INTERFERENCE

Q.1) State and explain the Principle of superposition of waves (Short Answer question).

Two or more waves can traverse the same space independently of one another. In the region where they meet, the displacement of the particles of the medium is the algebraic sum of their displacements due to individual waves alone. This process of vector addition of the displacements of a particle is called the principle of superposition.

When two waves are displaced through an integral number of wavelengths, constructive interference takes place. When two waves are displaced with respect to each other by an odd number of half-wavelengths, destructive interference results.

Q.2) State and explain the Principle of superposition of waves.

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Principle of superposition is valid when the equations describing the wave motion are linear i.e., when the wave amplitudes are small. If the equations describing the wave motion are not linear, superposition principle fails. Shock waves produced by violent explosions do not obey the principle of superposition since the equation describing the wave motion is quadratic.

Using the superposition principle it is possible to analyse a complicated wave motion as a combination of simple waves.

Q.3) Define Interference? (Short Answer question)

Two or more light waves of the same frequency travelling approximately in the same direction with constant phase difference can combine to give rise to redistribution of energy in the form of maxima and minima. This type of redistribution of energy due to superposition is called interference. The series of alternate maxima and minima is called an interference pattern.

Interference provides the most convincing evidence that light is a wave.

The formation of bright and dark fringes is in accordance with the law of conservation of energy. The energy which apparently disappears at minima has actually been transferred to the maxima where the intensity is greater than that produced by the two beams acting separately.

Q.4) Discuss the techniques for producing interference of light. (Short Answer question).

To produce a pair of coherent beams of light, two techniques are used. One is the division of wave front and the second is the division of amplitude.

(1) Division of wave front:

The incident wave front is divided into two parts by using the phenomenon of reflection, refraction or diffraction. They travel unequal distances and reunite at small angle to produce interference fringes. Here point sources of light should be used.

Young's double slit experiment, Fresnel's Bi-prism, Lloyd's mirror etc., are examples for this method. Here the waves spread out by diffraction at the point sources.

(2) Division of amplitude:

The amplitude of incoming beam is divided into two parts either by partial reflection or refraction which reunite after travelling along different paths and produce interference. Here extended source of light should be used.

Thin film interference such as Newton's rings, Michelson's interferometer etc., come under this method.

Q.5) What are the essential conditions for producing interference?

(1) Conditions for sustained interference:

(a) The two sources should be coherent. Sources derived from a single source are in phase with each other or maintain a constant phase difference. Coherent beams of light produce a steady interference pattern.

(b) The two interfering waves must be of the same wavelength and periodic time and propagate approximately in the same direction.

(c) Planes of polarization of the waves must be the same. Waves polarized in perpendicular planes cannot produce interference effects.

(2) Conditions for observation:

(a) The separation between the two sources should be small. Large separation leads to smaller fringe width with loss of visibility.
(b) The distance between the sources and screen should be large. If this distance is small then the fringe width will be very small and the fringes will not be separately visible.
(c) The background should be dark. If the source is not strong, the fringes happen to have low intensity losing clarity against bright background.

(3) Conditions for good contrast:

(a) The two sources should be very narrow. A broad source may be thought of as a group of sources with different frequencies / wavelengths so that superposition of light from any pair cannot give an interference pattern.

(b) The sources should be monochromatic. The fringe width β depends upon the wavelength of light. If the source is monochromatic, β will be constant and hence fringes of good intensity can be observed. If the source used is emitting white light, it is equivalent to an infinite number of monochromatic sources. This results in overlapping of fringes due to different wavelengths, and thus only a few colored fringes with poor contrast are visible. When the path difference is large, it results in uniform illumination.

(c) The amplitudes of the interfering waves should be preferably equal. If a_1 and a_2 are the amplitudes of the interfering beams, then Intensity of maxima is $(a_1 + a_2)^2$ and Intensity of minima is $(a_1 - a_2)^2$. If the difference between the amplitudes a_1 and a_2 is very large, then the intensity of minima will be practically the same as that of the maxima and hence the contrast will be poor. For a good contrast $a_1 \approx a_2$, so that the minima have a low intensity.

Q.6) What are the conditions for producing sustained interference of light? (Short Answer question)

(a) The two sources should be coherent. Sources derived from a single source are in phase with each other or maintain a constant phase difference. Coherent beams of light produce a steady interference pattern.

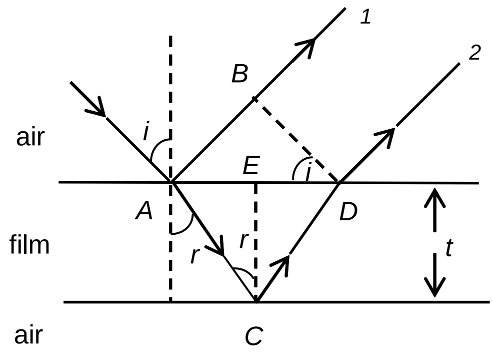
(b) The two interfering waves must be of the same wavelength and periodic time and propagate approximately in the same direction.

(c) Planes of polarization of the waves must be the same. Waves polarized in perpendicular planes cannot produce interference effects.

Q.7) Give the theory of interference of light incident on a thin film.

Thin-film interference is an example of interference by division of amplitude. The striking colours of soap bubbles, oil slicks, peacock feathers, throats of humming birds, Newton's rings, interference patterns in Michelson interferometer are some examples of thin film interference. The condition of coherence is satisfied in thin film interference because the rays are derived from the same ray incident on the film. A thin film has two surfaces, the upper surface and the lower surface of the film. The rays reflected or transmitted from these surfaces participate in the interference process. The interfering waves combine either to enhance or to suppress certain colours in the spectrum of the incident sun light. This selective enhancement or suppression of selected wavelengths has several applications.

The figure shows a film of uniform thickness t, index of refraction μ. Let light be incident at A. Part of the light is reflected towards B and the other part is reflected at C and emerges at D and is parallel to the first part. The condition of coherence is satisfied here because rays 1 and 2 are derived from a single incident ray. At normal incidence, the path difference Δx between rays 1 and 2 is twice the optical thickness of the film.



$\Delta x = 2\mu t$ where μ is the refractive index of the film. At oblique incidence, the optical path differnce is $\Delta x = \mu(AC + CD) - AB$

In the Δle ABD, $Sin\, i = \dfrac{AB}{AD} = \dfrac{AB}{2AE}$ or $AB = 2(AE) Sin\, i$

In the Δle AEC, $Tan\, r = \dfrac{AE}{t}$ or $AE = t\, Tan\, r$

$\therefore AB = 2t\, Tan\, r\, Sin\, i = 2t\, Tan\, r\, \mu\, Sin\, r$. Since $AC = \dfrac{t}{Cos\, r}$ and $AC + CD = \dfrac{2t}{Cos\, r}$, we have

$$\therefore \Delta x = \frac{2\mu t}{Cos\, r} - 2\mu t\, Tan\, r\, Sin\, r = 2\mu t\left(\frac{1}{Cos\, r} - Tan\, r\, Sinr\right) = 2\mu t\left(\frac{1 - Tan\, r\, Sin\, r\, Cos\, r}{Cos\, r}\right)$$

$\Delta x = 2\mu t\left(\frac{1 - Sin^2 r}{Cos\, r}\right)$ or $\Delta x = 2\mu\, t\, Cos\, r$, where μ is the refractive index of the medium between the surfaces. Since for air $\mu = 1$, the path difference between rays *1* and *2* is given by

$$\Delta x = 2\, t\, Cos\, r$$

However, this is only the apparent path differnce. To calculate the real path difference, one should also consider the change in phase brought in by reflection. According to electromagnetic theory of light, whenever reflection occurs at an interface backed by a denser medium, a phase of change of π or a path difference of $\frac{\lambda}{2}$ is additionally introduced in the reflected component. Hence the real path difference is $\Delta x = 2\mu\, t\, Cos\, r + \lambda/2$

Hence, the condition for maxima for the thin film to appear bright is

$$2\mu\, t\, Cos\, r + \frac{\lambda}{2} = m\lambda \text{ or } 2\mu\, t\, Cos\, r = m\lambda - \frac{\lambda}{2} = (2m-1)\frac{\lambda}{2}$$, where $m = 0, 1, 2, \ldots\ldots$

The film will appear dark in the reflected light when

$$2\mu\, t\, Cos\, r + \frac{\lambda}{2} = (2m+1)\frac{\lambda}{2} \text{ or } 2\mu\, t\, Cos\, r = m\lambda$$, where $m = 0, 1, 2, 3, \ldots\ldots.$

If the film is viewed in the transmitted light, it is easy to derive the conditions for maxima and minima following similar geometry. The condition for maxima in this case is

$2\mu\, t\, Cos\, r = m\lambda$ and the condition for minima is $2\mu\, t\, Cos\, r = (2m+1)\frac{\lambda}{2}$.

One cannot observe any interference pattern in thick films. For observing the interference pattern, the thickness of the film should be comparable with the wavelength of light.

Q.8) Explain the conditions for constructive and destructive superposition of waves. (Short Answer question)

When two waves are displaced through an integral number of wavelengths, constructive interference takes place. In case of reflected light from thin film, the condition for maxima for the thin film to appear bright (constructive interference) is

$$2\mu\, t\, Cos\, r + \frac{\lambda}{2} = m\lambda \text{ or } 2\mu\, t\, Cos\, r = m\lambda - \frac{\lambda}{2} = (2m-1)\frac{\lambda}{2}$$, where $m = 0, 1, 2, \ldots\ldots$

When two waves are displaced with respect to each other by an odd number of half-wavelengths, destructive interference results. In case of reflected light from thin film, the condition for minima for the thin film to appear dark (destructive interference) is

$$2\mu t \, Cos\, r + \frac{\lambda}{2} = (2m+1)\frac{\lambda}{2} \quad \text{or } 2\mu t \, Cos\, r = m\lambda \text{ , where } m = 0, 1, 2, 3, \ldots\ldots$$

Q.9) Write a short note on colours in thin films. (Short Answer question)

The colours exhibited in reflection by thin films of oil, mica, soap bubbles are due to interference of light from an extended source such as sky. The reflected rays from the top and bottom surfaces of the film are very close to each other and are in a position to interfere. The optical path difference between the interfering rays is $\Delta = 2\,\mu\, t\, Cos\, r - \lambda/_2$. It is seen that the path difference depends upon the thickness t of the film, the wavelength λ and the angle r, which is related to the angle of incidence of light on the film.

White light consists of a range of wavelengths and for specific values of thickness t and the angle r, waves of certain wavelengths (colours) constructively interfere. Therefore, only those colours are present in the reflected light. The other wavelengths interfere destructively and hence are absent from the reflected light. Hence, the film at a particular point appears coloured. As the thickness and the angle of incidence vary from point to point, different colours are intensified at different places. The colours seen are not isolated colours, as at each place there is a mixture of colours.

Q.10) Describe the formation of Newton's rings in reflected light and derive expressions for the radii of bright and dark rings.

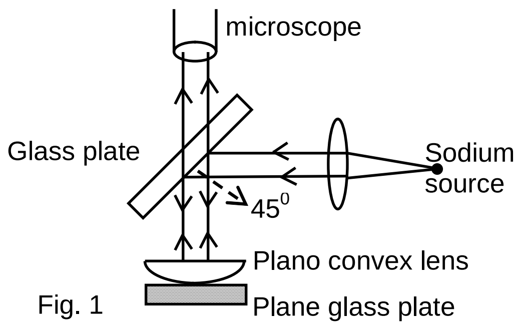


Fig. 1

Newton's rings are classic example of thin film interference by division of amplitude. The experimental arrangement for observing Newton's rings is shown in Figure 1. When a plano-convex lens of long focal length with its convex surface is placed on a plane glass plate, an air film of gradually increasing thickness is formed between the two. The film thickness at the point of contact is zero. If monochromatic light is allowed to fall normally, and the film is viewed in the reflected light, concentric bright and darks rings around the point of contact are seen. These circular fringes were discovered by Newton and are called Newton's rings. When the film is viewed in the reflected light, dark spot is formed at the point of contact of the lens with the glass plate. The circular fringes are localized and are of equal thickness and get crowded away from the point of contact.

The ray diagram in the formation of Newton's rings is shown in Figure 2. The ray is incident normally on the lens-plate system. Ray *1* and Ray *2* are the rays reflected from top and bottom surface of the air film. Ray *1* undergoes no phase change but ray *2* acquires a phase change of π upon reflection, because it is reflected from air-glass interface. Rays *1* and *2* are coherent because they are derived from the same incident ray. The conditions for the bright and dark rings are governed by the following relations:

$$2\mu t \cos r = (2m+1)\frac{\lambda}{2} \qquad \text{(Bright rings) and}$$

$$2\mu t \cos r = m\lambda \qquad \text{(Dark rings)}$$

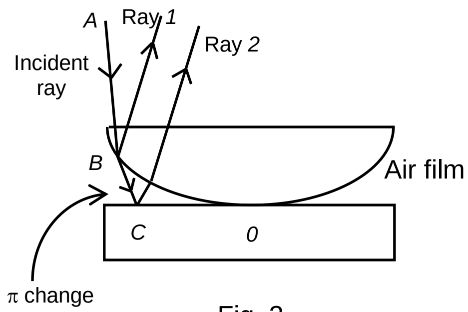


Fig. 2

For normal incidence $\cos r = 1$ and for the air film $\mu = 1$

$$\therefore 2t = (2m+1)\frac{\lambda}{2} \qquad \text{(Bright fringes) and } 2t = m\lambda \qquad \text{(Dark fringes)}$$

Dark spot is observed at the point of the contact *0* of the lens with the glass plate. The air film at the point of contact is only a few molecules thick and is very small compared to a wavelength ($t \ll \lambda$). The path difference introduced between the interfacing waves is zero, i.e, $2t = 0$. But the wave reflected from the glass plate suffers a phase change of π which is equivalent to a path difference of $\frac{\lambda}{2}$. Consequently, the interfering waves at the centre are out of phase and interfere destructively and produce a dark spot.

In Newton's ring arrangement, the thickness of the air film at the point of contact is zero and gradually increases as we move outward. The locus of points where the air film has the same thickness then fall on a circle whose centre is the point of contact. Thus, the thickness of air film is constant at points on any circle having the point of lens-glass plate contact as the centre. The fringes are therefore circular.

Theory of Newton's rings :

In the reflected monochromatic light, Newton's rings are alternate bright and dark circles with a central dark spot. Refer to Fig. 3. Let *R* be the radius of curvature of the lens.

At Q, let the thickness of the film $PQ = t$ satisfies the condition for a dark ring to form by interference. Let it be an m^{th} dark ring with a radius $OQ = r_m$. By the theorem of intersecting chords,

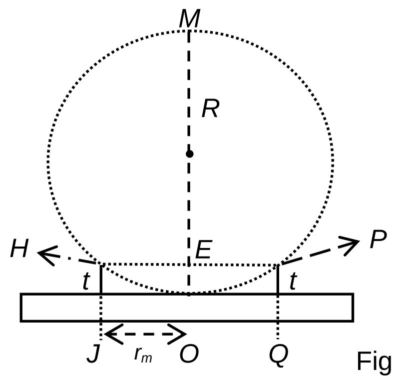


Fig. 3

$(EP) \times (HE) = (OE) \times (EM)$. But $EP = OQ = HE = r_m$; $OE = PQ = t$ and

$(EM) = (OM – OE) = (2R - t)$

$\therefore r_m^2 = t(2R - t)$ or $r_m^2 = 2Rt - t^2$. As $2Rt >> t^2$, t^2 can be neglected. $\therefore r_m^2 = 2Rt$.

For dark rings, the governing relation is $2t = m\lambda$. $\Rightarrow r_m^2 = m\lambda R$ or $r_m = \sqrt{m\lambda R}$

The diameter of the dark ring is therefore given by $D_m = 2\sqrt{m\lambda R}$. The radii of the dark rings can be found by taking $m = 0, 1, 2, 3$……. It can be seen that

$r_0 = 0,\ r_1 = \sqrt{\lambda R},\ r_2 = \sqrt{2\lambda R},\ r_3 = \sqrt{3\lambda R},....$ and so on. Thus, the radii (also diameters) of the dark rings are proportional to the square root of natural numbers.

Considering bright rings, let us suppose that a bright ring is located at the point Q. The radius of the m^{th} bright ring is given by $r_m^2 = 2Rt$. For bright rings, the governing relation is

$$2t = (2m + 1)\frac{\lambda}{2}$$

$$\therefore r_m^2 = \frac{(2m+1)\lambda R}{2} \quad \text{or} \quad r_m = \sqrt{\frac{(2m+1)\lambda R}{2}}$$

. The radii of different bright rings can be found by putting $m = 0, 1, 2, 3,$…………… in above equation. It is seen that $r_0 = \sqrt{\frac{\lambda R}{2}},\ r_1 = \sqrt{\frac{3\lambda R}{2}},\ r_2 = \sqrt{\frac{5\lambda R}{2}}$ and so on. It is clear that the radii (also diameters) of bright rings are proportional to the square root of the odd natural numbers.

Q. 11) Describe the formation of Newton's rings in reflected light and describe how the wavelength of sodium light can be determined by forming Newton's rings.

Newton's rings are classic example of thin film interference by division of amplitude. The experimental arrangement for observing Newton's rings is shown in Figure 1. When a plano-convex lens of long focal length with its convex surface is placed on a plane glass plate, an air film of gradually increasing thickness is formed between the two. The film thickness at the point of contact is zero. If monochromatic light is allowed to fall normally, and the film is viewed in the reflected light, concentric bright and darks rings around the point of contact are seen. These circular fringes were discovered by Newton and are called Newton's rings. When the film is viewed in the reflected light, dark spot is formed at the point of contact of the lens with the glass plate. The circular fringes are localized and are of equal thickness and get crowded away from the point of contact.

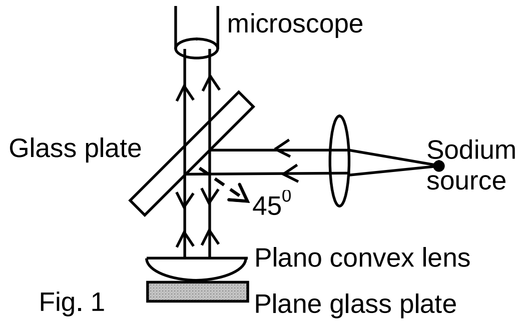


Fig. 1

The ray diagram in the formation of Newton's rings is shown in Figure 2. The ray is incident normally on the lens-plate system. Ray *1* and Ray *2* are the rays reflected from top and bottom surface of the air film. Ray *1* undergoes no phase change but ray *2* acquires a phase change of π upon reflection, because it is reflected from air-glass interface. Rays *1* and *2* are coherent because they are derived from the same incident ray. The conditions for the bright and dark rings are governed by the following relations:

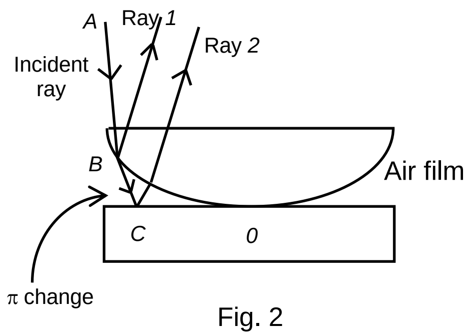


Fig. 2

$$2\mu t \,Cos\, r = (2m+1)\frac{\lambda}{2} \qquad \text{(Bright rings) and } 2\mu t\, Cos\, r = m\lambda \qquad \text{(Dark rings)}$$

For normal incidence $Cos\ r = 1$ and for the air film $\mu = 1$

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Dark spot is observed at the point of the contact *0* of the lens with the glass plate. The air film at the point of contact is only a few molecules thick and is very small compared to a wavelength ($t << \lambda$). The path difference introduced between the interfacing waves is zero, i.e, $2t = 0$. But the wave reflected from the glass plate suffers a phase change of π which is equivalent to a path difference of $\frac{\lambda}{2}$. Consequently, the interfering waves at the centre are out of phase and interfere destructively and produce a dark spot.

In Newton’s ring arrangement, the thickness of the air film at the point of contact is zero and gradually increases as we move outward. The locus of points where the air film has the same thickness then fall on a circle whose centre is the point of contact. Thus, the thickness of air film is constant at points on any circle having the point of lens-glass plate contact as the centre. The fringes are therefore circular.

The wave length of incident monochromatic light can be determined by forming Newton's rings and measuring the diameters of the dark rings using travelling microscope. For the m^{th} dark ring, $D_m^2 = 4m\lambda R$. For the n^{th} dark ring, $D_n^2 = 4n\lambda R$.

$$\therefore D_m^2 - D_n^2 = 4(m-n)\lambda R \quad or \quad \lambda = \frac{D_m^2 - D_n^2}{4(m-n)R}.$$

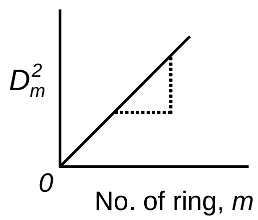

Fig. 4

In practice, the diameters of successive dark rings are measured with a travelling microscope and a plot is drawn between D_m^2 and m. The plot is a straight line as shown in Fig.4.

The slope of the line gives the value of $4\lambda R$. Thus $\lambda = \frac{\text{Slope}}{4R}$.

The radius of curvature R of the lens is measured using a spherometer and λ is determined using the above equation.

Q.12) How will you measure the refractive index of a liquid using Newton’s rings ?

The liquid whose refractive index is to be determined is filled in the gap between the lens and plane glass plate. The condition for interference may then be written as

$2\,\mu\, t\, Cos\, r = m\,\lambda$ (Darkness) where μ is the refractive index of the liquid.

For normal incidence the equation becomes $2\,\mu\, t = m\,\lambda$.

The diameter of m^th dark ring is given by $(D_m^2)_L = \frac{4\,m\,\lambda\,R}{\mu}$.

Similarly, the diameter of the (m + p)^th ring is given by $\left(D_{m+p}^2\right)_L = \frac{4\,(m+p)\,\lambda\,R}{\mu}$.

Subtracting the above two equations, we get $\left(D_{m+p}^2\right)_L - (D_m^2)_L = \frac{4\,p\,\lambda\,R}{\mu}$.

But we know that $\left(D_{m+p}^2\right)_{air} - (D_m^2)_{air} = 4\,p\,\lambda\,R$.

$$\mu = \frac{\left(D_{m+p}^2\right)_{air} - (D_m^2)_{air}}{\left(D_{m+p}^2\right)_L - (D_m^2)_L}$$

Q.13) What are Newton's rings and how they are formed ? (Short Answer question)

Newton's rings are classic example of thin film interference by division of amplitude. When a plano-convex lens of long focal length with its convex surface is placed on a plane glass plate, an air film of gradually increasing thickness is formed between the two. If monochromatic light is allowed to fall normally, and the film is viewed in the reflected light, concentric bright and darks rings around the point of contact are seen. These circular fringes were discovered by Newton and are called Newton's rings. When the film is viewed in the reflected light, dark spot is formed at the point of contact of the lens with the glass plate. The circular fringes are localized and are of equal thickness and get crowded away from the point of contact.

The condition of coherence is satisfied in thin film interference because the rays are derived from the same ray incident on the film. A thin film has two surfaces, the upper surface and the lower surface of the film. The rays reflected or transmitted from these surfaces participate in the interference process.

Q.14) What are the important applications of Newton's rings phenomena? (Short Answer question)

1. Newton's rings find application in testing the surface finish of lenses and other optical components used in telescopes and other optical instruments like cameras etc.

2. The refractive indices of liquids and gases can be conveniently measured.

3. Small displacements such as those produced by compression or elongation of crystals can be measured.

4. To determine the wavelengths of monochromatic light radiation.

Q.15) Discuss the applications of interference.

The applications of interference phenomenon are wide and varied. Some of the applications are :

1. Interference is used for making precision measurement. For example, the wavelength of light can be measured using Michelson's interferometer up to an accuracy eight significant digits. Interferometer methods are used to determine and redefine the length standard,

namely, the metre. The resolution between two closely spaced spectral lines such as D_1 and D_2 lines of a sodium doublet can be accurately determined.

2. Double-slit interference method is used to determine the angular separation of double stars and the diameter of fixed stars.

3. The refractive indices of liquids and gases can be conveniently measured using interference methods.

4. Interference method is used for measuring small displacements such as those produced by compression or elongation of a metal rod, crystals etc.

5. Newton's rings find application in testing the surface finish of lenses and other optical components used in telescopes and other optical instruments.

6. Wedge-film interference is effectively employed in testing the planeness of glass plates and extremely thin metallic plates.

7. Dielectric transparent thin films are often coated on optical components, solar cells etc. Multiple beam interference method is used to determine the thickness of such films coatings.

8. Thin-film interference is used to enhance or suppress certain colors in the spectrum of the incident sunlight. This selective enhancement or suppression of selected wavelengths has many applications. When light falls on an ordinary glass surface, for example, about 4 % of the incident light is reflected, thus weakening the transmitted beam by that amount. This unwanted loss of light can be a real problem in optical systems with many components. A thin transparent film deposited on the optical surface, can largely suppress the reflected light (and thus enhance the transmitted light) by destructive interference. Such transparent thin film coating are called antireflection coatings or AR coatings. Camera lenses appear slightly bluish because of the presence of such coatings.

9. Optical interference coatings are also used sometimes to enhance the reflectivity of a surface. The coated surface acts like a mirror. Such interference coatings are electrically non -conducting and hence are called dielectric mirrors.

In fact, an interference stack of a number films, with differing thickness and indices of refraction, can be designed to give almost any desired wavelength profile for a reflected or transmitted light. For example, windows can be provided with coatings that have a high reflectivity in the infrared, thus admitting the visible component of sunlight but reflecting its infrared or heating component.

SOLVED PROBLEMS

Example 1: Two coherent sources whose intensity ratio is *81:1* produce interference fringes. Deduce the ratio of maximum intensity to minimum intensity?

Since intensity *I* is square of the amplitude *a*,

$$\frac{I_1}{I_2} = \frac{a_1^{\ 2}}{a_2^{\ 2}} = \frac{81}{1} \Rightarrow \frac{a_1}{a_2} = \frac{9}{1} \Rightarrow a_1 = 9a_2$$

$$\frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \frac{(9a_1 + a_2)^2}{(9a_1 - a_2)^2} = \frac{(10a_2)^2}{(8a_2)^2} = \frac{100}{64} = \frac{25}{16} \Rightarrow I_{max} : I_{min} = 25:16$$

Example 2 : A monochromatic beam of light travels through a medium of refractive index *1.33* and thickness *0.75* μm. Calculate its optical path ?

Optical path = $\mu \times$ geometrical path = $1.33 \times 0.75 = 0.998\ \mu m$.

Example 3 : A parallel beam of light of wavelength $5890\ A^0$ is incident on a thin glass plate ($\mu = 1.5$) such that the angle of refraction into the plate is 60^0. Calculate the smallest thickness of the glass plate which will appear dark in reflected light.

The condition is given by $2\mu\, t\ Cos\ r = m\lambda$. Taking $m = 1$, the smallest thickness of plate that causes destructive interference is

$$t = \frac{\lambda}{2\mu\, Cos\, r} = \frac{5890 \times 10^{-10}}{2 \times 1.5 \times Cos\ 60^0} = 0.39\,\mu m$$

Example 4: A Newton's ring arrangement is used with a source emitting two wavelengths $\lambda_1 = 6 \times 10^{-5}\, cm$ and $\lambda_2 = 4.5 \times 10^{-5}\, cm$. It is found that n^{th} dark ring due to λ_1 coincides with $(n+1)^{th}$ dark ring for λ_2. If the radius of curvature of the curved surface is *90cm*, find the diameter of n^{th} dark ring for λ_1.

$r = \sqrt{m\lambda R}$, $m = 0,1,2,....$ for dark ring $\Rightarrow r_n = \sqrt{n \times 6 \times 10^{-5} \times 90}$

$r_{n+1} = \sqrt{(n+1) \times 4.5 \times 10^{-5} \times 90}$ and $n \times 6 \times 10^{-5} \times 90 = (n+1)4.5 \times 10^{-5} \times 90$

$$6n = 4.5(n+1) \Rightarrow 6n = 4.5n + 4.5 \Rightarrow 1.5n = 4.5 \Rightarrow n = 4.5/1.5 = 3$$

$$r_n = \sqrt{3 \times 6 \times 10^{-5} \times 90} = \sqrt{1620 \times 10^{-5}} = \sqrt{0.0162} = 0.127cm$$

Diameter of n^{th} ring = *0.127 x 2 = 0.254cm*.

Example 5: Newton's rings are observed in the reflected light of wavelength 5900 A^0. The diameter of 10^{th} dark ring is 0.5 cm. Find the radius of curvature of lens used.

Wavelength of reflected light, $\lambda = 5900\ A^0 = 5900 \times 10^{-10}\ m$

Diameter of 10^{th} dark ring, $D_{10} = 0.5\ cm = 5 \times 10^{-3}\ m$

Diameter of n^{th} dark ring, $D_n = 2\sqrt{n\,\lambda\,R}$

$$R = \frac{D_n^2}{4\,n\,\lambda} = \frac{(5 \times 10^{-3})^2}{4 \times 10 \times 5900 \times 10^{-10}} = 1.059\ m$$

Radius of curvature of the lens = 1.059 m

Example 6: In Newton's rings experiment, the diameter of 15^{th} ring was found to be 0.59 cm and that of 5^{th} ring 0.336 cm. The radius of curvature of the lens is 100 cm. Find the wavelength of light.

The diameter of 15^{th} ring, $D_{15} = 0.59\ cm = 5.9 \times 10^{-3}\ m$

The diameter of 5^{th} ring, $D_5 = 0.336\ cm = 3.36 \times 10^{-3}\ m$

The radius of curvature of the lens, $R = 100\ cm = 1\ m$

The expression for wavelength of light is, $\lambda = \frac{D_m^2 - D_n^2}{4\,(m-n)\,R}$

$$\lambda = \frac{(5.9 \times 10^{-3})^2 - (3.36 \times 10^{-3})^2}{4 \times 10 \times 1} = 0.588 \times 10^{-6}\ m$$

Example 7: In Newton's rings experiment the diameter of the 12^{th} ring changes fro 1.45 cm to 1.25 cm when a liquid is introduced between the lens and the plate. Calculate the refractive index of the liquid.

For n^{th} ring in air, $(D_n^2)_{air} = 4\,n\,\lambda R$

For n^{th} ring in liquid, $(D_n^2)_{liquid} = \frac{4\,n\,\lambda\,R}{\mu}$

$$\mu = \frac{(D_n^2)_{air}}{(D_n^2)_{liquid}} = \frac{(1.45)^2}{(1.25)^2} = 1.3456$$