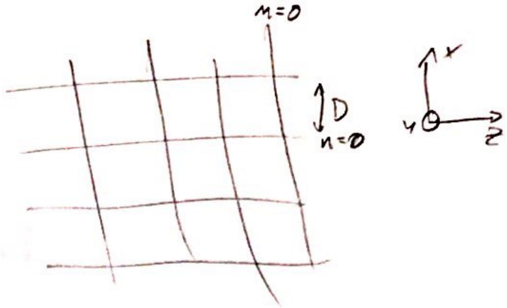
$$V(\vec{r}) = \underbrace{\sum_n V_1(x-nD, y)}_{V_n(x,y)} + \underbrace{\sum_m V_1(y, z-mD)}_{V_m(y,z)}$$



$$\langle \vec{k}' | V(\vec{r}) | \vec{k} \rangle = \frac{1}{V} \int V(\vec{r}) e^{-i\vec{q}\cdot\vec{r}} d\vec{r}$$

$$= \frac{1}{V} \int \left(V_n(x,y) + V_m(y,z) \right) e^{-i\vec{q}\cdot\vec{r}}$$

$$= \frac{1}{V} \int V_n(x,y) e^{-i\vec{q}\cdot\vec{r}} d\vec{r} + \frac{1}{V} \int V_m(y,z) e^{-i\vec{q}\cdot\vec{r}} d\vec{r}$$

$$= \frac{2\pi \delta(q_z)}{V} \iint V_n(x,y) e^{-i(q_x x + q_y y)} dx\,dy + \frac{2\pi \delta(q_x)}{V} \iint V_m(y,z) e^{-i(q_y y + q_z z)} dy\,dz$$

$$= \frac{2\pi \delta(q_z)}{V} \tilde{V}_n(q_x, q_y) + \frac{2\pi \delta(q_x)}{V} \tilde{V}_m(q_y, q_z)$$

$$|\langle \vec{k}' | V(\vec{r}) | \vec{k} \rangle|^2 = \frac{(2\pi)^2}{V^2} \left| \delta(q_z) \tilde{V}_n(q_x, q_y) + \delta(q_x) \tilde{V}_m(q_y, q_z) \right|^2$$

$$= \frac{(2\pi)^2}{V^2} \Big(\delta^2(q_z) |\tilde{V}_n(q_x, q_y)|^2 + \delta(q_z)\delta(q_x) \tilde{V}_n \tilde{V}_m^* + \delta(q_z)\delta(q_x) \tilde{V}_n^* \tilde{V}_m + \delta^2(q_x) |\tilde{V}_m(q_y, q_z)|^2 \Big)$$

$$W_{\vec{k},\vec{k}'} = \frac{2\pi}{\hbar} |\langle \vec{k}' | V(\vec{r}) | \vec{k} \rangle|^2 \delta(\hbar\omega - \hbar\omega')$$

$$= \frac{(2\pi)^3}{V^2 \hbar^2} \Big[\underset{①}{\delta^2(q_z) |\tilde{V}_n(q_x, q_y)|^2} + \underset{②}{\delta(q_z)\delta(q_x) \tilde{V}_n \tilde{V}_m^*} + \underset{③}{\delta(q_z)\delta(q_x) \tilde{V}_n^* \tilde{V}_m} + \underset{④}{\delta^2(q_x) |\tilde{V}_m(q_y, q_z)|^2} \Big] \delta(\omega - \omega')$$

$$\Gamma(\vec{k}) = L_x L_y L_z \iiint \frac{d^3 k'}{(2\pi)^3} W_{\vec{k},\vec{k}'} (1 - \hat{k}\cdot\hat{k}')$$

① Note: $\delta^2(q_i) = \frac{L_i \delta(q_i)}{2\pi}$

Split up $\Gamma(\vec{k})$ for ①, ②, ③, and ④

① $$\Gamma_1(\vec{k}) = \frac{L_x L_y L_z}{(L_x L_y L_z)^2} \iiint \frac{d^3\vec{k}'}{(2\pi)^3} \frac{(2\pi)^3}{\hbar^2} \frac{L_z\, \delta(q_z)}{2\pi} |\tilde{V}_n|^2 (1-\hat{k}\cdot\hat{k}')\, \delta(\omega-\omega')$$

$$\boxed{n_{ij} \equiv \frac{1}{L_i L_j}}$$

$$\boxed{\Gamma_1(\vec{k}) = \frac{n_{xy}}{2\pi\hbar^2} \iiint d^3\vec{k}'\, \delta(q_z)\, \delta(\omega-\omega')\, |\tilde{V}_n|^2 (1-\hat{k}\cdot\hat{k}')}$$

② $$\Gamma_2(\vec{k}) = \frac{L_x L_y L_z}{(L_x L_y L_z)^2} \iiint \frac{d^3\vec{k}'}{(2\pi)^3} \frac{(2\pi)^3}{\hbar^2} \delta(q_z)\delta(q_x)\, \delta(\omega-\omega')\, \tilde{V}_n \tilde{V}_m^* (1-\hat{k}\cdot\hat{k}')$$

$$\boxed{\Gamma_2(\vec{k}) = \frac{n_{xyz}}{\hbar^2} \iiint d^3\vec{k}'\, \delta(q_z)\, \delta(q_x)\, \delta(\omega-\omega')\, \tilde{V}_n \tilde{V}_m^* (1-\hat{k}\cdot\hat{k}')}$$

③ $$\boxed{\Gamma_3(\vec{k}) = \frac{n_{xyz}}{\hbar^2} \iiint d^3\vec{k}'\, \delta(q_z)\, \delta(q_x)\, \delta(\omega-\omega')\, \tilde{V}_n^* \tilde{V}_m (1-\hat{k}\cdot\hat{k}')}$$

④ $$\Gamma_4(\vec{k}) = \frac{L_x L_y L_z}{(L_x L_y L_z)^2} \iiint \frac{d^3\vec{k}'}{\cancel{(2\pi)^3}} \frac{\cancel{(2\pi)^3}}{\hbar^2} \frac{L_x\, \delta(q_x)}{2\pi}\, \delta(\omega-\omega')\, |\tilde{V}_m|^2 (1-\hat{k}\cdot\hat{k}')$$

$$\boxed{\Gamma_4(\vec{k}) = \frac{n_{yz}}{2\pi\hbar^2} \iiint d^3\vec{k}'\, \delta(q_x)\, \delta(\omega-\omega')\, |\tilde{V}_m|^2 (1-\hat{k}\cdot\hat{k}')}$$

and $$\boxed{\Gamma(\vec{k}) = \Gamma_1(\vec{k}) + \Gamma_2(\vec{k}) + \Gamma_3(\vec{k}) + \Gamma_4(\vec{k})}$$

Treating Γ_1 :

$$\tilde{V}_n = \iint V_n\, e^{-i(q_x x + q_y y)}\, dx\, dy = \iint \sum_{n=-\infty}^{\infty} V_1(x-nD, y)\, e^{-i(q_x x + q_y y)}\, dx\, dy$$

substitute
$x' = x - nD$
$x = x' + nD$

$$\tilde{V}_n = \iint \sum_{n=-\infty}^{\infty} V_1(x', y)\, e^{-i(q_x(x'+nD) + q_y y)}\, dx'\, dy = \sum_n e^{-iq_x nD}\, \tilde{V}_1(q_x, q_y)$$

Poisson Summation Formula

$$\sum_n e^{-iq_x nD} = \frac{2\pi}{D} \sum_{n^*} \delta(q_x - q_{n^*}) \qquad q_{n^*} = \frac{2\pi n^*}{D}$$

$$|\tilde{V}_n|^2 = \frac{2\pi L_x}{D^2} \sum_{n^*} \delta(q_x - q_{n^*})\, |\tilde{V}_1(q_x, q_y)|^2$$

$$\boxed{\Gamma_1(\vec{k}) = \frac{n_y}{\hbar^2 D^2} \sum_{n^*} \iiint d^3 k'\, \delta(q_z)\, \delta(q_x - q_{n^*})\, \delta(\omega - \omega')\, |\tilde{V}_1(q_x, q_y)|^2 (1 - \hat{k}\cdot\hat{k}')}$$

Likewise for Γ_4:

$$\tilde{V}_m = \sum_m e^{-iq_z mD}\, \tilde{V}_1(q_y, q_z) \quad \text{and} \quad \sum_m e^{-iq_z mD} = \frac{2\pi}{D} \sum_{m^*} \delta(q_z - q_{m^*})$$

$$q_{m^*} = \frac{2\pi m^*}{D}$$

$$\tilde{V}_m = \frac{2\pi}{D} \sum_{m^*} \delta(q_z - q_{m^*})\, \tilde{V}_1(q_y, q_z)$$

$$|\tilde{V}_m|^2 = \frac{2\pi L_z}{D^2} \sum_{m^*} \delta(q_z - q_{m^*})\, |\tilde{V}_1(q_y, q_z)|^2$$

$$\boxed{\Gamma_4(\vec{k}) = \frac{n_y}{\hbar^2 D^2} \sum_{m^*} \iiint d^3 k'\, \delta(q_x)\, \delta(q_z - q_{m^*})\, \delta(\omega - \omega')\, |\tilde{V}_1(q_y, q_z)|^2 (1 - \hat{k}\cdot\hat{k}')}$$