DIFFRACTION

Q.1) Define diffraction? (Short Answer question)

When waves encounter obstacles or small apertures, they apparently bend round the edges of the obstacles if the dimensions of the obstacles are comparable to the wavelength of the waves. The apparent bending of waves around the edges of an obstacle (or aperture) is called diffraction.

Q.2) Define diffraction and explain diffraction phenomena?

When waves encounter obstacles or small apertures, they apparently bend round the edges of the obstacles if the dimensions of the obstacles are comparable to the wavelength of the waves. The apparent bending of waves around the edges of an obstacle (or aperture) is called diffraction.

Fresnel explained diffraction using Huygen's principle of secondary wavelets in conjunction with the principle of superposition. The diffraction phenomenon is due to mutual interference of secondary wavelets originating from various parts of a wave front which are not blocked off by the obstacle.

Diffraction sets a limit to the image formation ability of optical instruments. Diffraction phenomenon demonstrates wave behavior of light.

Q.3) Distinguish between Fresnel and Fraunhoffer classes of diffraction. (Short Answer question)

Fresnel's diffraction:

In this type of diffraction, the source of light or screen or both are at finite distances from the obstacle or aperture. The incident wave front is either spherical or cylindrical. As a result, the phase of secondary wavelets is not the same at all points in the plane of the aperture. No lenses are used to make the rays parallel or convergent. The treatment of Fresnel diffraction is mathematically complex.

Fraunhoffer diffraction:

In this class of diffraction, the source of light and the screen are effectively placed at infinite distances from the aperture. This may be achieved by using two convex lenses. The incident wave front is plane. As a result, the secondary wavelets are in the same phase at every point in the plane of aperture. Fraunhoffer diffraction is a special case of the more general Fresnel diffraction and is easier to handle mathematically.

Q.4) Distinguish between interference and diffraction. (Short Answer question).

Interference	Diffraction
1) Interference is the result of interaction of light coming from two different wave fronts originating from the same source.	1) Diffraction is the result of interaction of light coming from different parts of the same wave front.
2) Interference fringes may or may not be of the same width.	2) Diffraction fringes are not of the same width.
3) Points of minimum intensity may or may not be perfectly dark.	3) Points of minimum intensity are not perfectly dark.
4) All bright bands are of uniform intensity.	4) All bright bands are not of the same intensity.

Q.5) Explain the diffraction due to a single slit. Establish the condition for minimum intensity.

Fig. 1 shows a plane wave falling at normal incidence on a long narrow slit of width *a*. At the central point P_0 of the screen *C*, the parallel rays extending from the slit have the same optical paths. They are converged to the point P_0 by the lens. Since they are in phase at the plane of the slit, they will be in phase at P_0 and the central point of the diffraction pattern that appears on the screen *C* has a maximum intensity and is called zero order central maxima.

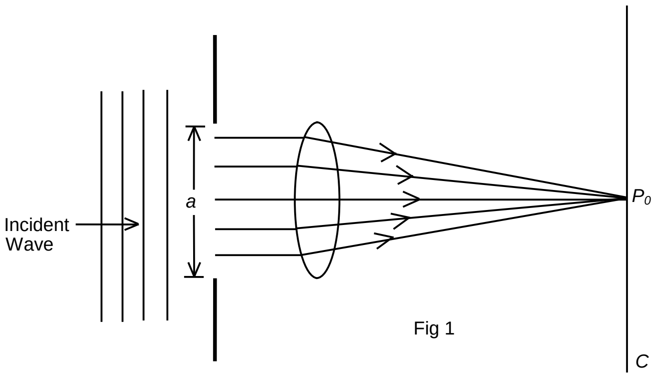


Fig 1

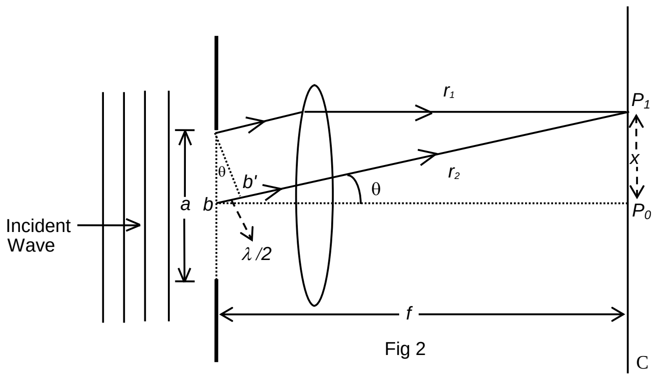


Fig 2

Consider another point P_1, on the screen as shown in Fig. 2. To study the intensity at the point P_1, let us divide the width of the slit into two halves. Let ray r_1 originate from the top of the upper half and the ray r_2 originate from the top of the lower half of the slit.

The path difference between these two rays and hence the phase difference will decide the intensity at P_1. If P_1 is so chosen, that is, If θ is so chosen that the distance bb' is a half wavelength, r_1 and r_2 will be out of phase and will produce no effect at P_1. Likewise, every ray from the upper half of the slit will be cancelled by a corresponding ray from the lower half. For the point P_1, the first minimum of the diffraction pattern, the condition for zero intensity is

$$\frac{a}{2} Sin\,\theta = \frac{\lambda}{2} \quad \Rightarrow a Sin\,\theta = \lambda$$

In Fig. 3, the slit is divided into four equal zones, with a ray leaving the top of each zone. Let θ be chosen so that the distance bb' is one half a wavelength. Rays r_1 and r_2 will then cancel at P_2. Rays r_3 and r_4 will also be half a wavelength out of phase and will also cancel. Hence in this manner one can proceed across the entire slit and conclude that no light reaches P_2, which means that we have located a second point of zero intensity.

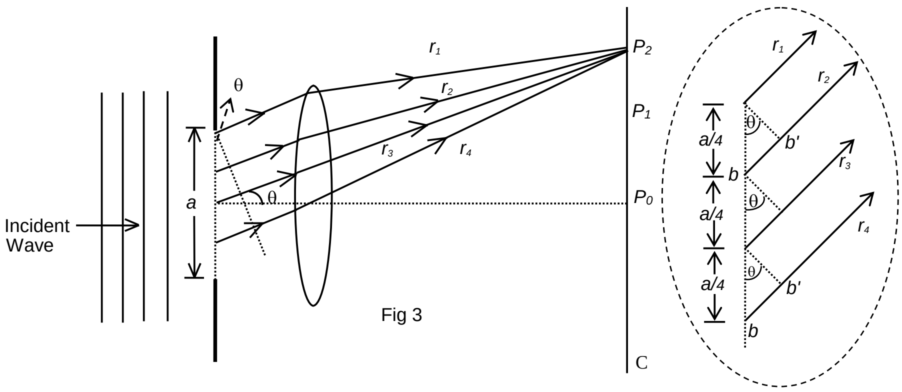


Fig 3

$$\therefore \frac{a}{4} Sin\,\theta = \frac{\lambda}{2} \quad \Rightarrow \quad a\, Sin\,\theta = 2\lambda$$

Arguments similar to the above show that a point of zero intensity occurs whenever

$Sin\,\theta = \frac{2\lambda}{a}, \frac{3\lambda}{a}, \frac{4\lambda}{a}, \frac{5\lambda}{a}$ …… etc. They are known as second order minimum, third order minimum, etc.

Hence, by extension the general formula for the minima in the single slit diffraction pattern can be written as $a\, Sin\,\theta = m\lambda,\ m = 1,2,3$ ……

In addition to the central maximum there are secondary maxima, which lie in between the secondary minima on either side of the central maximum. Since the first order minima occurs at an angle $\sin^{-1}\left(\frac{\lambda}{a}\right)$ and the second order minima at an angle $\sin^{-1}\left(\frac{2\lambda}{a}\right)$, it is natural to expect first order maxima half way between them, at $\theta = \sin^{-1}\left(\frac{3\lambda}{a}\right)$. To arrive at this condition, we imagine the wave front to be divided into three equal parts. The waves from the extreme ends of the upper two parts will have a phase difference of $\lambda/2$. The waves from these two parts produce darkness. The waves from the third portion are not cancelled and thus produces a weak maximum. As θ increases, still weaker maxima with rapidly falling of intensity are observed at $5\,\lambda/2\,a$,$7\,\lambda/2\,a$ …….. etc.

In general, the secondary maxima are given by

$$Sin\,\theta = \left(\frac{2\,m+1}{a}\right)\frac{\lambda}{2}$$

Thus, the diffraction pattern due to a single slit consists of a central bright maximum flanked by secondary maxima & minima on both sides.

Q.6) Derive an expression for the intensity distribution at a point on a screen due to Fraunhoffer diffraction at a single slit?

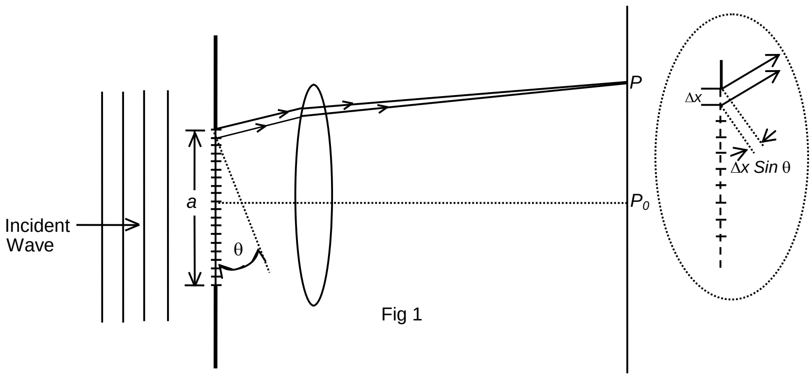


Fig 1

Fig. 1 shows a slit of width *'a'* divided into *N* parallel strips of width Δx. Each strip acts as a radiator of Huygen's secondary wavelets and produces a characteristic wave disturbance at point *P*. Each strip contributes an electric field of amplitude ΔE_0 at the point *P*. The wave disturbances from adjacent strips have a constant phase difference $\Delta\phi$ between them at *P* given by $\Delta\phi = \frac{2\pi}{\lambda}(\Delta x\, Sin\,\theta)$ where $(\Delta x\, Sin\,\theta)$ is the path difference for the rays originating at the top edges of adjacent strips.

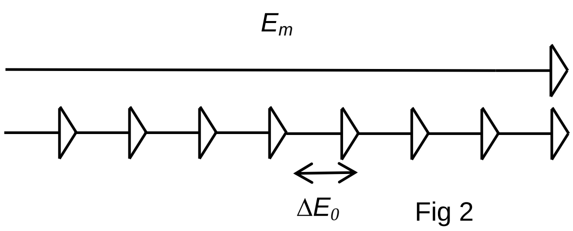


Fig 2

At the point *P*, *N* vectors with the same amplitude ΔE_0, the same wavelength λ and the same phase difference $\Delta\phi$ between adjacent strips, combine to produce a resultant disturbance. The resultant disturbance is found by representing the individual wave disturbances ΔE_0 by phasors (rotating vectors) and calculating the resultant phasor amplitude.

At the center of the diffraction pattern $\theta = 0$ and the phase shift between adjacent strips is also zero. Fig. 2 shows the phasor arrows in this case and the amplitude of the resultant has its maximum value E_m. This corresponds to the center of the central maximum.

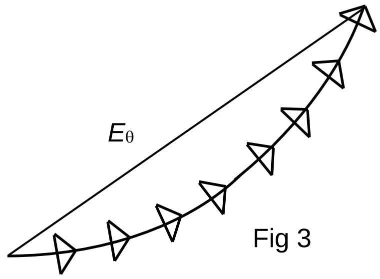


Fig 3

For any value of θ other than zero, $\Delta\phi$ assumes a definite non-zero value and the array of arrows is as shown in Fig. 3. The resultant amplitude E_θ is less than E_m.

As θ increases further, the phase shift continues to increase, and the chain of arrows curls around through 360^0, the tip of the last arrow touching the foot of the first arrow as shown in Fig. 4. This corresponds to $E_\theta = 0$, that is, to the first minimum. For this condition, the ray from the top of the slit is 180^0 out of phase with the ray from the center of the slit. The length of the arc of small arrows in all the figures shown here is the same.

As θ increases further, the phase shift continues to increase, and the chain of arrows curls around through an angle greater than 360^0. Proceeding in this way, one can reach the first maximum beyond the central maximum. This maximum is much smaller than the central maximum. In all the figures shown here, the arrows marked E_θ correspond to the amplitudes of the wave disturbances and not to the intensities. The amplitudes must be squared to obtain the corresponding relative intensities.

The equation $a \, Sin\, \theta = m\, \lambda$, $m = 1, 2, 3, \ldots$.. tells us how to locate the minima of the single-slit diffraction pattern on the screen as a function of the angle θ. The angle θ is the position locator, every screen point P being associated with a definite value of θ. Let us now find out an expression for the intensity I of the pattern as a function of θ.

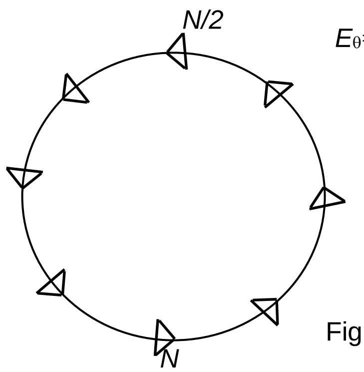
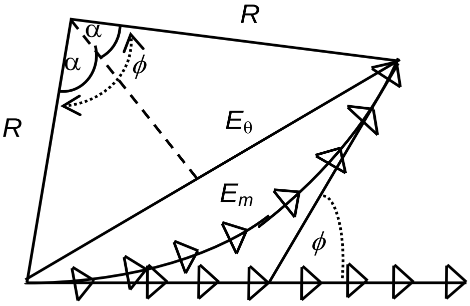


Fig 4

The arc of small arrows in Fig. 5 shows the phasors representing in amplitude and phase, the wave disturbances that reach an arbitrary point *P* on the screen, corresponding to a particular angle θ. The resultant amplitude at *P* is E_θ. If the slit is divided into infinitesimal strips of width Δx, the arc of arrows in Fig. 5 approaches the arc of a circle of radius *R* as shown. The length of the arc is E_m.

Fig 5



The angle ϕ at the bottom of Fig. 5 is the difference in phase between the infinitesimal vectors at the left and the right ends of the arc E_m. This means that ϕ is the phase difference between the rays from the top and the bottom of the single slit. From geometry we see that ϕ is also the angle between the two radii marked *R* in Fig. 5. From this figure we can write $Sin\,\frac{\phi}{2} = \frac{E_\theta/2}{R} \Rightarrow E_\theta = 2\,R\,Sin\,\frac{\phi}{2}$. In radian measure, $\phi = \frac{E_m}{R}$. Combining we get

$E_\theta = \frac{E_m}{\phi/2}\,Sin\,\frac{\phi}{2} \Rightarrow E_\theta = E_m\left[\frac{Sin\,\frac{\phi}{2}}{\phi/2}\right]$. In terms of intensity,

$$I_\theta = I_m\left(\frac{Sin\,\frac{\phi}{2}}{\phi/2}\right)^2$$

A graph of function the above function is depicted in Fig. 6. The relative intensities of secondary maxima can be obtained from the above equation.

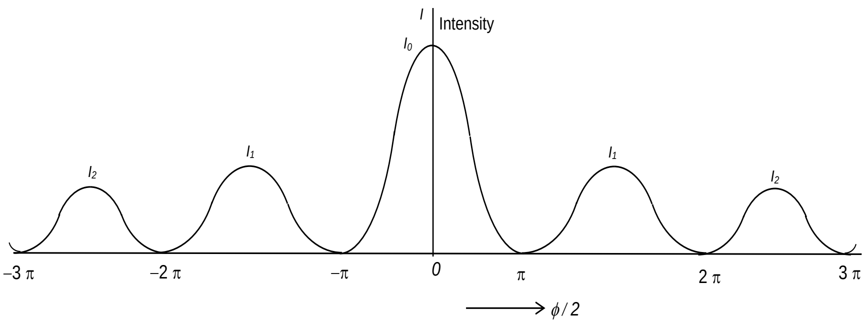
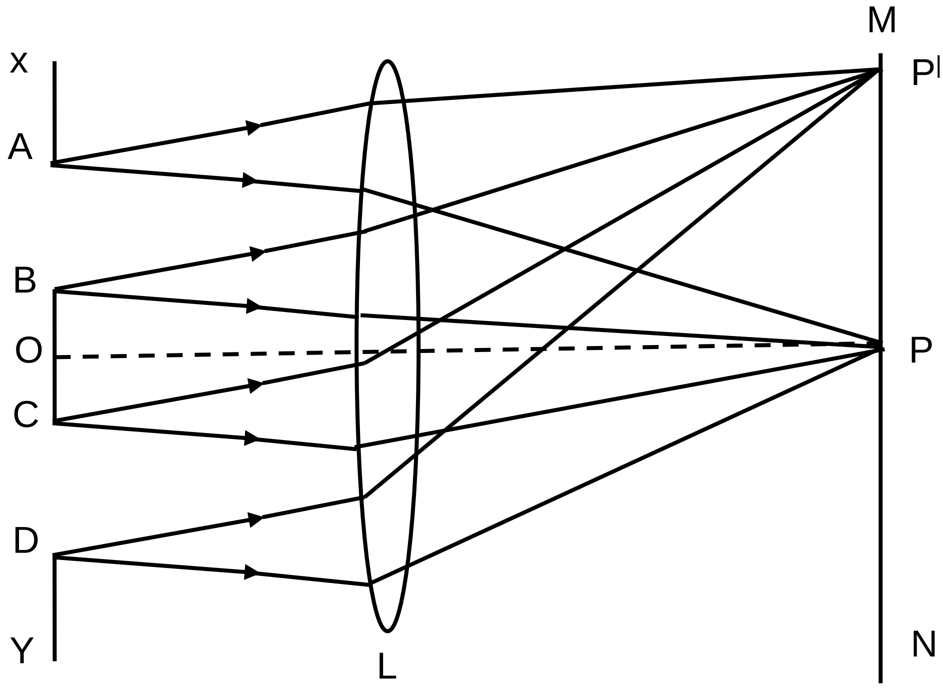


Fig 6

Q.7) Discuss the Fraunhoffer diffraction at double slit.



In the above figure, AB and CD are two rectangular slits parallel to one another and perpendicular to the plane of the paper. The width of each open portion is ‘a’ and the width of the opaque portion is ‘b’. L is a collecting lens and MN is a screen such that OP is perpendicular to the screen. Let a plane wave front be incident on the surface of XY. All the secondary waves traveling in a direction parallel to OP come to focus at P. Therefore, P corresponds to the position of the central bright maximum.

In this case, the diffraction pattern has to be considered in two parts.

1) The interference phenomenon due to the secondary waves emanating from the corresponding points of the two slits and

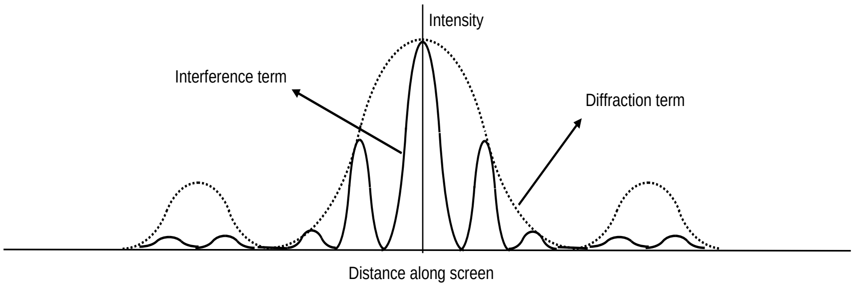
2) The diffraction pattern due to the secondary waves from the two slits individually.

The resultant intensity at a point P| is given by, $I = 4\, I_0 \left(\frac{Sin^2\, \alpha}{\alpha^2}\right) Cos^2\, \beta$

Thus, the resultant intensity at any point depends on two factors.

1) The factor $I_0 \left(\frac{Sin^2\, \alpha}{\alpha^2}\right)$ is the same as that derived for a single slit Fraunhoffer diffraction. It represents the intensity variation in the diffraction pattern due to any individual slit.

2) The factor $Cos^2\,\beta$ gives the interference pattern due to waves overlapping from the two slits.



Intensity distribution due to the Fraunhoffer diffraction at two parallel slits is shown in the above figure. In the above figure, the dotted curve represents the intensity distribution due to diffraction pattern due to double slit and the thick line curve represents the intensity distribution due to interference between the light from both the slits. The pattern consists of diffraction maxima and minima and equally spaced interference maxima and minima within each diffraction maximum.

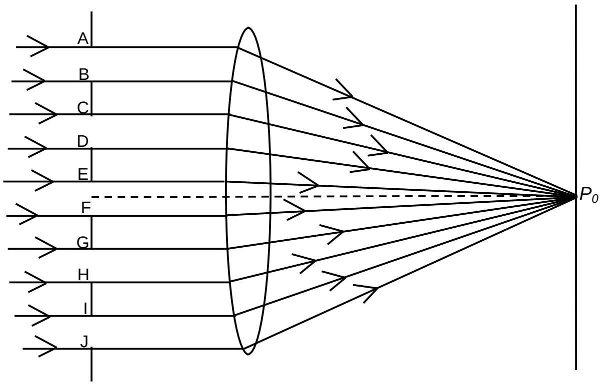
Q.8) Discuss the distinction between single slit and double slit diffraction patterns.

The single slit diffraction pattern consists of a central bright maximum with secondary maxima and minima of gradually decreasing intensity. The double slit diffraction pattern consists of equally spaced interference maxima and minima within the central maximum. The intensity of the central maximum in the diffraction pattern due to a double slit is four times that of the central maximum due to diffraction at a single slit.

In the above arrangement, if one of the slit is covered with an opaque screen, the pattern observed is similar to the one observed with a single slit. The spacing of the diffraction maxima and the minima depends on the width of the slit. The spacing of the interference maxima and minima depends on the width of the slit and width of the opaque spacing between the two slits.

Q.9) What is diffraction grating ?

When there is a need to separate light of different wavelengths with high resolution, then a diffraction grating is most often the tool of choice. A large number of parallel, closely spaced slits constitutes a diffraction grating. A device consisting of a large number of parallel slits of equal width and separated from one another by equal opaque spaces is called a diffraction grating. The distance between the centres of the adjacent slits is known as grating period.



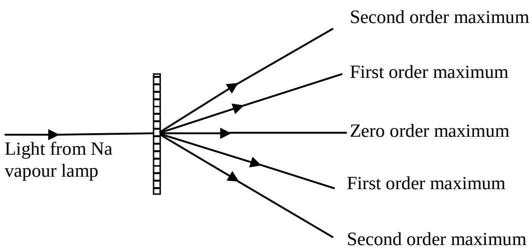
Grating is a polished surface, usually glass or metal, having a large number of very fine parallel grooves or slits, and used to produce optical spectra by diffraction of reflected or transmitted light.

The position of mth maxima is given by

$(a+b)\,Sin\,\theta_m = m\,\lambda$ where m = 0, 1, 2, 3 etc., and angles $\theta_1, \theta_2, \theta_3$ etc., correspond to the directions of the principal maxima.

The position of mth minima is given by $(a+b)Sin\,\theta_m = (2m+1)\frac{\lambda}{2}$

There will be minimum intensity in between the central maximum and the first maximum and so on. Similar maxima and minima are obtained on the other side of central maximum. Thus, on each side of the central maximum, principal maxima and minimum intensity are observed due to diffracted light.



Q.10) What is diffraction grating? (Short Answer question).

When there is a need to separate light of different wavelengths with high resolution, then a diffraction grating is most often the tool of choice. A large number of parallel, closely spaced slits constitutes a diffraction grating. A device consisting of a large number of parallel slits of equal width and separated from one another by equal opaque spaces is called a diffraction grating. The distance between the centres of the adjacent slits is known as grating period.

Q. 11) Write a shirt notes on the maximum number of orders formed by a grating. (Short Answer question).

The principal maxima in a grating satisfy the condition

$(a+b)\,Sin\,\theta \quad = m\,\lambda$ or $m = \frac{(a+b)\,Sin\,\theta}{\lambda}$

The maximum angle of diffraction is 90^0, hence the maximum possible order is given by

$$m_{max} = \frac{(a+b)\,Sin\,90^0}{\lambda} = \frac{(a+b)}{\lambda}$$

For example, consider a grating having grating element which is less than twice the wavelength of the incident light, then $(a+b) < 2\lambda$

$\therefore\ m_{max} < \frac{2\lambda}{\lambda} < 2$ i.e., only the first order is possible.

Q. 12) Write a shirt notes on missing order of diffraction in grating spectrum. (Short Answer question).

Sometimes it happens that the first order spectrum is clearly visible, second order is not visible and third order is again visible, i.e., the second order is absent, and so on. This happens when for a given angle of diffraction θ, the path difference between the diffracted rays from the two extreme ends of one slit is equal to an integral multiple of λ. Suppose the path difference is λ, then each slit can be considered to be made up of two halves, the path difference the secondary waves from the corresponding points in the two halves will be ${\lambda}/{2}$. Now they will cancel one another resulting zero intensity.

We know that, in case of a grating the principal maxima are obtained in the directions given by

$(a+b)\,Sin\,\theta \quad = m\,\lambda$

Also, in case of a single slit, the minima are obtained in the directions given by

$a\,Sin\,\theta = n\,\lambda$

If both the conditions are satisfied simultaneously, a particular maximum of order m will be missing in the grating spectrum. Dividing above equations

$\frac{a+b}{a} = \frac{m}{n}$, which is the condition of absent spectra.

If the width of the ruling is equal to the width of the slit, $\because a = b$ then $\frac{a+a}{a} = 2 \rightarrow m = 2n$, the second order spectrum will be missed.

Q.13) What is Rayleigh's criterion for resolving power of an optical instrument? (Short Answer question).

To express the resolving power of an optical instrument as a numerical value, Rayleigh proposed an arbitrary criterion. According to him, two nearly images are said to be resolved if the position of the central maximum of one coincides with the first minimum of the other and vice versa.

Q.14) Define resolving power of a grating? (Short Answer question).

The resolving power of a grating is its ability to show two neighbouring lines in a spectrum as separate. If we consider two very close spectral lines of wavelength λ and $\lambda + d\lambda$, its spectral resolution is given by

Spectral resolving power = $\frac{\lambda}{d\lambda}$, where $d\lambda$ is the smallest difference in wavelengths that can be resolved by the grating and viewed separately.

Hence the resolving power of a diffraction grating may also be defined as the ratio of the wavelength of any spectral line to its difference of wavelengths between this line and a neighbouring line such that the two spectral lines can be just seen as separate.

$\frac{\lambda}{d\lambda} = Nn$. This expression measures the resolving power of a grating. Thus the resolving power of a grating is directly proportional to the order of the spectrum and the total number of lines on the grating surface

Q.15) Define Dispersive power of a grating? (Short Answer question).

Dispersive power is the change in the angle of diffraction per unit change in wave length.

The diffraction of m^{th} order principal maximum is given by

$(a + b)\, Sin\, \theta_m = m\, \lambda$ where m = 0, 1, 2, 3 etc., and angles $\theta_1, \theta_2, \theta_3$ etc., correspond to the directions of the principal maxima.

Differentiating this equation, we get

$$(a + b)\, Cos\, \theta\, d\theta = m\, d\lambda$$

$$\frac{d\theta}{d\lambda} = \frac{m\, N}{Cos\, \theta}$$

SOLVED PROBLEMS

Example 1: Calculate the angular separation between the first order minima on either side of central maximum when the width of the slit is 1 x 10 $^{-4}$ cm and the light illuminating has wave length 6000 A^0.

The width of the slit, a = 1 x 10 $^{-4}$ cm = 1 x 10 $^{-6}$ m

The wave length of light = $\lambda = 6000\ A^0 = 6000 \times 10^{-10} m$

Order number, m = 1

The condition for minima is $a\ Sin\ \theta = m\ \lambda$

$$\theta = \sin^{-1}\left[\frac{m\ \lambda}{a}\right] = \sin^{-1}\left[\frac{1 \times 6000 \times 10^{-10}}{1 \times 10^{-6}}\right] = \sin^{-1}[0.6]$$

$\theta = 36^0 52'$

Angular separation between the first order minima on either side of central maximum = 2 θ

$2\ \theta = 73^0 44'$

Example 2: Find the highest order that can be seen with a grating having 15000 lines/inch. The wave length of light used is 600 nm.

The number of lines on grating = N = 15000 lines/inch = 5906 lines/cm = 5906 x 100 lines/m

Wave length of light = $600\ nm = 600 \times 10^{-9}\ m$

We know that, $m = \frac{(a+b)\ Sin\ \theta}{\lambda} = \frac{(a+b)}{\lambda}$

For highest order, $m_{max} = \frac{(a+b)\ Sin\ 90}{\lambda} = \frac{(a+b)}{\lambda} = \frac{1}{N\ \lambda} = \frac{1}{5906 \times 100 \times 600 \times 10^{-9}}$ = 2.82

Since m_{max} has to be integer, the highest order that can be seen is 2.

Example 3: A grating has 5 cm of surface, ruled with 6000 lines/cm. What is the resolving power of the grating in the first order?

The length of ruled surface of grating = 5 cm

Number of lines on the grating = 6000/cm

The total number of lines on the grating surface = N = 5 x 6000 = 30000

Order of the spectrum, n = 1

We know that the resolving power of grating, $\frac{\lambda}{d\lambda} = Nn$

$$\frac{\lambda}{d\lambda} = 1 \times 30000 = 30000$$

The resolving power of grating in the first order = 30000.