Algorithms and Data structures

2024 2025

Lecture 2

ملاحظة

جميع السلايدات مأخوذة من المرجع رقم 1

1.1 Algorithms

Informally, an ***algorithm*** is any well-defined computational procedure that takes some value, or set of values, as ***input*** and produces some value, or set of values, as ***output*** in a finite amount of time. An algorithm is thus a sequence of computational steps that transform the input into the output.

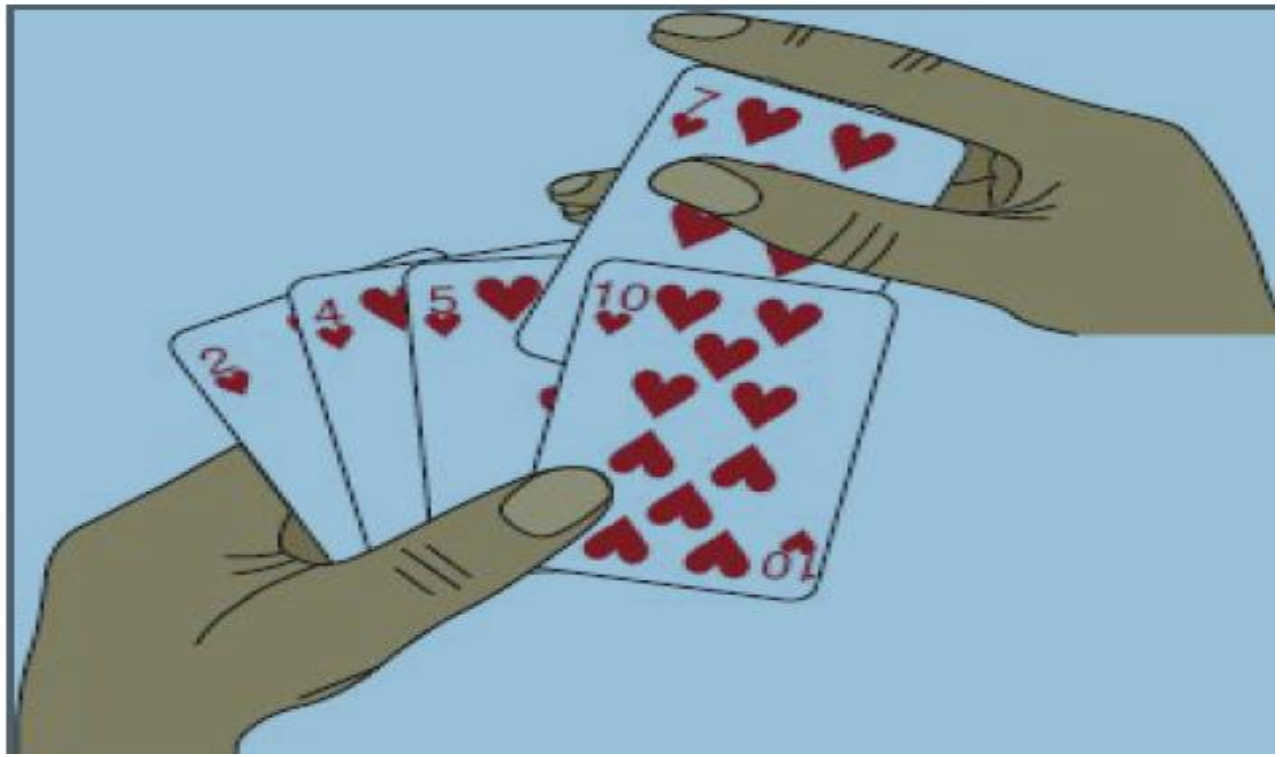
2.1 Insertion sort

Our first algorithm, insertion sort, solves the ***sorting problem*** introduced in Chapter 1:

Input: A sequence of n numbers $\langle a_1, a_2, \ldots, a_n \rangle$.

Output: A permutation (reordering) $\langle a'_1, a'_2, \ldots, a'_n \rangle$ of the input sequence such that $a'_1 \leq a'_2 \leq \cdots \leq a'_n$.

Insertion Sort



pseudocode

```
INSERTION-SORT(A, n)
1 for i = 2 to n
2     key = A[i]
3     // Insert A[i] into the sorted subarray A[1 : i – 1].
4     j = i – 1
      while j > 0 and A[j] > key
6         A[j + 1] = A[j]
7         j = j – 1
8     A[j + 1] = key
```

The pseudocode for insertion sort is given as the procedure INSERTION-SORT on the facing page. It takes two parameters: an array A containing the values to be sorted and the number n of values of sort. The values occupy positions $A[1]$ through $A[n]$ of the array, which we denote by $A[1 : n]$. When the INSERTION-SORT procedure is finished, array $A[1 : n]$ contains the original values, but in sorted order.

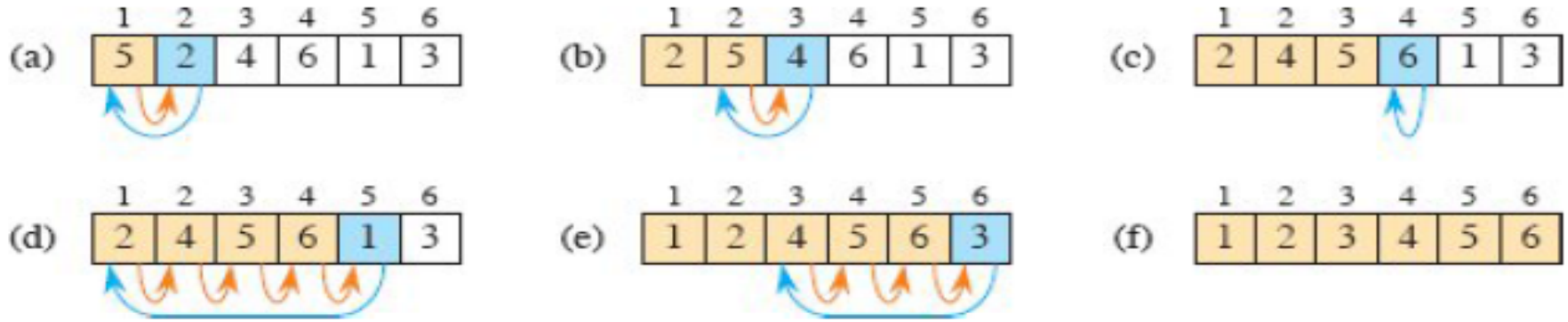


Figure 2.2 The operation of INSERTION-SORT(A, n), where A initially contains the sequence $\langle 5, 2, 4, 6, 1, 3 \rangle$ and $n = 6$. Array indices appear above the rectangles, and values stored in the array positions appear within the rectangles. **(a)–(e)** The iterations of the **for** loop of lines 1–8. In each iteration, the blue rectangle holds the key taken from $A[i]$, which is compared with the values in tan rectangles to its left in the test of line 5. Orange arrows show array values moved one position to the right in line 6, and blue arrows indicate where the key moves to in line 8. **(f)** The final sorted array.

correctness of insertion sort

loop invariant:

> At the start of each iteration of the **for** loop of lines 1–8, the subarray $A[1 : i - 1]$ consists of the elements originally in $A[1 : i - 1]$, but in sorted order.

Loop invariants help us understand why an algorithm is correct. When you're using a loop invariant, you need to show three things:

Initialization: It is true prior to the first iteration of the loop.

Maintenance: If it is true before an iteration of the loop, it remains true before the next iteration.

Termination: The loop terminates, and when it terminates, the invariant —usually along with the reason that the loop terminated—gives us a useful property that helps show that the algorithm is correct.

Let's see how these properties hold for insertion sort.

Initialization: We start by showing that the loop invariant holds before the first loop iteration, when $i = 2$.[2] The subarray $A[1 : i - 1]$ consists of just the single element $A[1]$, which is in fact the original element in $A[1]$. Moreover, this subarray is sorted (after all, how could a subarray with just one value not be sorted?), which shows that the loop invariant holds prior to the first iteration of the loop.

Maintenance: Next, we tackle the second property: showing that each iteration maintains the loop invariant. Informally, the body of the **for** loop works by moving the values in $A[i - 1]$, $A[i - 2]$, $A[i - 3]$, and so on by one position to the right until it finds the proper position for $A[i]$

(lines 4–7), at which point it inserts the value of $A[i]$ (line 8). The subarray $A[1 : i]$ then consists of the elements originally in $A[1 : i]$, but in sorted order. ***Incrementing*** i (increasing its value by 1) for the next iteration of the **for** loop then preserves the loop invariant.

A more formal treatment of the second property would require us to state and show a loop invariant for the **while** loop of lines 5–7. Let's not get bogged down in such formalism just yet. Instead, we'll rely on our informal analysis to show that the second property holds for the outer loop.

Termination: Finally, we examine loop termination. The loop variable i starts at 2 and increases by 1 in each iteration. Once i's value exceeds n in line 1, the loop terminates. That is, the loop terminates once i equals $n + 1$. Substituting $n + 1$ for i in the wording of the loop invariant yields that the subarray $A[1 : n]$ consists of the elements originally in $A[1 : n]$, but in sorted order. Hence, the algorithm is correct.

Asymptotic notation

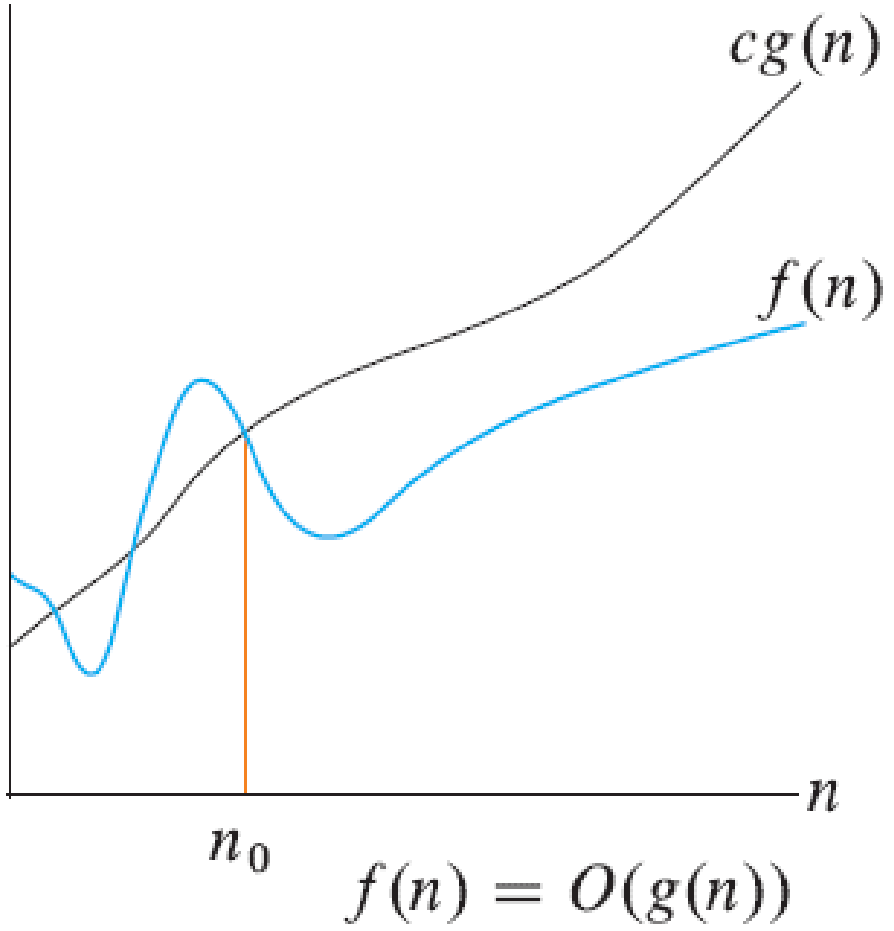
- O-notation describes an asymptotic upper bound.

Here is the formal definition of O-notation. For a given function $g(n)$, we denote by $O(g(n))$ (pronounced "big-oh of g of n" or sometimes just "oh of g of n") the *set of functions*

$$O(g(n)) = \{f(n) : \text{there exist positive constants } c \text{ and } n_0 \text{ such that } 0 \le f(n) \le cg(n) \text{ for all } n \ge n_0\}.^{1}$$

A function $f(n)$ belongs to the set $O(g(n))$ if there exists a positive constant c such that $f(n) \le cg(n)$ for sufficiently large n. Figure 3.2(a) shows the intuition behind O-notation. For all values n at and to the right of n_0, the value of the function $f(n)$ is on or below $cg(n)$.

- O-notation describes an ***asymptotic upper bound***.



The running time of insertionsort is $O(n^2)$. Why?