COLLABORATIVE FILTERING ALGORITHM IMPLEMENTATION

Gradient Descent with Probabilistic Assumptions

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Alternating Least Squares

GROUP 7

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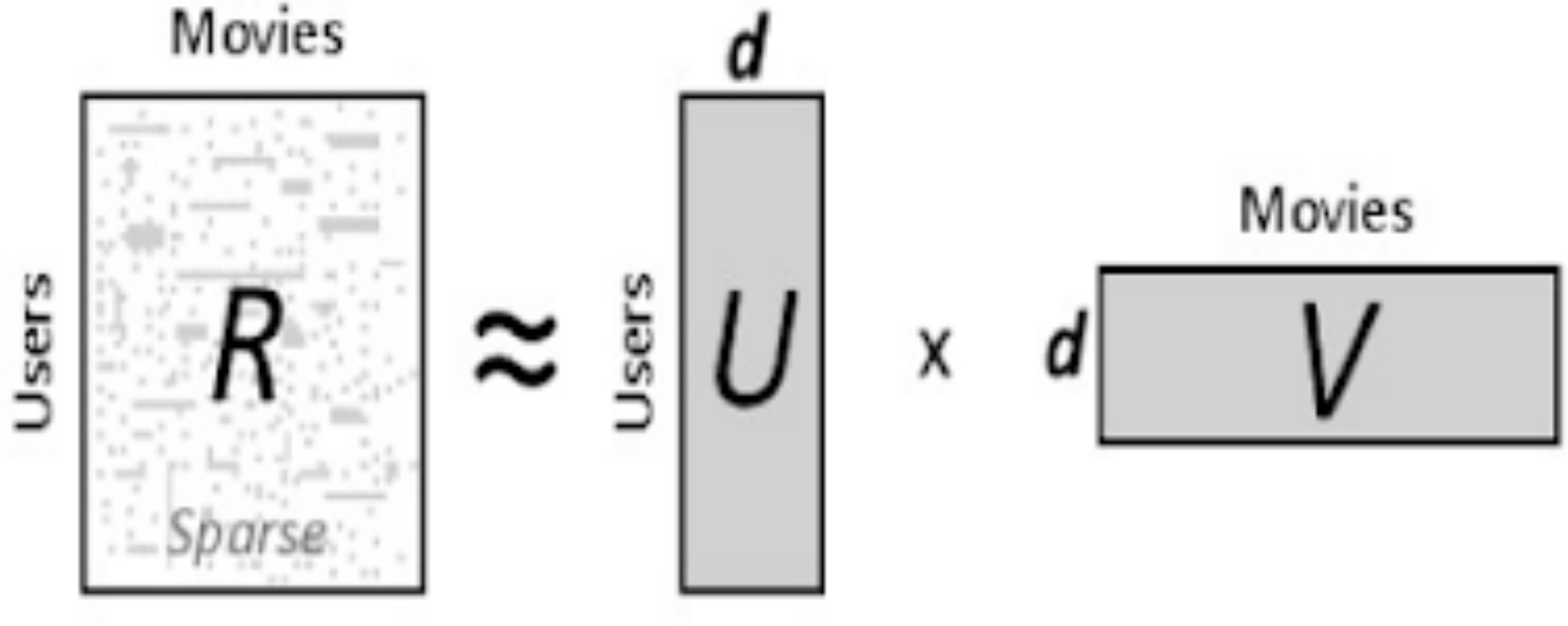
COLLABORATIVE FILTERING

- **FILTERING: Making automatic predictions**

- **COLLABORATING: Collecting preference information from many users**

MATRIX FACTORIZATION TECHNIQUES

$$R_{ij} = Q_j^T P_i$$



OBJECTIVE

- **Gradient Descent with Probabilistic Assumption**
  - **Probabilistic Matrix Factorization**
- **Alternating Least Square**
- **Post-Processing:**
  - **SVD with KNN**

PROBABILISTIC MATRIX FACTORIZATION

We have M movies and N users, and denote Q and P as latent movie and user feature matrices. Adopt a probabilistic linear model with Gaussian observation noise, we define the conditional distribution over the observed ratings:

$$p(R|P,Q,\sigma^2) = \prod_{i=1}^{N}\prod_{j=1}^{M}[\mathcal{N}(R_{ij}|Q_i^T P_j, \sigma^2)]^{I_{ij}}$$

with

$$p(P|\sigma_P) = \prod_{j=1}^{M}\mathcal{N}(P_j|0, \sigma_P^2\mathbf{I})$$

$$p(Q|\sigma_Q) = \prod_{i=1}^{N}\mathcal{N}(Q_i|0, \sigma_Q^2\mathbf{I})$$

I_{ij} is the indicator function that is equal to 1 if user i rated movie j and 0 otherwise

PROBABILISTIC MATRIX FACTORIZATION

Maximizing the log-posterior over movie and user features with hyperparameters is equivalent to minimizing the sum of squared errors objective function with quadratic regularization terms:

$$E = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{M} I_{ij}(R_{ij} - Q_i^T P_j)^2 + \frac{\sigma^2}{2\sigma_Q^2} \sum_{i=1}^{N} ||Q_i||_F^2 + \frac{\sigma^2}{2\sigma_P^2} \sum_{j=1}^{M} ||P_j||_F^2$$

GRADIENT DESCENT WITH PROBABILISTIC ASSUMPTION

We are going to update each iteration with the following:

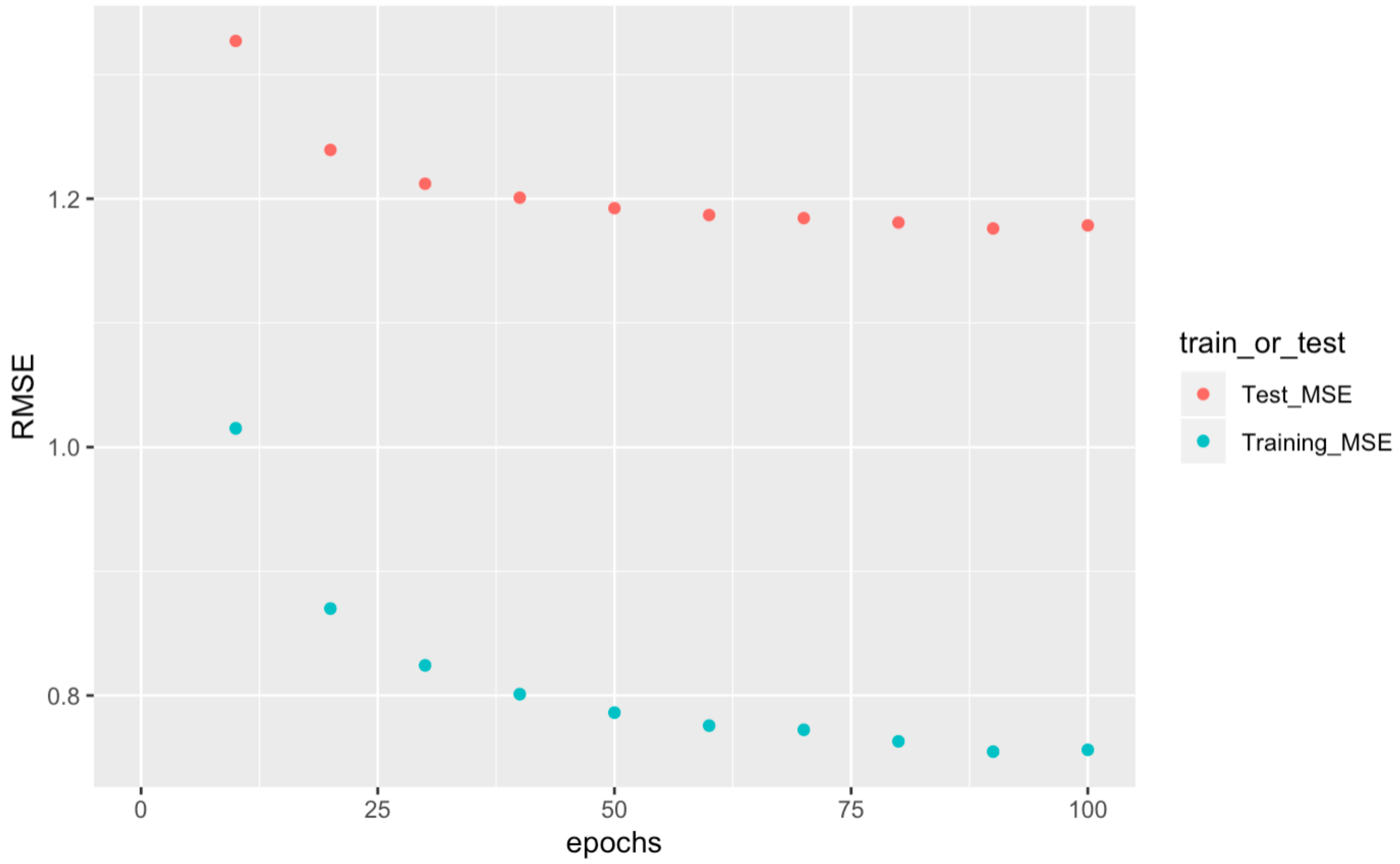
$$P_j^{(t+1)} = P_j^{(t)} + \text{lrate} \sum_{i=1}^{N} I_{ij}(R_{ij} - Q_i^{T(t)} P_j^{(t)}) Q_i^{T(t)} - \frac{\sigma^2}{\sigma_P^2} P_j^{(t)}$$

$$Q_i^{(t+1)} = Q_i^{(t)} + \text{lrate} \sum_{j=1}^{M} I_{ij}(R_{ij} - Q_i^{T(t)} P_j^{(t)}) P_j^{(t)} - \frac{\sigma^2}{\sigma_Q^2} Q_i^{(t)}$$

PROBABILISTIC MATRIX FACTORIZATION

- **Tune parameters through cross-validation**

- **f = 5**
- **sigma_p = 0.5**
- **sigma_q = 0.5**
- **lrate = 0.01**



ALTERNATING LEAST SQUARE

ALS with Tikhonov regulation (penalizes large parameters)

$$\min_{Q,P} f(Q,P) = \sum_{i,j \in K} (R_{ij} - Q_i^T P_j)^2 + \lambda(\sum_i n_{Q_i} ||Q_i||^2 + \sum_j n_{P_j} ||P_j||^2)$$

During each iteration, fix Q to solve P, and fix P to solve Q by:

$$P_i = (Q_{I_i} Q_{I_i}^T + \lambda n_{P_i} E)^{-1} Q_{I_i} R^T(i, I_i)$$

$$Q_j = (P_{I_j} P_{I_j}^T + \lambda n_{Q_j E})^{-1} P_{I_j} R(I_j, j)$$

n_{P_i} and n_{Q_j} are the numbers of ratings of user i and movie j respectively

E is the $n_f \times n_f$ identity matrix

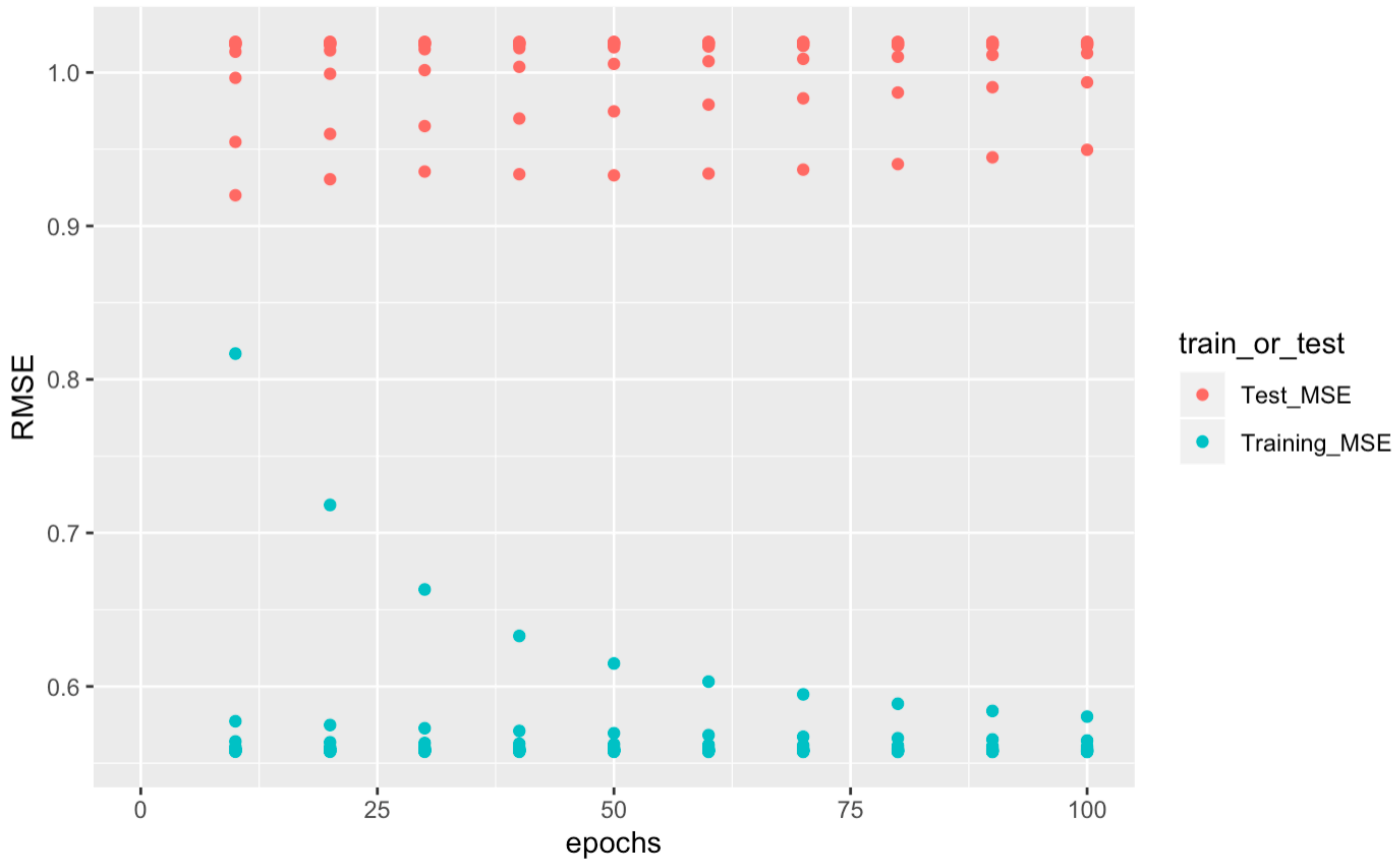
Q_{I_i} is the sub-matrix of M where columns $j \in I_i$ are selected

$R(i, I_i)$ is the row vector where columns $j \in I_i$ of the i-th row of R is taken

$R(I_j, j)$ is the column vector where rows $i \in I_i$ of the j-th column of R is taken

ALTERNATING LEAST SQUARE

- **Tune parameters through cross-validation**
- f = 5
- lambda = .1



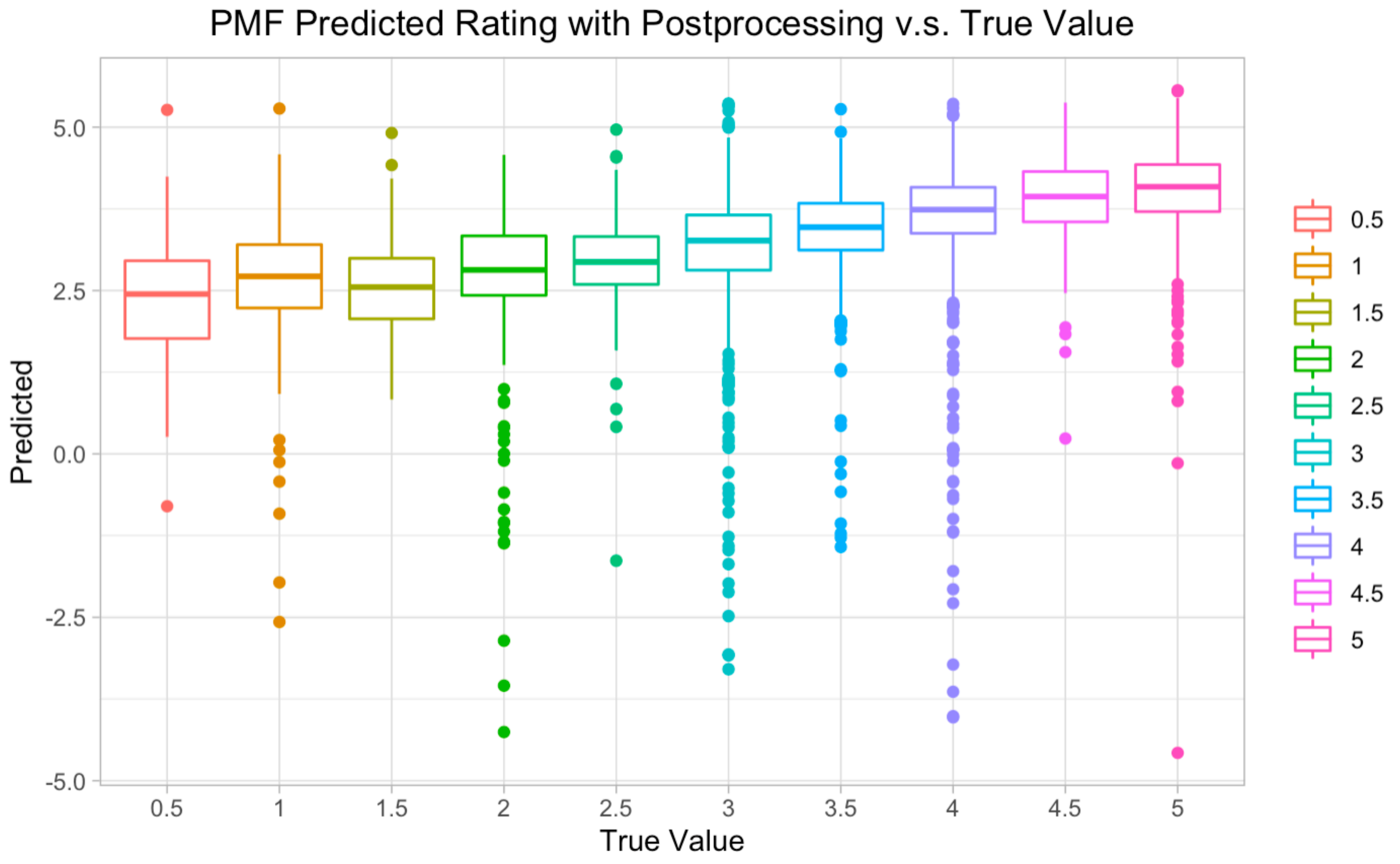
POSTPROCESSING

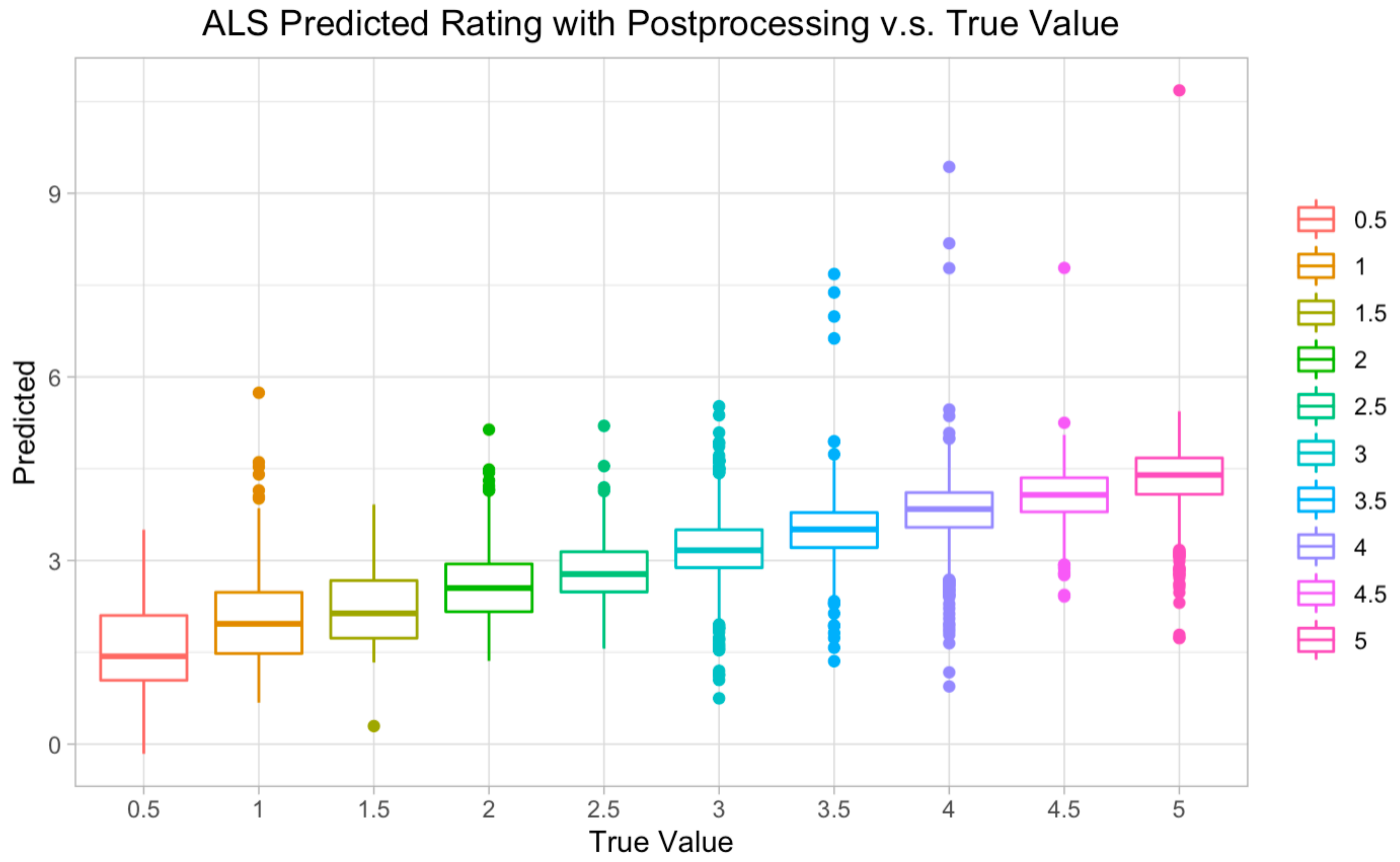
SVD with KNN

Define similarity between movies j and j_2 as cosine similarity between vectors v_j and v_{j_2} as

$$s(v_j, v_{j_2}) = \frac{v_j^T v_{j_2}}{||v_j|| ||v_{j_2}||}$$

POSTPROCESSING RESULTS





RESULTS

	Gradient Descent with Probabilistic Assumptions	Alternating Least Squares
RMSE before postprocessing	Train 0.756 Test 1.179	Train 0.557 Test 1.017
RMSE after postprocessing	Test 0.985	Test 0.664

THANK YOU