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Fair Value Measurement in Inactive Crypto Asset Markets

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Abstract

This article proposes a new dynamic method, the Principal Path Method (PPM), for pricing crypto asset against a primary or functional (fiat) currency in situations where these assets do not trade directly against the functional currency or trade at volumes that prevent resulting pricing information to qualify as Level 1 (ASC 820) for financial reporting. We base our method on the guidance provided in ASC 820, IFRS 13, and IAS 21. Our method is designed to extract prices from "compliant" markets that result in reliable inputs to the valuation process. We believe that our methodology improves the current techniques used to value thinly traded crypto assets such as using the last observable transaction price, creating a weighted-average price across multiple markets, or using data on comparable tokens, if available. Furthermore, we present empirical evidence that suggests pricing information generated by our method for non-exchangeable, thinly traded, or illiquid crypto assets better reflects the fundamental qualitative characteristics of useful information, relevance and faithful representation, and results in more reliable inputs used in the valuation process. Unlike methods currently used in practice, our method ensures the integrity of the valuation data employed by selecting prices from compliant markets.

Keywords

crypto asset, fair value measurement, thinly traded, inactive markets

Introduction

The evolution of digital assets and blockchain technology since 2009 and the advent of non-fungible tokens (NFTs) and Web 3.0 technology since 2020, have redefined the meaning of assets, markets, and economies. Moving well beyond the status of payment tokens or

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stores of value, digital assets can now be works of art, virtual real estate, or entertainment assets. Early issuance of access tokens such as MATIC,[1] UNI,[2] and GRT,[3] was not only instrumental in developing new economic ecosystems but also created unique possibilities for raising investment capital such as crowd sales, initial coin offerings (ICO), and initial exchange offerings (IEO). However, these new assets present ever-increasing challenges for financial reporting, both conceptual, such as the classification of virtual real estate, and methodological, such as auditing and reporting transactions on decentralized exchanges. Many of the fundamental concepts must be reevaluated, reinterpreted, and expanded to the realities of digital asset ecosystems.

In this article, we propose an expansion of the notion of *fair value* pricing for financial reporting. Fair value is a market-based estimate of the price of an asset, namely, an estimate of the price an asset could attain on the market in an orderly transaction. It is primarily geared toward valuation of fungible assets traded on public exchanges against a fiat currency.

ASC 820 (Financial Accounting Standards Board [FASB], 2011) and IFRS 13 (International Accounting Standards Board [IASB], 2011) provide guidance for fair value measurement and disclosure, outlining a three-level fair value hierarchy used to rank the reliability of the inputs used in the fair value measurement. The levels, from high to low, are as follows:

Level 1: Quoted prices in active markets for identical assets or liabilities that the entity can access at the measurement date.

Level 2: Pricing inputs other than Level 1, observable directly or indirectly, including quoted prices for similar assets and prices derived from or corroborated by observable markets.

Level 3: Pricing using unobservable inputs.

The classification in the hierarchy is driven by the lowest level of input reliability.

To expand fair value to digital assets, we revisit the notions of markets and transaction and expand them to digital assets within the framework of IFRS 13 and ASC 820. First, we show, as result of a world view focusing on decentralization to which many of the blockchain pioneers adhere, these markets are inherently fragmented. Consequently, the notion of principal market, central to the fair value measurement, must be subsumed by a new construct that could be applied to digital assets. Second, we show that as result of geographical disparity, globalization, technological legacy and the nature of financing and value appropriation, the market has become partitioned into multiple enclaves using different digital assets as vehicles for value. Consequentially, there is no single unit of measure universally applicable to all transactions in digital assets, and most transactions involve exchanges of digital assets for other digital assets. This situation closely resembles global portfolio across assets denominated in different currencies. However, foreign currency conversions fall under a different set of guidelines in which they are not considered separate transactions. This implies that exchange rates used to translate fair value measurements to a functional currency are not considered additional measurements and therefore do not impact level in the fair value hierarchy. Crypto asset conversions, however, are considered transactions and fall under fair value guidelines, and as such a conversion of digital assets going through one or more vehicles would be considered a chain of transactions. An estimation of value obtained through such a chain would therefore require multiple measurements. Currently, IFRS 13 and ASC 820 offer no guidelines for situations where value estimation requires multiple measurements.

Market fragmentation has been previously addressed in the literature, in particular a paper by Beigman et al. (2021) (BBHS, hereafter) extensively address the issue of fragmentation in crypto markets, offering a methodology for selecting principal market in a dynamic setting. Multiple measurements have so far received only limited attention within the fair value framework; however, similar issues had been addressed in discussions included in amendments to IAS 21 (IASB, 2001, 2021) focusing on situations where there is a lack of exchangeability in foreign currency. In this article, we adapt key concepts from these two lines of literature and apply them to the case of digital asset, offering a comprehensive methodology for pricing digital asset by fiat currency within the framework of IFRS 13 and ASC 820.

This article is organized as follows. In the next section, we present a brief review of current practices and recommendations for pricing crypto assets that do not directly trade against fiat. We then discuss fragmented markets and provide an overview of the fundamental pricing assumptions used in Beigman et al. (2021) in section ‘‘Fragmented Markets.’’ In section ‘‘Inactive Markets and Lack of Exchangeability,’’ we discuss currency conversion (the discussion around lack of exchangeability) and how it relates to inactive markets. We then employ the notion of a ‘‘path of assets’’ as a new construct used for pricing. In section ‘‘The Principal Path Method (PPM),’’ we expand these ideas and develop a methodology for comprehensive evaluation of digital assets. In section ‘‘PPM: An Empirical Demonstration,’’ we will demonstrate the methodology numerically and empirically. Finally, we present our conclusions, limitations, and suggestions for future research in section ‘‘Conclusions.’’

A Review of Current Practice

This paper contributes a novel approach to the valuation of crypto assets. Few papers related to this issue have been published in the academic literature to date (2023). However, there is much debate regarding the use of fair value to measure crypto assets in practice.

In its publication, *Accounting for and Auditing of Digital Assets* (Association of International Certified Professional Accountants [AICPA], 2022), the AICPA takes the position that digital assets are intangible assets and should be valued at cost. Fair value should be used only the case of impairment. Nevertheless, it does emphasize the need to use as much observable data as available. However, more recently standard setters have supported the use of fair value in measuring crypto assets. Specifically, in March 2022, the SEC issued Staff Accounting Bulletin (SAB) 121 which indicates that it would be appropriate for an entity that has an obligation to protect crypto assets held on its platform to record and asset and related obligation at fair value (U.S. Securities and Exchange Commission [SEC], 2022). In addition, recent decision by the FASB recommends the use of fair value through earnings to measure cryptocurrency (FASB 2022a, 2022b). As a result, the need to value crypto assets at fair value has expanded and assumes a central role in accounting for such assets, whether actively or thinly traded.

Current techniques used to value thinly traded crypto assets include methods such as using the last observable transaction price, creating a weighted-average price across multiple markets, or using data on comparable tokens, if available. In general, firms are using a simple management judgment-based approach identifying assets that are not actively traded via an analysis of observable transactions and then using price discovery from the market the entity transacted on most frequently. These techniques generally result in a Level 2 classification.[4] Although not specifically included in the current guidance, the use of multiple markets may be justified from a risk management standpoint. In addition, a 2019 publication from PwC discussed the accounting considerations for cryptographic asset transactions and included situations where an active market for a cryptographic asset is not likely to exist (PricewaterhouseCoopers [PwC], 2019). It used the term ''trading pairs,'' which uses some of the elements found in our method. Specifically, the PwC monograph provides an example that considers a cryptographic asset (CA ABC) that cannot be directly converted into fiat currencies but can convert into other cryptographic assets (CA DEF). PwC concludes that the price of the CA ABC would likely be the product of the exchange rate of [Fiat/(CA DEF)] by the exchange rate of [(CA DEF)/(CA ABC)]. Moreover, PwC indicates that

> in practice, there are exchanges that do not offer the possibility for crypto to fiat trades at all. In this instance, an entity might exchange a cryptographic asset for another cryptographic asset, and then exchange the second cryptographic asset into fiat at another exchange. This means that it is possible for a market to exist for a cryptographic asset in which there is frequency and volume of trades, but that this market is not an active market under IFRS 13.

Our methodology improves upon the techniques used in current practice by proposing a dynamic approach that relies on compliant markets for price discovery. This method will result in more reliable and faithfully represented valuations for thinly traded crypto assets, even in an ecosystem that includes fragmented markets. We discuss fragmented markets further in the following section of this article.

Fragmented Markets

Although in some cases assets may be traded over multiple exchanges or exchange alternatives (market makers, dark pools, or OTC), classical markets are designed to promote a single centralized exchange as the primary source of pricing information, termed the *principal market*. This designation may be difficult to apply when dealing with certain crypto asset markets. There are a multitude of centralized exchanges operating in this space (e.g., Coinbase, Kraken, and Binance) spanning diverse geographical regions and sovereignties. Beyond that there are different types of decentralized market institutions, such as decentralized exchanges (Uniswap, IDEX, and Sushiswap), automated market makers (Bancor and Pancakeswap), and other trading platforms (dXdY, Metcha, and Balancer to name just a few). The fragmentation this creates in the market, as well as the lack of centralization within many of these entities, is not an arbitrary side effect, but part of an ideology of decentralization that many of the pioneers in this space follow. At the heart of this ideology is the concept of ''code is law,'' namely, a belief that code and protocol can replace intermediary third-party institution as a source of trust in peer-to-peer transactions. This ideology is in direct contrast to fair value as a price observation on a designated centralized market which in this context serves as a third-party intermediary.

In their paper, Beigman et al. (2021) addressed the issue of fragmentation in crypto markets. In their setting, digital assets are traded over multiple public exchanges in volume sufficient to qualify as Level 1; however, none of these exchanges have a designation that would qualify them as *principal* market. Moreover, key relevance indicators, such as volume and price discovery, tend to fluctuate drastically throughout the day such that there is no one exchange that would be a natural focal point. The BBHS method enhances the classical notion of principal market with a corresponding ephemeral construct dependent on multiple factors, including exchange compliance, quality, volume, and freshness of the data, used for pricing.[5]

This article employs elements of the BBHS method, including a new ephemeral construct used to develop a methodology for pricing digital assets for which the asset to fiat market is inactive, either do not exist or trade at volumes that are insufficient to qualify as Level 1, but trade against other assets over public exchanges in sufficient volume.

Inactive Markets and Lack of Exchangeability

When Bitcoin first launched in 2009, it operated as a single-asset platform dedicated almost exclusively to financial transfers. The underlying scripting language was too limited for most other uses. This fundamentally changed with the introduction of Ethereum and the Ethereum Virtual Machine (EVM), a Turing complete on-chain computing device supported by Solidity and Viper, full-fledged programming languages, opening the way for smart contracts and the ERC-20 standard for fungible tokens. This in turn led to the proliferation of new class of digital assets on top of the Ethereum blockchain and later, on other EVM compatible blockchains like Polygon and Binance. Each of these platforms requires access to a native token which is used to pay for gas, the resource required for transactions on the blockchain. Moreover, many of the new ERC-20 tokens are access tokens to a whole ecosystem built on top of these blockchains, such as APE,[6] MANA,[7] CAKE,[8] and dXdY.[9] In many cases, these digital assets may not be directly exchangeable into a fiat currency. To illustrate, we take a closer look at the ApeCoin,[10] the governance and utility token used within the APE ecosystem. This ecosystem includes a wide range of products and services using the token, including the Bored Ape Yacht Club NFT series by Yuga Labs, as well as additional NFT series and a metaverse. ApeCoin tokens are used for accessing different NFTs, as governance tokens and as currency. NFT assets are traded on ApeCoin marketplace, quoted in either ETH or APE. Transactions are either in wrapped ETH (an ERC-20 token pegged to ETH, for technical reasons ETH cannot be used in the Ape ecosystem) or APE and will come with a fee denoted in APE. Thus, to obtain a fair value measurement in terms of a fiat currency such as the USD, will require a path of at least two ‘‘hops’’ (NFT-APE-USD or NFT-WETH-USD) across at least two exchanges, and in practice more likely will be at least three ‘‘hops’’ (NFT-APE-ETH-USD, NFT-APE-USDT-USD, etc.) across three exchanges. Some centralized exchanges such as Binance (the largest exchange by volume), support multiple stablecoins pegged to USD and non-USD fiat, as well as non-USD currencies, offering many different paths through which to estimate price versus functional.

The immediate question is, of course, how should fair value be estimated in such settings, and the level assigned to these estimates within the fair value hierarchy? As noted earlier, conversion of one digital asset to another is considered a transaction and is not the equivalent of foreign currency conversions from the accounting perspective. However, pricing digital assets through valuations of other digital assets is not fundamentally different than pricing foreign currency denominated assets through currency conversions. We therefore use analogous concepts found in the amendments to IAS 21 (IASB, 2014, 2021) which focus on the lack of exchangeability.

IFRS 13 (IASB, 2011) states that assets that are quoted in a currency different from the functional currency, should translate the quote to the functional currency, and will qualify as Level 1 if the exchange rate is observable. IAS 21 addresses situations where an asset is priced in a currency that is not directly convertible to the functional currency. Initially, the focus was on temporary non-exchangeability and was later expanded to settings where currencies are permanently non-exchangeable. The reality in the FX market is that 85% of the volume involves USD pairs (Gourinchas et al., 2019), even though the U.S. economy accounts for less than a quarter of the global economy. If we add to USD the EUR and JPY volume, this accounts for over 95% of the volume. Moreover, the two established principal markets for FX, EBS and Refinitiv both support only pairs that include at least one of the vehicle currencies: USD, EUR, or JPY. Given there are over 150 currencies worldwide, lack of direct conversion is norm rather than the exception. Research (Goldberg & Tille, 2008; Somogyi, 2022) indicates that up to 40% of the USD volume results from indirect transactions. Both US GAAP and IFRS treat FX as a separate asset class, and as such, ASC 820 and IFRS 13 do not apply. IFRS 13 does not differentiate between direct and indirect observable data. Nevertheless, in the accounting guidance that considers the lack of exchangeability, the standards indicate that a path, (i.e., a chain of multiple assets that are traded over an active market), can be used to price the assets at the ends of the path. Agenda proposal for foreign currency accounting issued by the Korean Accounting Standards Board and submitted to the IASB discussed whether such evaluations qualify as single or multiple measures and what exchange rate constraints on the chain must be satisfied (IASB, 2014). In addition, the issue of whether low volumes and high volatility could disqualify a market from consideration is discussed. These issues were partly addressed in a recent Exposure Draft ED/2021/4 *Lack of Exchangeability* (IASB, 2021). The proposed guidelines specify the requirements for reporting exchange rates when a pair is lacking exchangeability. Specifically, it provides a procedure through which it is determined when the exchange rate should be priced through direct or indirect observations and when model-based pricing should be preferred. The proposal emphasizes estimating exchange rates through an observable exchange rate as the estimated spot exchange rate when that observable exchange rate would have applied to an orderly transaction between market participants and a rate that faithfully reflects the prevailing economic conditions (IASB, 2021).

Accountants face a similar situation where there is lack of exchangeability or thinly traded crypto assets. Our objective is to develop a pricing method that improves upon current practice by extracting the most reliable data from compliant exchanges and allows for the valuation of crypto assets that do not directly trade or are thinly traded against fiat. We adapt the IAS 21 implicit idea of pricing an asset through a path or chain of assets, if any link in this chain is a pair of assets that are supported by an active market and transactions are observable, public, and orderly. We explicitly take the position that each conversion estimate is a measurement, and thus, the overall price is not a direct quote but a derivation of multiple observed inputs and should therefore fall primarily into Level 2 in the fair value hierarchy.

IAS 21 does not include any recommendations regarding the path used for FX valuation, but given the volume involving USD pairs, virtually any asset can be converted directly to USD or at most via EUR, thus virtually all paths include one vehicle with a small fraction including two vehicles.

The situation is quite different for digital assets, while Bitcoin and Ethereum are supported almost universally, the choice of fiat currencies, stable coins, and exchange tokens available on each exchange differ significantly from one exchange to the other.[11] In some cases, stable coins and exchange tokens command a significant portion of the volume. As such, these crypto-to-crypto markets are critical in the price discovery process and should not be ignored. We propose the *Principal Path Method* (PPM, hereafter) as a method to systematically choose a path for pricing crypto assets. By this method, price is determined by the exchange rates of assets on a path, as in FX pricing; however, due to the fragmentation in crypto markets, each link is priced independently through the BBHS method.[12] Moreover, the chain used for pricing is not fixed as it typically is in FX, but dynamically chosen based on market conditions. We designate this ephemeral construct the *principal path*. Using data from both centralized and decentralized exchanges, we will demonstrate that most thinly traded crypto assets have short chains connecting them to functional, fiat currencies, with sufficient volume and activity to qualify as a principal path, rarely with length greater than three links or ''*hops*.'' However, our method accounts for chains of all lengths when determining a dynamic principal path. Chains with a significant number of ''hops'' is in line with the new realities on the ground where in many centralized and decentralized exchanges technology-enabled trading bots are used to discover possible superior liquidity provisions through longer chains of assets.

The determination of whether a crypto asset pair is trading in inactive market will be made based on a volume threshold[13] as well as additional characteristics of the market. The use of a volume threshold for thinly traded, inactive, or illiquid assets has precedent as it is recommended by the SEC in its analysis of the national market system (NMS) for equity shares (SEC, 2018). Specifically, the SEC (2018) used the average daily share volume (ADV) as the criterion to differentiate liquid from illiquid stocks. This approach was also supported by the U.S. Department of the Treasury (SEC, 2019; the U.S. Department of the Treasury, 2017). Although there is no specific accounting guidance provided to differentiate between the liquid and illiquid crypto assets, our method can be employed under a designated volume threshold. Our proposed PPM intentionally does not propose explicit thresholds; however, once sufficient experience has been gained, with input from practitioners, market data can be used to establish a threshold.

The Principal Path Method (PPM)

Once a pair of crypto assets is determined to be thinly traded or lacking exchangeability, our proposed PPM will conceptually follow the elements of the BBHS method by scoring each possible path or chain of indirect conversion based on compliance, volume, and freshness of data, and designating the exchange with the highest score as the *principal path*. It should be noted that although we use the key elements of the BBHS method in our valuation, we are not attempting to identify a principal market with the use of the PPM. Based on the current standards, the resulting valuation cannot be considered as extracted from a principal market. The guidance (e.g., IFRS 13) indicates that while the market predominantly used to trade crypto currency should be considered the principal market, a market that does not exchange crypto to fiat cannot be considered the principal market. Similarly, PwC (2019) advises that a crypto asset that does not convert directly into fiat in an active market does not fulfill the criteria for a Level 1 asset. Although the definition of an active market does not refer to fiat currency, the presumption of this interpretation is that, to qualify as a Level 1 fair value measurement, the transaction should be measured in a fiat currency. The fair value of a cryptocurrency that is convertible into a fiat currency through another cryptocurrency is likely to qualify for a Level 2 asset in the fair value hierarchy. A Level 3 classification may result if the asset is not readily convertible into a fiat currency (PwC, 2019).

PPM: Method Development

Specifically, the BBHS method dynamically identifies compliant potential markets with a volume-based approach consistent with the accounting standards (ASC-820-10-35-5A-6A

& IFRS 13, 17-19), for actively traded crypto asset pairs across all available exchanges. The method determines its fair value from the exchange that is most compliant and exhibiting the largest volume with the least decay. Specifically, the BBHS method applies a scoring mechanism (Base Exchange Score or BES) that considers several exchange characteristics, including oversight, microstructure, and technology which assure only compliant markets are used, and then adjusts for volume and frequency of trades. Selecting only compliant markets when any fair value modeling is critical in the crypto eco-system as research has established that wash and fake trading, misreporting, price manipulation, and other misconduct are widely pervasive in the crypto ecosystem in both centralized and decentralized exchanges, and in particular in smaller and exchanges in jurisdictions that do not have KYC/AML requirements (Aloosh & Li, 2019; Amiram et al., 2020; Chen et al., 2022; Cui & Gao, 2022; von Wachter et al., 2022).

We make several assumptions in the development of our method. First, we assume that markets are highly fragmented and decentralized, a pair of assets like BTC/USDT, ETH/USD or even USDT/USD could trade over multiple exchanges in parallel, dispersed geographically and across sovereignties. Next, vast majority of active pairs are not supported by active markets, and in particular, there are many assets that do not trade against USD or other fiat in active markets, thus there is a need to price assets to USD (or other fiat) based on trade against other crypto assets. Finally, we assume that liquid markets with higher volume are more likely to support estimated prices in real-life transaction than markets with lower volume. By applying the bottleneck methodology, we are able to identify the path that is more likely to support the estimated price in real-life transactions than other paths.

The method is making some trade-offs. For example, using a path, we are reconstructing a price rather than using a direct trade, thus dropping to a lower level of reliability (to Level 2). In addition, we are adding a significant computational burden of reporting. Identifying principal path requires a significant amount of computations and knowledge of algorithms.

The BBHS method is used in this article to dynamically identify compliant markets (credibility and quality) that would provide more reliable information, for a certain pair, than other markets for the same pair, but not to determine a principal market.

To rank the credibility and quality of each exchange, a score that incorporates the key characteristics for each exchange is assigned through the following steps:

Step 1: Assign a Base Exchange Score (BES) for each exchange for the targeted crypto asset pair based on the static exchange characteristics.
Step 2: Adjust the score by the relative monthly volume of the exchange Volume-Adjusted Path Score (VAPS).
Step 3: Decay the adjusted score based on the time passed since the last trade on exchange Decay Volume-Adjusted Path Score (DVAPS).

We take a similar approach for the indirect pricing of chains for non-exchangeable pairs. For each *chain*, we look at the static characteristics, volume, and freshness for each *link*, then, for each attribute separately, in we take the *minimal* score across all links as the score of the chain for the given attribute. We then compute an adjusted score for the chain and select the chain with the highest score as the principal path. Note that this a min-max score and not a simple max volume.

The mathematical specification of the method is as follows, let $A = \{a_0, \ldots, a_k\}$ be the set all of assets (XBT, ETH, USDT, USDC, BAT, MCO, DOGE, CRO, etc.). We say that a pair (a_i, a_j) *exchangeable* if the pair a_i/a_j or a_j/a_i is traded on at least one exchange and priced by the BBHS model let E be the set of exchangeable pairs in A. Thus, (XBT, USDT) is exchangeable since this pair is traded on multiple exchanges, while (DOGE, APE) is not exchangeable as of the time of this writing. We shall call $G = (A, E)$ the *asset graph*. A *path* $\sigma_{a_0,\ldots,a_k}$ in the asset graph is a sequence $a_0, \ldots, a_k$ such that $a_i \in A$ for $0 \leq i \leq k$ and $(a_i, a_{i+1}) \in E$ for $1 \leq i \leq k-1$ this corresponds to a chain of pairs a_i/a_{i+1} where each link is an exchangeable pair $a_i, a_j \in A$.

For any link $a_i, a_j \in A$, let $S^{BES}_{a_i,a_j}$ be the *base exchange score (BES)* of the BBHS market for the pair, we define the *path base score (PBS)* to be

$$S^{PBS}_{\sigma_{a_0,\ldots,a_k}} = min\{S^{BES}_{a_i,a_{i+1}} \,|0 \leq i \leq k-1\}.$$

The link (a_i, a_{i+1}) on which this minimum is attained is the path *score bottleneck*. In the same manner, we define the *path volume* to be

$$Vol_{\sigma_{a_0,\ldots,a_k}} = min\{Vol_{a_i,a_{i+1}} \,|0 \leq i \leq k-1\},$$

where, as before, Vol_{a_i,a_j} is the volume on the BBHS path for (a_i, a_j) as measured in some common numeraire, typically either USD or XBT, using the BBHS method. Let $Vol_{a_0a_k}$ be the volume passing through the graph with a_0 as source and a_k as sink,[14] then we define the *volume-adjusted path score (VAPS)* to be

$$S^{VAPS}_{\sigma_{a_0,\ldots,a_k}} = S^{PBS}_{\sigma_{a_0,\ldots,a_k}} \times \frac{Vol_{\sigma_{a_0,\ldots,a_k}}}{Vol_{a_0a_k}}.$$

Similarly, we define

$$\tau_{\sigma_{a_0,\ldots,a_k}} = max\{\tau_{a_i,a_{i+1}} \,|0 \leq i \leq k-1\},$$

where τ_{a_i,a_j} is the time elapsed since the last trade of the pair on the BBHS market. Note that the greater the time lapse, the lower the score. Finally, we define the *decayed volume-adjusted path score (DVAPS)*

$$S^{DVAPS}_{\sigma_{a_0,\ldots,a_k}} = e^{-\kappa \bullet \tau_{\sigma_{a_0,\ldots,a_k}}} \times S^{VAPS}_{\sigma_{a_0,\ldots,a_k}}.$$

We then define the *principal path* as the path $\sigma_{a^*_0,\ldots,a^*_k}$ such that $\sigma_{a^*_0,\ldots,a^*_k}$ = *argmax* $\{B_{\sigma_{a_0,\ldots,a_k}} \cdot S^{DVAPS}_{\sigma_{a_0,\ldots,a_k}}$—for every path $a_0, \ldots, a_k$ in $G\}$,[15] where $B_{\sigma_{a_0,\ldots,a_k}}$ depends on and is descending on k, the latter reflects a higher score for Level 1 over Level 2 and for shorter paths over longer ones. The fair value exchange rate of a_k to a_0 is then

$$a_k/a_0 = a^*_k/a^*_{k-1} \times a^*_{k-1}/a^*_{k-2} \times \cdots \times a^*_1/a^*_0.$$

One limitation of this method is that reconstructing the trades on each link may imply transfer of assets between exchanges, transfers that may be costly in both time and gas. Crypto assets traded in centralized exchanges are typically held in the exchange wallet and credited to client accounts, thus buying, and selling crypto assets on such exchanges is not registered on the blockchain and does not require block confirmation. Transferring assets between exchanges requires transfer of assets from one wallet to another; these transfers are registered on the blockchain and require block confirmation. Blocks are added at fixed time intervals depending on the specific blockchain and entail corresponding latency,[16] as well as a fee, typically referred to as *gas price*, to the miner for including a transaction in the block. For modern blockchain platforms such as Avalanche and Solana, the latency is a fraction of a second, but with older platforms such as the pre-merge PoW Ethereum, latency was around 10 s, and for Bitcoin, at time of writing, is about 10 min. Another significant factor is the volatility of gas prices, the process through which a slot on a block is allocated to a transaction is essentially an auction, thus in times when there is a high volume of on-chain transactions, the gas prices can go up significantly. All this suggests, of course, that PPM is often not easy to attain, however this is not unlike many other asset classes such as equities where there may be significant transaction costs that are accounted for in ASC 820 guidelines for fair value.[17] Regarding latency, the IAS 21 includes a provision by which ''normal administrative delay in obtaining the other currency does not preclude a currency from being exchangeable into that other currency'' (IASB, 2021).

Moreover, we should recall that many of the more exotic assets, including many NFT's and specialized tokens, are traded on exchanges supporting only assets contained in a particular ecosystem. In these cases, traders use local exchanges for on/off ramping their fiat-to-crypto and execute crypto-to-crypto trades on more liquid exchanges, thus multi-exchange paths are quite common in practice.

PPM: An Illustration

In this section, we provide a basic example to illustrate the PPM. The illustration assumes that we would need to value MCO in terms of the USD for financial reporting purposes. However, MCO is not directly paired and traded with the USD but transacted with other crypto assets. To measure MCO at fair value for financial reporting purposes, we will use PPM to find the pairs of transactions with MCO that ultimately lead to the USD valuation.

In applying PPM, the first step is to identify ''links'' (crypto-crypto or crypto-fiat pairs) in all observed chains/paths from the targeted crypto asset (MCO) to the destination fiat currency[18] (i.e., USD, for financial reporting purposes). Figure 1 shows the complete ''asset graph'' for MCO-USD. As indicated in Figure 1,[19] MCO was mainly traded with other crypto assets including XBT, ETH, and USDT on major exchanges worldwide. Hence, three observed paths with one intermediary crypto asset would be MCO-XBT-USD, MCO-ETH-USD, and MCO-USDT-USD. In addition, some observed paths have two intermediary crypto assets such as MCO-XBT-USDT-USD, MCO-ETH-USDT-USD, MCO-USDT-XBT-USD, and MCO-USDT-ETH-USD. It is also possible to identify observed paths with more than two intermediary crypto assets. The more crypto assets or fiat currencies that the targeted crypto asset could be traded against, the more complex the map of observed paths would become—our method can address paths of any length. We know from data observations and input from experts that most paths used for both centralized and decentralized exchanges are short and consist of only two to three hops—this generally reflects the behavior of a typical crypto market participant. For illustration purposes, we will employ only the observed paths with one and two intermediary crypto assets.

Table 1 summarizes the simulated information for the illustration, including the principal path, the base exchange score, the fair value, the minute transaction volume (in USD),

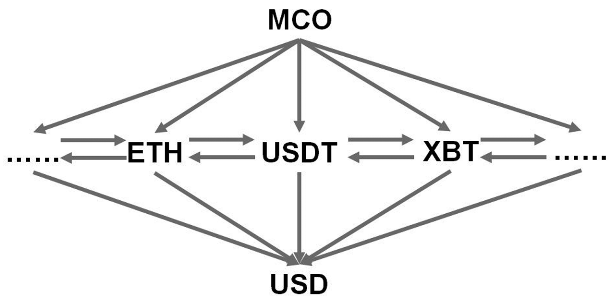

Figure 1. Illustration of Selected Observed Path Candidates for MCO-USD.

Table 1. Simulated Information for the Illustration of the Principal Path Method (PPM).

Pairs/Links	Market	Base exchange score	Fair value	Minute transaction volume (USD)	Time of the last trade
MCO-ETH	Exchange C	74.9889	0.0295	0.33	16:05:31.273
MCO-USDT	Exchange A	92.1846	4.689	597.17	16:05:52.207
MCO-XBT	Exchange A	94.9548	0.00054	606.64	16:05:52.143
ETH-USDT	Exchange A	96.2627	161	355,471.40	16:05:59.870
XBT-USDT	Exchange A	93.2820	8,698.97	7,409,847.00	16:05:59.988
ETH-XBT	Exchange A	96.3611	0.0185	64,553.04	16:05:59.620
ETH-USD	Exchange B	91.7013	160.7	79,671.79	16:05:57.405
USDT-USD	Exchange D	75.9979	0.99775	1,901.71	16:05:59.268
XBT-USD	Exchange B	86.3616	8,684.33	873,888.00	16:05:59.841

Note. The illustration is supposed to implement PPM at 16:06:00.000 of the day. For each pair, there is a BBHS principal market. The base exchange score for each pair is derived from the BBHS method. The fair value for each pair is derived from the BBHS method and presented on a basis of the first currency of each pair. For example, the fair value for XBT-USD is 8,684.33, meaning 8,684.33 USD/Bitcoin. The minute transaction volume (USD) for each link is the sum of transaction volume from 16:05:00.000 to 16:05:59.999 on the BBHS principal market. The time of the last trade for each link is derived from the BBHS principal market. PPM = Principal Path Method.

and the time of the last trade for each link (crypto-to-crypto or crypto-to-fiat pairs). The principal paths and base exchange scores of the links in all observed chains are individually determined and obtained using the BBHS method. For this part of the illustration, we implemented PPM at 16:06:00 of the day.[20] PPM will determine the *PBS*, which is the *minimum* base exchange score among links in each observed chain, for each observed chain. The result of the determination of the PBS for seven selected observed chains is presented in Panel A of Table 2.

Panel B of Table 2 shows the determination process of the *path volume*, which is the *minimum* transaction volume among links in each observed chain. The *volume-adjusted path score (VAPS)* will be determined after obtaining the volume weights for each chain, indicated in the Panel C of Table 2. Finally, the *decayed volume-adjusted path score (DVAPS)* will be calculated after identifying the *maximum* time elapsed since the last trade among all links for each observed chain. The observed chain with the highest DVAPS would be designated as the *principal path* at that moment in time. As indicated in the

Table 2. Determination of the Path Base Score (PBS), the Path Volume, and the Volume-Adjusted Path Score (VAPS) Under the Principal Path Method (PPM). Panel A. Path Base Score (PBS) in PPM; Panel B. Path Volume in PPM; Panel C. Determination of the Volume-Adjusted Path Score (VAPS) under the Principal Path Method (PPM); Panel D. Determination of the Decayed Volume-Adjusted Path Score (DVAPS) under the Principal Path Method (PPM).

Panel A

	The BBHS model (2021)—Base Exchange Score									
Observed path	MCO-ETH	MCO-USDT	MCO-XBT	ETH-USDT	XBT-USDT	ETH-XBT	ETH-USD	USDT-USD	XBT-USD	Path Base Score (PBS)
MCO-XBT-USD			94.9548						**86.3616**	**86.3616**
MCO-ETH-USD	**74.9889**						91.7013			**74.9889**
MCO-USDT-USD		92.1846						**75.9979**		**75.9979**
MCO-XBT-ETH-USD			94.9548			96.3611	**91.7013**			**91.7013**
MCO-XBT-USDT-USD			94.9548		93.2820			**75.9979**		**75.9979**
MCO-ETH-XBT-USD	**74.9889**					96.3611			86.3616	**74.9889**
MCO-ETH-USDT-USD	**74.9889**			96.2627				75.9979		**74.9889**
MCO-USDT-ETH-USD		92.1846		96.2627			**91.7013**			**91.7013**
MCO-USDT-XBT-USD		92.1846			93.2820				**86.3616**	**86.3616**

Note. The *path base score (PBS)* under PPM is the minimum of base exchange scores for links in each observed chain, indicated with the bold numbers. The concept and calculation of the Base Exchange Score are derived from Beigman, Brennan, Hsieh, and Sannella (BBHS, 2021).

Panel B

	Minute transaction volume (USD) on the observed path determined by the BBHS model (2021)									
Observed path	MCO-ETH	MCO-USDT	MCO-XBT	ETH-USDT	XBT-USDT	ETH-XBT	ETH-USD	USDT-USD	XBT-USD	Path Volume
MCO-XBT-USD			**606.64**						873,888	**606.64**
MCO-ETH-USD	**0.33**						79,671.79			**0.33**
MCO-USDT-USD		**597.17**						1,901.71		**597.17**
MCO-XBT-ETH-USD			**606.64**			64,553.04	79,671.79			**606.64**
MCO-XBT-USDT-USD			**606.64**		7,409,847			1,901.71		**606.64**
MCO-ETH-XBT-USD	**0.33**					64,553.04			873,888	**0.33**
MCO-ETH-USDT-USD	**0.33**			355,471.4				1,901.71		**0.33**
MCO-USDT-ETH-USD		**597.17**		355,471.4			79,671.79			**597.17**
MCO-USDT-XBT-USD		**597.17**			7,409,847				873,888	**597.17**

Note. Path volume under PPM is the *minimum* of transaction volume (USD) on the BBHS market for the links in each observed chain, indicated with the **bold** numbers.

Table 2. (continued)

Panel C

Observed path	Path Base Score (PBS)	Volume weight	Volume-Adjusted Path Score (VAPS)
MCO-XBT-USD	86.3616	606.64/1,204.14[a]	43.5087
MCO-ETH-USD	74.9889	0.33/1,204.14	0.0202518
MCO-USDT-USD	75.9979	597.17/1,204.14	37.6899
MCO-XBT-ETH-USD	91.7013	606.64/1,204.14	46.1989
MCO-XBT-USDT-USD	75.9979	606.64/1,204.14	38.2875
MCO-ETH-XBT-USD	74.9889	0.33/1,204.14	0.0202518
MCO-ETH-USDT-USD	74.9889	0.33/1,204.14	0.0202518
MCO-USDT-ETH-USD	91.7013	597.17/1,204.14	45.4777
MCO-USDT-XBT-USD	86.3616	597.17/1,204.14	42.8296

[a]1,204.14 (= 606.64 + 597.17 + 0.33) is "the overall volume passing through the graph." As we explained in section "Fragmented Markets," the path bottleneck volume would be identified for each path and the VAPS would be calculated based on the volume weight in which the denominator is the sum of all *unique* path bottleneck volume. That's the reason that 606.64, 597.17, and 0.33 are included only *once* when calculating the denominator (sum).

Panel D

Observed path	Volume-Adjusted Path Score (VAPS)	Time of the last trade	Decayed Volume-Adjusted Path Score (DVAPS)	Computed price
MCO-XBT-USD	43.5087	16:05:52.143	43.1682	4.68954
MCO-ETH-USD	0.0202518	16:05:31.273	0.0196783	4.74065
MCO-USDT-USD	37.6899	16:05:52.207	37.3973	4.67845
MCO-XBT-ETH-USD	46.1989	16:05:52.143	**45.8373**	**4.69070**
MCO-XBT-USDT-USD	38.2875	16:05:52.143	37.9878	4.68687
MCO-ETH-XBT-USD	0.0202518	16:05:31.273	0.0196783	4.73947
MCO-ETH-USDT-USD	0.0202518	16:05:31.273	0.0196783	4.73881
MCO-USDT-ETH-USD	45.4777	16:05:52.207	45.1247	4.68026
MCO-USDT-XBT-USD	42.8296	16:05:52.207	42.4971	4.68111

Note. The time of the last trade for each observed chain is derived from the trade with the longest time lag among all links of each observed chain.

Panel D of Table 2, the path *MCO-XBT-ETH-USD* is designated as the principal path because it generated the highest DVAPS (45.8373) among all observed chains.

After determining the principal path, the fair value price or exchange rate for MCO-USD would be the *product* of the BBHS exchange rates (ER) for the links in the chain. Therefore, as the designated principal path is MCO-XBT-ETH-USD, the fair value price for MCO-USD would be

$$FV_{MCO-USD} = ER_{MCO-XBT} \times ER_{XBT-ETH} \times ER_{ETH-USD} = 0.00054 \times \left(\frac{1}{0.0185}\right) \times 160.7 = 4.6907.$$

PPM: An Empirical Demonstration

In this section, we demonstrate the proposed PPM, determining the principal path and fair value measures for digital assets that are thinly traded or inactive against USD. The empirical demonstration uses actual market data. In addition, we will compare the fair value measures generated by the method with the actual prices of trades of the pairs, if they are available.

Specifically, we test eight different thinly traded crypto assets, including Basic Attention Token (BAT), Dogecoin (DOGE), STASIS EURO (EURS), Ankr (ANKR), Bytecoin (BCN), Crypto.com Coin (CRO), MCO (MCO), and NEM (XEM). The first three crypto assets had direct trades to the USD available on at least one exchange, while the remaining five crypto assets did not have any direct USD trades during the test period covering January 1, 2019, to June 30, 2020 (totaling 787,680 minutes).

Figure 2[21] shows the minute transaction volume from thinly traded crypto assets to other mainstream crypto assets and USD (if available) during the test period. Volumes from BAT, DOGE, EURS, ANKR, BCN, CRO, MCO, and XEM to other crypto assets and the USD are displayed in Panels A, B, C, D, E, F, G, and H, respectively. For BAT (Panel A) and DOGE (Panel B), the transaction volume to the USD is much lower compared with the volume for XBT, ETH, USDT, or USDC. Interestingly, the transaction volume for the USD is the same level as the volume for USDT for EURS (Panel C). Although the remaining crypto assets (Panel D-H) had no direct trades against the USD, they had frequent trades and sufficient transaction volume with XBT, ETH, USDT, USDC, or BNB.

In the empirical demonstration, PPM is tested at the end of each minute for the period from January 1, 2019, to June 30, 2020 (totaling 787,680 min). Table 3 presents the number (percentage) of minutes available for the fair value measures from PPM and the prices of direct trades to the USD for the eight selected thinly traded crypto assets. For instance, the BAT-USD trades were accessible on four exchanges; however, they are infrequently traded. Specifically, only 28,079 (3.56%), 19,219 (2.44%), 25,117 (3.19%), and 5,698 (0.72%) min had trades on the four exchanges for BAT-USD during the test period, while PPM could generate fair value measures for BAT-USD in 757,923 (96.22%) min. Furthermore, DOGE-USD could only be transacted on Exchange A in 6,735 (0.86%) min, but there were 783,579 (99.48%) min having fair value measures from PPM during the test period. For ANKR, BCN, CRO, MCO, and XEM—crypto assets with no direct trades against the USD—PPM could still determine fair value measures in terms of USD in 339,567 (43.11%), 254,746 (32.34%), 623,970 (79.22%), 667,862 (84.79%), and 777,042 (98.65%) min, respectively. PPM can provide timely, reliable, and frequent fair value measures for those thinly traded crypto assets with few or even no direct trades against the USD.

Figure 3 shows the minute fair value measures of PPM for thinly traded crypto assets and prices of trades from thinly traded crypto assets to the USD (if available) during the test period. The fair value measures and prices (if available) from BAT, DOGE, EURS, ANKR, BCN, CRO, MCO, and XEM to other crypto assets and the USD (if available) are displayed in Panels A through H, respectively.

The fair value measures and the actual prices for BAT-USD were well matched with each other for most minutes. However, PPM had one fair value outlier at 7:35 AM on January 15, 2020. This outlier might result from the real-time, atypical transaction behavior in the determined principal path or a possible data integrity issue on exchanges. Moreover, there were some price outliers (large abnormal price jumps comparing the adjacent

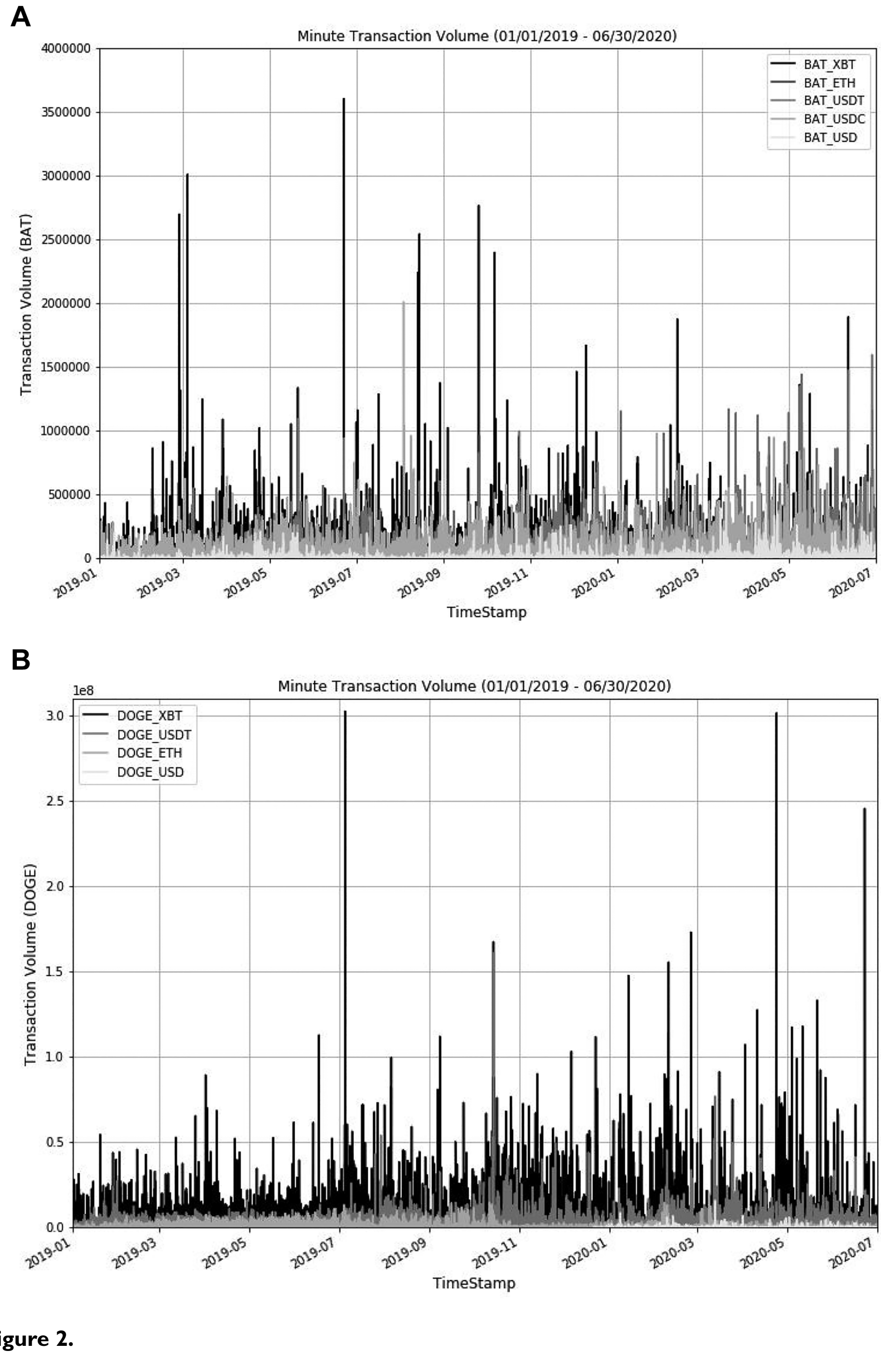

Figure 2.

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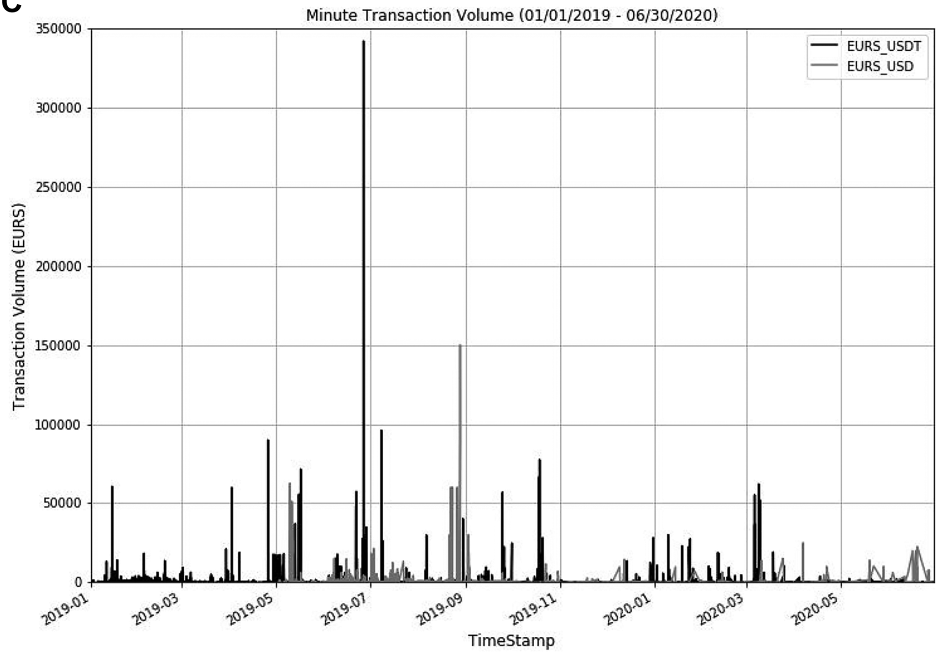

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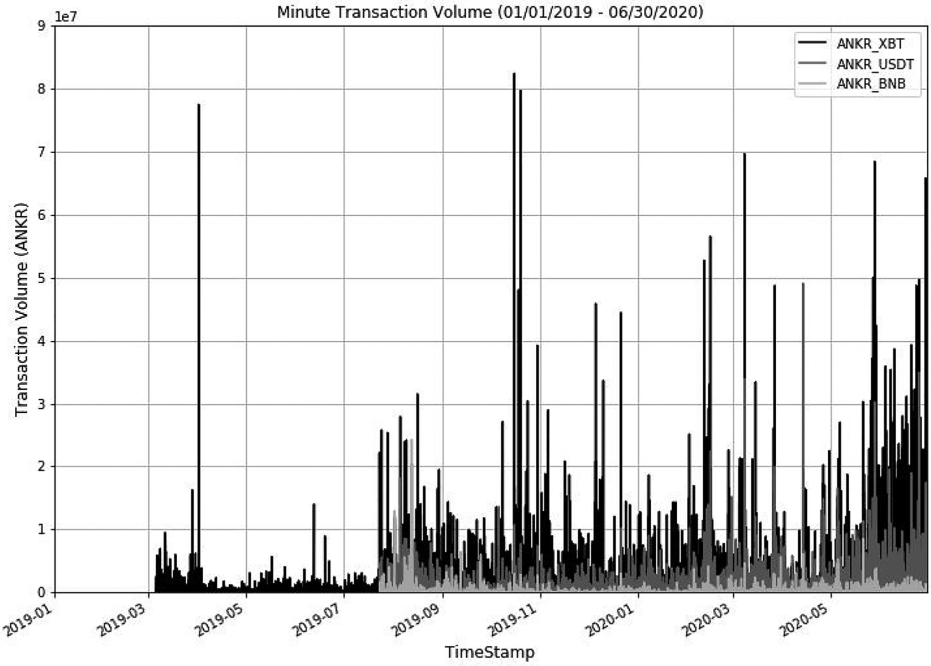

Figure 2.

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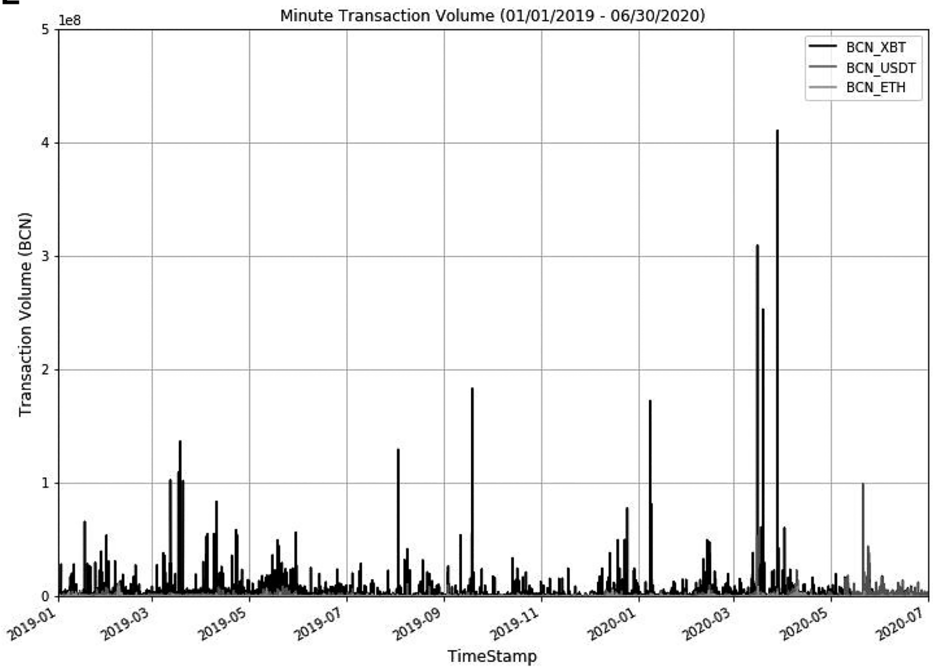

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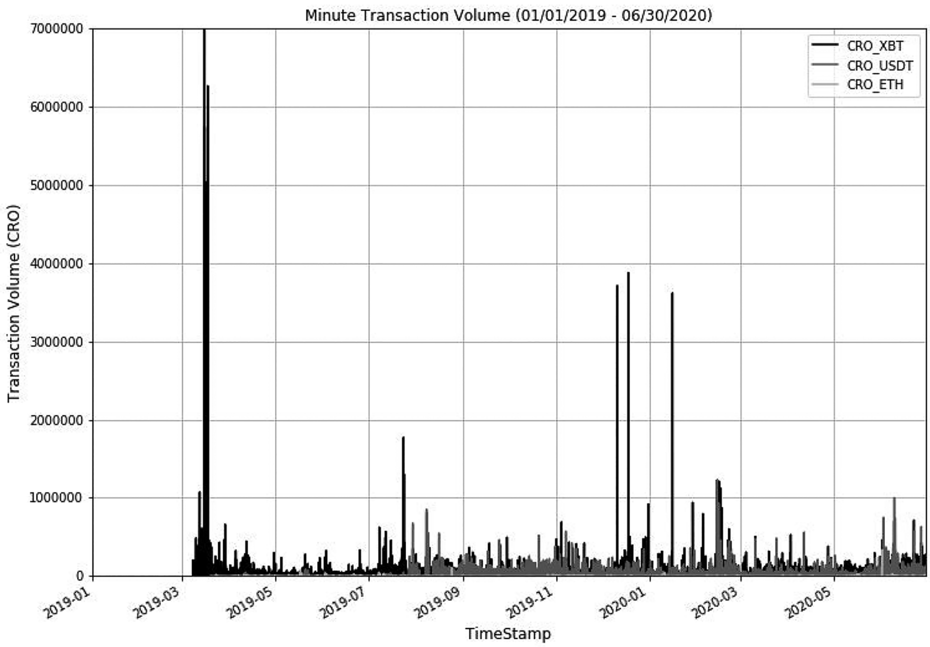

Figure 2.

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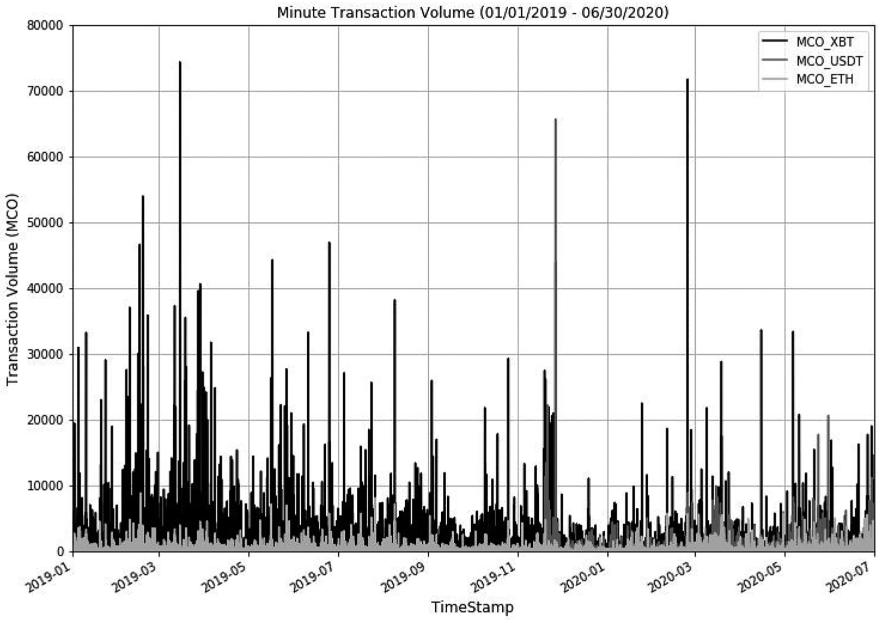

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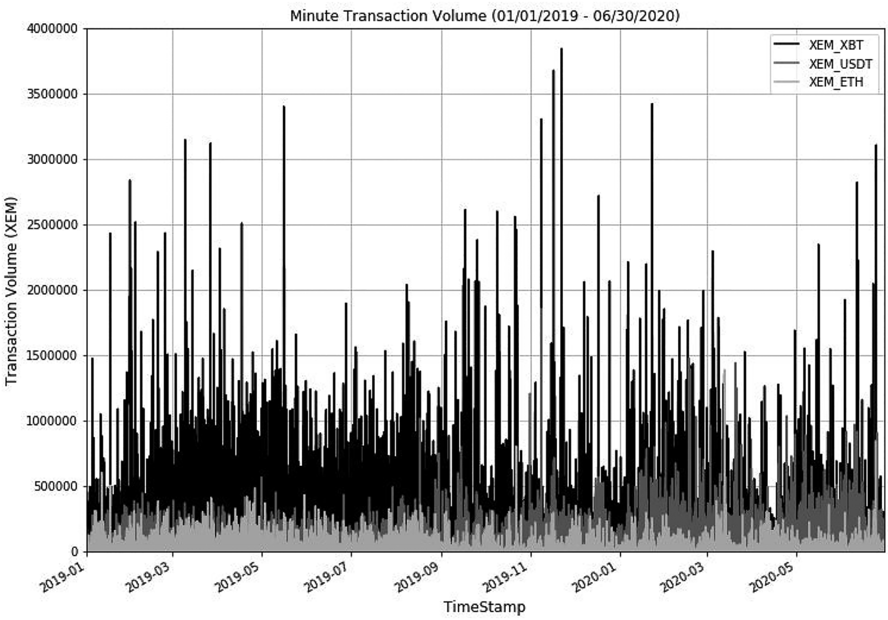
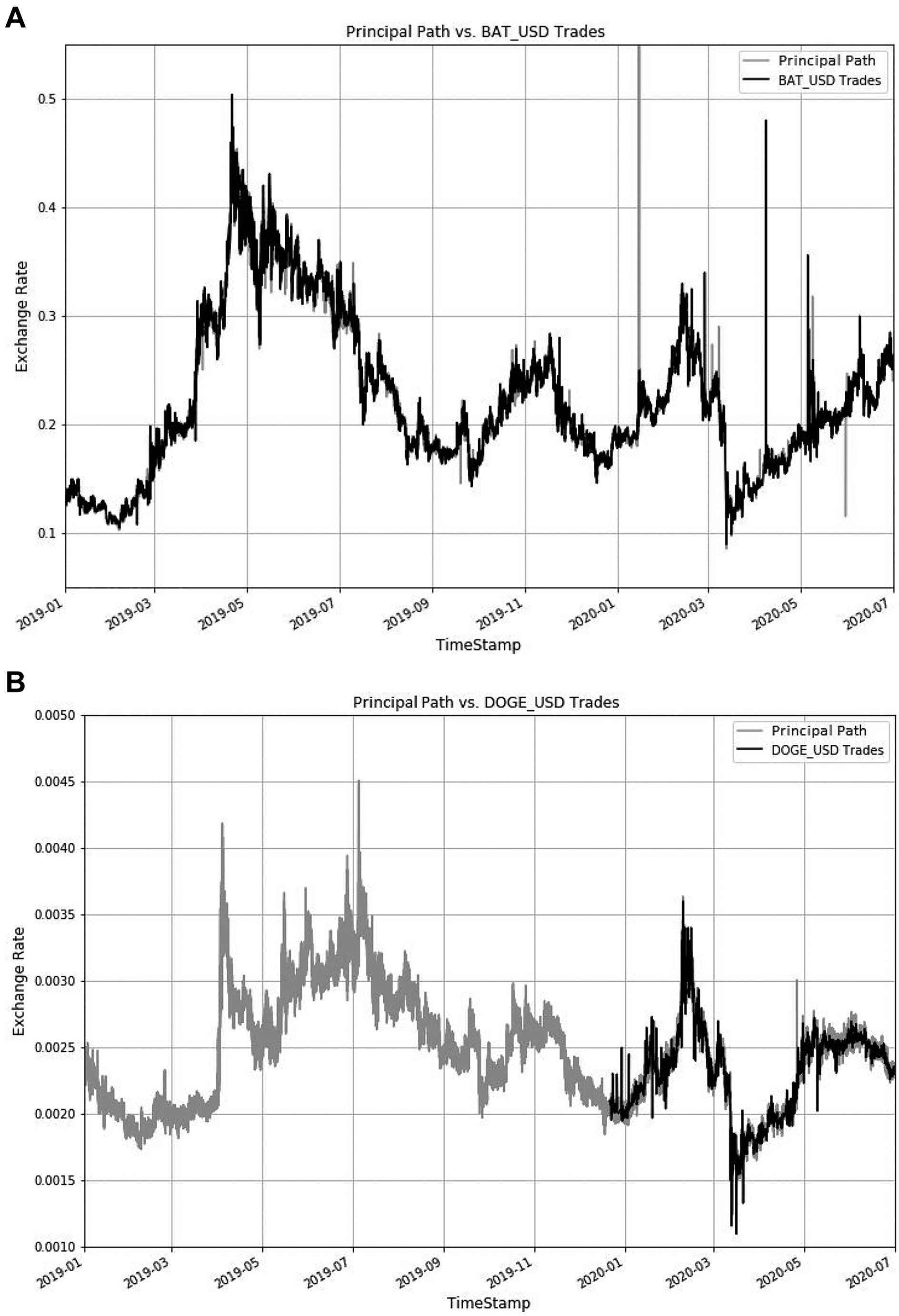


Figure 2. Minute Transaction Volume from Thinly Traded Crypto Assets to Other Mainstream Crypto Assets and the USD (If Available).

Note. This figure presents the minute transaction volume from thinly traded crypto assets to other mainstream crypto assets and the USD (if available) from January 1, 2019, to June 30, 2020. Volumes from Basic Attention Token (BAT), Dogecoin (DOGE), STASIS EURO (EURS), Ankr (ANKR), Bytecoin (BCN), Crypto.com Coin (CRO), MCO (MCO), and NEM (XEM) to other crypto assets and the USD (if available) are displayed in Panels A, B, C, D, E, F, G, and H, respectively. Panel A: Basic Attention Token (BAT), with available trades for BAT-USD. Panel B: Dogecoin (DOGE), with available trades for DOGE-USD. Panel C: STASIS EURO (EURS), with available trades for EURS-USD. Panel D: Ankr (ANKR), without available trades for ANKR-USD. Panel E: Bytecoin (BCN), without available trades for BCN-USD. Panel F: Crypto.com Coin (CRO), without available trades for CRO-USD. Panel G: MCO (MCO), without available trades for MCO-USD. Panel H: NEM (XEM), without available trades for XEM-USD.



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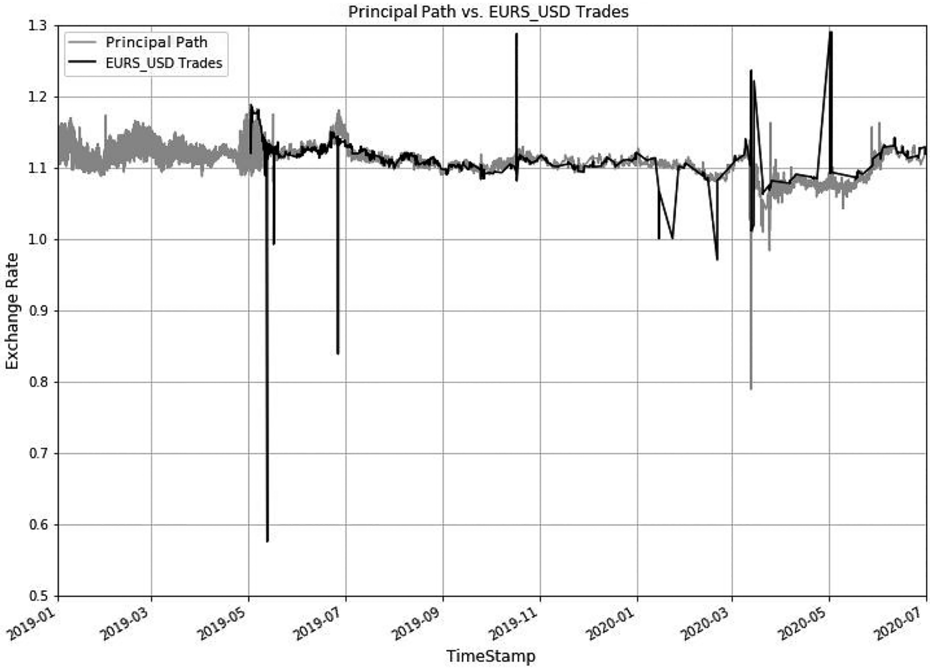

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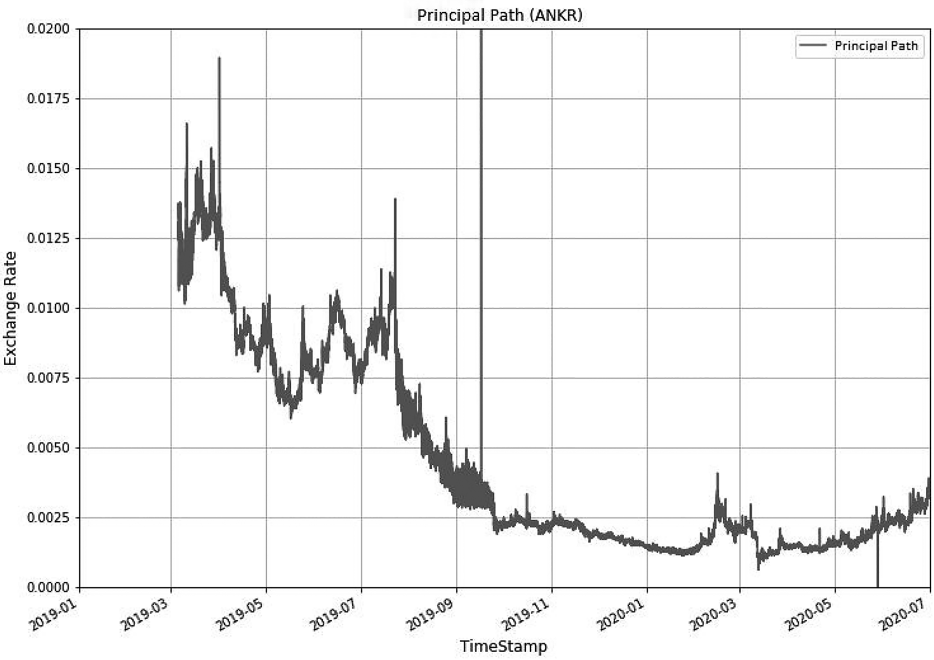

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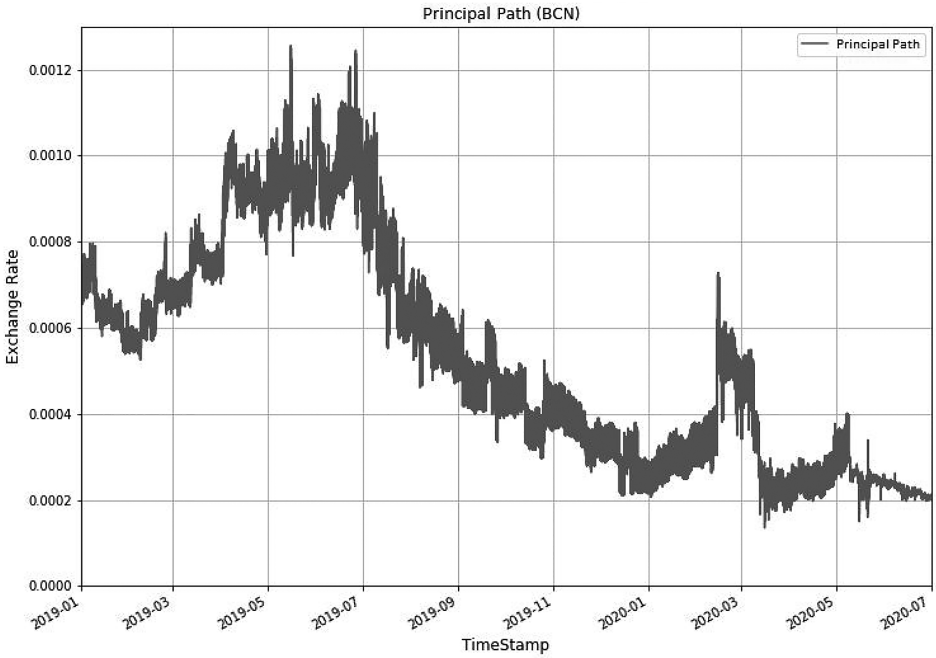

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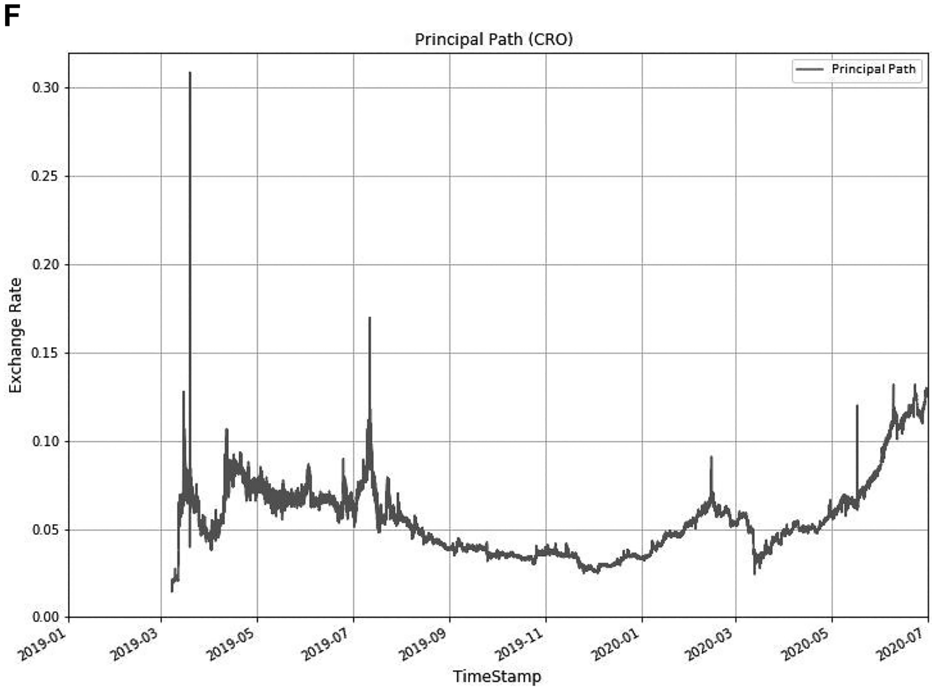

Figure 3.

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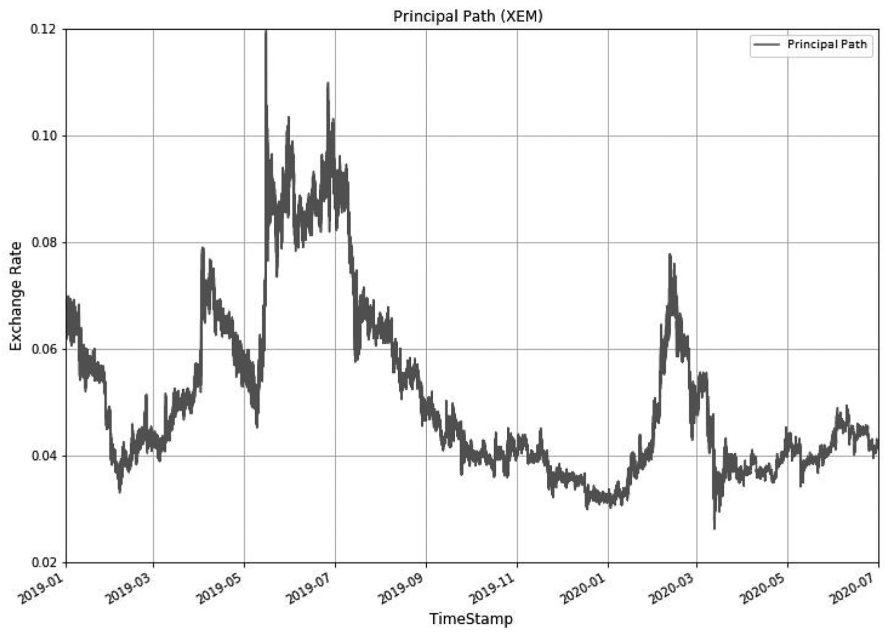

Figure 3. Minute Fair Value Measures From the Principal Path Method (PPM) for Thinly Traded Crypto Assets and Prices From Thinly Traded Crypto Assets-USD Trades (If Available).

This figure presents the minute fair value measures of PPM for thinly traded crypto assets and prices of trades from thinly traded crypto assets to the USD (if available) from January 1, 2019, to June 30, 2020. The fair value measures and prices (if available) from Basic Attention Token (BAT), Dogecoin (DOGE), STASIS EURO (EURS), Ankr (ANKR), Bytecoin (BCN), Crypto.com Coin (CRO), MCO (MCO), and NEM (XEM) are displayed in Panels A, B, C, D, E, F, G, and H, respectively. Panel A: Basic Attention Token (BAT), with available trades for BAT-USD. Panel B: Dogecoin (DOGE), with available trades for DOGE-USD. Panel C: STASIS EURO (EURS), with available trades for EURS-USD. Panel D: Ankr (ANKR), without available trades for ANKR-USD. Panel E: Bytecoin (BCN), without available trades for BCN-USD. Panel F: Crypto.com Coin (CRO), without available trades for CRO-USD. Panel G: MCO (MCO), without available trades for MCO-USD. Panel H: NEM (XEM), without available trades for XEM-USD.

Table 3. The Number of Minutes With Available Fair Value Measure Under the Principal Path Method (PPM) With Available Prices of Trades to the USD From January 1, 2019, to June 30, 2020.

Crypto assets	Principal Path Method	Direct trade to USD			
		Exchange A	Exchange B	Exchange C	Exchange D
Basic Attention Token (BAT)	757,923 (96.22%)	28,079 (3.56%)	19,219 (2.44%)	25,117 (3.19%)	5,698 (0.72%)
Dogecoin (DOGE)	783,579 (99.48%)	6,7235 (0.86%)	N/A	N/A	N/A
STASIS EURO (EURS)	16,373 (2.08%)	N/A	2,267 (0.29%)	N/A	N/A
Ankr (ANKR)	339,567 (43.11%)	N/A	N/A	N/A	N/A
Bytecoin (BCN)	254,746 (32.34%)	N/A	N/A	N/A	N/A
Crypto.com Coin (CRO)	623,970 (79.22%)	N/A	N/A	N/A	N/A
MCO (MCO)	667,862 (84.79%)	N/A	N/A	N/A	N/A
NEM (XEM)	777,042 (98.65%)	N/A	N/A	N/A	N/A

Note. We perform the proposed PPM in each minute from January 1, 2019, to June 30, 2020, totaling 787,680 min. The names of available exchanges were anonymized as Exchanges A to D. Exchanges A to D in this table are different from the ones in Table 1.

minutes) of real trades for BAT-USD at 8:10 AM on April 8, 2020, and from 5:00 to 6:00 PM on May 5, 2020. As presented in Figure 4, the real trades of BAT-USD experienced abnormal price jumps during the one-hour period; however, PPM constantly generated relatively stable fair value measures in those 60 minutes.

Some thinly traded crypto assets, such as DOGE and EURS, would be initially traded with other mainstream crypto assets rather than fiat currencies. As displayed in Panels B and C of Figure 3, PPM could identify the principal path and determine the fair values for DOGE-USD and EURS-USD even if there were no direct trades from DOGE and EURS against the USD at the beginning of the test period. Furthermore, Panel C of Figure 3 revealed that the real trades of EURS-USD experienced many significant abnormal price jumps, but the fair value measures from PPM were relatively stable, and therefore, more reliable.

Panels D to H of Figure 3 show the fair value measures from ANKR,[22] BCN, CRO,[23] MCO, and XEM to the USD. At time of writing, none of these assets are traded against USD on any exchange and can only be priced through PPM or some other methodology.

Conclusions

Innovative valuation methodologies for crypto assets are a critical component of the ongoing acceptance and adoption of these emerging economic phenomena built on blockchain/distributed ledger technology. Commercial market-to-market methodologies for actively traded crypto assets in orderly markets exist and are offered in commercially available software products, but dynamic standards–aligned valuation models for thinly traded, or Level 2, crypto assets are needed to provide more comprehensive pricing and asset valuation. In addition, we believe that our methodology improves on current techniques used to value thinly traded crypto assets such as using the last observable transaction price, creating a weighted-average price across multiple markets, or using data on comparable tokens, if available. Unlike methods currently used in practice, our method ensures the

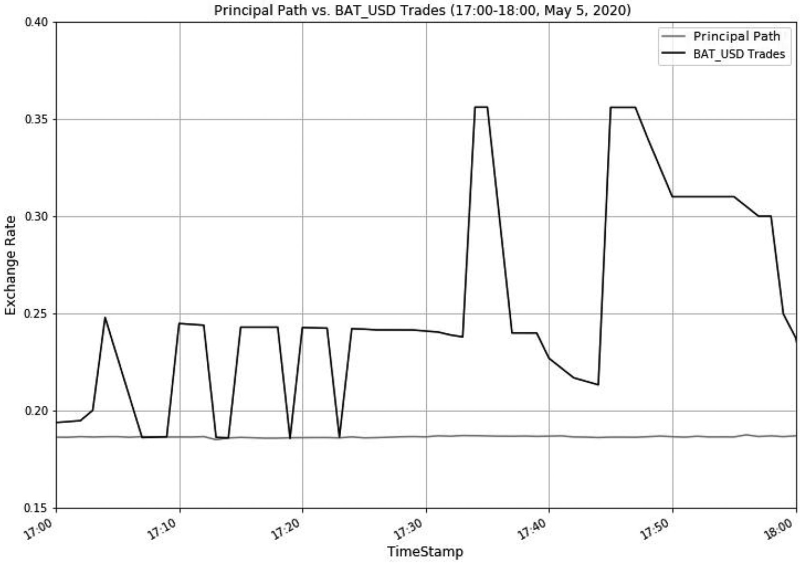

Figure 4. Minute Fair Value Measures From the Principal Path Method (PPM) and Prices From Real Trades for BAT-USD From 17:00 to 18:00 on May 5, 2020.
This figure presents the minute fair value measures from the Principal Path Method (PPM) and prices from real trades for BAT-USD from 17:00 to 18:00 on May 5, 2020. The real trades of BAT-USD experienced abnormal price jumps during the 1-hr period. However, PPM constantly generated stable fair value measures during the period.

integrity of the valuation data by selecting prices from compliant markets to ensure reliability and faithful representation.

This article addresses these needs by proposing the PPM, as an approach to determining fair value for non-exchangeable, thinly traded, or illiquid asset pairs. The PPM enables the dynamic identification of the principal path for the pairs by scoring the static characteristics, transaction volume, and the time elapsed from the last trade for all observed paths. Only compliant exchanges are considered in establishing fair value for thinly traded crypto pairs and transaction volume-informed fair value measures are derived by multiplying the crypto asset prices of links in the determined principal path. Empirical evidence is also provided, supporting the assertion that PPM can derive more timely and reliable fair value measures for crypto to fiat currency than standard model-based and other methods used in practice. While non-exchangeable or thinly traded crypto assets represent a small part of the total trading volume, they account for a significant part of the total crypto transaction activity in absolute number of trades and many crypto assets may move between valuation levels as they emerge, mature, or decline. Our exchange-compliant method will ensure broader valuation coverage in the rapidly expanding crypto asset space and may be applied to a wide variety of crypto asset types and pairs.

There are some limitations in our approach. As noted earlier in our paper, one limitation of this method is that reconstructing the trades on each link may imply the transfer of assets between exchanges, transfers that could potentially be costly in time, fee, and gas. We discuss penalizing longer paths through a path-dependent coefficient $B_{\sigma_{a_0,\ldots,a_k}}$, but do not provide any guidance regarding how this penalty should be implemented if appropriate. In addition, market factors have changed over time and may impact the assumptions used in our method. Several years ago, the best way to attain the PPM conversion rate would be to use multiple centralized exchanges, transferring assets directly from one exchange wallet to another. In this setting, we incur additional costs for gas required for the on-chain transaction, as well as latency for confirmation. Currently, a significant amount of trading is done on decentralized exchanges. These are markets not structured as limit order book markets, applying their own logic for matching through smart contracts. While transactions and all relevant information are recorded on the blockchain, there are currently no guidelines regarding the use of on-chain information. Moreover, these exchanges live on a particular blockchain platform, moving between exchanges requires moving between blockchain platforms, typically done through special bridges, which have their own costs and risks. This technology is still in its infancy, and it might take some time before it is clear how it should be used but these issues merit revisiting in future research.

Further research is also needed to explore the development of models for valuing assets or asset pairs where there are no observable transactions resulting in a ''Level 3'' classification in the fair value hierarchy.[24] There is also a need to determine how to define ''thinly traded markets.'' As discussed in this article, a level like the ADV of 100,000 for NMS stocks (SEC, 2018) could be employed. The threshold established would be based on the evaluation of available market data with input and recommendations from a cross-section of valuation experts in industry, accounting firms, and academic researchers' familiar with crypto assets and the supporting ecosystem.

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Declaration of Conflicting Interests

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Data Availability

Contact the correspondence author.

Notes

1. Native token for the Polygon layer 2 sidechain, ranked #10, market cap US$10.9B.
2. Utility token for Uniswap decentralized exchange (DEX), ranked #18, market cap US$5.5B.
3. Governance and utility token for The Graph ecosystem, ranked #52 with market cap US$981.9M.

4. Information about the pricing techniques currently used in practice was obtained from extensive conversations with accounting and other valuation professionals. As noted, the resulting valuations would yield a Level 2 valuation. An exception would be in those cases where discounted cash flow is used or if an observed transaction price is discounted for significant decay. A Level 3 valuation would probably be used is such cases.
5. The BBHS method refers to the fair value measurement method developed by Beigman et al. (2021).
6. Utility token used in the APE ecosystem developed from the *Bored Ape Yacht Club* project, ranked #30, market cap 2.2B, 5% transactions vs. fiat.
7. Utility token used for purchasing virtual real estate on in Decenterland, ranked #41, market cap 1.44B, 6.5% transactions vs. fiat.
8. Utility and governance token, used to reward liquidity provision for automated market makers in the PancakeSwap platform, ranked #61, market cap 768M, does not trade against fiat.
9. Governance token, used for staking and trading on the dXdY Layer 2 decentralized exchange, ranked #81, market cap 495M, only 0.44% vs. fiat.
10. More information is available at: https://apecoin.com/.
11. There is significant diversity within the class of stablecoins. While all are pegged to one fiat or another, the pegging mechanism can differ substantially, ranging from fully backed, partially backed or algorithmic stable coins. This diversity was in full display in the recent events around TerraUSD (UST). The rapid pace at which a stable coin such as UST can move away from peg target is one of the motivations for our layered approach for path scoring accounting for both monthly volumes and minute to minute decay.
12. Note that the BBHS method is only used to identify compliant markets at each link of the chain. The exchanges identified at each link are not considered principal markets. This approach not only ensures the inclusion of compliant markets, but it also enhances the reliability of the inputs used to value the crypto asset held. We are using elements of the BBHS method to address the need for price discovery for Level 2 assets only and are using a ''mark to model'' approach. We are not using this method to identify a principal market. FASB ASC-820 and IFRS-13 does not require the use of a principal or most advantageous market to price the crypto assets considered in this article.
13. The threshold established would be based on the evaluation of available market data with input and recommendations from a cross-section of valuation experts in industry, accounting firms, and academic researchers' familiar with crypto assets and the supporting ecosystem.
14. Vol is a solution to the max flow problem which can be solved efficiently (namely, in polynomial time) with various algorithms. For details, see Matousek and Gärtner (2007) and Papadimitriou and Steiglitz (1998).
15. If there are more than two observed paths with the same highest *DVAPS*, then the PPM will take the path with the *least number of links* to be the *principal path*.
16. Proposed paragraph A5 in the Exposure Draft ED/2021/4 *Lack of Exchangeability* (IASB, 2021) reflects the Board's conclusion that a normal administrative delay in obtaining the other currency does not preclude a currency from being exchangeable into that other currency. Ignoring normal administrative delays would, in the Board's view, lead to entities inappropriately concluding that exchangeability is lacking when a currency would, in effect, be exchangeable into that other currency. The Board decided not to propose application guidance on what would constitute a 'normal administrative delay'—this assessment would depend on facts and circumstances (for example, the jurisdiction in which an exchange transaction occurs and the type of exchange mechanism) (IASB, 2021).
17. It should be noted that transactions costs would only play a role in the fair value process if we were considering the most advantageous market. The most advantageous market is used when a principal market cannot be identified. Because the principal path subsumes the principal market when the latter is not relevant, the most advantageous market or path and consideration of

transactions costs are not pertinent to our method as in classical market situations where this could allow for possible to cherry picking and methodological inconsistency. In addition, prices observed in illiquid markets tend to be discounted (i.e., an implied fee). Nonetheless, the *B* factor/coefficient penalizes length of chains, it is somewhat arbitrary as of time of writing, but as with the threshold for thinly traded, when sufficient experience on crypto trading is attained this should be revisited. In addition to penalizing longer paths, we are also decaying the price for latency. We realize that most transactions have short paths simply because that reflects the behavior of a typical market participant trying to convert crypto to a fiat currency or other crypto. The short path would imply immaterial transactions costs. There may be many possible reasons for a longer path, and it may only occur with the use of automated trading bots. In addition, there may not a fiat offramp on the exchange the participant is trading on. A market participant would generally only need one hop to find an exchange with a fiat off-ramp or another crypto they want to trade. A review of current practice indicates that multiple markets are often considered when computing a weighted-average price for thinly traded cryptocurrencies. The use of multiple markets might be justified from a risk management standpoint although not specifically included in the current guidance.

18. The destination currency is not limited to fiat currencies and can be other crypto assets based on the customized needs.
19. The MCO may be transacted with additional crypto assets or fiat currencies (represented by the dotted lines in Figure 1), making the observed paths more various. Additional observed paths may be possible in the future when MCO becomes more popular.
20. Users can implement PPM in any time to fit their specific purposes. We randomly selected 16:06:00 of the day because the thinly traded pairs (MCO-XBT, MCO-USDT, and MCO-ETH) had trades reported in the previous minute (from 16:05:00.000 to 16:05:59.999).
21. The names of exchanges used in the empirical demonstration section were anonymized as Exchange A to D. The names of the actual exchanges were used in an earlier version of our paper. However, one of the exchanges asked us not to disclose the name of the exchanges, and we complied.
22. The ANKR trade data were not available until March 5, 2019.
23. The CRO trade data were not available until March 7, 2019.
24. It should be noted that even if crypto assets continue to be classified as indefinite-lived intangible assets, a current FASB decision supports the valuation of cryptocurrency at fair value through earnings. In addition, the SEC in SAB 121 indicates that crypto assets held in a custodial capacity, along with the related liability be measured at fair value. The valuation technique developed in our paper is needed to measure the fair value of thinly traded crypto currency for these purposes.

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