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A20506653

1. Linked List and Seperation logic

1.1

$x_0 := !(head + \top);$

$x_1 := !(x_0 + \top);$

$head2 := !(x_1 + \top);$

$!(x + \top) := nil;$

$head1 := head;$

1.2

$\{list(a_1 ; a_2 ; a_3, head, m+3)\}$

$\{head \mapsto a_1, i_1 * i_1 \mapsto a_2, i_2 * i_2 \mapsto a_3, i_3 * list(L_2, i_3, m)\}$

$x_0 := !(head + \top);$

$\{(x_0 = i_1 \wedge head \mapsto a_1, x_0) * i_1 \mapsto a_2, i_2 * i_2 \mapsto a_3, i_3 *$
$list(L_2, i_3, m)\} \Rightarrow \{head \mapsto a_1, x_0 * x_0 \mapsto a_2, i_2 * i_2 \mapsto$
$a_3, i_3 * list(L_2, i_2, m)\}$

$x_1 := !(x_0 + 1);$

$\{head \mapsto a_1, x_0 * (x_1 = i_2 \wedge x_0 \mapsto a_2, x_1) * i_2 \mapsto a_3, i_3 *$

$list(L_2, i_3, m)\} \Rightarrow \{head \mapsto a_1, x_0 * x_0 \mapsto a_2, x_1 * x_1 \mapsto$

$a_3, i_3 * list(L_2, i_2, m)\}$

$head2 := !(x_1 + 1);$

$\{head \mapsto a_1, x_0 * x_0 \mapsto a_2, x_1 * (x_1 \mapsto a_3, head2 \wedge head2 = i_3) *$

$* list(L_2, i_3, m)\} \Rightarrow \{head \mapsto a_1, x_0 * x_0 \mapsto a_2, x_1 * x_1$

$\mapsto a_3, head2 * list(L_2, head2, m)\}$

$!(x_1 + 1) = nil;$

$\{head \mapsto a_1, x_0 * x_0 \mapsto a_2, x_1 * x_1 \mapsto a_3, nil * list(L_2, head2, m)\}$

$head1 := head$

$\{head_1 \mapsto a_1, x_0 * x_0 \mapsto a_2, x_1 * x_1 \mapsto a_3, nil * list(L_2, head2, m)\}$

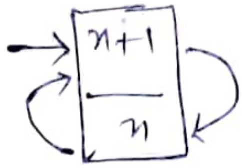
$\Rightarrow \{list(a_1; a_2; a_3, head1, 3) * list(L_2, head2, m)\}$

2. Resource logic

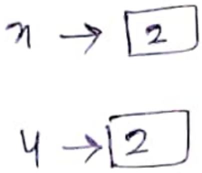
2.1

a) $x \mapsto x$

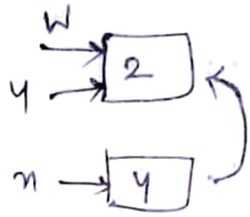
c) $x \mapsto x+1, x$



b) $x \mapsto 2 * y \mapsto 2$



d) $w \mapsto 2 * (x \mapsto y \wedge y = w)$



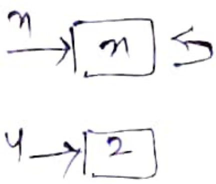
$(y = w)$

e) $(\exists_{x.y} \mapsto x) \wedge y \mapsto 2$

2.2

a. $x \hookrightarrow x$

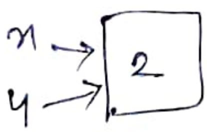
(i)



(ii)

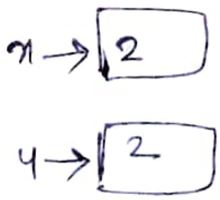
b. $x \hookrightarrow 2 \wedge y \hookrightarrow 2$

(i)



$(x = y)$

(ii)



c. $x \mapsto \_$

(i) $x \to \boxed{1}$

(ii) $x \to \boxed{2}$

2.3

a. $(x \mapsto 2 * y \mapsto 2) * z \mapsto 2 \Rightarrow (z \mapsto 2 * y \mapsto 2) * x \mapsto 2$

This is provable

$(x \mapsto 2 * y \mapsto 2) * z \mapsto 2$

$\Rightarrow z \mapsto 2 * (x \mapsto 2 * y \mapsto 2)$ — ~~Using rule~~ — 1.

$\Rightarrow z \mapsto 2 * (y \mapsto 2 * x \mapsto 2)$ — 2

$\Rightarrow (z \mapsto 2 * y \mapsto 2) * x \mapsto 2$ — 3

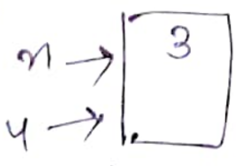
1. Using rule $P_1 * P_2 \Leftrightarrow P_2 * P_1$
2. Using rule $P_1 * P_2 \Leftrightarrow P_2 * P_1$
3. Using rule $(P_1 * P_2) * P_3 \Leftrightarrow P_1 * (P_2 * P_3)$

b) $(x \hookrightarrow 3 \wedge y \hookrightarrow 3) \Rightarrow x = y$

This is not provable

Since the above is imprecise assertion, it can be below 2,

(i)



$(x = y)$

(ii) $x \to \boxed{3}$

$y \to \boxed{3}$

By this figure, we can see $x \neq y$

Hence it is not provable

c) $emp * ((\exists_x . y \mapsto x) * w \mapsto 2) \Rightarrow \exists_x . (y \mapsto x * w \mapsto 2)$

This is provable

$\Rightarrow emp * ((\exists_x . y \mapsto x) * w \mapsto 2)$

$\Rightarrow ((\exists_x . y \mapsto x) * w \mapsto 2) * emp$ → ①

$\Rightarrow (\exists_x . y \mapsto x) * w \mapsto 2$ → ②

$\Rightarrow \exists_x . (y \mapsto x * w \mapsto 2)$ → ③

1. Using rule $P_1 * P_2 \Leftrightarrow P_2 * P_1$
2. Using rule $P * emp \Leftrightarrow P$
3. Using rule $(\exists x . P_1) * P_1 \Leftrightarrow \exists_x . (P_1 * P_2)$ when x is not free in P_2

d. $(\exists_x . y \mapsto x) * x \mapsto 2 \Rightarrow \exists_x . (y \mapsto x * x \mapsto 2)$

This is not provable

We can use the rule $(\exists_x . P_1) * P_1 \Leftrightarrow \exists_x . (P_1 * P_2)$ only when x is not free in P_2.

However, in the given question x is ~~not~~ been present as P_2 is $x \mapsto 2$ hence it can not be used.

3. Non determinism

3.1

a)

$$S_1 \triangleq \text{while } \{x \geq 0 \rightarrow y := x/y \ \square\ x \leq 0 \rightarrow y := y * x\}$$

on the state $\sigma \triangleq \{x = 0, y = 1\}$

$$\langle S, \{x = 0, y = 1\} \rangle$$

$$\rightarrow \langle y := x/y ; S, \{x = 0, y = 1\} \rangle$$

$$\xrightarrow{2} \langle S, \{x = 0, y = 0\} \rangle$$

$$\rightarrow \langle y := y * x ; S, \{x = 0, y = 0\} \rangle$$

$$\xrightarrow{2} \langle S, \{x = 0, y = 0\} \rangle$$

$$\rightarrow \langle y := y * x ; S, \{x = 0, y = 0\} \rangle$$

$$\xrightarrow{2} \langle S, \{x = 0, y = 0\} \rangle$$

b)

$$M(S, \sigma) = \{\} \text{ or } \perp$$

4.

4.1

$$S_2 \triangleq [x := y ;\ y := y+\bar{1} \parallel y := y+\bar{3}],\quad \sigma \triangleq \{x=3,\ y=3\}$$

$$\langle [x := y,\ y := y+\bar{1} \parallel y := y+\bar{3}],\ (\{x=3,\ y=2\}, h)\rangle$$

$$\downarrow$$

$$\langle [y := y+\bar{1} \parallel y := y+\bar{3}],\ (\{x=2,\ y=2\}, h)\rangle \qquad \langle [x := y,\ y := y+\bar{1} \parallel skip],\ (\{x=3,\ y=5\}, h)\rangle$$

$$\downarrow$$

$$\langle [skip \parallel y := y+\bar{3}],\ (\{x=2,\ y=3\}, h)\rangle \qquad \langle [y := y+\bar{1} \parallel skip],\ (\{x=2,\ y=5\}, h)\rangle \qquad \langle [y := y+\bar{1} \parallel skip],\ (\{x=5,\ y=5\}, h)\rangle$$

$$\downarrow$$

$$\langle [skip \parallel skip],\ (\{x=2,\ y=6\}, h)\rangle \qquad \langle [skip \parallel skip],\ (\{x=5,\ y=6\}, h)\rangle$$

5.

5.1

It took me 4 hours to finish this.