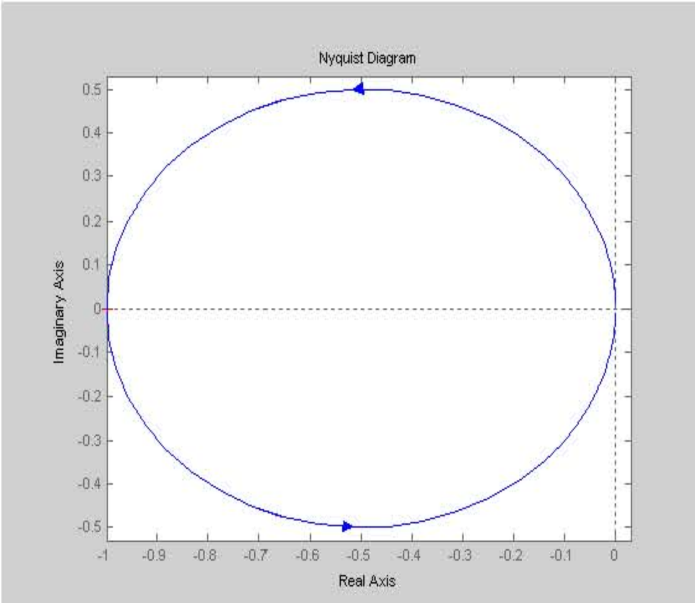
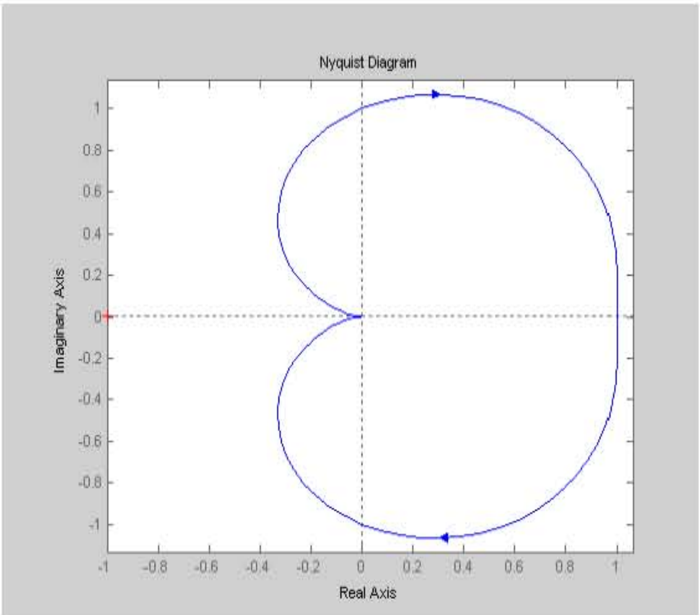
7.1 Apply Theorem 7.2 and use example 7.2~4

(1)

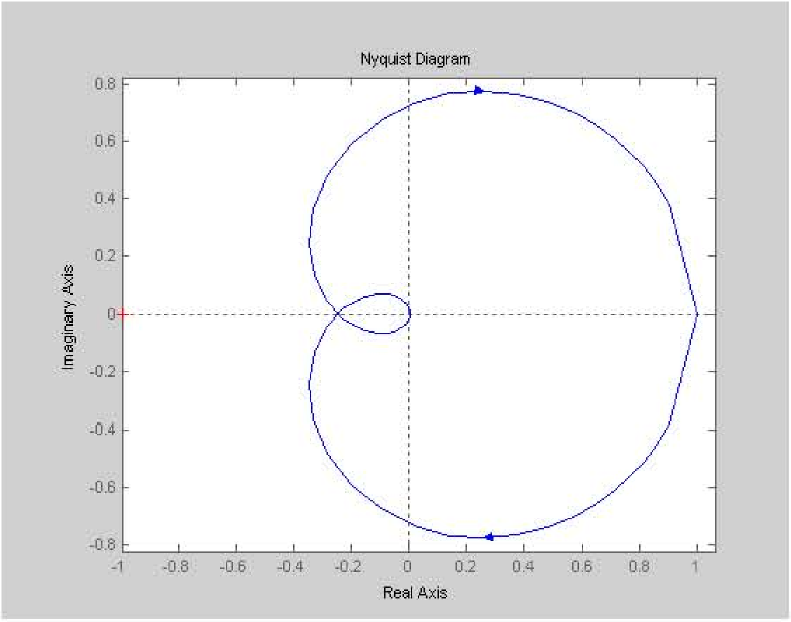


(3)



7.1

(7) Apply Theorem 7.2



7.2

$$(1) z(s) = 1 + G(s) = \frac{s^2 + 3s + 1}{s^2 + s + 1}$$

$$\mathrm{Re}(z(j\omega)) = \frac{(1-\omega^2)^2 + 3\omega^2}{(1-\omega^2)^2 + \omega^2} > 0$$

$(2)\psi(y) = sat(y)$ has nonlinearities in the sector $[0,1]$.

$\Rightarrow$ eq. point is globally uniformly asymptotically stable.

$\Rightarrow$ no limit cycle.

7.11

(3) $$\psi(a)=\frac{4}{\pi a}$$

$$G(j\omega)=\frac{-(\omega^2-1)(\omega^4-14\omega^2+1)}{\omega^2+6\omega^{10}+15\omega^8+20\omega^6+15\omega^4+6\omega^2+1}$$

$$-j\frac{6\omega(\omega^4-3.333\omega^2+1)}{\sim}$$

$$\Rightarrow \omega=1.73205 \text{ or } 0.577735$$

$$\psi(a)=\frac{4}{\pi a}=-\frac{1}{\text{Re}[G(j\omega)]}=-64 \text{ or } 2.37$$

$$\therefore \text{periodic solution w ith } a\approx 0.54,\ \omega=0.577$$

(5) $$\psi(a)=\frac{5}{8}a^4$$

$$G(j\omega)=\frac{-\omega^2}{\omega^4-\omega^2+1}-j\frac{\omega(\omega^2-1)}{\omega^4-\omega^2+1}$$

$$\Rightarrow \omega=1$$

$$\psi(a)=\frac{5}{8}a^4=-\frac{1}{\text{Re}[G(j\omega)]}=1 \quad\Rightarrow\quad a=\left(\frac{8}{5}\right)^{1/4}$$

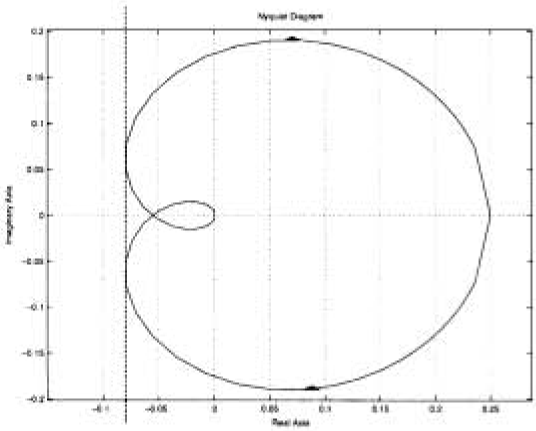
$$\therefore \text{ periodic solution with } a=\left(\frac{8}{5}\right)^{1/4},\ \omega=1$$

(9)

The same method above problems

7.14

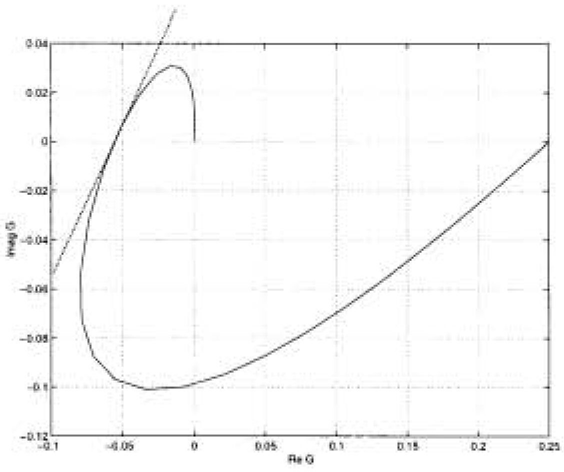
(a)



case 2 of Theorem 7.2

$$-\frac{1}{\beta} = -0.08 \Rightarrow b \approx 12.5$$

(b)



$$-\frac{1}{b} \approx -0.055 \Rightarrow b \approx 18$$

7.14

(c)

$$\psi(a)=\begin{cases} b & \text{if } 0\le a\le \dfrac{1}{b} \\ \dfrac{2b}{\pi}\left[\sin^{-1}\left(\dfrac{1}{ab}\right)+\dfrac{1}{ab}\sqrt{1-\left(\dfrac{1}{ab}\right)^2}\right] & \text{if } 1<ab \end{cases}$$

$$\begin{cases} \operatorname{Im} G(j\omega)=0 \rightarrow \omega=\sqrt{2} \\ \operatorname{Re} G(j\omega)\big|_{\omega=\sqrt{2}}=-\dfrac{1}{18} \end{cases}$$

$$1+\psi(a)\operatorname{Re} G(j\omega)\big|_{\omega=\sqrt{2}}=1-\frac{1}{18}\psi(a)=0 \Rightarrow \psi(a)=18$$

$$\text{if } \ 0\le a\le \frac{1}{b} \rightarrow b=18$$

$$\text{if } \ \frac{1}{ab}<1 \rightarrow \frac{2b}{\pi}\left[\sin^{-1}\left(\frac{1}{ab}\right)+\frac{1}{ab}\sqrt{1-\left(\frac{1}{ab}\right)^2}\right]=18$$

$$\frac{1}{ab}=1 \rightarrow b=18, \quad a=\frac{1}{18}$$

System oscillates for $b\ge 18$ with $\omega=\sqrt{2}$, $a=\dfrac{1}{18}$.

7.15

Refer to the solution of 7.14.

10.10

(1) $\dot{x}_1 = \varepsilon x_2$

$\dot{x}_2 = -\varepsilon(1+2\sin t)x_2 - \varepsilon(1+\cos t)\sin x_1,\quad$ where $\varepsilon << 1$

$$f_{av}(x) = \frac{1}{2\pi}\int_0^{2\pi}\begin{bmatrix} x_2 \\ -(1+2\sin t)x_2 - (1+\cos t)\sin x_1 \end{bmatrix} dt = \begin{bmatrix} x_2 \\ -x_2 - \sin x_1 \end{bmatrix}$$

$$\dot{\bar{x}} = \varepsilon\begin{bmatrix} \bar{x}_2 \\ -\bar{x}_2 - \sin \bar{x}_1 \end{bmatrix}$$

The averaged system has an equilibrium point at the origin. The linearization yields

$$\left.\frac{\partial f_{av}}{\partial \bar{x}}\right|_{\bar{x}=0} = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} \quad \text{which is Hurwitz.}$$

$\Rightarrow$ the origin is exponentially stable.

(3)

$$\mathrm{f}_{\mathrm{av1}} = \frac{1}{2\pi}\int_0^{2\pi}(-\mathrm{x}\sin^2 \mathrm{t} + \mathrm{x}^2 \sin \mathrm{t})\mathrm{dt} = -\frac{1}{2}\mathrm{x}$$

$$\mathrm{f}_{\mathrm{av2}} = \lim_{\mathrm{T}\to\infty}\frac{1}{\mathrm{T}}\int_0^{\mathrm{T}}\mathrm{x}\mathrm{e}^{-\mathrm{t}}\mathrm{dt} = 0$$

$$\therefore \dot{\bar{\mathrm{x}}} = -\frac{1}{2}\bar{\mathrm{x}} \rightarrow \text{Hurwitz}$$

$\therefore$ exponentially stable

11.10

(a) $\cdot$ Set $\varepsilon = 0$ $(z + xt)(z - 2)(z - 4) = 0$

$$\begin{cases} z = -xt \\ z = 2 \\ z = 4 \end{cases} \qquad \therefore \text{ possible three}$$

(b) i) $h(t,x) = -xt$ $(t = t_0$ (0 for convenience), $x(0) = \xi_0)$

$$\frac{\partial y}{\partial \tau} = -y(y-2)(y-4)$$

$$V(y) = \frac{1}{2}y^2 \qquad \frac{\partial V}{\partial y}g = -y^2(y-2)(y-4) \ \rightarrow \text{exp. stable}$$

(RoA : $y < 2$)

ii) $h(t,x) = 2$

$$\frac{\partial y}{\partial \tau} = -y(y+2)(y-2) \ \rightarrow \text{unstable}$$

iii) $h(t,x) = 4$

$$\frac{\partial y}{\partial \tau} = -y(y+2)(y+4)$$

$$V(y) = \frac{1}{2}y^2 \qquad \frac{\partial V}{\partial y}g = -y^2(y+2)(y+4) \ \rightarrow \text{exp. stable}$$

(RoA : $y > -2$)

11.10

(c) i) $h(t,x) = -xt$

$\dot{x} = -x,\ x(0) = 1 \Rightarrow \bar{x}(t) = e^{-t}$

Since RoA: $y < 2$ and $y(0) = z(0) - h(0, x(0)) = a$

$$\begin{cases} x(t,\varepsilon) = e^{-t} + O(\varepsilon) \\ z(t,\varepsilon) = -te^{-t} + \hat{y}(\frac{t}{\varepsilon}) + O(\varepsilon) \quad a \in [-2, 2) \end{cases}$$

ii) unstable

iii) $h(t,x) = 4$

$\dot{x} = \dfrac{x^2 t}{4},\ x(0) = 1$

Since RoA: $y > -2$ and $y(0) = z(0) - h(0, x(0)) = a - 4$

$$\begin{cases} x(t,\varepsilon) = \bar{x}(t) + O(\varepsilon) \\ z(t,\varepsilon) = 4 + \hat{y}(\frac{t}{\varepsilon}) + O(\varepsilon) \quad a \in (2, 6] \end{cases}$$