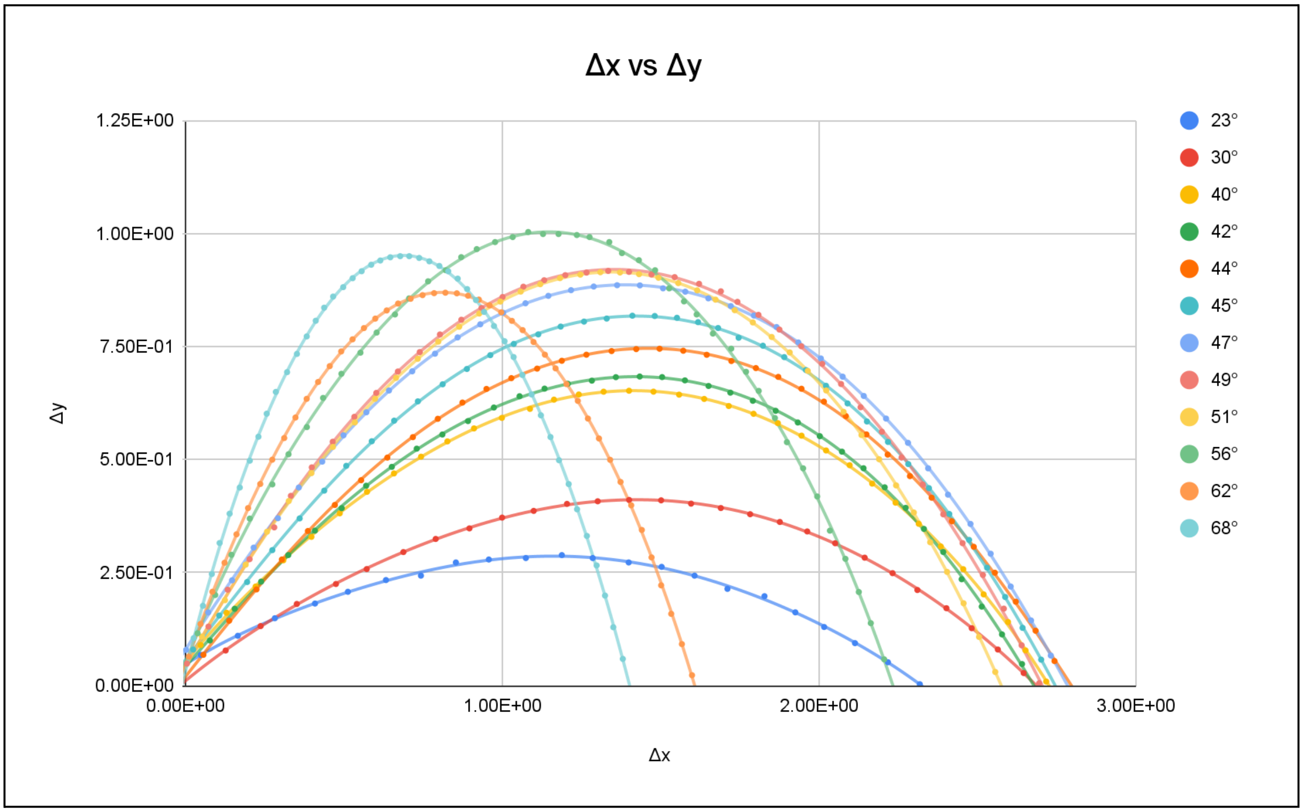




Graph and Data

Primary Task Experimental Data

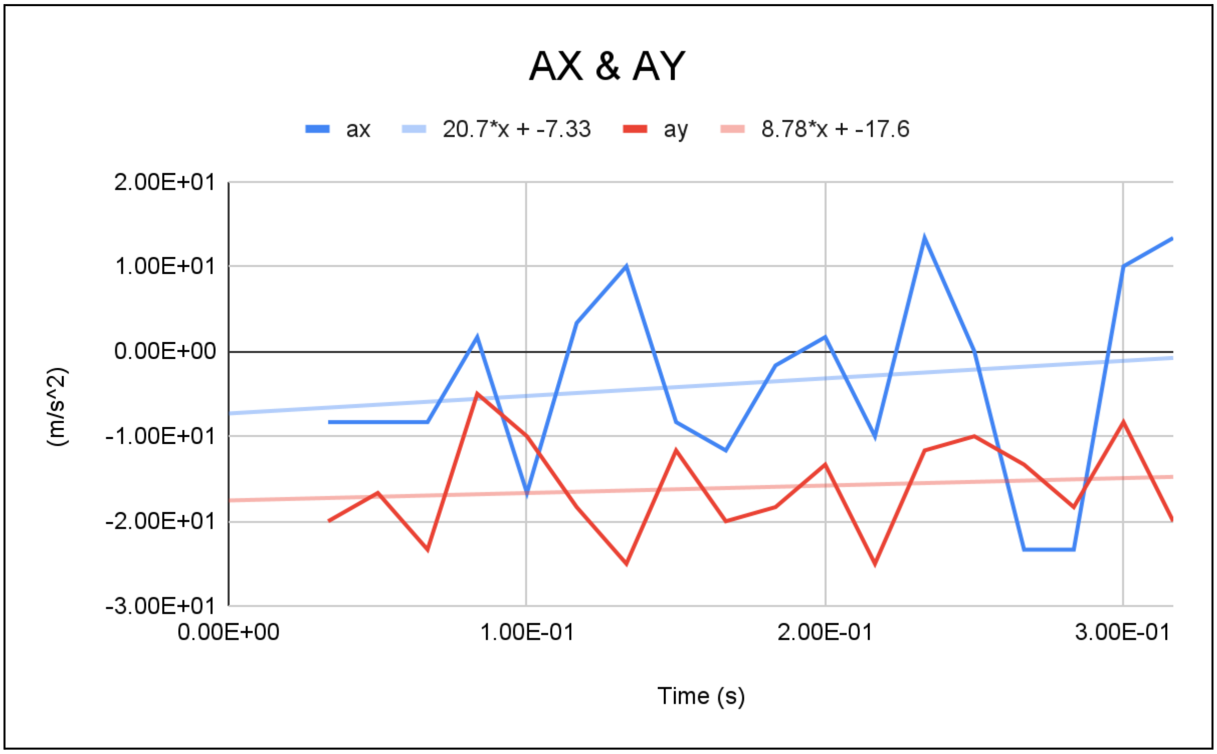
Figure 1. Graph showing the horizontal and vertical range (Δx and Δy) from each trial (angle) for first launch speed (≈6.0 m/s).



Based on the graph the ideal launch angle to optimize for range (Δx) is 44° ± 2°, as indicated by the bright orange flight path. The ping-pong ball travelled at a total horizontal distance of approximately 2.74m.

Primary Task Theoretical Data

Figure 2. Graphs showing a change in horizontal and vertical acceleration (ax and ay) over time.



The first observation made in the trials was that there was an acceleration on the x-component of the motion (**Figure 2**). This is something that would not be present in the motion of an ideal projectile. Because there is an acceleration, we can affirm it is caused by a force, the force of air resistance.

Figure 3. Graph showing horizontal acceleration (ax) over horizontal velocity (ay).

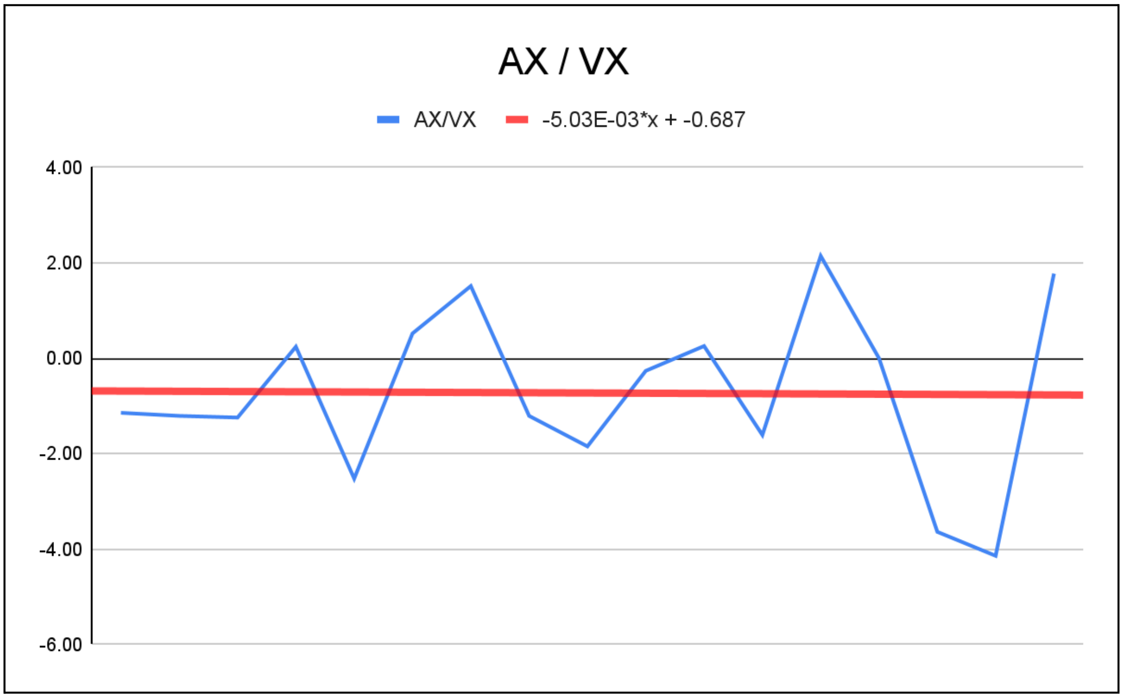
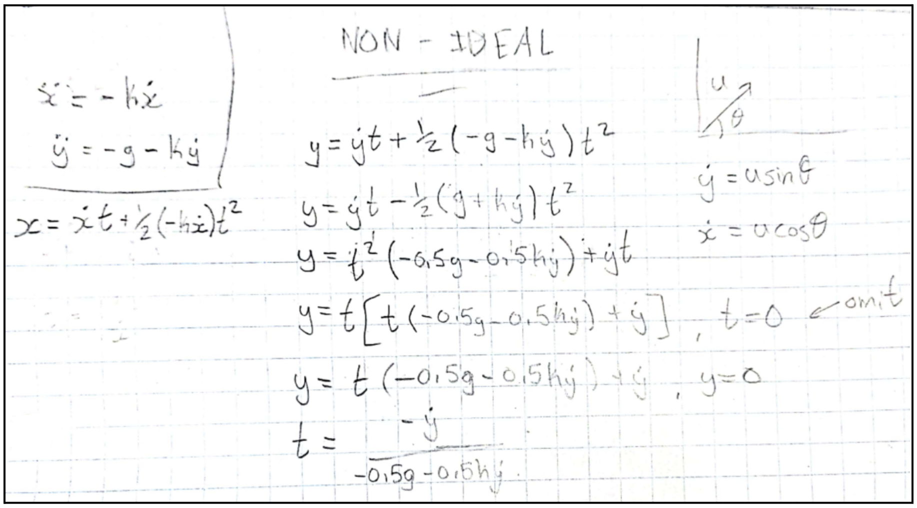


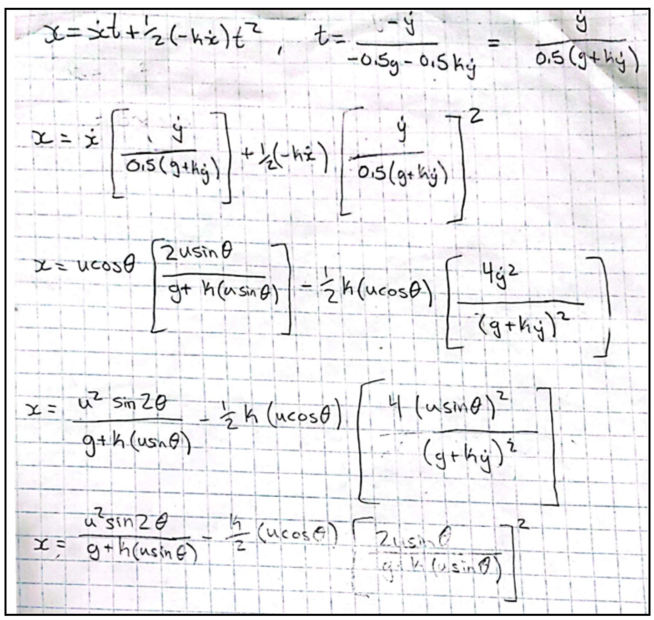
Figure 4. Table showing average data samples used to calculate constant, K, for air resistance.

CALCULATING AIR RESISTENCE PROPORTION (k)				
ax	-ax	vx		k=-ax/vx
-7.33	7.33	7.01E+00		1.05E+00
-7.27	7.27	6.73E+00		1.08E+00
-3.19	3.19	6.20E+00		5.14E-01
-1.05	1.05	5.20E+00		2.02E-01
-1.67	1.67	4.85E+00		3.44E-01
-0.812	0.812	5.02E+00		1.62E-01
-2.71	2.71	5.10E+00		5.31E-01
0.924	-0.924	4.34E+00		-2.13E-01
1.85	-1.85	3.85E+00		-4.81E-01
			AVERAGE	3.54E-01
				≈0.4

Analysis of the acceleration was done on the x-component, where the force of air resistance was isolated. An observation made about the acceleration on the x-component was that it is directly proportional to the velocity on the x-component. This is shown in **Figure 3** as the trendline has a slope ≈0. This means that the greater the speed of the ball, the greater the acceleration to slow it down. We also know that the direction of the force of air resistance is opposite to the direction of the velocity. With this, the proportionality constant, K, was calculated using the average of all data samples (**Figure 4**).

Figures 5.1 / 5.2. Derivation of the formula that will predict the range of the projectile given air resistance constant (K), launch speed, and angle.





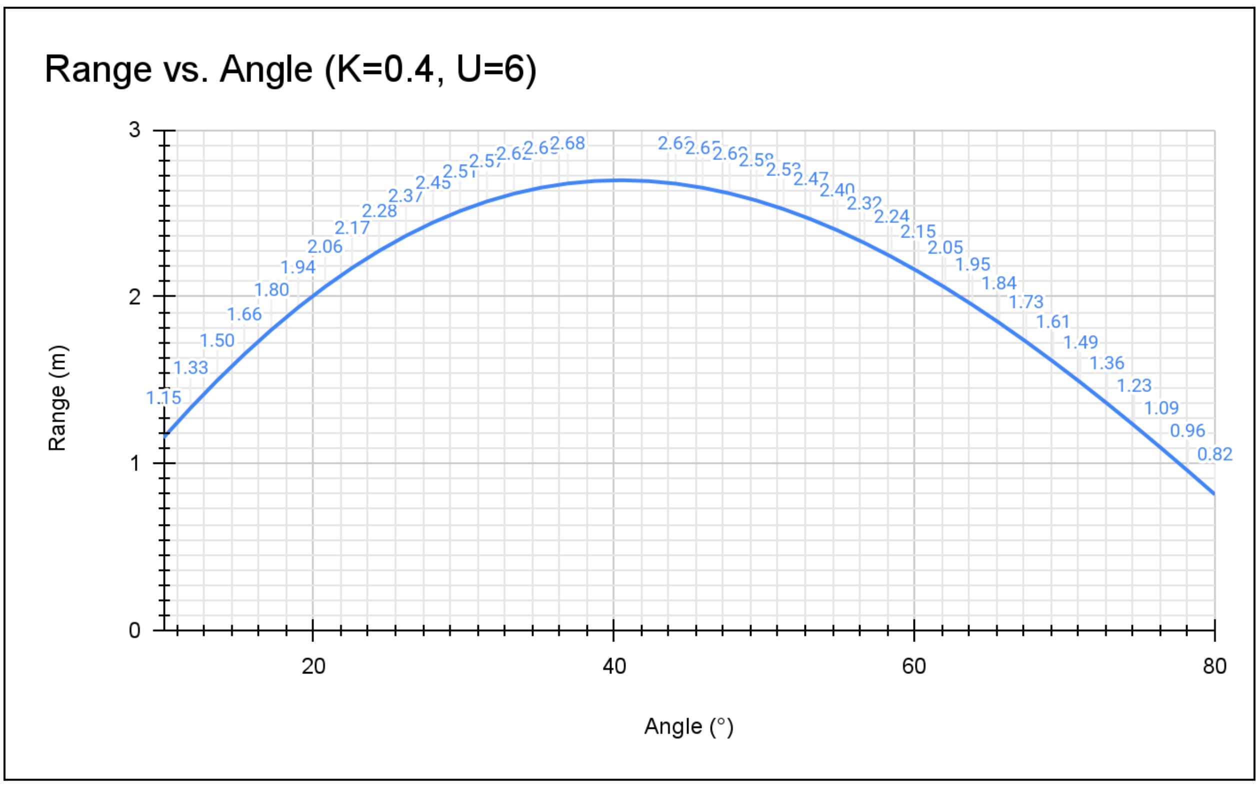
Now that we have defined the magnitude and direction of the force of air resistance, it is possible to quantify it. Using foundational theory of projectile motion, and the findings thus far, an equation for the range of the projectile was derived (**Figure 5**).

Figure 6. Code used to generate the predicted projectile range based on its angle, K, and launch speed. Data is outputted into a spreadsheet file.

```
// Constants for output
public static double AIR_K = 0.4;
public static double NUM_SHOTS = 40;
public static double START_ANGLE = 10;
public static double END_ANGLE = 80;
public static double LAUNCH_SPEED = 8.2;

public static void main(String[] args) throws IOException {

    // Create file
    f = createFile();

    // Create header line
    f.write("Angle,Range\n");

    // Loop for every angle
    double delta = (END_ANGLE-START_ANGLE) / (NUM_SHOTS-1);
    for (int i = 0; i < NUM_SHOTS; i++) {
        double angle = START_ANGLE + delta*i;

        // Calculate the range
        double range = calcRangeFromAngle(angle);

        // Add to csv
        f.write("" + angle + "," + range + "\n");

    }

    // Close stream
    f.close();
}

// Calculate the range based on angle
public static double calcRangeFromAngle(double a) {
    double rads = Math.toRadians(a);

    double temp1 = Math.pow(LAUNCH_SPEED, 2) * Math.sin(2 * rads);
    temp1 /= 9.81 + AIR_K * LAUNCH_SPEED * Math.sin(rads);

    double temp2 = -0.5 * AIR_K * LAUNCH_SPEED * Math.cos(rads);

    double temp3 = 4 * Math.pow(LAUNCH_SPEED * Math.sin(rads), 2);
    temp3 /= Math.pow(9.81 + AIR_K * LAUNCH_SPEED * Math.sin(rads), 2);

    double out = temp1 + (temp2*temp3);

    return out;

}
```

The process of finding the ideal launch angle given a velocity and K would involve calculating the range for every angle and choosing the highest one. To automate this process and help visualize the data, a computer program was written to output the results as a spreadsheet file. The program solves the equation derived previously for all angles within a range. (**Figure 6**).

Figure 7. Graphs displaying data generated by the program to show the predicted range of the projectile for any given angle, including the optimized launch angle.



The theoretical data (**Figure 7**) suggests that the optimized launch angle is 43°, which is within the margin of error for our experimental results.

Secondary Task Theoretical/Experimental Data

Figure 8. Computer-generated graph displaying the predicted range of the projectile at any given angle (including ideal launch angle) at an increased launch speed (8.2m/s).

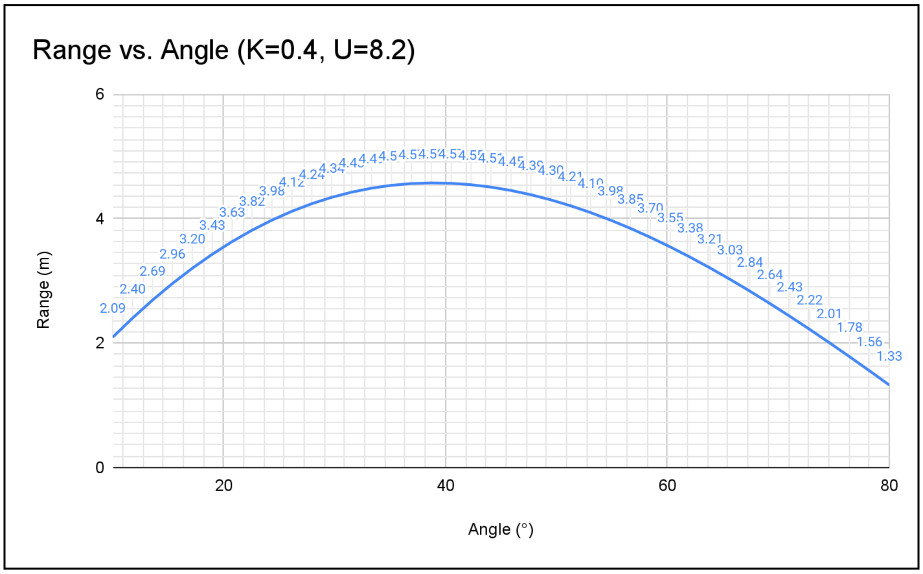
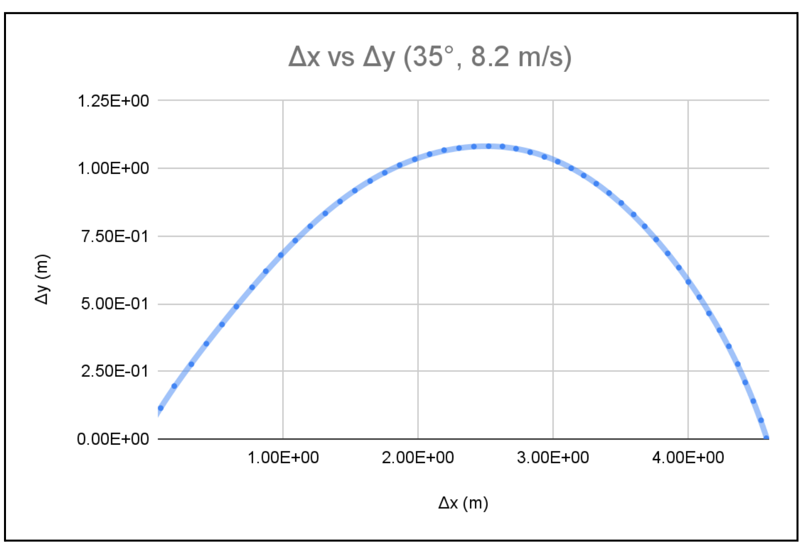


Figure 9. Graph showing ball trajectory (Δx vs Δy) at 35° and at an increased launch speed of 8.2 m/s.



Based on **Figure 8**, the predicted angle to optimize for range at this increased launch speed is 37°. Through various experimental trials at this velocity, the most optimal launch angle was found to be 35° ± 2° (**Figure 9**). Therefore, the mathematical model proved to be accurate when compared to measured values.

Primary Goal

Through testing and analysis of the x and y position data the angle to optimize for the range of the ping-pong ball is 42° ± 2°. The theoretical data was found to be accurate as it predicted the launch range for all trials within 20cm (0.2m).

Errors present during the experiment include

- Camera not being perfectly parallel to the ball launch, causing a distorted view
- *Physics Tracker*
    - Setting a consistent origin
    - Plotting the points on the ball's motion
    - Accurately setting the calibration length

Secondary Goal

Using the mathematical model, the launch angle to optimize for range (Δx) at an increased launch speed (8.2m/s) was calculated to be 37°. This angle was confirmed through experimental testing and analysis where the optimal angle was 35° ± 2°. The theoretical data was again found to be accurate as it predicted the launch range within

20cm (0.2m). Errors present during the experiment remain consistent with those present during the primary task.

Conclusion

In conclusion, the ideal angle for our ping pong ball (non-ideal projectile) was found to be approximately 43°. This value was proven experimentally and was reaffirmed by its consistency with our mathematical model (**Figure 5**), based on the force of air resistance being opposite in direction and proportional in magnitude to velocity. Using this model (**Figure 5**), we were also able to successfully predict the optimal angle to launch the object at a higher launch speed. When an object is affected by air resistance, a greater velocity or higher constant K, will result in a lower optimized launch angle. The analysis of non-ideal projectiles is relevant as real-world projectiles deviate from the ideal conditions that are assumed in many physics equations and models.