TTK4150 Nonlinear Control Systems
Department of Engineering Cybernetics
Norwegian University of Science and Technology
Fall 2014 - Assignment 4
Due date: Monday 27 October at 12.00.

1. Exercise 4.10 in Khalil.

2. Let

$$
\begin{aligned}
V_1(x_1, x_2, t) &= x_1^2 + \left(1 + e^t\right) x_2^2 \\
V_2(x_1, x_2, t) &= \frac{x_1^2 + x_2^2}{1 + t} \\
V_3(x_1, x_2, t) &= \left(1 + \cos^4 t\right)\left(x_1^2 + x_2^2\right)
\end{aligned}
$$

   For each of the functions $V_i(x_1, x_2, t)$, $i \in \{1, 2, 3\}$ investigate the properties of positive definite and decrescent.

3. Consider the system

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -x_1 - c(t)\, x_2
\end{aligned}
$$

   where the function $c(t)$ is continuous differentiable and satisfies

$$k_1 \leq c(t) \leq k_2 \text{ and } |\dot{c}(t)| \leq k_3 \ \forall t \geq 0$$

   and k_i are constants and $k_1 > 0$. Use the Lyapunov function candidate

$$V(x) = \frac{1}{2}\left(x_1^2 + x_2^2\right)$$

   to show that the origin is uniformly stable and that $x_2 \to 0$ as $t \to \infty$.

4. Exercise 4.54 in Khalil.
   Hint: If a system is ISS, then:

   (a) for $u(t) \equiv 0$ the origin is globally asymptotically stable.

   (b) for a bounded input $u(t)$, every solution $x(t)$ is bounded.

   If one of these is not satisfied, the system can **not** be ISS.

5. Exercise 4.55 no. (1), (2), (4) and (5) in Khalil.
   Hint for part (2): Read example 4.27 before doing this exercise.
   Hint for part (4): For $u(t) \equiv 0$ an ISS system needs to have a globally asymptotically stable origin. This requires the absence of other equilibria.

6. Exercise 4.56 in Khalil.
7. Exercise 5.3 in Khalil.
8. Exercise 5.4 in Khalil.
9. Exercise 5.20 in Khalil.
10. Exercise 6.2 in Khalil.
11. Exercise 6.4 in Khalil.
12. Consider again the Duckmaze system from the previous assignments.
    (a) Consider the transformed system from Assignment 2 (Exercise 1b):

$$\dot{\tilde{x}}_1 = \tilde{x}_2 \tag{1}$$
$$m\dot{\tilde{x}}_2 = -f_3\left[(\tilde{x}_1 + x_{1d})^3 - x_{1d}^3\right] - f_1\tilde{x}_1 - d\tilde{x}_2 + \tilde{u} \tag{2}$$

    Define the output

$$y = \tilde{x}_2 \tag{3}$$

    As in Assignment 3 (Exercise 1a), use $V = \frac{1}{2}\left(\tilde{x}_1^2 + m\tilde{x}_2^2\right)$ as Lyapunov function candidate.
    Outline a control law that makes the system passive from the new control input v to the output y (in Khalil this technique is described as feedback passivation).
    Note: This topic (Chapter 14.4: Passivity-based Control) will be covered in the lectures later on - this problem is however an easy introduction which you will be able to solve.

    (b) Is the system zero state observable?

    (c) Explain why the origin can be globally stabilized. Derive a controller that globally stabilizes the origin.

    (d) An unknown constant disturbance w is acting on the system, i.e. the system equations are changed to

$$\dot{x}_1 = x_2 \tag{4}$$
$$\dot{x}_2 = -\frac{f_3}{m}x_1^3 - \frac{f_1}{m}x_1 - \frac{d}{m}x_2 - g + \frac{u}{m} + \frac{w}{m} \tag{5}$$

    Do we still have $x_1^* = \lim_{t\to\infty} x_1 = x_{1d}$?

When investigating passivity for interconnected systems, the first step is often to try a storage function as a sum of the storage functions for the interconnected systems. This is illustrated in Exercise 12.

13. Exercise 6.6 in Khalil.
    (Note: In this problem, for output strictly passivity you can assume that $y_i^T\rho_i(y_i) \geq \delta_i y_i^T y_i$ for some positive δ_i)
14. Exercise 6.1 in Khalil.