IIT CS536: Science of Programming

Homework 2: State and IMP

Prof. Stefan Muller

Out: Thursday, Sept. 7
Due: Thursday, Sept. 21, 11:59pm CDT

Updated Sept. 18

This assignment contains 7 written task(s) for a total of 65 points.

Logistics

Submission Instructions

Please read and follow these instructions carefully.

- Submit your homework on Blackboard under the correct assignment by the deadline (or the extended deadline if taking late days).
- You may submit multiple times, but we will only look at your last submission. Make sure your last submission contains all necessary files.
- Email the instructor and TAs ASAP if
  - You submit before the deadline but then decide to take (more) late days.
  - You accidentally resubmit after the deadline, but did not intend to take late days.

  Otherwise, you do not need to let us know if you're using late days; we'll count them based on the date of your last submission.
- Submit your written answers in a single PDF or Word document. Typed answers are preferred (You can use any program as long as you can export a .pdf, .doc or .docx; LaTeX is especially good for typesetting logic and math, and well worth the time to learn it), but *legible* handwritten and scanned answers are acceptable as well.
- Your Blackboard submission should contain only the file with your written answers. Do not compress or put any files in folders.

Collaboration and Academic Honesty

This homework is to be completed individually. Read the policy on the website and be sure you understand it.

1 State

Task 1.1 (Written, 8 points).

Let $\sigma = \{x = 5, y = 2, z = 1, a = [8; 2; 5]\}$.
For questions where the answer is a state, please give it as a set of variable-value pairs (e.g., $\{x = 5, y = 2, z = 1, a = [8; 2; 5]\}$, not as a state update).

a) What is $\sigma[x \mapsto 3][x \mapsto 5]$?

b) What is $\sigma[w \mapsto 4](w)$?

c) What is $\sigma[y \mapsto 7][w \mapsto 8]$?

d) What is $|\sigma(a)|$?

Task 1.2 (Written, 12 points).

For each of the following, say whether the satisfaction holds or not. If not, why?

a) $\{x = 0\} \vDash \forall y \in \mathbb{Z}.x \leq y^2$

b) $\{x = 2, y = 4\} \vDash \exists x \in \mathbb{Z}.x > y$

c) $\{x = 1, y = 2\} \vDash \forall z \in \mathbb{Z}.z > x \to y \cdot z > 0$

d) $\{x = 5\} \vDash \exists y \in \mathbb{Z}.2 \cdot y = x$

Task 1.3 (Written, 8 points).

For each of the situations below, fill in the blanks to describe when the situation holds.
Fill in _____ with "some", "all" or "no".

a) $\vDash \exists x \in \mathbb{Z}.\forall y \in \mathbb{Z}.p$ if for _____ states σ, it is true that $\sigma[x \mapsto \alpha_1][y \mapsto \alpha 2] \vDash p$ for _____ $\alpha_1 \in \mathbb{Z}$ and _____ $\alpha_2 \in \mathbb{Z}$.

b) $\vDash \neg(\forall x \in \mathbb{Z}.\exists y \in \mathbb{Z}.q)$ if for _____ states σ, it is true that $\sigma[x \mapsto \alpha_1][y \mapsto \alpha 2] \vDash q$ for _____ $\alpha_1 \in \mathbb{Z}$ and _____ $\alpha_2 \in \mathbb{Z}$ (*Updated 9/18*: Typo fix).

2 IMP Syntax and Semantics

Task 2.1 (Written, 15 points).

Evaluate each of the following expressions with the state

$$\sigma = \{x = 5, y = 2, z = 1, w = T, v = F, a = [8; 2; 5]\}$$

a) $\sigma(x * y)$

b) $\sigma(\mathsf{if}\ x > y\ \mathsf{then}\ x - z\ \mathsf{else}\ y - z)$

c) $\sigma(a[z] + x)$

d) $\sigma(w \vee v)$

e) $\sigma(a[size(a) - z])$

Task 2.2 (Written, 8 points).

Write a program (statement) in the syntax of IMP that "truncates" an array a to length x, setting any elements after that to 0.

- You can assume that both a and x are in the state at the beginning of the program.
- If S is your program, and $\langle S, \sigma\rangle \to^* \langle \mathsf{skip}, \sigma'\rangle$, then for all $0 \le i < x$, we should have $\sigma'(a[i]) = \sigma(a[i])$ and for all $x \le i < |\sigma(a)|$, we should have $\sigma'(a[i]) = 0$.
- As an example, if $\sigma = \{a = [1; 2; 3; 4; 5], x = 3\}$, then $\sigma'(a) = [1; 2; 3; 0; 0]$.
- If $x \ge |\sigma(a)|$ (i.e., x is bigger than the length of a), then the array is not changed.
- If $x \le 0$, then $\sigma'(a)$ should have all 0s.
- It's OK if your program changes the value of x.
- Your program should never access an out-of-bounds array element for any proper state (a state that contains values for both x and a).

Task 2.3 (Written, 14 points).

Consider the program $S = \mathsf{while}\ x > y\ \mathsf{do}\ x := y\ \mathsf{od}$, in the state $\sigma = \{x = 3, y = 2\}$.

a) Evaluate the program using the **small-step operational semantics** we saw in class, i.e., give a series of configurations such that $\langle S, \sigma\rangle \to \langle S', \sigma'\rangle \to^* \langle \mathsf{skip}, \sigma''\rangle$.

b) What is $M(S, \sigma)$? Explain how you know (potentially by showing your calculation, but other explanations are also acceptable).

3 One more wrap-up question

Task 3.1 (Written, 0 points).

How long (approximately) did you spend on this homework, in total hours of actual working time? Your honest feedback will help us with future homeworks.