Homework - 2

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1. State

1.1) Let $\sigma = \{x = 5, y = 2, z = 1, a = [8; 2; 5]\}$

a) what is $\sigma[x \to 3][x \to 5]$

$\sigma[x \to 3] = \{x = 3, y = 2, z = 1, a = [8; 2; 5]\}$

$\sigma[x \to 3][x \to 5] = \{x = 5, y = 2, z = 1, a = [8; 2; 5]\}$

b) what is $\sigma[w \to 4](w)$

$\sigma[w \to 4] = \{x = 5, y = 2, z = 1, a = [8; 2; 5], w = 4\}$

$\sigma[w \to 4](w) = 4$

c) what is $\sigma[y \to 7][w \to 8]$

$\sigma[y \to 7] = \{x = 5, y = 7, z = 1, a = [8; 2; 5]\}$

$\sigma[y \to 7][w \to 8] = \{x = 5, y = 7, z = 1, a = [8; 2; 5], w = 8\}$

d) what is $|\sigma(a)|$

$|\sigma(a)| = 3$

Task 1.2

a. True.

This statement states that for all x, it is less than or equal to y^2. Since $x \geq 0$ it is also less than y.

b. True

Because x is not essential that it is 8. It is possible that there is x which is greater than y. Hence it is true.

c. True

Because the statement states that if z is greater than x then product of z & y is greater than 0. So if $z \geq 2$ then as $y \geq 2$ the product is always greater than 0.

d. False.

The statement states that for any integer of y if it is multiplied by 2 then it is equal to x (where $x = 5$). There is no integer which can be multiplied by 2 to get 5. Hence False

Task 1.3

a) all , some , all

b) no , all , some

2. IMP Syntax & Semantics

Task 2.1 $\sigma = \{x=5, y=2, z=1, w=T, v=F, a=[5;2;5]\}$

a. $\sigma(x * y)$

$= \sigma(x) * \sigma(y) = 5 * 2 = 10$

b) σ (if $x > y$ then $x - z$ else $y - z$)

$= \sigma(x - z)$ since $x > y$

$= \sigma(x) - \sigma(z) = 5 - 1 = 4$

c) $\sigma(a[z] + x)$

$= \sigma(a[z]) + \sigma(x) = \sigma(a[1]) + \sigma(x)$

$= \sigma(2) + 5 = 2 + 5 = 7$

d) $\sigma(w \vee v)$

$= \sigma(w) \vee \sigma(v)$

$= T \vee F = T$

e) $\sigma(a[size(a) - z])$

$= \sigma(a[\sigma(size(a)) - \sigma(z)])$

$= \sigma(a[3-1]) = \sigma(a[2]) = 5$

Task 2.9

```
n := if n < 0 then 0 else n fi;
while n < Size(a)
do
    n := n + 1
    a[n] := 0;
od
```

Task 2.3

S = while $x > y$ do $x := y$ od in state $\sigma = \{x=3, y=2\}$

a)

→ $\langle$ if $x > y$ then $x := y$; S else skip fi, $\{x=3, y=2\}\rangle$

→ $\langle x := y ; S, \{x=3, y=2\}\rangle$

→ $\langle S, \sigma[x \mapsto 2, y=2]\rangle$ → $\langle S, \{x=2, y, 2\}\rangle$

→ $\langle$ while $x > y$ do $x := y$ od, $\{x=2, y=2\}\rangle$

→ $\langle$ if $x > y$ then $x := y$; S else skip fi, $\{x=2, y=2\}\rangle$

→ $\langle$ skip, $\{x=2, y=2\}\rangle$

b) $M(S, \sigma)$ is the final stage of execution

$M(S, \sigma) = \{\sigma'\}$

From the above execution of the program we know

$\Sigma_0 = \{x=3, y=2\}$

$\Sigma_1 = \{x=2, y=2\}$ is where the program ends,

So, $M(s, \sigma) = \{m = 2, \quad y = 2\}$.

8. It took me 4-5 hours to finish the assignment.