Binomial Coefficient Laws & Stars and Bars

1. Analytic (factorial) formula: C(n, k) = n! / (k! * (n-k)!)

2. Recurrence (Pascal’s rule): C(n, k) = C(n-1, k-1) + C(n-1, k), and C(n, k) = 0 if n < k

3. Symmetry rule: C(n, k) = C(n, n-k)

4. Multiplicative (factoring in) formula: C(n, k) = (n / k) * C(n-1, k-1)

5. Sum over k: Sum_{k=0 to n} C(n, k) = 2^n

6. Sum over n: Sum_{m=0 to n} C(m, k) = C(n+1, k+1)

7. Sum over n and k: Sum_{k=0 to m} C(n+k, k) = C(n+m+1, m)

8. Sum of squares: Sum_{k=0 to n} [C(n, k)]^2 = C(2n, n)

9. Weighted sum: Sum_{k=1 to n} k * C(n, k) = n * 2^(n-1)

10. Fibonacci connection: Sum_{k=0 to n} C(n-k, k) = F_{n+1}

Stars and Bars Theorem:

The number of ways to put n identical balls into k distinct boxes (allowing empty boxes) is: C(n+k-1, k-1). If no box can be empty, the formula is: C(n-1, k-1).