13.7

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + D\dot{q} + g(q) = u$$

let $q = x_1$, $\dot{q} = x_2$

then

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = M(x_1)^{-1}\{-C(x_1,x_2)x_2 - Dx_2 - g(x_1)\} + u$$

$\therefore$ feedback linearizable

13.11

$$\dot{x}_1 = x_1 + x_2$$

$$\dot{x}_2 = 3x_1^{\,2}x_2 + x_1 + u$$

$$y = -x_1^{\,3} + x_2$$

(a) $\dot{y} = -3x_1^{\,3} + x_1 + u \rightarrow$ input-output linearizable

(b) relative degree $= 1$

Let $z = \begin{bmatrix}\eta \\ \xi\end{bmatrix}$. from $\dfrac{\partial \eta}{\partial x} g(x) = 0 \rightarrow$ try $\eta = x_1$

then $z = \begin{bmatrix}\eta \\ \xi\end{bmatrix} = \begin{bmatrix} x_1 \\ -x_1^{\,3} + x_2\end{bmatrix}$

normal form $\Rightarrow \begin{cases} \dot{\eta} = \eta + \xi + \eta^3 \\ \dot{\xi} = -3\eta^3 + \eta + u \\ y = \xi \end{cases}$

$z = T(x)$ is a global diffeomorphism : valid region $= R^2$

$$\left(\frac{\partial T}{\partial x} = \begin{bmatrix} 1 & 0 \\ -3x_1^{\,2} & 1 \end{bmatrix}, \ \lim_{\|x\|\to\infty} \|T(x)\| = \infty \right)$$

13.11

(c) zero-dynamics $\dot{\eta} = \eta + \eta^3$

$\therefore$ non-minimum phase system

(d)
$$\begin{cases} ad_f g = \begin{bmatrix} -1 \\ -3x_1^{\ 2} \end{bmatrix} \Rightarrow \begin{bmatrix} g & ad_f g \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & -3x_1^{\ 2} \end{bmatrix} \quad \therefore \text{ rank} = 2 \\ D = \text{span}\{g\} \text{ is involutive} \end{cases}$$

$\therefore$ feedback linearizable

(e) Try $z = \begin{bmatrix} x_1 \\ x_1 + x_2 \end{bmatrix}$

then

$\dot{z}_1 = z_2$

$\dot{z}_2 = z_1 - 3z_1^{\ 3} + 3z_1^{\ 2} z_2 + z_2 + u$

13.14 See example 13.14

13.21

(a) $\dot{y} = \dot{x}_1 - \dot{x}_2 = x_3$

$\ddot{y} = \dot{x}_3 = x_1 x_2 - x_2^{3} + u$

$\Rightarrow$ relative degree : 2

$\dot{\eta} = \frac{\partial \phi}{\partial x}[f(x) \quad g(x)u] \longrightarrow \frac{\partial \phi}{\partial x} g(x) = 0, \quad g(0) = 0.$

$\frac{\partial \phi}{\partial x} g(x) = \frac{\partial \phi}{\partial x_1} + \frac{\partial \phi}{\partial x_2} + \frac{\partial \phi}{\partial x_3} = 0.$

$\psi_1 = h(x) = x_1 - x_2 \longrightarrow \xi_1 = x_1 - x_2$

$\psi_2 = L_f h = x_3 \longrightarrow \xi_2 = x_3.$

$T(x) = [\phi(x) \quad x_1 - x_2 \quad x_3]^T, \quad z = [\eta, \ \xi_1, \ \xi_2]^T$

Let $\phi(x) = x_3 - x_1 = \eta.$

$f_0(\eta, \xi) = \frac{\partial \phi}{\partial x} f(x) \Big|_{x = T^{-1}(z)}$

$f_0(\eta, \xi) = \xi_1 - \eta^3$

$\therefore \dot{\eta} = \xi_1 - \eta^3$

$$\begin{cases} \dot{\xi}_1 = \xi_2 \\ \dot{\xi}_2 = \xi_1 \xi_2 + u \\ \dot{\eta} = \xi_1 - \eta^3 \end{cases}$$

(b) zero dynamics : $\dot{\eta} = -\eta^3$

Let $V = \eta^2$

13.25 See chapter 13.4 and example 13.21

13.26

$$\dot{x}_1 = x_2 + x_1 \sin x_1$$
$$\dot{x}_2 = x_1 x_2 + u$$
$$y = x_1$$
$$r(t) = \sin t$$

Let

$$e_1 = x_1 - r$$
$$\dot{e}_1 = x_2 + x_1 \sin x_1 - \dot{r} = e_2$$
$$\dot{e}_2 = x_1 x_2 + (x_2 + x_1 \sin x_1)(\sin x_1 + x_1 \cos x_1) - \ddot{r} + u$$
$$u = \ddot{r} - \{x_1 x_2 + (x_2 + x_1 \sin x_1)(\sin x_1 + x_1 \cos x_1)\} + k_1 e_1 + k_2 e_2$$