

TTK4150 Nonlinear Control Systems
Department of Engineering Cybernetics
Norwegian University of Science and Technology
Fall 2015 - Assignment 5

Due date: Monday 20 November at 6.00.

1. Exercise 14.43 in Khalil (Hint: Theorem 14.4 in Khalil).
2. Exercise 13.1 in Khalil.
3. Exercise 13.2 in Khalil.
4. Given is the system

$$\begin{array}{lcl} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} & = & \begin{bmatrix} -x_1 + e^{x_2}u \\ x_1x_2 + u \\ x_2 \end{bmatrix} \\ y & = & x_3 \end{array}$$

- (a) Find the relative degree of the system and specify the region on which this relative degree holds.
 - (b) Show that the system is input-output linearizable. Specify the region on which it is input-output linearizable.
 - (c) Find a coordinate transformation $z = T(x)$ that transforms the system into the normal form. (Note: $T(x)$ must be a *diffeomorphism*¹ over the region of interest and $T(0) = 0$)
 - (d) Express the system in normal form. Determine all functions and constants involved in the normal form. Which part of the normal form counts for the internal dynamics?
 - (e) Find the zero dynamics and show that it has a globally asymptotically stable equilibrium at the origin.
 - (f) Choose an input u to solve the stabilization problem for the entire system (asymptotically stable equilibrium in the origin).
 - (g) Choose an input u to solve the tracking problem for the entire system (asymptotically stable equilibrium at the origin).
5. Exercise 13.25 in Khalil.

¹A function $y = f(x)$ is a *diffeomorphism* over a domain \mathcal{D} if $f(x)$ and $\frac{\partial f}{\partial x}(x)$ are continuous over \mathcal{D} and there exist the inverse of f , $f^{-1}(y)$ such that $f^{-1}(y)$ and $\frac{\partial f^{-1}}{\partial y}(y)$ are continuous over $\bar{\mathcal{D}} = \{y = f(x) | x \in \mathcal{D}\}$.
(Example: $f(x) = x^3$ is a diffeomorphism over $\mathbb{R}_+ = \{x : x > 0\}$ since $f(x)$ and $\frac{\partial f}{\partial x}(x) = 3x^2$ are continuous over \mathbb{R}_+ , $f^{-1}(y) = \sqrt[3]{y}$ and $\frac{\partial f^{-1}}{\partial y}(y) = \frac{1}{3}y^{-\frac{2}{3}}$ are continuous over \mathbb{R}_+).