Convertible bonds or bonds with warrants: an approach using sequential financing

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Abstract

The context for this article is the problem of funding through the issuing of debt associated with a call option on shares to be issued: convertible bonds and bonds with warrants. The securities studied are generally considered to be the same in the literature. The aim of this article is to propose a study of a sequential financing model that integrates one of the objective differences between these two securities, the difference observed between the cash flow sequences resulting from the issue. We will deduce the characteristics that lead companies to issue either convertibles or bonds with warrants.

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Introduction

A company in search of funding through complex securities that combine simple debt with a call option on shares to be issued has the possibility of choosing between bonds with warrants (which may or may not be refundable) and convertible bonds.

The theoretical literature that focuses on financial contracts design concentrates more particularly on the motives that may lead companies to issue this type of bonds, especially when they are complex. Since Modigliani and Miller [10], we know that justifying the existence and use of a particular form of contract means identifying imperfections in the financial market by means of which the contract proposed becomes an optimal solution to the problem of financing the company.

Research into the reasons why companies issue debt instruments accompanied by a call option on owner's equity has taken a wide range of different avenues.

Of these, we can quote :

- modification in risk following the issue, (Jensen and Meckling [7], Green [5]);
- estimation of risk, (Brennan [1], [2]);
- the asymmetry of information, (Constantinides [4], Stein [12]);
- the context of sequential financing, (Mayers [9]).

These models, however, deal exclusively with convertibles, or only take generic securities into account, with no particular formalization. The problem with having to choose between convertibles and bonds with warrants is thus not dealt with directly[1].

Certain authors have however tried to compare the two types of securities by means of an empirical study (Long [8]). Most of the works identified were published in the United States and focus on the American flotation markets and euro-flotations.

In France, the studies available deal with market reactions to the issue, as in Hachette [6], or Burlacu [3] who tests the Myers model [11] through an events study of convertible issues over the period 1981-1997. He concludes that the hypothesis of a pecking order is reinforced.

[1] Green makes a formal differentiation between the two types of security, but both can solve the problem stated.

In this paper, we propose to study one of the objective differences[2], the sequence of the flow generated by the issuance of a convertible and by a bond with warrants. This difference appears in a multi-period context which will be described in detail in section 2, and these circumstances are illustrated by means of an example in section 3. Finally, we will provide a discussion of the proposals to which this model leads in section 4.

1 Highlighting the cash flow differential

An objective difference between issuing convertibles or bonds with warrants is linked to the fact that, in the case of the latter, exercising the option is accompanied by a cash flow to the issuer, that is, payment of the option strike price by the subscribers. Exercising the option in the case of convertibles results only in savings on cash outflows. The chronological cash flow sequences that result from issuing convertibles or bonds with warrants for the issuing company are thus different.

In theory, however, if the present values of the two sequences are identical, the issuer should be indifferent to the choice. In practice, this is not necessarily the case as the issuer may want to match the flow of resources that he hopes to obtain from his investments with the cash outflows associated with serving the debt (interests to be paid and repayment of the capital). If this is not possible, particularly when the service of the debt is greater than the flow of resources, the issuer must resort to other sources of funds. However, these other sources will, on the one hand, result in further issuing costs, and, on the other, modify the overall cost of the funding obtained. In this way, the chronological profile of the cash flows can be of importance when choosing the type of issue. This section focuses on developing the circumstances under which the convertibles vs bonds with warrants cash flow differential can appear.

1.1 Characteristics of the bonds

Let us imagine a company that must choose between issuing convertibles and bonds with warrants in order to finance an investment. The choice is only between these two types of security for reasons that we will not go into at the present time but which will be dealt with in the following section.

To be able to compare them in a valid manner, solely in relation to their cash flow profiles, the two issues envisaged must be equivalent in terms of the initial

[2] there are many others, such as, for example, separating the debt from the call option (the warrant) immediately after issuing in the case of the bonds with warrants, or the regulatory framework for each security.

amount E collected and invested, the interest rate (nominal, which will determine the coupons), and the fraction of their equity (and thus their future cash-flows) attributed to the subscribers who will become shareholders in case of exercise.

The model described is composed of two periods, or three moments:

- 0, corresponds to the issuance of the securities (subscription) and to the investment of the proceeds of the issue;
- 1 is the date of exercise of the (European) call option, that is, the possibility of converting the convertible or exercising the warrants ;
- 2 is the normal repayment date for the debt in both cases.

In practice, the convertibles or the bond part of bonds with warrants are sometimes repaid with a premium. Similarly, the warrants in bonds with warrants are sometimes refundable if they are not exercised [3].

Here, we consider that, without loss of generality, there is no repayment premium and that the warrants are not refundable.

The bonds pay the coupon rate c. Given the tax deductibility of the interest and a tax rate on profits equal to τ, the real cost for the issuer is $c' = c(1 - \tau)$.

Bonds with warrants are designed in such a way that exercising all the warrants is done at a price such that the amount collected by the company is equal to the nominal amount of the debt, E. We can, however, revoke this rather simplified assumption without modifying the conclusions of the model.

Finally, we put forward the assumption that the call options are always European. This assumption will be discussed in section 4.

Both bonds are thus identical with regard to:

- the amount collected by the issuer E and the nominal value of the bond;
- the value and dates of payment of the coupons;
- the fraction of equity obtained from the exercise.

[3] Added to this additional repayment, often symbolic, is the tax incidence: the low value of the repayment of the warrant makes it possible to externalize a capital loss that generates tax savings

Issue	Convertibles	Bonds with warrants
Nominal	E	E
Coupon after tax savings Dates of the coupons	c' 1 and 2	c' 1 and 2
Repayment of the debt	2	2
Call option	European in 1 1 share per bond	European in 1 1 warrant per bond 1 share per warrant at subscription price E

Table 1: Characteristics of the securities

1.2 Cash flows profile

In all cases, the issuer receives the sum corresponding to the issue at time 0, or E, minus the issuing costs which we suppose to be identical (and thus not discriminatory) in both cases.

We consider that the subscriber will always wait for the payment of the coupon before deciding to exercise the option he holds: the accrual date for the shares obtained is always the start of the following period. The issuer thus pays the interest at time 1 (regardless of the type of bond and the subscriber's decision with regard to the option), at a rate of c on the unpaid balance E, representing an interest flow of $-Ec'$, taking into account the tax saving.

We examine the cash flows generated, first in the case where the call option on the equity is not exercised, then in the case where it is (exercise of warrants or conversion of convertibles).

1.2.1 The option on the equity is not exercised

If the option is not exercised at time 1, at the end of period 2 the issuer pays interest again on period 2, or $-Ec'$, and refunds the debt when it becomes due E. The total flow is thus $-E(1+c')$, taking into account the tax saving on the interest payment. Table 2 describes a cash flow profile that is strictly identical between the two financing methods.

1.2.2 Exercising the option on the equity

In the case of convertibles, exercising the option corresponds to converting the bond, which occurs at time 1, just after payment of the coupon, which is still (after

Date	0	1	2
Bond with warrant	E	$-Ec'$	$-E(1+c')$
Convertible	E	$-Ec'$	$-E(1+c')$
Difference	0	0	0

Table 2: Option on the equity not exercised

tax savings) equal to $-Ec'$. The conversion does not result in any particular cash flow for the issuer at time 1, but the bond disappears, and the cash flow for period 2 is thus null.

In the case of bonds with warrants, the warrants are separated from the bond, and the payments associated with the bonds are independent of exercising (or not) the option. These payments are thus: $-Ec'$ in 1 and $-E(1+c')$ in 2. If we consider exercising the warrants in time 1, the company will receive a cash flow of E, corresponding to the exercise price. Finally, the flows are $-Ec' + E = E(1-c')$ in 1 and $-E(1+c')$ in 2.

These results are summarized in Table 3, which also highlights the flow differential between the two bonds in each case.

Date	0	1	2
Bond with warrants	E	$E(1-c')$	$-E(1+c')$
Convertible	E	$-Ec'$	0
Difference	0	E	$-E(1+c')$

Table 3: Exercising the call option on the equity

The company thus has at its disposal an additional cash flow, E, during one period (between time 1, when the flow is received, and time 2, when the flow is repaid) in the bond with warrants case. This resource costs the company c'.

For what can this flow be used?

We can imagine two possibilities:

- at time 2, the company does not have any investment opportunities with a positive NPV. It can however choose to repay in advance the debt part of the bonds with warrants, for example, or invest with a negative NPV;
- the company has at least one investment opportunity with a positive NPV and chooses to reinvest E for one period of time.

We will now examine these hypotheses in a context similar to the Mayers [9]

model.

2 A model in the context of sequential financing

The differential highlighted in the cash flow sequences for convertibles and bonds with warrants leads us to favor the analysis framework for sequential financing, going back over the characteristics of the Mayers model [9]. Although Mayers only considers convertibles as an alternative to simple debt, here we will adapt the model, adding the possibility of funding through bonds with warrants. The aim is to demonstrate that there are cases in which the issuing company can use the cash flow advantage of the bonds with warrants, and will thus tend to favor this type of security over convertibles.

The universe described by Mayers [9] has many analogies with that presented in section 1. In both cases, the company must choose a means of finding funds, and the model covers two periods. Mayers added the destination of the funds in the form of investment opportunities in periods 1 and 2. The need to successively finance these investment opportunities creates the framework for sequential financing.

Let us suppose that a company has an immediate investment opportunity, and an investment opportunity in the future that will also require funds. We consider first that the opportunity for investing immediately has a positive NPV, and that this information is common knowledge. We will see the consequences of abandoning this assumption in the context of the discussion section 4. However, the first investment project will not produce sufficient cash flow to finance the second project if this latter project is profitable: self-financing is impossible at the start of the second period.

The company always has opportunities to invest in projects with negative NPV, which will produce incentives for the managers of these projects to over-invest. The investment projects last only one period.

The only possibility at the disposal of the shareholders for controlling the managers is the nature of the financial contract (security) that will be chosen.

The value of the option to invest at the start of the second period is unknown until its maturity.

In short, the company is at the start of the first period in a universe composed of two periods. It finds itself in the context of sequential financing because it requires funding for:

- a profitable investment project that will start immediately and the revenue

from which is certain;

- an option to invest that will become due at the start of the second period and the value of which is unknown at this time.

The financing possibilities are:

- convertibles: the bond can be converted at the end of the first period and will be repaid at the end of the second if there is no conversion;
- bonds with warrants, the bond of which is refundable *in fine* at the end of the second period, and the warrants can be exercised at the end of the first period;
- finally, the company still has the possibility of going into debt for a certain period in order to top up the funding needed.

Issuing new securities is always costly and the structure of the issuing costs (fixed and variable costs) makes repeated issues of small amount less attractive than a single issue of a larger amount.

The options on equity (converting the convertibles or exercising the subscription warrants) are European for reasons of simplification. The consequences of using American options will be discussed in section 4.

It should be noted that the convertibles or bonds with warrants that companies have the possibility of issuing are identical in terms of their characteristics to those described in section 1.

The two key factors in the financing problem are thus:

- the issuing costs and their characteristics;
- the problems of managers over-investment. They are always able to invest any funds available, unless those are needed for repayments or other obligatory payments.

It is important to remember that in this context, issuing simple debt, whether it is over two periods or in a sequential manner, is less than optimal for the reasons developed by Mayers. Actually, debt that is sufficient to finance both projects can lead to over-investment if the option to invest turns out not to be profitable. On the contrary, going into debt in a sequential manner results in higher issuing costs.

The options on the equity of convertibles and bonds with warrants are designed in such a way as to be in the money if the investment opportunity in 1 is profitable, out of the money if the opposite is true.

We will consider that if the investment opportunity revealed in 1 is not profitable, the call option is not exercised. The value of the underlying asset is considerably less than that of the exercise price. The managers may be encouraged to over-invest using the cash flow from the first project, but simple mechanisms can prevent them from doing so: anticipated repayment of the debt, for example.

If, on the contrary, the investment opportunity is profitable, the call option is exercised: conversion of the convertibles, or exercise of the subscription warrants. However, as we demonstrated in the previous section, only this latter hypothesis provides the additional funds for the company in period 1. The contribution to the funding of the second project is thus greater than that obtained in the case of convertibles, and there are two possibilities:

- the funding obtained is less than what is required by the company, and the savings in issuing costs are thus higher than in the case of convertibles;
- the funding obtained exceeds requirements, the question of reinvestment arises again, and that of over-investment, as the two projects studied are the only projects likely to have a positive NPV.

There are thus cases where, under circumstances similar to those described by Mayers, the company would be better to issue bonds with warrants than convertibles.

One illustration of the model in an example is proposed in the following section. The various different consequences of these observations will then be discussed in section 4.

3 Practical example

Let us imagine a company whose initial balance sheet in market values[4] is as follows:

Assets		Liabilities	
Former assets	1,000	Equity	1,000

Balance sheet (a)

[4] All the balance sheets in the example are in market values.

For reasons of simplification, the risk free rate is null and there are no issuing costs. The cost of the equity for the company is $k_c = 20\%$, and the assets in place generate a cash flow of 200 to infinity. The market value of the assets (and thus of the company) is thus indeed:

$$\frac{200}{0.20} = 1,000$$

The company's equity is represented by 1,000 shares, the market price of which is 1.

The level of uncertainty is the same for everyone (shareholders and managers): the flows produced by the projects will be revealed when they occur (at the end of the period). The decisions are made in a risk-neutral environment, considering expected cash flows only.

3.1 First period

The company has an immediate investment opportunity which requires a payment of 500. This investment may produce a cash flow of 720, or a flow of 552 at the end of the period and will then disappear. Both events have the same probability of occuring.

In order to remove the impact of the chosen financing on the cost of the resources, we use the adjusted net present value method, and for the moment we only calculate the NPV of the economic cash flows for the project.

The net present value of the economic flows for the project is thus, in the favorable case:

$$-500 + \frac{720}{1.20} = 100$$

or, in the unfavourable case:

$$-500 + \frac{552}{1.20} = -40$$

As both elements are equally probable, the expected NPV is positive and its value is 30.

For this reason, as soon as the investment has been made in the project, the equity increases by the same amount and the company's balance sheet becomes:

Assets		Liabilities	
Former assets	1,000	Equity	1,030
Project assets	530	Funding 1	500

Balance sheet (b)

The company can fund this investment in two ways.

The first means of financing considered is to issue 400 convertibles at 1.25, thus covering the financing requirements of 500. Each of these bonds can be converted into one share at the end of the first period.

The second possible means of financing is to issue bonds with warrants. Again, 400 bonds would be issued, each carrying a warrant giving the right to buy shares at 1.25 at the end of the first period.

We can see that both issues are similar in terms of the funding collected and the option: in both cases, 500 is collected immediately and 400 new shares will be created if the option is exercised, which presupposes a minimum growth in the stock price that will be identical in each case.

Finally, we know that at the end of the first period, the company will have a new investment opportunity. This will require a payment of 1,200 and may turn out to be profitable or not. Effectively, the NPV of the project's economic flows may be 200 or -200, and both possibilities are equally probable. The uncertainty may be removed at the start of the second period, or not. In the latter case, we assume the managers will invest and the uncertainty will only be removed at the end of the second period. Should the NPV be positive, the flow generated by this project at the end of the second period will be 1,704, as:

$$-1,220 + \frac{1,704}{1.20} = 200$$

We consider that the existence of this option to invest is common knowledge from the start of the first period, but as the expected NPV for this option is zero, it has no influence on the balance sheet at this date.

At the end of the first period, and before the new investment, the company receives the cash flow from the first project, and the assets associated with it disappear. The cash flow is considered to be available to finance a new investment. As we have already seen, this flow can be either 720 or 552.

3.2 Second period

Three cases can be identified:

- the option to invest (the second project) at the start of the second period turns out not to be profitable;
- the option to invest (the second project) at the start of the second period turns out to be profitable;

- the uncertainty remains with regard to the second project until the end of the second period.

3.2.1 The second project is not profitable

Let us first suppose that the NPV for the second project, as revealed at the start of the second period, is negative.

We have already seen that at the end of the first period, the company has 720 or 552 available, and the managers are encouraged to invest this available cash in any projects, including some with a negative NPV, as they take advantage of the size of the company. However, this problem of over-investment can be partially avoided if the debt associated with the funding for project 1, of a value of 500, should immediately be repaid: the over-investment is thus limited. This problem of over-investment will be discussed in section 4.

In this case, the options attached to the convertibles and bonds with warrants are never exercised. Indeed, in the "best" situations (that is, even if there is no investment with a negative NPV and project 1 is profitable), the value of the equity is 1,220, that is, the present value of the former assets, increased by 220 ($720 - 500$) provided by project 1. The share price is thus 1.22 per share.

The strike price of the option is 1.25, it is still out of the money. In this case, we thus limit any over-investing as the company's debt remains in all cases. There is no fundamental difference (for the issuer) under these circumstances between convertibles and bonds with warrants.

3.2.2 The second project is profitable

Let us now suppose that the second project does turn out to be profitable (its NPV will be positive). In this case, the company invests and the available cash left over from the first project will contribute to its funding.

The assets from project 2 have a present value of 1,420, that is, the investment amount (1,220) plus the NPV (200).

In a worst case scenario, project 1 was not profitable and only generated a flow of 552. This sum is reinvested in project 2. For this reason, the funding needed is 668, or 1,220 - 552. The company's equity is thus worth 1,252. Before investment, its value was effectively the same as that of the former assets, 1,000, increased by 52 (552 - 500). Project 2 further increases it by its NPV, or 200.

The balance sheet at the start of the second period *before the influence of the funding* thus becomes:

Assets		Liabilities	
Former assets	1,000	Equity	1,252
Project 2	1,420	Funding 1	500
		Funding required	668

Balance sheet (c)

In the most favorable case, project 1 results in a flow of 720, and the balance sheet is:

Assets		Liabilities	
Former assets	1,000	Equity	1,420
Project 2	1,420	Funding 1	500
		Funding needed	500

Balance sheet (d)

These balance sheets highlight the need for the company to obtain, depending on the circumstances, 500 or 668 in additional funds. However, in the case of either convertibles or bonds with warrants, the option on the company's equity is exercised as the value of the equity is, in the worst case scenario, 1,252 and thus the share price is set at 1.252. Exercising the option creates 400 new shares and increases the value of the equity by 500, or 400×1.25. The minimum share price after exercising the option (dilution included) is thus set at:

$$\frac{1,252 + (400 \times 1.25)}{1000 + 400} = 1.25$$

In the most favorable case (project 1 was profitable), this share price is:

$$\frac{1420 + (400 \times 1.25)}{1000 + 400} = 1.37$$

It is always in the interests of the subscribers of the option to exercise it.

In the case where the company is financed by means of convertibles at the start of period 1, funding 1 in balance sheet (b) is the debt corresponding to the convertible. If the company invests at the start of period 2, because the opportunity to do so is profitable, the conversion will result in this debt being transformed into equity, although it will not provide the company with new funds. The balance sheet at the start of the second period will then be (in the least favorable scenario, that is, when the first investment has produced only a flow of 552):

Assets		Liabilities	
Former assets	1,000	Equity	1,752
Project 2	1,420	Funding 1 (converted bond)	0
		Funding needed	668

Balance sheet (e)

The need for additional funding of 668 persists. In the favorable scenario, this need for additional funding is still there, but is only 500.

If the company is financed by means of bonds with warrants at the start of period 1, funding 1 on balance sheet (b) is also the debt corresponding to the bond. But exercising the option (exercising the warrants) at the start of period 2 does not bring about the bond's disappearance. It allows the company to have an additional amount of equity at its disposal, to the tune of a sum corresponding to the price obtained from exercising the warrants, here 500, or 400×1.25.

The balance sheet at the start of the second period, still in the unfavorable case where project 1 produced a flow of only 552, is as follows:

Assets		Liabilities	
Former assets	1,000	Equity	1,752
Project 2	1,420	Funding 1 (bond)	500
		Funding needed	168

Balance sheet (f)

The additional funding needed is greatly reduced, resulting in savings in terms of issuing costs. In the most favorable case (that is, when the first project results in a flow of 700), the need for additional funding can even disappear completely.

3.2.3 The uncertainty regarding project 2 is maintained

If the uncertainty is maintained on the second project, the call options on the equity will never be exercised. Effectively, in the best case scenario (project 1 was profitable, the cash flow at the end of period 1 is 720), the equity value is 1,220, or the value of the former assets, increased by 220 for the first project (see above), and by 0 for the second project (its expected NPV). The share price prior to dilution is 1.22. If it were exercised (dilution included), this price would be:

$$\frac{1,220 + (400 \times 1.25)}{1000 + 400} = 1.23$$

The option is out of the money and will never be exercised.

4 Discussion

On the basis of the developments and illustrations in the previous sections, it is now possible to analyze the circumstances in which a company may choose to issue bonds with warrants rather than convertibles. These circumstances concern:

- the characteristics of the governance of the company (the relations between shareholders and managers, and the incitation to over-invest of the latter);
- the characteristics of the corporate environment (structure of the issuing costs, contracts available);
- the characteristics of the company's projects (NPV, investment horizons, the size of the project to be funded, the nature of the uncertainty and the date at which this uncertainty will disappear);
- the characteristics of the funding proposed (maturity, type of option on the equity (American or European), presence of a *call provision* on the bond).

4.1 Environment and governance of the company

Both these elements are dealt with together as it is their simultaneous presence that is required in order to reach the conclusions of the previous section. Indeed, the problem with sequential financing for investment projects could be resolved in a simple manner through the repeated use of external funding, or a single call at the start of the first period for a sum large enough to finance both projects if necessary. However, the first possibility is rendered sub-optimal because of the structure of the issuing costs, which are characterized by economies of scale, and the second possibility is also rendered sub-optimal by the impossibility of controlling the tendency of managers to over-invest otherwise than through the design of financial contracts (securities) issued.

It is also possible to show that this structure for the issuing costs plays a part in the advantage of bonds with warrants, as can be seen in the Tables in section 1. To this, we can add the influence of the issuing costs for each time external financing is required[5]. The costs are composed of a fixed part F and a variable part, which is a fraction f of the funding obtained. Thus, for a funding of E, the total costs will be $F + fE$. It can be noted that these costs make it possible to make the economies of scale mentioned above: it will always be less costly to issue E in a single operation, than $E/2$ twice successively. The costs incurred in the case of two successive issuing operations of $E/2$ would be:

$$\left[F + f\frac{E}{2}\right] \times 2 = 2F + fE$$

And we always have:

$$F + fE < 2F + fE$$

[5]The costs that may possibly be incurred by the company during the conversion of bonds or the exercise of warrants are not issuing costs and are considered to be negligible. In addition, introducing them into the model would have no impact on our conclusions.

Let us suppose without loss of generalities, that the investment needed for project 1 is $I_1 = E - (F + fe)$, which is the same as saying that the issuer chooses the size of the issue in relation to his investment projects. In addition, and still without loss of generality, the investment needed for project 2 is $I_2 = E(1 - c')$.

Tables 4 and 5 summarize Table 3 from section 1 (the case of exercising the call option), adding the impact of the issuing costs. They show clearly that it is better to issue a bond with warrant at the start of period 1 to limit the issuing costs later, still in the hypothesis in which both projects are profitable. This leads us to examine the influence of the characteristics of the projects on the conclusions obtained from the model.

Date	0	1	2
Bond with warrants	$E - (F + fE)$	$E(1 - c')$	$-E(1 + c')$
Investment	$E - (F + fE)$	$E(1 - c')$	0
Funding needed	0	0	-
Associated costs	0	0	-

Table 4: Both investment projects have a positive NPV, case of the bond with warrants

Date	0	1	2
Convertibles	E	$-Ec'$	0
Investment	$E - (F + fE)$	$E(1 - c')$	0
Funding needed	0	E	-
Associated costs	0	$F + fE$	-

Table 5: Both investment projects have a positive NPV, case of the convertibles

4.2 Characteristics of the projects

4.2.1 Performances of the projects

With regard to the projects, the example shows that the problem of over-investment cannot always be completely avoided in cases where the NPV for the second project turns out to be negative at the start of the second period. This problem is only eliminated or limited in cases where the bond must be repaid at the end of the first period. This thus brings us back to the hypotheses proposed by Mayers[6], and the advantage of the bonds with warrants disappears — which is normal:

[6] In the Mayers model, convertible bonds arrive at maturity at the same time as the conversion option, at the end of the first period.

there is no advantage to obtaining cash over a period of time if this cash cannot be invested in a positive NPV. As a result, the bond part of a bond with warrants will only have a maturity that falls after the maturity of the subscription warrants if the issuer is confident of the potential of the portfolio of options to be invested at his disposal. *A contrario*, if the life of the bond is limited to that of the option that accompanies it, the issuer is "prudent": he only has limited confidence in the future investment possibilities at his disposal. But it is not in the issuer's interests to issue bonds whose warrants reach maturity at the same time as the debt: this security is similar to a convertible, and the latter has additional advantages in such cases, such as the possibility of adding a call provision (see below, section 4.3).

The two following propositions can be made as a result of these observations:

Proposition 1 The warrants attached to the bonds with warrants have a shorter maturity than the bonds with which they have been issued.

Proposition 2 The conversion options for the convertibles have the same maturity than the bond with which they have been issued.

4.2.2 Uncertainty

The uncertainty in the investment projects is relatively low in the model proposed: the NPV for a project is always revealed at the time of the investment. Effectively, the NPV for project 1 is known at the start of period 1, and that of project 2 at the start of project 2. There is only uncertainty at the start of period 1 concerning project 2 (which is only an investment option at that point).

It is possible to discuss the conclusions obtained from the model if we increase the level of uncertainty by revealing the value of the projects only once they have been completed (*ex-post*), and this:

- both for the managers and for those who provide resources;
- or only for those who provide resources. In the latter case, the managers have privileged information regarding the nature of the projects. We have thus introduced an information asymmetry between the managers and the resource providers.

Adding uncertainty *ex-ante* to the first project, at least for the resource providers, raises the question of subscription to the financing. Effectively, the option on the equity that is offered to subscribers will only be exercised if the first project turns

out to have a positive or zero NPV. The subscription then depends on the profile of the possible subscriber. We could thus think that the subscriber will only subscribe if he expects a yield that is at least equal to that of the company's ordinary debt. We can also, however, imagine that he will be more demanding and that the minimum yield must be that of the equity, or a yield calculated on the basis of the cost of the bond part and the optional part of the security issued. In the first case, the uncertainty in the first project does not have any effect on the subscription of convertibles or bonds with warrants. On the contrary, if the subscriber demands *ex-ante* a yield that is comparable to that of the equity of the company, he will only subscribe if he thinks that project 1 will have a positive NPV. If the managers are informed of this (the uncertainty only concerns the resource providers), they can use the issue itself as a signal of the value of the project (as, for example, in Brennan [1]).

Once the issue has occurred, it is in the company's interests that the warrants be exercised at the end of the first period, if there is a need for funding for project 2, and thus if the investment option turns out to be profitable. But for the warrants to be exercised, they must be in the money, which supposes that the first project has had a positive NPV.

It is thus when both projects turn out to be profitable that bonds with warrants are more interesting than convertibles: the additional cash flow obtained from exercising the warrants (which occurs because the first project has had a positive NPV and the warrants are in the money) is invested with positive NPV in the second project. Thus, companies whose projects are highly correlated will have the greatest tendency to use bonds with warrants to obtain financing. The projects are correlated if the companies have activities that involve little diversification and higher volatility (risk). It is thus possible to write:

Proposition 3 Companies that issue bonds with warrants have activities that are, in relative terms, less diversified than companies that issue convertibles (and thus the risk for their assets is higher).

4.3 Characteristics of the options

Another restrictive hypothesis for the models is the timetable for the options (option to invest and option on the equity). In the model, these options are European and are supposed to reach their maturity at the same time, at the end of the first period. What would happen if the options were American (as is generally the case) and were thus out of synch? It should be specified that this time-lag does not apply to the total duration of the period: the case where period 2 disappears or when the maturity for the debt and the option coincide has already been discussed above. Effectively, in the latter case, the cash flow advantage described in section 1 disappears. Instead, we consider that the possible period for exercising can be divided

into shorter sub-periods, and we shall now examine two possible cases: the option to invest is due to reach maturity before the option on the equity is exercised, or the contrary.

The option to invest can mature before the option on the equity is exercised. More specifically, the option to invest arrives at maturity and turns out to be profitable, yet this does not trigger the exercise of the option on the equity. One of the reasons generally invoked for such behavior by subscribers is a form of speculation based on uncertainty: waiting seems to them to be more promising than exercising their option immediately (whereas the option is in the money). The company must then finance the option to invest to the best of its ability, and, still taking into account the observations made in the previous section, the best choice would be for the subscribers to exercise their option. Mayers thus justifies the presence of a *call provision* on the convertibles, which makes it possible to force the conversion as soon as the option is in the money. This possibility does not exist for the bonds with warrants: it is very difficult, if not impossible, to force the exercise of the warrants, even if they are in the money[7]. The issuer thus finds himself in a situation where he will never obtain the funding he requires at the time of the investment: he can force the conversion if he has issued convertibles, but that will not generate an immediate cash flow and there is nothing he can do about it (other than wait until the warrants are exercised) if he has issued bonds with warrants.

In cases where the option on equity is exercised before the maturity of the option to invest, the managers of a company that has issued bonds with warrants have at their disposition a certain cash flow before they have the opportunity to invest it in a positive NPV project. They will thus invest it in a negative NPV project. In this context, it would be better for the company to have issued convertibles: exercising the call option on equity does not produce extra cash, and over-investment is avoided. However, this example can easily be avoided by the issuer: the fact that the call option which accompanies the bond is American does not prevent:

- either setting as early a date as possible for the exercise;
- or varying the exercise conditions in relation to the period.

Given the remarks above, this issuer should try above all to avoid having an option to invest that matures before the option on equity as this case is never favorable, and will always oblige him to seek additional funding. He will thus issue American options and seek relatively short maturitys. However, in this case, and in order to protect himself from possible over-investment, he will rather choose convertibles which do not produce new cash if converted.

[7] One possibility is to issue warrants at an exercise price that is both variable and dependent on the company's performance, but this solution is not used very often because of its complexity.

Proposition 4 Issuers prefer convertible bonds for issues with short maturities. For this reason, convertibles should have shorter maturities than bonds with warrants.

Conclusion

Issuing debt accompanied by a call option on the equity of the issuer has been the subject of a considerable amount of literature. However, this literature is less abundant when it is a question of separating convertibles from bonds with warrants: both these securities are frequently seen to be interchangeable.

In practice, this is not the case. The main reason is that, in the case of bonds with warrants, there is a total separation of the call option from its bond support, which occurs right after the issue. The main consequence of this characteristic is a means of operating that is very different from that of convertibles when the options are exercised. In the case of bonds with warrants, the subscriber pays a strike price, which produces a cash flow for the issuer. This flow does not exist in the case of convertibles.

The advantage of the cash flow in bonds with warrants only occurs under very particular conditions, as shown in section 2. However, it is reasonable to believe that these circumstances are not exceptional and can be encountered in practice. They have been illustrated by a simple example following on from the formal description of the model.

The discussion (section 4) of these circumstances makes it possible to a certain extent to make propositions concerning the reasons behind the choices made by issuers. These propositions are formulated in such a way as to produce testable hypotheses.

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