TTK4150 Nonlinear Control Systems
Department of Engineering Cybernetics
Norwegian University of Science and Technology
Fall 2015 - Assignment 6
Due date: Thursday 26 November at 11.00.

1. Exercise 13.1 in Khalil.

2. Exercise 13.2 in Khalil.

3. Given is the system

$$\begin{aligned}\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} &= \begin{bmatrix}-x_1 + e^{x_2}u\\ x_1x_2 + u\\ x_2\end{bmatrix}\\ y &= x_3\end{aligned}$$

   (a) Find the relative degree of the system and specify the region on which this relative degree holds.

   (b) Show that the system is input-output linearizable. Specify the region on which it is input-output linearizable.

   (c) Find a coordinate transformation $z = T(x)$ that transforms the system into the normal form. (Note: $T(x)$ must be a *diffeomorphism*[1] over the region of interest and $T(0) = 0$)

   (d) Express the system in normal form. Determine all functions and constants involved in the normal form. Which part of the normal form counts for the internal dynamics?

   (e) Find the zero dynamics and show that it has a globally asymptotically stable equilibrium at the origin.

   (f) Choose an input u to solve the stabilization problem for the entire system (asymptotically stable equilibrium in the origin).

   (g) Choose an input u to solve the tracking problem for the entire system (asymptotically stable equilibrium at the origin).

4. Exercise 13.25 in Khalil.

5. Exercise 14.31 in Khalil.

[1] A function $y = f(x)$ is a *diffeomorphism* over a domain $\mathcal{D}$ if $f(x)$ and $\frac{\partial f}{\partial x}(x)$ are continuous over D and there exist the inverse of f, $f^{-1}(y)$ such that $f^{-1}(y)$ and $\frac{\partial f^{-1}}{\partial y}(y)$ are continuous over $\bar{\mathcal{D}} = \{y = f(x) | x \in \mathcal{D}\}$.
(Example: $f(x) = x^3$ is a diffeomorphism over $\mathbb{R}_+ = \{x : x > 0\}$ since $f(x)$ and $\frac{\partial f}{\partial x}(x) = 3x^2$ are continuous over $\mathbb{R}_+$, $f^{-1}(y) = \sqrt[3]{y}$ and $\frac{\partial f^{-1}}{\partial y}(y) = \frac{1}{3}y^{-\frac{2}{3}}$) are continuous over $\mathbb{R}_+$).

6. Use backstepping method to stabilize

$$\begin{aligned}\dot{x}_1 &= x_1 x_2 + x_1^2 \\ \dot{x}_2 &= u\end{aligned}$$