

Develop the Grammar for the following language, Convert to CNF and then use the grammar as input into your code.

① The grammar for the language $\{a^n b^n \mid n \geq 0\}$

$$L = \{\epsilon, ab, aabb, \dots\}$$

CFG :- $S \rightarrow aSb \mid \epsilon$

→ $\left. \begin{array}{l} S_0 \rightarrow S \\ S \rightarrow aSb \mid \epsilon \end{array} \right\}$ Add $S_0 \rightarrow S$

→ $\left. \begin{array}{l} S_0 \rightarrow S \\ S \rightarrow aX_1 \mid \epsilon \\ X_1 \rightarrow Sb \end{array} \right\}$ chop down long rules

→ $\left. \begin{array}{l} S_0 \rightarrow S \mid \epsilon \\ S \rightarrow aX_1 \\ X_1 \rightarrow Sb \mid b \end{array} \right\}$ eliminate ϵ -rules

→ $\left. \begin{array}{l} S_0 \rightarrow aX_1 \mid \epsilon \\ S \rightarrow aX_1 \\ X_1 \rightarrow Sb \mid b \end{array} \right\}$ eliminate unit rules

→ $\left. \begin{array}{l} S_0 \rightarrow X_a X_1 \mid \epsilon \\ S_1 \rightarrow X_a X_1 \\ X_1 \rightarrow S X_b \mid b \\ X_a \rightarrow a \\ X_b \rightarrow b \end{array} \right\}$ fix rhs of length 2, to get CNF.

Q2

Given

$$L = \{a^i b^j c^k \mid i=j \text{ or } j=k\}$$

as there's 'or' in the question, we could design the grammar as follows:-

Now,

Case: When $i=j$
let, $i=j=n$
then $a^n b^n c^k$

CFG:-

$$\begin{aligned} P &\rightarrow RT \\ R &\rightarrow aRb \mid ab \\ T &\rightarrow Tc \mid c \end{aligned}$$

Case: when $j=k$
let, $j=k=m$
then $a^i b^m c^m$

CFG

$$\begin{aligned} Q &\rightarrow xy \\ x &\rightarrow ax \mid a \\ y &\rightarrow byc \mid bc \end{aligned}$$

~~Combining~~

Combining P & Q:-

$$\begin{aligned} S &\rightarrow P \mid Q \\ P &\rightarrow RT \\ R &\rightarrow aRb \mid ab \\ T &\rightarrow Tc \mid c \\ Q &\rightarrow xy \\ x &\rightarrow ax \mid a \\ y &\rightarrow byc \mid bc \end{aligned}$$

Cfg to cnf:-

steps:

→ no, so \rightarrow s rule

→ Chop down long rules

$S \rightarrow p|Q$

$P \rightarrow RT$

$R \rightarrow aRb|ab \rightarrow$

$T \rightarrow Tc|c$

$Q \rightarrow xy$

$x \rightarrow ax|a$

$y \rightarrow byc|bc$

$S \rightarrow p|Q$

$P \rightarrow RT$

$Q \rightarrow xy$

$R \rightarrow aZ_1|ab$

$T \rightarrow Tc|c$

$x \rightarrow ax|a$

$y \rightarrow bZ_2|bc$

$Z_1 \rightarrow Rb$

$Z_2 \rightarrow yc$

→ No, nullable variables

→ next, eliminating unit rules

$S \rightarrow RT|xy$

$P \rightarrow RT$

$R \rightarrow aZ_1|ab$

$T \rightarrow Tc|c$

$Q \rightarrow xy$

$x \rightarrow ax|a$

$y \rightarrow bZ_2|bc$

$Z_1 \rightarrow Rb$

$Z_2 \rightarrow yc$

→ fix subs to get final cnf

$$S \rightarrow P\Phi | XY$$

$$P \rightarrow RT$$

$$R \rightarrow Xaz_1 | Xaxb$$

$$T \rightarrow TXc | C$$

$$\Phi \rightarrow xy$$

$$X \rightarrow Xax | a$$

$$Y \rightarrow Xbz_2 | XbXc$$

$$z_1 \rightarrow RXb$$

$$z_2 \rightarrow YXc$$

$$Xa \rightarrow a$$

$$Xb \rightarrow b$$

$$Xc \rightarrow c.$$

(3) The grammar for the language
 $\{x_1 \# x_2 \# \dots \# x_k \mid k \geq 1, \text{ each } x_i \in \{a, b\}^*$
 and for some i and j , $x_i = x_j^R\}$

CFG:-

$$X \rightarrow Y \mid Z \# Y \# Z \mid Z \# Y \mid Y \# Z$$

$$Y \rightarrow aYa \mid bYb \mid \# \mid \#Z\#$$

$$Z \rightarrow aZ \mid bZ \mid \#Z \mid \epsilon$$

Steps:-

$$\rightarrow X \rightarrow Y \mid Zx_1 \mid Zx_2 \mid YZ_3$$

$$X_1 \rightarrow \#x_4$$

$$x_4 \rightarrow Yx_5$$

$$x_5 \rightarrow \#Z$$

$$x_2 \rightarrow \#Y$$

$$x_3 \rightarrow \#Z$$

$$Y \rightarrow aXb \mid bXa \mid \# \mid \#x_8$$

$$x_6 \rightarrow Ya$$

$$x_7 \rightarrow Yb$$

$$x_8 \rightarrow Z\#$$

$$Z \rightarrow aZ \mid bZ \mid \#Z \mid \epsilon$$

Chop down
long
rules

$$\Rightarrow X \rightarrow Y | Z X_1 | Z X_2 | Y Z_3 | X_1 X_2$$

$$X_1 \rightarrow \# X_4$$

$$X_4 \rightarrow Y X_5$$

$$X_5 \rightarrow \# Z | \#$$

$$X_2 \rightarrow \# Y$$

$$X_3 \rightarrow \# Z | \#$$

$$Y \rightarrow a X_6 | b X_7 | \# | \# X_8$$

$$X_6 \rightarrow Y a$$

$$X_7 \rightarrow Y b$$

$$X_8 \rightarrow Z \# | \#$$

$$Z \rightarrow a Z | b Z | \# Z | a | b | \#$$

eliminate
ε-rules

$$\Rightarrow X \rightarrow a X_6 | b X_7 | \# X_8 | \# | Z X_1 | Z X_2 | Y X_3 | \# X_4 | \# Y$$

$$X_1 \rightarrow \# X_4$$

$$X_4 \rightarrow Y X_5$$

$$X_5 \rightarrow \# Z | \#$$

$$X_2 \rightarrow \# Y$$

$$X_3 \rightarrow \# Z | \#$$

$$Y \rightarrow a X_6 | b X_7 | \# | \# X_8$$

$$X_6 \rightarrow Y a$$

$$X_7 \rightarrow Y b$$

$$X_8 \rightarrow Z \# | \#$$

$$Z \rightarrow a Z | b Z | \# Z | a | b | \#$$

eliminate
unit rules

fin sibs to get final crf

$$X \rightarrow X_a X_6 \mid X_b X_7 \mid X_{\#} X_8 \mid \# \mid Z X_1 \mid \\ Z X_2 \mid Y X_3 \mid X_{\#} X_4 \mid X_{\#} Y$$

$$X_1 \rightarrow X_{\#} X_4$$

$$X_4 \rightarrow Y X_5$$

$$X_5 \rightarrow X_{\#} Z \mid \#$$

$$X_2 \rightarrow X_{\#} Y$$

$$X_3 \rightarrow X_{\#} Z \mid \#$$

$$\cancel{X_6} \\ Y \rightarrow X_a X_6 \mid X_b X_7 \mid \# \mid \cancel{X_{\#}} X_{\#} X_8$$

$$X_6 \rightarrow Y X_a$$

$$X_7 \rightarrow Y X_b$$

$$\cancel{X_8} \rightarrow \cancel{Z X_{\#}} \mid \#$$

$$X_8 \rightarrow Z X'_{\#} \mid \#$$

$$Z \rightarrow X_a Z \mid X_b Z \mid X_{\#} Z \mid a \mid b \mid \#$$

$$X_a \rightarrow a$$

$$X_b \rightarrow b$$

$$X_{\#} \rightarrow \#$$

→ X ——— ✓.

q4

Given grammar

$$S \rightarrow TT|U$$

$$T \rightarrow OT|TO| \#$$

$$U \rightarrow OUOO| \#$$

Ans:-

$$\begin{aligned} \textcircled{1} \quad & S \rightarrow TT|U \\ & T \rightarrow OT|TO| \# \\ & U \rightarrow OX_1| \# \\ & X_1 \rightarrow UOO \end{aligned}$$

no, $S \rightarrow S$ rule

Chop down long rules

$$\begin{aligned} \textcircled{2} \quad & S \rightarrow TT|U \\ & T \rightarrow OT|TO| \# \\ & U \rightarrow OX_1| \# \\ & X_1 \rightarrow UX_2 \\ & X_2 \rightarrow OO \end{aligned}$$

no, nullable variables

$$\begin{aligned} \textcircled{3} \quad & S \rightarrow TT|OX_1| \# \\ & T \rightarrow OT|TO| \# \\ & U \rightarrow OX_1| \# \\ & X_1 \rightarrow UX_2 \\ & X_2 \rightarrow OO \end{aligned}$$

} eliminate unit rules.

$$\begin{aligned} \textcircled{4} \quad & S \rightarrow TT|X_0X_1| \# \\ & T \rightarrow X_0T|TX_0| \# \\ & U \rightarrow X_0X_1| \# \\ & X_1 \rightarrow UX_2 \\ & X_2 \rightarrow X_0X_0 \\ & X_0 \rightarrow O \\ & \text{final} \end{aligned}$$

} fix lhs to get final ans.