

# Convective Thermal Metamaterials: Exploring High-Efficiency, Directional, and Wave-Like Heat Transfer

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**Convective thermal metamaterials are artificial structures where convection dominates in the thermal process. Due to the field coupling between velocity and temperature, convection provides a new knob for controlling heat transfer beyond pure conduction, thus allowing active and robust thermal modulations. With the introduced convective effects, the original parabolic Fourier heat equation for pure conduction can be transformed to hyperbolic. Therefore, the hybrid diffusive system can be interpreted in a wave-like fashion, reviving many wave phenomena in dissipative diffusion. Here, recent advancements in convective thermal metamaterials are reviewed and the state-of-the-art discoveries are classified into the following four aspects, enhancing heat transfer, porous-media-based thermal effects, nonreciprocal heat transfer, and non-Hermitian phenomena. Finally, a prospect is cast on convective thermal metamaterials from two aspects. One is to utilize the convective parameter space to explore topological thermal effects. The other is to further broaden the convective parameter space with spatiotemporal modulation and multi-physical effects.**

## 1. Introduction

Almost all types of energy will eventually release into the ambience as heat. In light of growing issues of carbon emission, global warming, and energy crisis, a more rational thermal management is necessary and urgent. However, due to the limitations of naturally existing materials and theoretical design tools, traditional heat transfer manipulation methods still lack sufficient efficiency and flexibility.

Thanks to the pioneering metamaterials in wave physics such as optics and acoustics,<sup>[1–3]</sup> the concept of thermal metamaterial

emerges, paving new avenues for heat flow control.<sup>[4]</sup> Metamaterials are artificial structures whose physical properties can be tailored unprecedentedly through reasonable design and arrangement of subwavelength unit cells.<sup>[5]</sup> In recent years, exotic phenomena and novel devices powered by wave metamaterials further boosted the development of their diffusive counterpart.<sup>[6–8]</sup>

Generally, heat transfer has three forms, conduction, convection, and radiation. These three forms usually intercouple with each other and seldom stand alone. As the primary heat transfer mode in solids following the Fourier law, heat conduction has been well studied in the past decades. Relying on the form-invariance of the Fourier equation under coordinate transformations, the notion of thermal cloak originated from the optical invisibility cloak based on transformation optics.<sup>[9,10]</sup> The

anisotropic thermal conductivity required by a thermal cloak can be experimentally realized with tailored layered structures.<sup>[11,12]</sup> A typical thermal cloak possesses two properties, that is, a zero temperature gradient in its interior and suppressed perturbations on its exterior temperature field. The experimental demonstrations of thermal cloak further promote various macroscopic thermal meta-devices via engineering the thermal conductivity distributions in solids, such as thermal concentrators,<sup>[13–15]</sup> thermal rotators,<sup>[16,17]</sup> thermal illusion,<sup>[18,19]</sup> and thermal camouflage.<sup>[20]</sup> Not limited to the macro scale, various devices used for thermal information processing at the microscale, such as thermal

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storage,<sup>[21,22]</sup> thermal logic gates,<sup>[23]</sup> and thermal diodes,<sup>[24,25]</sup> are investigated based on phonon transport and scattering theory.<sup>[26]</sup>

Despite the above achievements, some challenges restrict state-of-the-art conductive thermal metamaterials. For example, its effective thermal conductivity cannot exceed the limitations of naturally occurring materials, and the heat flow can only be driven passively by temperature gradient. In particular, the features of conductive metamaterials cannot be flexibly modified without changing the external ambience or the device structures.

Facing these challenges, more and more recent studies focus on introducing convection to design thermal metamaterials, aiming at removing the restrictions of pure thermal conduction and opening new avenues for this burgeoning realm. In general, convection has three effects that differ from conduction.

First, convection can change the heat transfer efficiency; second, convection exhibits a preference for heat flux direction; and finally, convection could carry oscillating temperature fields. By introducing convection, the original parabolic diffusion equation describing the macroscopic thermal conduction can be modified into the hyperbolic convection-diffusion equation,<sup>[27]</sup> hence the temperature field carried by convection could imitate many physical phenomena in wave systems. Comprehensive understandings of these emerging works are indispensable for more advanced heat manipulation. Upon the different types of convective effects, herein, we classify the research progress of convective thermal metamaterials into several sub-categories, including enhancing heat transfer, porous-media-based thermal effects, nonreciprocal heat transfer, and non-Hermitian phenomena (Figure 1).

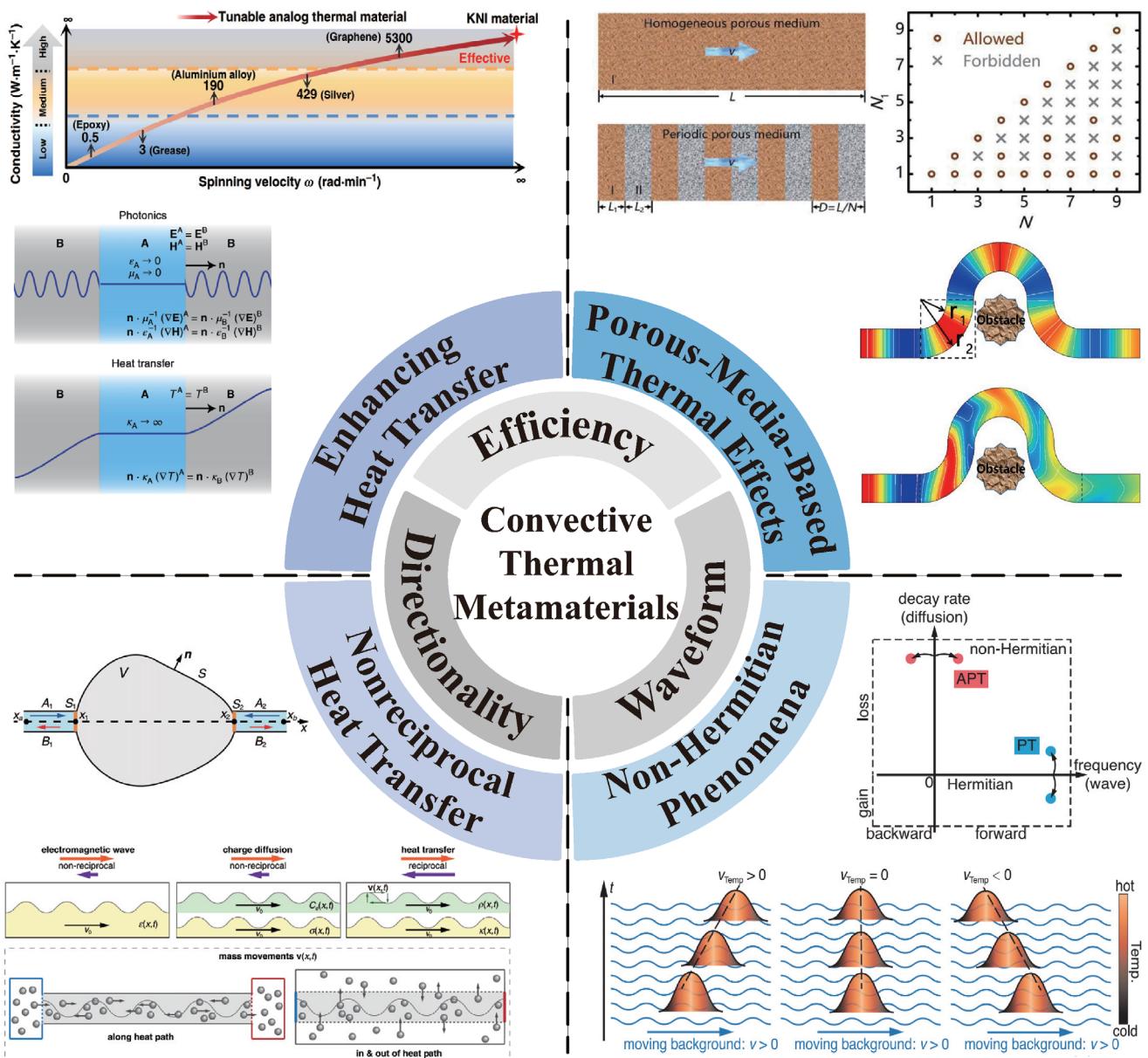


Figure 1. Convective thermal metamaterials.

## 2. Enhancing Heat Transfer

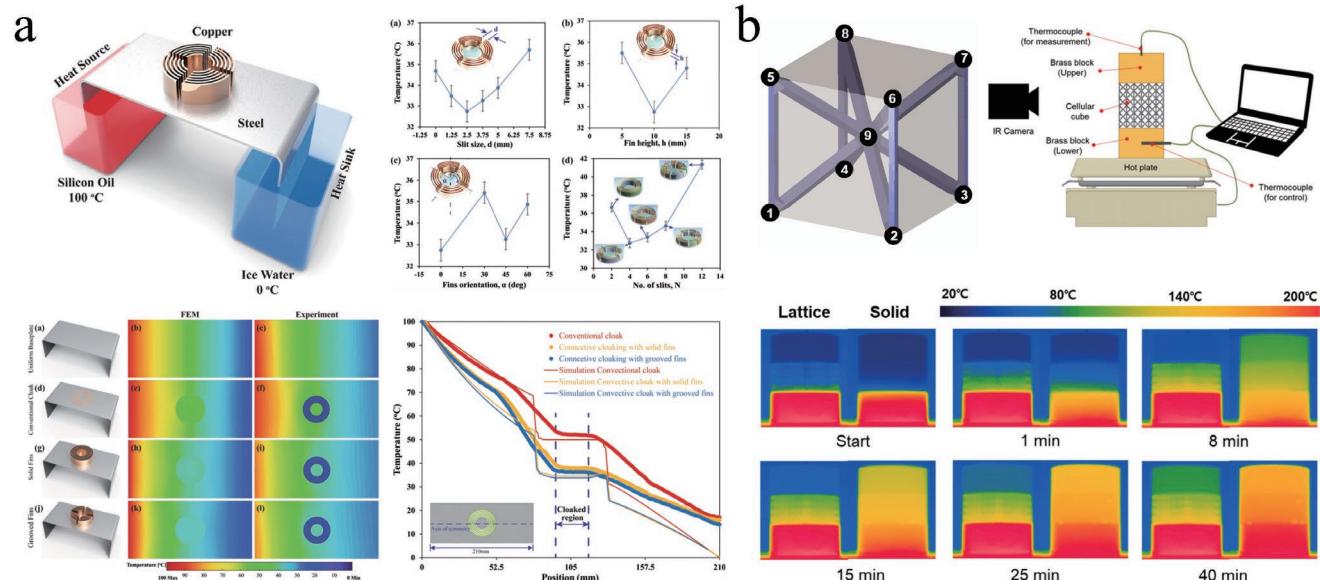
Convective heat transfer is the most efficient and widely used method for cooling or heating between solids and fluids. This process can be described by Newton's law of cooling  $q = hA(T_\infty - T)$ ,<sup>[28,29]</sup> where  $q$  is the heat flux magnitude flowing into the effective surface area  $A$ ,  $h$  is the heat transfer coefficient,  $T_\infty$  and  $T$  are the fully developed fluid temperature and solid wall temperature, respectively. In this regard, the enhanced heat transfer between solid and fluid is achieved by changing the value of the heat transfer coefficient  $h$ .

Convection possesses three forms, that is, natural convection, forced convection, and mixed convection. Mixed convection combines natural convection and forced convection, added with a precondition that natural-convection currents are in the same order of magnitude as forced flow velocities.<sup>[30]</sup> Natural convection is a physical phenomenon caused by inhomogeneous fluid temperatures.<sup>[31]</sup> Recently, a study took natural convection as a cooling tool for designing and optimizing a thermal cloaking device.<sup>[32]</sup>

As clarified above, the function of a thermal cloak is to produce a cloaked region with zero temperature gradient while simultaneously eliminating perturbations on the surroundings for concealment. However, though the thermal field outside the cloak region can be kept free from distortion under rational design, it remains a problem that the temperature within the cloaked region keeps increasing over time. This property of a conventional thermal cloak is undesirable for heat-sensitive components and devices that are inevitably required to work continuously for long periods. From this point, to enhance heat dissipation via natural convection, some researchers attach an

array of passive cooling surface fins illustrated in Figure 2a to the center of a convection cloak so that a lower steady-state temperature in the cloaked region can be achieved.<sup>[32]</sup> In detail, the introduced solid fins are manufactured with high thermal conductivity materials to lift the efficiency of heat dissipation. On this basis, grooved fins with slit gaps rather than solid fins are introduced, and geometry parameters, including height, thickness, and slit size, are taken for combinatorial optimization. Two reasons account for this strategy. On the one hand, the core of this structure is natural convection where a smooth airflow path is critical, thus thicker and higher fins may be a disadvantage. On the other hand, a bigger fin surface brings a more significant effective convective area according to Newton's law of cooling, so a trade-off must be made for maximum efficiency. As shown in the corresponding simulated and experimental results of different cloaks (Figure 2a, bottom), a 15 °C steady-state temperature drop in the cloaked region compared to the traditional thermal cloak is reported. As an analogy, this work governed by natural convection cooling is similar to the related research on radiative cooling. They are both passive processes without external energy inputs and can be effectively employed to remove the adverse consequences of high temperatures. As a ubiquitous process that carries away energy through photon heat flow, radiative cooling emerges at the forefront of renewable energy,<sup>[33–35]</sup> which motivates similar studies of solar cells. Readers interested in this subject may consult refs. [36–39] for more details. Passive cooling governed by natural convection without external actuation is a concept with far-reaching applications.

Despite the remarkable energy-saving advantage of natural convection cooling, forced convection is far more potent. In this case, a type of metamaterial acting as a thermal insulator



**Figure 2.** Convective thermal metamaterials for cooling. a) Thermal metamaterial designed for temperature control at the cloaked region. Schematic of the experimental setup and parameter optimization process (top). Comparison of experimental and simulation results between uniform baseplate without cloak, conventional cloak, convective element cloak with solid fins, and convective element cloak with grooved fins (bottom). Reproduced under the terms of the CC-BY Creative Commons Attribution 4.0 International license (<https://creativecommons.org/licenses/by/4.0/>).<sup>[32]</sup> Copyright 2020, The Authors, published by Springer Nature. b) The unit cell configuration (top left), experimental setup (top right), and stepwise temperature distributions during the heating stage (bottom) of the thermal metamaterial with high thermal resistance and cooling capability. Reproduced with permission.<sup>[40]</sup> Copyright 2021, Elsevier.

with high cooling performance under forced convection is presented.<sup>[40]</sup> Usually, a typical thermal insulator is bound to have a low cooling capacity, and insulators made of foamy or fibrous materials are restricted by their low rigidity and strength. In contrast, ceramic with great stiffness and thermal resistance is difficult to shape into a complicated geometry.<sup>[41–43]</sup> To overcome this dilemma, a cellular lattice structure using a body-centered cubic unit cell is conceived (Figure 2b). This metamaterial acts as a thermal insulator and a heat exchanger by letting forced coolant flow travel through the hollow interior gaps among lattice.<sup>[40]</sup> To compare the insulation effects, an experiment where two such structures, respectively, inserted into two brass cylinders is performed. After heating them simultaneously, the lattice structure showcases a thermal resistance 4.5–11.5 times higher than the solid one, positively correlated with the structure diameter of the lattice. While during the cooling process, water could circulate through the structure owing to the lattice. By this means, a thermal metamaterial for realizing the simultaneous optimization of two contradictory goals, namely high thermal resistance and high cooling performance, has been designed, which meanwhile has certain rigidity for load-bearing.

The above works are metamaterials for cooling at the macro level. The enhancement of the convective cooling effect is particularly essential for its broad applications, from macroscopic to microscopic structures, from cooling the core of nuclear reactors to cooling electronic devices.<sup>[44–47]</sup> Moreover, with the ever-increasing processing capacity and the ongoing efforts at miniaturization for better portability, the power density of electronics has risen as well, which remains a challenge for the heat dissipation of electronic products.<sup>[48,49]</sup> Considering that even though the state-of-the-art cutting-edge approaches still treat electronics and cooling separately, several researchers have developed a revolutionary liquid cooling system integrated into the chip.<sup>[50]</sup> Producing a monolithically integrated manifold microchannel cooling structure, they demonstrate great efficiency going beyond what was previously accessible. Thus, this work is of practical significance in further miniaturizing electronics and breaking Moore's law.

Herein, the contents discussed are some studies of convection-enhanced cooling effects of thermal metamaterials, while devices such as heat pipes and heat sinks are beyond the current scope because they directly use convection rather than resort to artificial structures.<sup>[51,52]</sup> Although convective cooling exists widely in natural environments, metamaterials using this effect to enhance heat transfer efficiency are relatively lacking. Conventionally, convection, especially natural convection, is always regarded as an external disturbance that should be ignored in the experimental process. As an example of mitigating the effects of air convection, many thermal experiments need to be carried out in vacuum chamber.<sup>[53,54]</sup> However, as the most fundamental effect of convection, cooling has broad prospects in thermal manipulations both at macroscopic and microscopic scale, where artificial thermal materials are worth further study.

As a part of heat transfer enhancement, the cooling effect focuses on introducing convection into metamaterials at the practical application level. Besides, from the theoretical level, the current research interest in convective thermal metamaterials

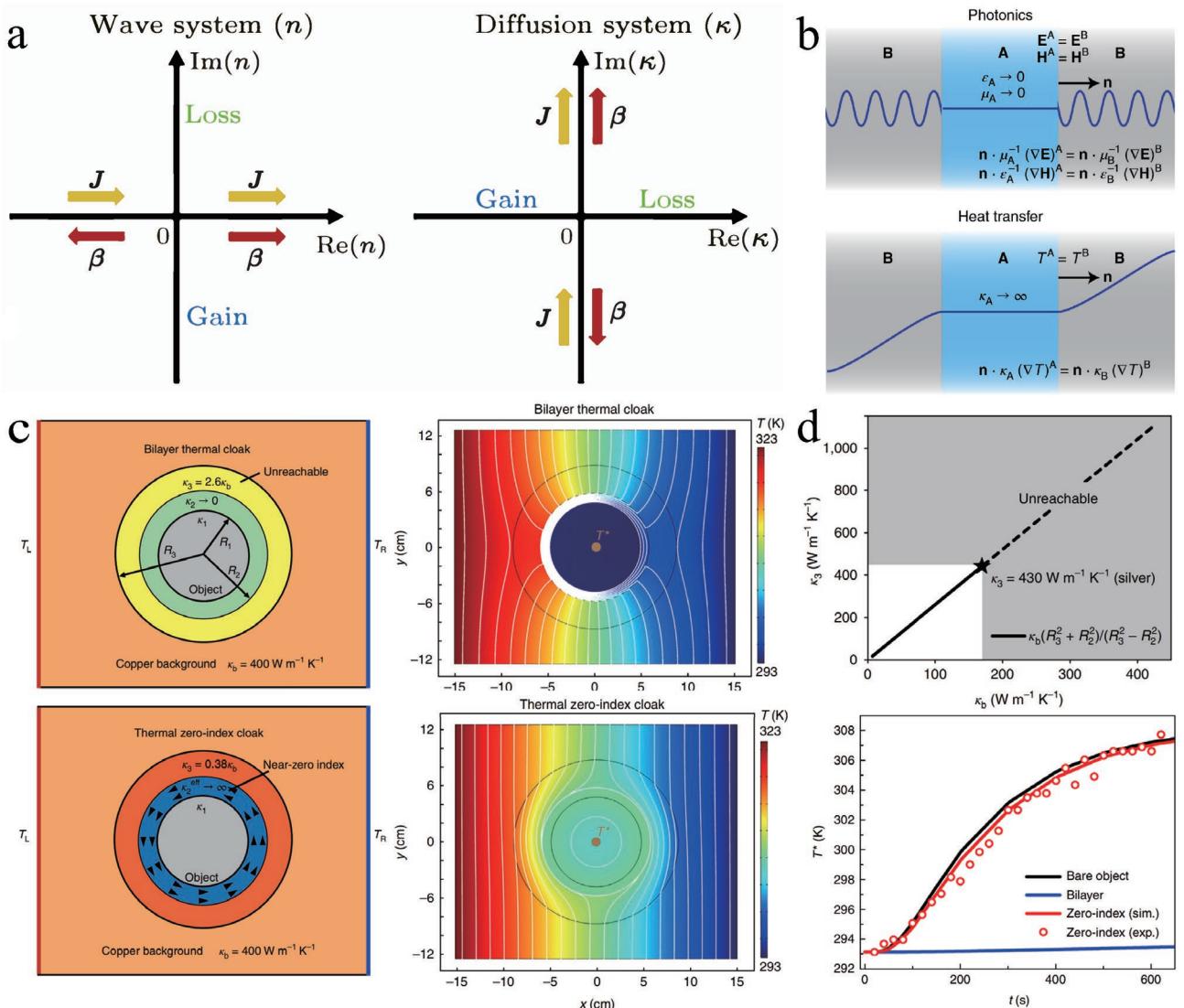
is not limited to introducing convection. Concerning physical parameters, researchers have attempted to integrate convection into the traditional theoretical framework of heat conduction by coupling hydrodynamics and thermodynamics. In these works, convection is expected to break the thermal conductivity limitation of natural materials to enhance heat transfer effectively. Thus, the concept of effective thermal conductivity arises.

With thermal convection, thermal energy transfer is not just driven by pure conduction but also by the directional mass flow. In this setting, heat flux can be carried into or out of the domain being studied at a rate  $-\nabla \cdot (\rho c v T)$ , where  $\rho$ ,  $c$ , and  $v$  are the density, specific heat capacity, and flow velocity of the fluid, respectively. Note that convection is not restricted to fluid motion, mechanical movements can also be regarded as forced convection. The essence of both is mass transfer, which is analogous to the ability to "drag" waves or heat. Although thermal convection has once been studied in isolation, understanding how thermal convection affects heat conduction does make sense in the heat transfer process. Therefore, it has been anticipated that the influence of thermal convection could be represented in some cases by a simple effective thermal conductivity so that it can then be incorporated into the initial theoretical framework of heat conduction.

To this end, it is pointed out that thermal conductivity may be studied in a complex plane,<sup>[55]</sup> and three models with the functions of cloaking, concentrating, and rotating thermal waves are demonstrated based on this method.<sup>[56,57]</sup> The core is to adopt a plane-wave temperature solution  $T = A_0 e^{i(\beta \cdot r - \omega t)} + T_0$  to the classical diffusion-convection equation  $\rho c \frac{\partial T}{\partial t} = \nabla \cdot (\kappa \cdot \nabla T) - \rho c v \cdot \nabla T$ . Then the dispersion relation can be derived as  $\omega = v \cdot \beta - i \frac{\kappa \beta^2}{\rho c}$ , where  $A_0$ ,  $\beta$ ,  $\omega$ , and  $T_0$  correspond

to the amplitude, wave vector, thermal wave frequency, and reference temperature of the wave-like temperature profile, respectively. Subsequently, with the relationship  $\nabla T = i \beta T$ , the complex thermal conductivity is  $\kappa^* = \kappa + i \rho c v \cdot \beta / \beta^2$ , where convection is mathematically regarded as a complex form of conduction. Here,  $\text{Re}(\kappa^*)$  and  $\text{Im}(\kappa^*)$  are related to dissipation and propagation, respectively, contrary to the complex refractive index (Figure 3a). Once the gain and loss are introduced, unique features in wave systems can also be observed in thermal conduction–convection systems. Moreover, a concept called negative thermal transport has been proposed, whose representative property is that the direction of energy flow (Poynting vector) and wave vector remain opposite.<sup>[55]</sup>

The above works give us a deeper understanding of the unification of conduction and convection in mathematical form. However, the effective thermal conductivity achieved through this method is a complex number, which is different from the conventional concept of thermal conductivity. Furthermore, though the effective medium theory has enabled the emergence of metamaterials with a wide range of effective parameters in photonics,<sup>[58–60]</sup> it still has limitations when applied to thermotronics due to the extreme narrower ranges of available thermal parameters in natural materials. Since the effective parameters of typical solid-state thermal metamaterials are their constituents average, it remains challenging to break the natural limitations.<sup>[17]</sup>



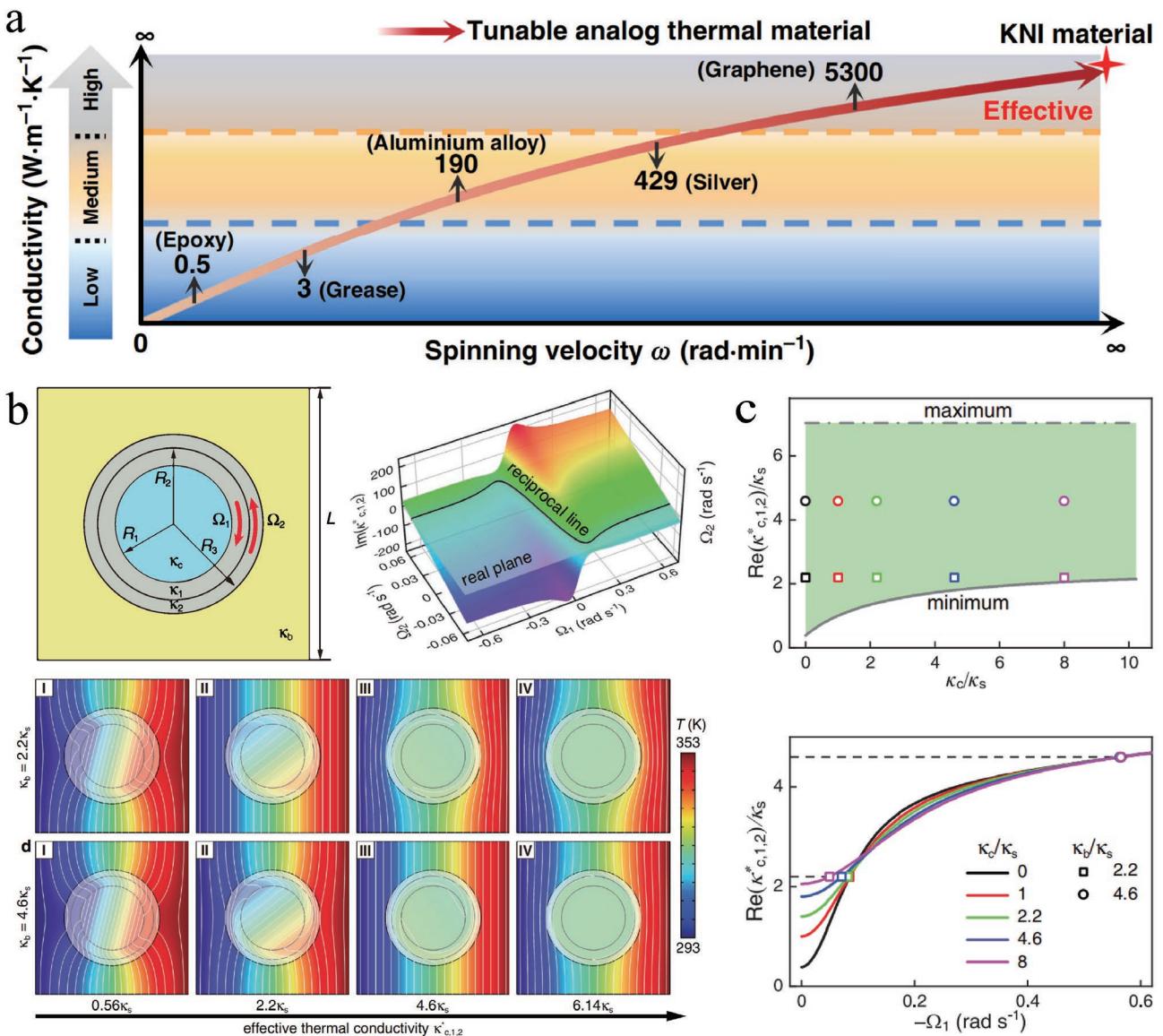
**Figure 3.** Complex and extreme effective thermal conductivity. a) Comparison between complex refractive index in wave system and complex thermal conductivity in diffusive system. Reproduced with permission.<sup>[55]</sup> Copyright 2020, IOP Publishing. b) The correspondence between  $\epsilon_A$ ,  $\mu_A$  of materials supporting electromagnetic wave transport (top), and  $\kappa_A$  of materials supporting heat transfer (bottom). c) A bilayer thermal cloak and a thermal zero-index cloak, both with a copper background (left). The corresponding transient temperature profiles at 500 s of the bilayer thermal cloak and thermal zero-index cloak (right). d) The range diagram of  $\kappa_b$  (thermal conductivity of background material) where a bilayer cloak can be realized using traditional material (top). Evolution of the cloak central temperature  $T^*$  with time (bottom). b-d) Reproduced with permission.<sup>[61]</sup> Copyright 2019, Springer Nature.

Concerning this practical problem, a thermal zero-index cloak composed of strong circulating fluid has been designed by demonstrating an exact equivalence between a zero index ( $n \rightarrow 0$ ) in photonics and an infinite thermal conductivity ( $\kappa \rightarrow \infty$ ) in heat transfer (Figure 3b).<sup>[61,62]</sup> In this device, the thermal property is insensitive to details of the fluid fields, so it requires no accurate adjustment of the flow field. While the traditional bilayer cloak with low conductivity inner layer enables uniform temperature distribution inside the cloak, this zero-index cloak can track ambient temperature changes from inside as quickly as the bare object (Figure 3c). This property allows it to be used as a temperature sensor (Figure 3d).

By introducing convection as an extra tool for regulation, the above work is the first to realize the effective  $\kappa$ -near-infinity metamaterial, where  $\kappa$  denotes thermal conductivity. However,

this model is only valid under the precondition of extremely strong convection. Namely, once the angular velocity of the rotation is moderate or slow, the Onsager reciprocity can no longer be retained due to the intrinsically distinct features of conduction and convection. Therefore, despite no requirement for precise fluid adjustment in this structure, there is also no potential in the sophisticated regulation of heat transfer, where the advantage might turn into a disadvantage on the opposite side.

To improve the above work, a tunable analog thermal metamatetrial whose effective conductivity can be modified in situ from near-zero to near-infinity has been fabricated.<sup>[63]</sup> Bridging the gaps of previously discrete digital values, it resembles an analog signal in electronics (Figure 4a). The device can be considered a close approximation of previously described



**Figure 4.** Tunable thermal conductivity. a) The current progress of thermal material, while KNI denotes the material with infinite thermal conductivity. Reproduced under the terms of the CC-BY Creative Commons Attribution 4.0 International license (<https://creativecommons.org/licenses/by/4.0/>).<sup>[63]</sup> Copyright 2020, The Authors, published by Springer Nature. b) The continuously tunable solid-like convective thermal disguiser. The structure (top left), the reciprocal line drawn by different combinations of rotation speeds (top right), and the corresponding simulation results (bottom). c) The range of  $\kappa_b$  (thermal conductivity of background material) where scattering cancellation can be realized (top). The according locations of scattering cancellation points (bottom). b,c) Reproduced with permission.<sup>[64]</sup> Copyright 2020, Wiley-VCH.

$\kappa$ -near-infinity material when extreme convection is employed, except that there is a near-zero temperature gradient in the center. Furthermore, this work indicates that the single spinning effect within a fluid only creates an effectively inhomogeneous conductivity described by an anisotropic tensor rather than a real number. Each conductive component of the anisotropic tensor can be modulated as a function of the angular velocity and spatial azimuth. Once the angular velocity is at a moderate value, there will be an incomplete homogenization in thermal profiles. However, this work solely discusses the effective thermal conductivity of the spinning fluid center, ignoring its impact on the external temperature profile. Then, the concept of ref. [61] is further explored. After adding another

spinning layer, the earlier model is upgraded to a convectively excited solid-like material whose effective thermal conductivity becomes a real number and dynamically covers a vast range.<sup>[64]</sup> Inspired by this model, an inverse-design method for adaptive thermal cloaking based on a rotating meta-device is proposed.<sup>[65]</sup>

Further designing a system with counter-rotating pieces, a particular combination of the rotation speeds called the reciprocal line is found to restore the Onsager reciprocity (Figure 4b). Due to the symmetric coupling between temperature gradient and heat flux, the effective thermal conductivity emerges as a real number or a symmetric tensor. The meta-device working on the reciprocal line can be regarded as a convective thermal

disguise because no rotation is visible on the temperature fields outside. Furthermore, the device can work on the reciprocal line by a proper combination of  $\Omega_1$  and  $\Omega_2$  for any combination of  $\kappa_b$  and  $\kappa_c$ , as long as the background thermal conductivity  $\kappa_b$  is in the proper range (Figure 4c, top). Remarkably, when the angular velocity of the inner layer reaches a relatively high value, the reciprocal lines quickly converge regardless of the material in the center (Figure 4c, bottom). In other words, any material in the center can be disguised as normal under this circumstance. Further to the previous investigations, the counter-rotating structure is extended to a rotating structure with multiple layers.<sup>[66]</sup> In essence, since the effective thermal conductivity of a rotating object is derived strictly based on its thermal scattering effect on the temperature field outside, the thermal conductivity of the structure can be rigorously described in a complex plane. According to this research, when only external effects on the background temperature field are considered, the effective thermal conductivity of the core-shell structure is a complex number, equivalent to an anti-symmetric tensor. However, once the temperature inside and outside the object is the target, the effective thermal conductivity becomes an asymmetric tensor rather than an anti-symmetric one that writes in complex form.

This section introduces the design and related research progress of convective thermal metamaterials for enhancing heat transfer. It includes cooling effects focusing on practical applications and tunable effective thermal conductivity. Relevant researches possess great potential in applications with strict requirements for heat transfer efficiency and response time, such as intelligent thermal camouflage and heat collection.<sup>[67–69]</sup> Moreover, the multi-functional integrated thermal metamaterials are realized thanks to the adjustable rotational speed of the multilayer structure. It innovates the traditional framework limited by conduction and integrates convection into the conduction framework. From this perspective, the inspired ideas are far beyond the aim of enhancing heat transfer, and may guide the future hardware preparation of intelligent thermal control.

### 3. Porous-Media-Based Thermal Effects

Forced convection can bring a variety of novel and even unprecedented functions besides enhancing heat transfer. However, despite the conspicuous progress in this field, it comes with its significant challenges: whether the transformation theory could be appropriately transferred to the design of convective thermal metamaterials. Unfortunately, while the convection-diffusion equation meets the indispensable form-invariance requirement, the momentum equations governing incompressible fluids, namely the famous Navier-Stokes equations, do not meet the requirement.<sup>[70–72]</sup> The thermal cloak can scarcely break the current conduction frame within such technological restrictions. To bypass this hindrance, porous media theory came into being, with which the governing equation simplifies and the form-invariance can be satisfied.

A simple view of the porous media theory will be taken in the following part. The Navier-Stokes equations for incompressible Newtonian flow are

$$\begin{cases} \rho \left( \frac{\partial v}{\partial t} + v \cdot \nabla v \right) = -\nabla P + \rho g + \mu \nabla^2 v \\ \nabla \cdot v = 0 \end{cases} \quad (1)$$

where  $\nabla P$  denotes the pressure gradient of fluid,  $\rho g$  denotes the gravity exerted, and  $\mu \nabla^2 v$  denotes the internal stress caused by viscosity.

Considering a flow possessing a small Reynolds number and constant velocity in a homogeneous and periodic porous media, the nonlinear and time-varying terms in Equation (1) can be omitted. Besides, the gravitational force  $\rho g$  can be lumped together with the pressure term  $-\nabla P$ . Therefore, an approximated Equation (1) can be expressed:

$$\frac{\mu}{\eta} v = -\nabla P \quad (2)$$

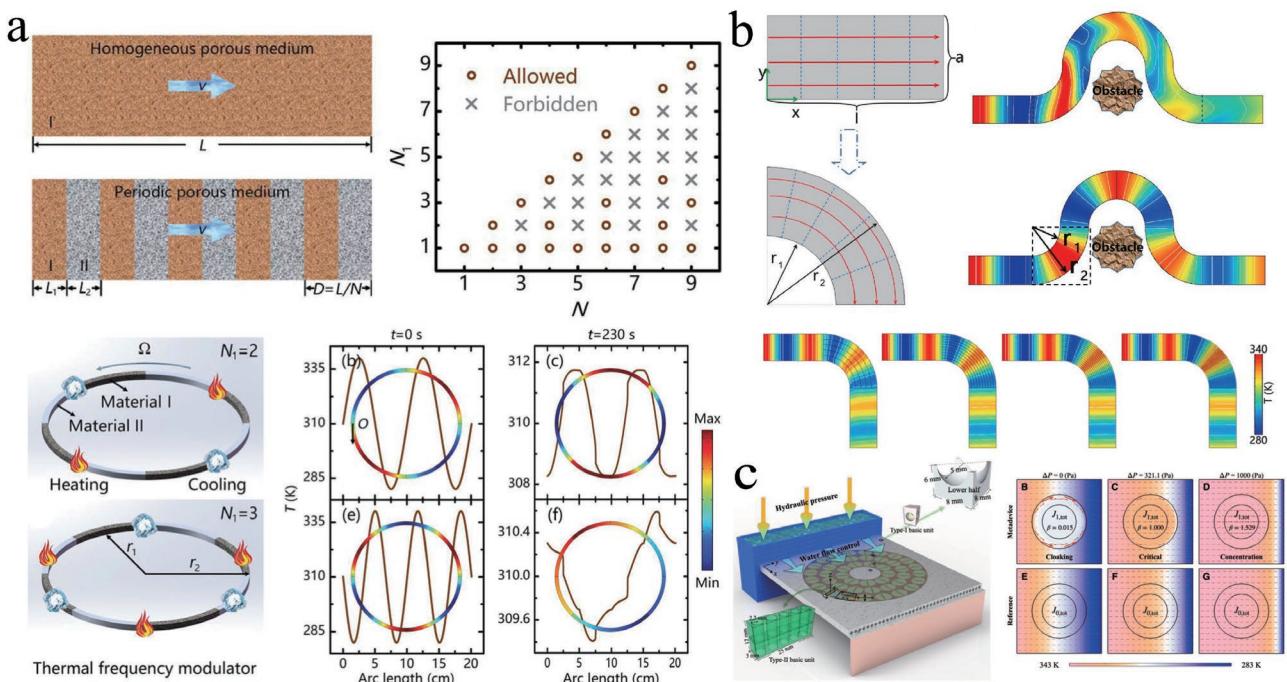
Equation (2) is the steady form of the Darcy equation, where  $\eta$  is permeability,  $\mu$  is dynamic viscosity, and  $P$  is pressure. In this regard, the whole term on the right side describes the drag caused by the fluid-solid surface forces in porous media with small permeability.<sup>[70]</sup> Yet, the above is just an intuitive grasp of Darcy's law rather than a strict process derived from Navier-Stokes equations. Detailed mathematical derivations can be found in refs. [71,72].

The Darcy law dominates the momentums in porous media. Herein, form invariance revives and lets the transform theory be compatible with thermal convection. In this regard, porous media can effectively perform as an ideal platform for hybrid conductive and convective heat transfer,<sup>[73–75]</sup> with the following governing equation:

$$\begin{cases} \rho_e c_e \frac{\partial T}{\partial t} = \kappa_e \nabla^2 T - \rho_f c_f v \cdot \nabla T \\ \rho_e c_e = \phi \rho_f c_f + (1-\phi) \rho_s c_s \\ \kappa_e = \phi \kappa_f + (1-\phi) \kappa_s \end{cases} \quad (3)$$

where  $\rho_f(\rho_s)$ ,  $c_f(c_s)$ , and  $\kappa_f(\kappa_s)$  correspond to the mass density, heat capacity, and thermal conductivity of the fluid (solid). Besides,  $\phi$  is a measurement index of the porosity of the material. With a delicate design of the material parameters, a large Peclet number (defined as  $Pe = vL/D$ , where  $v$  is convective velocity,  $L$  is characteristic length, and  $D$  is thermal diffusivity) can be achieved. In this case, convection dominates the heat transfer process, thereby the governing equation of the system exhibits hyperbolic features, where propagating thermal waves can be supported.

Combined with thermal waves dominated by convection, porous media exerts a transformative impact on the design approach of thermal metamaterials. Some recent studies have employed periodic porous media to realize thermal convection-diffusion crystal, which showcases great potential in modulating wave-like temperature profiles,<sup>[76]</sup> as a counterpart of photonic crystals in wave systems.<sup>[77,78]</sup> The model configuration comprises a homogeneous porous media of length  $L$  with periodic boundary conditions at both ends (Figure 5a, top). In this scenario, only the temperature profiles



**Figure 5.** Porous-media-based convective thermal metamaterials. a) Thermal convection-diffusion crystal for prohibition and modulation of wave-like temperature profiles. Structure schematics (top left), forbidden rule of the given thermal convection-diffusion crystal (top right), and simulation results of a thermal frequency modulator (bottom). Reproduced with permission.<sup>[76]</sup> Copyright 2020, AIP Publishing. b) The porous-media-based thermal waveguide. Schematic diagram of the coordinate transformation (top left). Temperature distributions in a practical application (top right). Porous-media-based thermal waveguides with different layers (bottom). Reproduced with permission.<sup>[79]</sup> Copyright 2021, Elsevier. c) Schematic diagram of a porous-media-based tunable hybrid thermal metamaterial with a topology transition (left). The simulated switch process between thermal cloaking and thermal concentration (right). Reproduced with permission.<sup>[80]</sup> Copyright 2023, National Academy of Sciences USA.

with wavelengths  $\lambda = L/N_1$  are allowed, whose wave numbers take  $k_1 = 2\pi/\lambda = 2\pi N_1/L$ , where  $N_1$  can be any positive integer. Furthermore, if two porous media with different parameters are arranged alternatively to form a thermal convection-diffusion crystal with a lattice constant  $D = L/N$ , only the wave numbers satisfying  $k_2 = 2\pi/N_2 D$  may survive. That is, the thermal wavelength must be an integral multiple of the lattice constant  $D$  to meet the periodic boundary condition  $k_1 = k_2$ , thereby requiring  $N_1 = N/N_2$ . Under such a strict constraint, only particular combinations of  $N_1$  and  $N_2$  can satisfy the requirement, thus exhibiting a promising ability in modulating wave-like temperature profiles. For verification, simulations are performed based on a ring structure with naturally satisfying periodicity utilizing period number  $N = 4$ . The initial wave profiles of  $N_1 = 2$  and  $N_1 = 3$  are both harmonics with different periods, but with time going by, the former can survive the modulator while the latter cannot, for  $N_1 = 2$  can be divisible by  $N = 4$  while  $N_1 = 3$  cannot (Figure 5a, bottom).

In the above scenario, though a ring structure naturally satisfies the periodic condition required by the proposed crystal, the ring needs to be thin enough to ensure the accuracy of the results. Specifically, the inner and outer radius of the ring must be close. This constraint comes from the fact that contrary to a fixed temperature, thermal wave features transient states and phase information. Therefore, different propagation distances inevitably lead to asynchronous phases. Thus, thermal waves through an irregularly shaped channel may be scattered

severely and loose the carried original information. To solve this problem, porous media and transformation theory are employed for designing a new metamaterial.<sup>[79]</sup> It is known that the cornerstone of transformation theory is the substitution of material transformations for space transformations. In this case, a similar strategy is revived. Based on the transformation method (Figure 5b), the initial cylindrical coordinate system ( $r, \theta$ ) in physical space is transformed into the rectangular one ( $x, y$ ) in virtual space. When thermal waves travel through a bent channel, the wave on the outer side will naturally endure a longer path than the wave on the inner side. Therefore, the media parameters ought to be modulated along the tangential direction according to the varying radius, so that fluid in the inner and outer circular channels may maintain the same phase regardless of their different traveling lengths. The scenario described above can be implemented with a combination of  $n$  layers of porous media, while each layer has isotropic and homogeneous parameters, respectively. Since the mismatching phase is attributed to different spatial distances, larger parameters in the outer layer lead to larger heat transfer efficiency and larger convective velocity. Therefore, the mismatching phase can be avoided in the curved part despite the different spatial distances. Intuitively, in terms of a channel with a specific width, a layer number signifies a smaller layer thickness, which can result in better performance. But 20 layers are enough for the specific parameters used in this example, whose simulation diagrams are depicted in Figure 5b.

Aiming at the single fixed functionality, the two thermal metamaterials based on porous media are not fully demonstrated with dynamic switching between different effects. Recently, a meta-device realizing a continuous switch between thermal cloaking and thermal concentration has been verified experimentally to exhibit a promising possibility of porous media.<sup>[80]</sup> Specifically, it comprises two basic solid–liquid thermal meta-material units arranged in-plane according to the generalized theory of transformation thermotrics, where the spatial profiles of the thermal conduction and convection properties could be pre-designed. As shown in Figure 5c, thermal convection and conduction share the same space in this device while allowing independent and local control. Under specific external tuning of the flow field, a topology transition in the virtual space will occur. Then the function of this porous-media-based meta-device could be transformed from one to the other (from thermal cloaking to thermal concentration or from thermal concentration to thermal cloaking, which depends on the preliminary design of virtual space). Although a topology transition can be achieved in the central part of this meta-device, the temperature profile around it has been altered unexpectedly because the introduced convection has disturbed the originally diffusive nature. Hence, a method that can realize such continuous tunability without disturbing the temperature profile of the surrounding part remains to be an inaccessible research concept.

Besides the above aspects, porous media is also quite useful in enhancing heat transfer. The so-called nanofluids could be formulated by adding suspending nanoparticles with high thermal conductivity in liquids. In general, the effective thermal conductivity of nanofluids increases with increasing particle volume fraction,<sup>[81–83]</sup> but specific conditions depend on the value of the Rayleigh number and materials.<sup>[84–86]</sup> By combining porous media with nanofluid, some researches have extended the study of porous media to heat transfer enhancement. One example is the study of a heat sink, which is partially fitted with a multilayered porous medium.<sup>[87]</sup> The corresponding investigation between a constant porosity model and a constant particle diameter model is conducted, thus indicating a reasonable trade-off between stronger heat transfer ability and lower pressure drop. Apart from that work, some researchers have investigated the impingement cooling effect of a porous metal CPU cooler saturated with nanofluid under a magnetic field, indicating that heat transfer performance could be enhanced with a higher Darcy number and a stronger magnetic field.<sup>[88]</sup>

For suiting the transformation theory, porous media theory has blazed a trail for developing novel convective thermal metamaterials. From the view of hydrodynamics, making fluids flow in a specific way is the goal of porous media. While from the perspective of thermodynamics, convection is just a tool, and the precise control of heat transfer plays an important role. Therefore, it is generally recognized that the study of porous-media-based thermal metamaterials is a multi-step thermal inverse design problem. First, the required thermal phenomena are envisioned, then the flow field is conceived, and finally, the porous media is designed according to the hydrodynamics. Many new hotspots have been enlightened, such as thermal wave detection,<sup>[89,90]</sup> thermal sensing,<sup>[91,92]</sup> and thermal wave modulation.<sup>[93–95]</sup> Besides, porous media is ideal for enhancing

heat transfer with large surface areas. Overall, porous-media-related topics with these unique properties are now emerging as a frontier in heat management, they are of non-negligible importance in the design of convective thermal metamaterials.

#### 4. Nonreciprocal Heat Transfer

As previously stated, the reciprocity of a pure diffusive system can be broken by the asymmetric effective thermal conductivity tensor induced by a mechanically rotating structure, thus effectively disguising a convective system as a pure conductive one.<sup>[64,66]</sup> In its most basic form, reciprocity asserts that the response function between two device points remains the same whether the source and receiver are swapped. Regarding wave transmission, reciprocity is a fundamental principle in nature considering reversed time. Despite the importance of reciprocity in analyzing wave systems, it can also be seen as a hindrance that needs to be overcome. Because, the presence of reciprocity excludes the possibility of asymmetric signal transmission between different directions in unidirectional energy aggregation or propagation without backscattering, which is a huge disadvantage for broad engineering and science areas.

According to the Onsager–Casimir theorem, a medium obeying the linearity, time-invariance, and microscopic reversibility hypothesis is proved to be a reciprocal one. From this standpoint, if one or more assumptions behind the Onsager–Casimir theorem are removed, reciprocity can be broken accordingly, and nonreciprocity is obtained.<sup>[96]</sup> Inspired by that, over the past decade, there has been substantial interest in developing nonreciprocity based on nonlinear materials, structured space-dependent or time-dependent constitutive properties, and mode-splitting methods.<sup>[97–99]</sup> Based on these ideas, devices such as acoustic one-way mirrors,<sup>[100,101]</sup> topological insulators,<sup>[102,103]</sup> thermal diodes,<sup>[104,105]</sup> and nonreciprocal solar cells are created.<sup>[106,107]</sup>

In recent years, achieving nonreciprocity in linear time-invariant materials has gained significant research attention. One remarkable example is the acoustic analog of the Zeeman effect studied in an acoustic circulator. Biased by a circulating air fluid, the compact linear device has demonstrated giant acoustic nonreciprocity. Essentially, the feasibility lies in applying the odd-symmetric external air flow under time reversal, which microscopically breaks the time-reversal symmetry of the time-invariant linear system, thus biasing the physical motion of acoustic waves. Different from wave motions that are usually time-reversal symmetry without loss, diffusive motions such as charge movement and heat conduction abide by the principle of time-reversal asymmetry, where dissipation is an inherent property. However, though macroscopically irreversible, constrained by the second law of thermodynamics, conductive heat transfer is still a microscopically reversible process because time-reversal symmetric equations govern the motion of microscopic particles. Out of this rationality, thermal convection can cause nonreciprocity similar to air circulation in acoustics because they are both time-odd external momentum biases employed to break the microscopic reversibility hypothesis of the Onsager–Casimir theorem. The physical quantity being studied in heat transfer can be heat flow or temperature

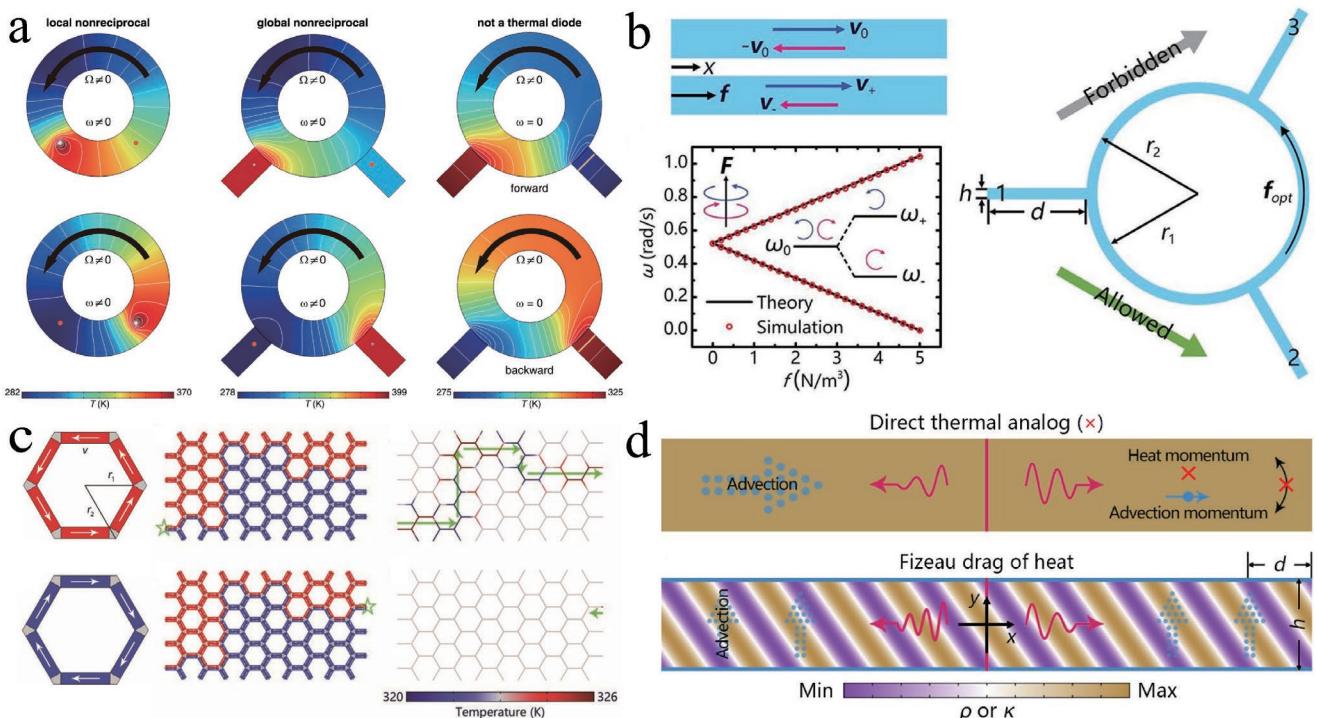
amplitude, whose asymmetry propagation is the role to be discussed in this section.

Recently, a new notion known as global reciprocity has been put forward. Local reciprocity means that the thermal conductivity tensor remains unchanged after interchanging the source and the observation points since the reference point can be chosen casually inside the material. In contrast, global nonreciprocity particularly refers to the transmission properties between the device's incoming and outgoing signals.<sup>[108,109]</sup> The scattering matrix of a reciprocal system is symmetric, whereas that of a nonreciprocal system is not, and its parameters directly relate to the output and input signals. Based on this definition, it is demonstrated that for a two-port rotating system, the global reciprocity is broken for nonzero frequencies while preserved at zero. Namely, global nonreciprocity can be obtained when the heat source is time-harmonic. Still, reciprocity holds when the heat source is a boundary with constant temperature. Corresponding temperature distributions attained by simulation are showcased in Figure 6a.

As an attractive concept, the convection-enabled thermal nonreciprocity has also been extended to the research on a thermal device with three ports, namely a thermal circulator. As shown in Figure 6b, the initial symmetric convection velocity fields between input and output ports are broken by applying a rotational volume force to the fluid of the circulator. Then nonreciprocal heat transfer between different ports can be achieved accordingly.<sup>[110]</sup> This approach toward nonreciprocal heat transfer avoids using nonlinear materials, thus the working temperature constraints posed by the inherent properties of materials

can be lifted naturally.<sup>[111,112]</sup> However, while acknowledging the progress of such an attempt, the defects of this work cannot be ignored. Separated from other pioneer researches on circulators that take advantage of scattering theory, this thermal circulator fails to derive the corresponding scattering matrix in theory. For this reason, in terms of the thermal circulator, we still anticipate a future exploration that fully studies the transmission characteristics of input signals and the dependence of nonreciprocity on the oscillation frequency. Furthermore, the experimental breakthrough of the thermal circulator is also a worthwhile direction, which has so far remained at the conceptual level.

With the mechanism of the thermal circulator proposed above, a robust one-way edge state is predicted in the thermal system accordingly, even though it is a phenomenon originally discovered in quantum mechanical and classical wave systems.<sup>[113]</sup> The model consists of two kinds of circulators rotating clockwise (marked by blue) and anticlockwise (marked by red). With boundaries conductively coupled well with each other, they are organized into a graphene-like lattice structure. In this scenario, as long as the relevant parameter space is chosen judiciously, decay rates in different directions will differ under the influence of convection. Then robust one-way edge states are expected to occur due to the interference-like behavior of opposite propagating temperature waves. Thermal waves that travel along the flow field are permitted, whereas thermal waves that travel against the flow field are prohibited (Figure 6c). These findings may enlighten the exploration of topological properties with diffusion dynamics, which is a research field in a new light.



**Figure 6.** Nonreciprocal thermal metamaterials induced by convection. a) Temperature profiles on rotating rings with different channels and heat sources. Reproduced with permission.<sup>[109]</sup> Copyright 2021, American Physical Society. b) The schematic diagram of modal splitting (left) and angular-momentum-biased thermal nonreciprocity (right). Reproduced with permission.<sup>[110]</sup> Copyright 2021, AIP Publishing. c) The thermal interface states based on two unit structures with clockwise and counterclockwise velocities. Reproduced with permission.<sup>[113]</sup> Copyright 2021, IOP Publishing. d) Conceptual graphs of diffusive Fizeau drag. Reproduced with permission.<sup>[114]</sup> Copyright 2022, American Physical Society.

Following the development of classic physical phenomena, a novel concept, diffusive Fizeau drag, has been put forward recently by resorting to the time-related advection in space-related inhomogeneous porous media.<sup>[114]</sup> Bypassing the problem that momentum is absent in thermal diffusion, this conceptual implementation demonstrates different propagating speeds of temperature fields in opposite directions, thus leading to nonreciprocal thermal profiles (Figure 6d). This work interestingly extends the famous phenomenon observed in the Fizeau flow test to convective thermal metamaterials, which may enrich our understanding of nonreciprocity studies from an interdisciplinary perspective.

To this end, thermal convection can be generated by the movement of either liquid or solid and can lead to thermal nonreciprocity. Remarkably, thermal convection is not even limited to the motion of matter. Interestingly, the “motion” of material parameters may also be promising in achieving nonreciprocity. Such an idea is inspired by the well-developed modulation studied in wave systems such as acoustics and optics.<sup>[115–118]</sup> The propagation of electromagnetic waves in two connected waveguides, for example, will be nonreciprocal once the electric permittivity  $\epsilon$  is modulated with a traveling wave.<sup>[119]</sup> Other devices, such as photonic topological insulators, can be created based on a similar principle.<sup>[120–122]</sup>

Note that the governing equation of diffusive systems contains the same Laplacian term as the wave equation, so relevant theory has evolved from wave system to diffusive system. Based on this idea, a work starts from Fick's diffusion equation in a 1D model and empirically realizes this concept for the first time.<sup>[123]</sup> The governing equation is written as

$$\frac{\partial q(x,t)}{\partial t} = \frac{\partial}{\partial x} \left( \sigma \frac{\partial}{\partial x} (gq(x,t)) \right) \quad (4)$$

where  $\sigma$  and  $g$  denote the conductivity and inverse of the capacity of the medium, respectively. Different from wave propagation by modulating only electric permittivity  $\epsilon$  for nonreciprocity, two parameters require to be modulated in diffusion as a function of time and space with the same modulation wavelength and frequency. For example,  $\sigma(x,t) = \sigma_0 + \sigma_m \sin(kx - \omega t)$  and  $g(x,t) = g_0 + g_m \sin(kx - \omega t + \phi)$ , and  $0 < \sigma_m < \sigma_0$ ,  $0 < g_m < g_0$ . Then, the effective diffusion equation can be obtained as

$$\frac{\partial \tilde{q}(x,t)}{\partial t} = \tilde{D} \frac{\partial^2}{\partial x^2} \tilde{q}(x,t) - \nu \frac{\partial \tilde{q}(x,t)}{\partial x} \quad (5)$$

where  $\tilde{q}(x,t)$  is the time-averaged charge density, and  $\tilde{D} = \sigma_0 g_0 + \frac{1}{2} \sigma_m g_m \cos(\phi)$  and  $\nu = \frac{1}{2} k \sigma_m g_m \sin(\phi)$  are the homogenized coefficients.

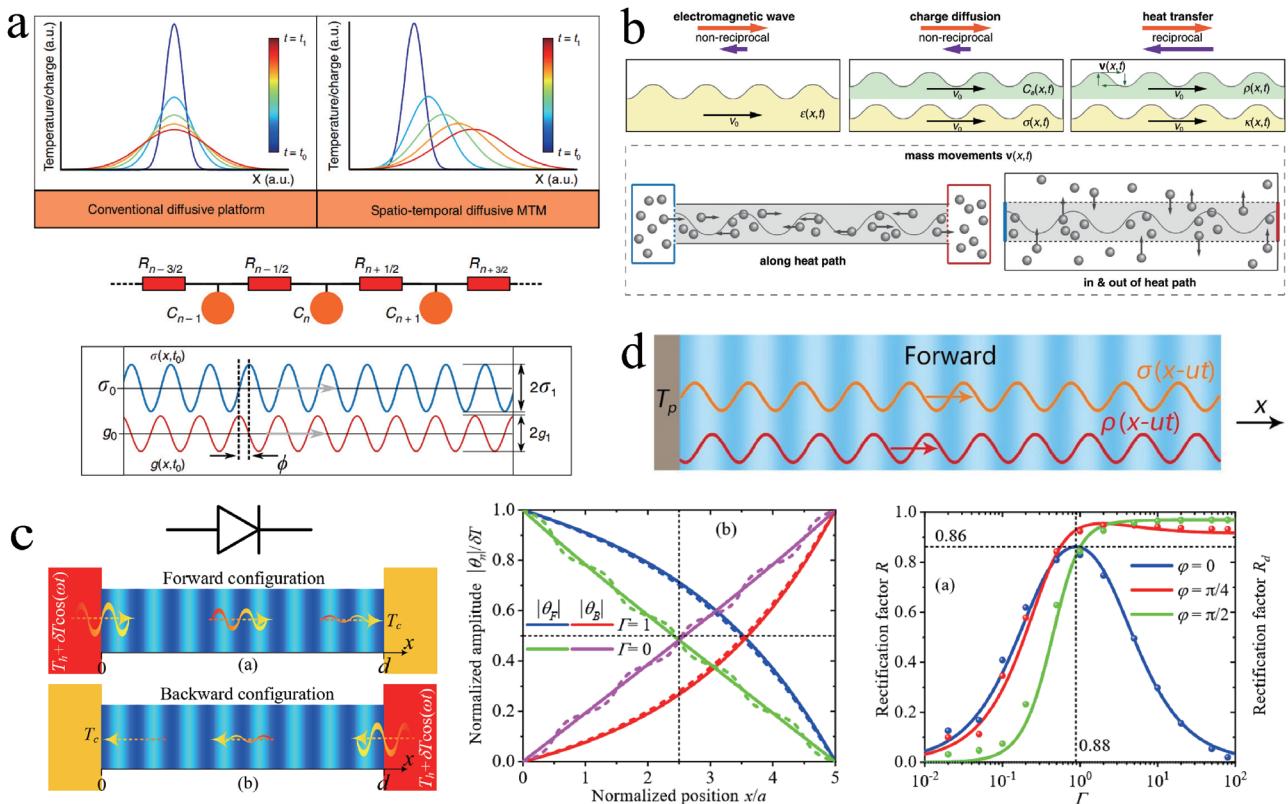
As shown in Equation (5) and the corresponding solution depicted in Figure 7a, even though there exists no motion of matter in the solid material, an internal convective term representing the “medium bias” appears effectively due to the modulation of parameters. Thus the system's spatial symmetry can be broken. The experiment setup is a finite chain with several resistors and capacitors arranged alternatively. In this setting, the direction of such equivalent flow velocity can be modulated forward or backward using a relative phase difference between one and the other. Consequently, the system will

impart a spatially directional preference to the charge diffusion, implying that under proper modulation, either charge accumulation or evacuation can occur at the domain boundary.

Inspired by this approach, many other attempts are made to investigate similar phenomena in thermal systems.<sup>[117,124]</sup> Among these explorations, one noticeable work presumes that electric conductivity resembles thermal conductivity, while electric capacity resembles the product of thermal capacity and density. Based on this analogy, it is anticipated that as long as these parameters modulate similarly to their counterparts in the electric system, the same phenomenon will also emerge in the thermal system.

From the perspective that all diffusive physical processes will tend to an equilibrium, as stipulated by the increasing law of entropy,<sup>[123]</sup> there appeared to be no problem in this scenario. However, constrained by the law of mass conservation, the matter that works as a thermal energy carrier cannot be created or destroyed at no cost. Therefore, as illustrated in the transport processes in dynamic materials under time modulation (Figure 7b), to modulate density with time and space, a mass movement is unavoidable even though there is no net directional mass flow. For this reason, time-modulated density and thermal conductivity are hardly feasible to achieve nonreciprocity since the effects of the “motion” of parameters and the inevitable mass flow will counterbalance each other.<sup>[125]</sup> Confronted with this dilemma, rather than density modulation, a tunable specific heat capacity may perform as a substitute because what matters is the product of the density and the heat capacity.

According to recent work, the thermal properties of a light-responsive material, such as azobenzene polymer,<sup>[126]</sup> will undergo a reversible transition under UV (375 nm) and green light (530 nm) illuminations in a few seconds at room temperature. Generally, the unique material parameters are sensitive to changes in external stimuli such as electric or magnetic fields. A layered media rather than a homogeneous one can be obtained by applying a periodically arranged illumination to such materials. Thus, the wave-like modulation of thermal parameters can formulate accordingly. As an application, a thermal wave diode with relatively tiny heat currents modulated in frequency is conceived.<sup>[127]</sup> By attaching two thermal baths (a time-harmonic one with temperature  $T_h + \delta T \cos(\omega t)$ , and a fixed one with temperature  $T_c$ ) to the left side and right side of the thermal diode, respectively, a temperature gradient  $T_h - T_c + \delta T \cos(\omega t)$  can be generated as a driven force of heat flow (Figure 7c). Under this setting, if the two thermal baths are exchanged, the heat flux magnitudes  $q_F$  and  $q_B$  in the forward and backward configurations are expected to be unequal, so nonreciprocal heat transport can be realized. For validation, numerical simulations accompanied by theoretical models in a 1D domain are performed. When the normalized speed of modulation  $\Gamma$  is zero, the temperature distributions are linear in both forward and backward directions. However, once a nonzero normalized speed of modulation  $\Gamma$  is applied, the temperature distributions are biased to one side of the testing sample, as if they were dragged by a convective term. There also exists an optimal speed at which the highest rectification factor (greater than 86%) can be obtained.<sup>[127]</sup> A similar investigation on thermal wave nonreciprocity enabled by spatiotemporal modulation is also reported (Figure 7d).<sup>[128,129]</sup> Since heat



**Figure 7.** Nonreciprocal metamaterials induced by spatiotemporal modulation. a) Conceptual representation of controlling diffusion with metamaterial platform (Electrical analogy). The spatial symmetry and effective spatial asymmetry in diffusive flow (top). Periodic spatiotemporal modulation of the two governing macroscopic material properties (bottom). Reproduced under the terms of the CC-BY Creative Commons Attribution 4.0 International license (<https://creativecommons.org/licenses/by/4.0/>).<sup>[123]</sup> Copyright 2020, The Authors, published by Springer Nature. b) Transport processes in dynamic materials under time modulation. Nonreciprocal propagation of electromagnetic wave, nonreciprocal diffusion of electric charges, and reciprocal heat transfer (top). Mass movements along and out of the heat transfer path (bottom). Reproduced under the terms of the CC-BY Creative Commons Attribution 4.0 International license (<https://creativecommons.org/licenses/by/4.0/>).<sup>[125]</sup> Copyright 2020, The Authors, published by Springer Nature. c) Scheme of a thermal wave diode operating in forward and backward configurations (left). The temperature amplitude profiles (middle) and the rectification factors for heat fluxes (right). Reproduced with permission.<sup>[127]</sup> Copyright 2021, American Physical Society. d) Schematic representation of the spatiotemporal modulation in thermal conductivity and mass density. Reproduced with permission.<sup>[128]</sup> Copyright 2021, American Physical Society.

waves could be utilized in thermal detection but need to be suppressed for thermal stability, nonreciprocity is crucial, and this young field calls for further development.

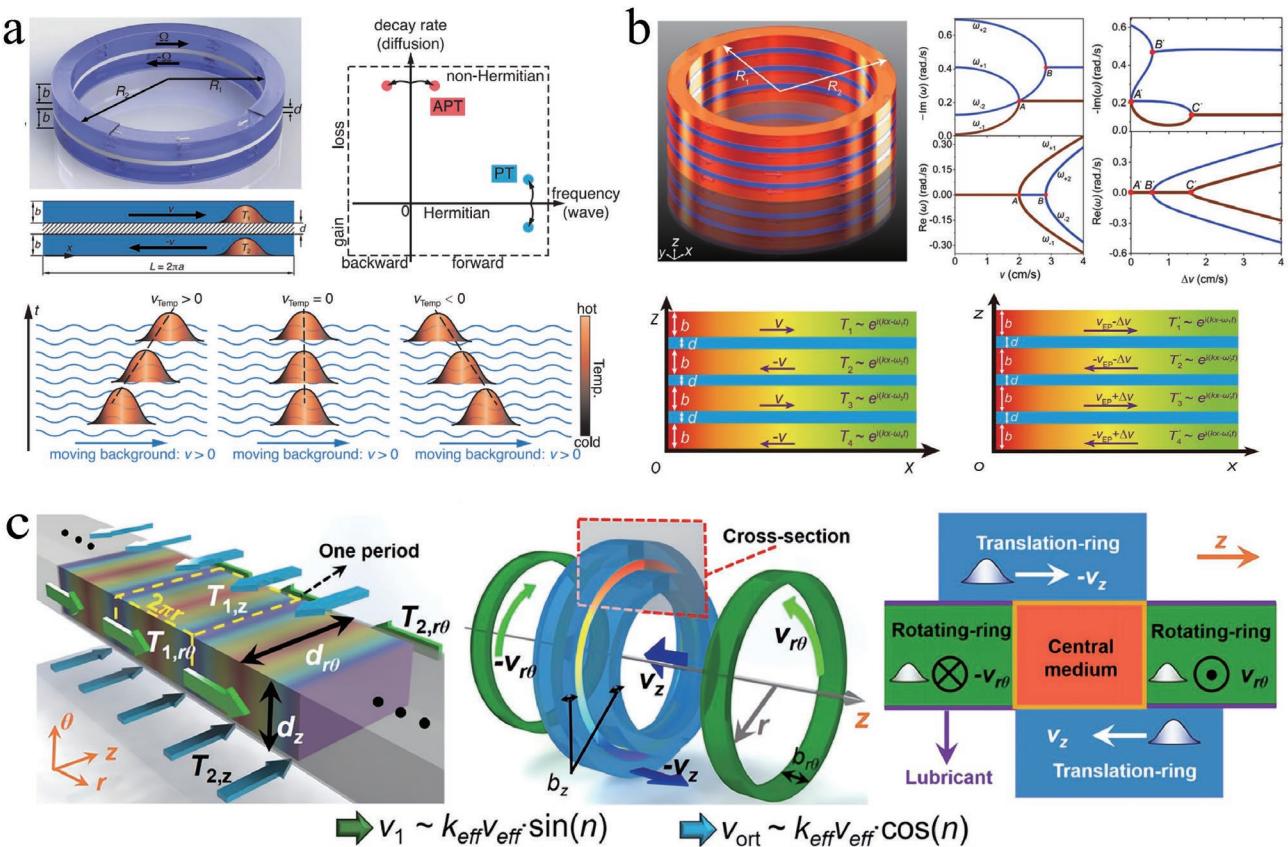
To conclude, reciprocity is a fundamental principle that is hard to break, since most natural materials are time-invariant, passive, and linear. As a convenient method to break the inherent microscopic time reversibility, convection and equivalent convection have proved to be promising in designing nonreciprocal thermal metamaterials that break the limitation of the working temperature range.

## 5. Non-Hermitian Phenomena

In the preceding sections, we review recent progress offered by thermal convection to manipulate heat transfer, revealing its potential for designing thermal metamaterials. The introduction of convection can also enrich our physical understanding of the heat transfer process. Therefore, this section reviews the relationship between convective thermal metamaterials and non-Hermitian physics.

In quantum physics, the Hermiticity of a Hamiltonian is a critical postulate, which guarantees the conservation of probability in an isolated system and ensures a total real-spectrum of eigenvalues.<sup>[130–132]</sup> However, in stark contrast with this important assumption, most physical systems in nature are coupled with the surrounding environment, where the presence of energy flow, particle motion, and information exchange can hardly be ignored. Thanks to its ability to effectively describe open systems, non-Hermitian physics has stimulated great research interest.<sup>[133–135]</sup>

As a versatile platform for exploring unconventional wave phenomena, the non-Hermitian description has been applied to various open systems with gain and loss, including optics, electronics, mechanics, and acoustics.<sup>[136–138]</sup> Enabled by unusual symmetry and topological characteristics, non-Hermitian physics enriches the topological phases outside the existing Hermitian framework.<sup>[139–141]</sup> For a long time, non-Hermitian physics has been predominantly studied in wave systems, where extensive research on unidirectional transparency,<sup>[141,142]</sup> coherent perfect absorption,<sup>[143]</sup> single-mode laser,<sup>[144]</sup> and non-Hermitian skin effect,<sup>[145,146]</sup> is progressing tremendously.



**Figure 8.** Applications of convection to study non-Hermitian phenomena in thermotics. a) APT symmetric diffusive system. The 3D model and the corresponding 2D model of an APT symmetric system (top left). Two routes to explore non-Hermitian physics: from a wave system with zero decay rate or from a diffusive system with zero frequency (top right). For heat convection, the temperature profile can follow, remain motionless, or move against the moving background (bottom). Reproduced with permission.<sup>[147]</sup> Copyright 2019, American Association for the Advancement of Science. b) High-order exceptional points in diffusive systems. Schematics of the 3D model (top left) and the corresponding 2D model (bottom). The imaginary part and real part of eigenvalues versus the background flow velocity. The imaginary part and real part of eigenvalues versus the flow velocity modulation  $\Delta v$  as a perturbation (top right). Reproduced with permission.<sup>[148]</sup> Copyright 2019, Engineered Science Publisher. c) Realization of the topological transitions in thermal diffusion. Schematic of the topological transitions in thermal diffusion (left). The multiple ring system coupled with two orthogonal pairs of counter-motional advective components (middle), and the cross-sectional view labeled in the red dashed border (right). Reproduced with permission.<sup>[150]</sup> Copyright 2021, American Physical Society.

However, a significant point that has long been neglected is that, since non-Hermiticity and dissipation are closely related, heat transfer processes can also be described by non-Hermitian physics. In other words, wave systems are inherently Hermitian without loss, while diffusive systems are inherently non-Hermitian with loss. Based on this fact, growing interests in bridging wave and diffusive systems are emerging.

One pioneer work reveals the anti-parity-time (APT) symmetry, which extends the non-Hermitian notions in diffusive systems.<sup>[147]</sup> As shown in Figure 8a, by constructing a system with two well-coupled solid rings with equal but opposite rotated velocities, the corresponding governing equations can be written as

$$\begin{cases} \frac{\partial T_1}{\partial t} = D \frac{\partial^2 T_1}{\partial x^2} - v \frac{\partial T_1}{\partial x} + h(T_2 - T_1) \\ \frac{\partial T_2}{\partial t} = D \frac{\partial^2 T_2}{\partial x^2} + v \frac{\partial T_2}{\partial x} + h(T_1 - T_2) \end{cases} \quad (6)$$

where  $D$  is the diffusivity of rings,  $h$  is the rate of heat exchange between two rings, and  $T_1$  and  $T_2$  are temperatures of the upper and lower rings, respectively.

In analogue to the Schrödinger equation, Equation (6) exhibits a parameter space that can find its counterpart in quantum mechanics. For instance, the convective term is a Hermitian component introduced into a non-Hermitian Hamiltonian, and the temperature distributions of the two rings represent quantum states. Through this approach, the corresponding eigenvalue of the effective Hamiltonian can be solved as

$$\omega_{\pm} = -i \left[ (k^2 D + h) \pm \sqrt{h^2 - k^2 v^2} \right] \quad (7)$$

where  $k$  is the wavenumber, and  $D$  and  $h$  have the same meaning as in Equation (6).

Then, an exceptional point  $v_{EP}$  satisfying  $h^2 = k^2 v_{EP}^2$  emerges. Depending on the convective velocity  $v$  compared with the critical value  $v_{EP}$ , the ordinarily stationary temperature profile can

either follow or even move against the convection velocity in the symmetry-broken phase, whose key point is the competence between thermal convection and coupled thermal conduction.

Diffusive APT symmetry has gained broad attention and further spurs research on high-order exceptional points in diffusive systems, which is robust against perturbation and phase oscillation.<sup>[148]</sup> The model is set to be a four-channel coupling thermal system, where the background flow velocities in adjacent channels are opposite. In this scenario, the eigen fields with high loss will decay rapidly in diffusive systems, and only the route with the minimum loss can eventually be observed. Therefore, the system will only experience the third-order EP at the critical velocity where the APT breaking phase transition occurs (Figure 8b with brown lines). Despite perturbation, the phases of steady-state temperature profiles in adjacent circulating channels can remain static as long as the strength is within the threshold, demonstrating the robustness of the APT symmetry. Besides, the phase oscillation is irrelevant to the initial conditions under the APT breaking condition.

Ref. [147] has extended the concepts of PT-symmetry beyond wave physics and inspired other research in non-Hermitian physics.<sup>[149,150]</sup> Most recently, a double-coupled convection pair (Figure 8c) is used to create an orthogonal convection space in the thermal diffusive system. Two pairs of oscillations are introduced to supplement the missing degrees of freedom in the thermal diffusive system. Thus, the dynamic encircling around the degeneracy in 2D thermal systems is realized, and the configurable topological phase transition of the thermal system is observed for the first time.<sup>[150]</sup>

## 6. Outlook

We have reviewed the research progress of convective thermal metamaterials in recent years and classify them into four categories based on different physical effects. From the perspective of exploration, convective thermal metamaterials not only developed novel functions absent in traditional conduction but also discovered new physical phenomena in diffusive systems. From the perspective of integration, two remarkable directions are worth mentioning. First, in terms of effective parameters, convective thermal metamaterials have integrated into the traditional conductive framework successfully. Second, in terms of equation form, convective thermal metamaterials transform the governing equation from the intrinsic parabolic form of the diffusive system to the hyperbolic one of the wave system, excavating their physical characteristics in common.

By its precise nature, the significance of introducing convection is that it provides a new view for designing thermal metamaterials and contributes to a new approach toward thermal regulations. More tunable parameters usually mean more applicable methods in the design of metamaterials. Herein, a thought may be raised that we could achieve more desired functionalities by resorting to variable parameters. However, the current artificial structure aims at satisfying the constant parameters for most types of convective thermal metamaterials. It thus limits the excavation of physical phenomena and their application in complicated ambience. To further promote the advancement of this field, we should make good use of the

convection freedom to excavate new applications and broaden the current barren parameter space for higher tunability.

In the contemporary energy crisis background where thermal management becomes increasingly important, the study of thermal topology has recently been an active direction. With convection, the thermal topology may bring new physical insights into the diffusive system. Toward this topic, heat-trapping is brought into practice via graded metamaterials where imitated advection is produced inside,<sup>[151]</sup> diffusive skin effects are observed via the asymmetric coupling double-channel model,<sup>[152]</sup> and localized heat diffusion is experimentally demonstrated.<sup>[153,154]</sup> Furthermore, resorting to the effective complex plane offered by two pairs of orthogonal advection, the original directionless restriction of thermal transfer can be sidestepped. It unveils deeper topological characteristics of thermal systems and allows a distinct mechanism for realizing a topologically robust thermal system immune to arbitrary perturbations.<sup>[155,156]</sup>

So far, we have disclosed that exploring new applications of the convective parameter space is very promising in the design of thermal metamaterials. In parallel, what makes equal sense is broadening the freedom of the current parameter space. One realistic way to realize this goal might be to bring the dimension of time and space into the parameter platform.<sup>[157]</sup>

As to temporal variation, the time-related convection has been within our reach under the drive of external force, which is ubiquitous in most convective thermal metamaterials and needless to state. However, compared to the universality of time-related convection, the temporal modulation of structural parameters is still relatively rare and should be the current research focus. According to the hint offered by existing research, the temporal modulation of material parameters can usually be achieved through two methods: periodically moving well-designed mechanical units,<sup>[125]</sup> and using materials whose parameters vary in response to external stimulation.<sup>[158]</sup> As for the specific form of the external field and the application possibility of the designed thermal metamaterial, it is worth studying both for metamaterial design and traditional material science.

In addition, temporal modulation alone sometimes cannot achieve the desired function,<sup>[114]</sup> so the spatial modulation of material parameters should also be considered. Usually, natural materials are homogeneous and isotropic, while the development of transformation theory breaks the above restrictions to a certain extent. One idea to step further along this path instead of solely modulating isotropic parameters in time or space is that a temporal change could be made to alter parameters from a single isotropic scalar value to an anisotropic tensor value. Though complicated, such exploration may shed light on new functionalities or physical phenomena.

Apart from spatiotemporal modulation, multi-physical field regulation could be another approach to broadening parameter space. For example, heat transfer can be affected by varying the magnitude or direction of the applied current in the thermoelectric material.<sup>[159,160]</sup> With higher-quality thermoelectric materials, this may be another promising approach to improving the tunability of thermal metamaterials.

In conclusion, given the academic research and the current calling for carbon neutrality, convective thermal metamaterials are bound to be a burgeoning research field. In the next few

years, the exploration of high-dimensional thermal metamaterials will be one of the trends. It may lay a solid foundation for the emerging research topic of thermal topology and provide a recipe for unveiling unexplored characteristics of the highly interdisciplinary area of research.

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## Conflict of Interest

The authors declare no conflict of interest.

## Keywords

enhancing heat transfer, non-Hermitian phenomena, nonreciprocal heat transfer, porous-media-based thermal effects, thermal convection, thermal metamaterials

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