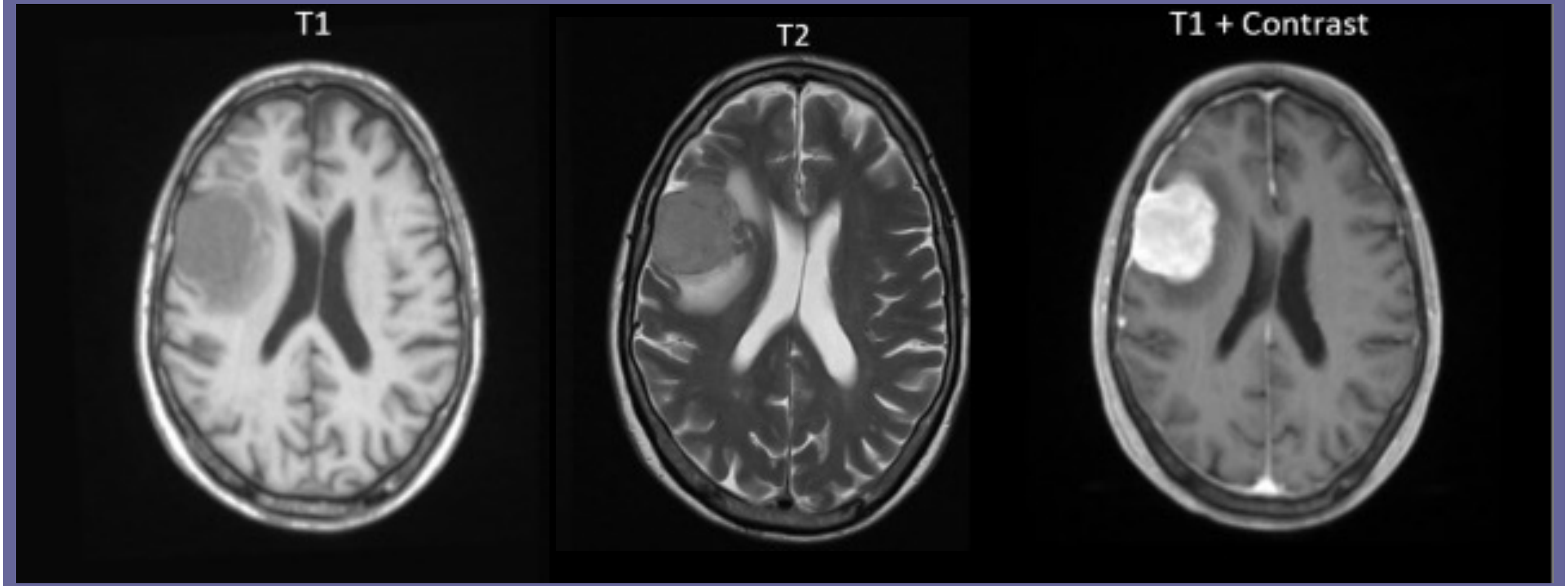
Relaxation time mapping in MRI

Quantitative and Functional Imaging

BME 4420/7450

Fall 2022

How are these different?



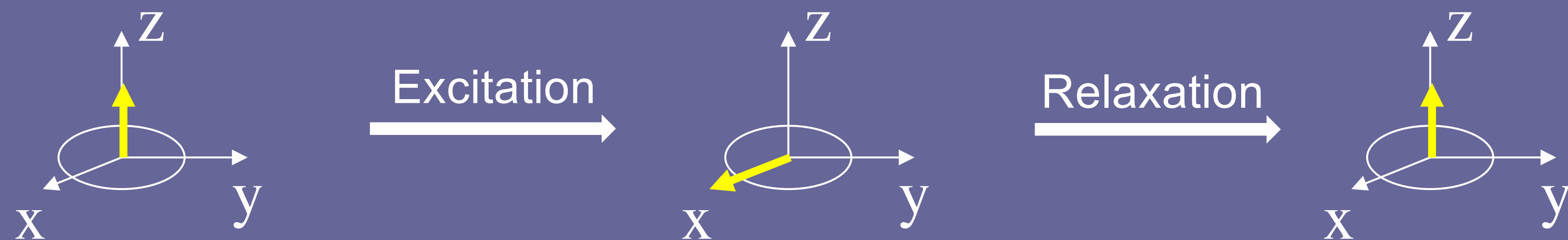
Geekymedics.com

MR relaxation: the most important source of MRI contrast

- Relaxation times vary among tissues
  - Depends on the environment of water—e.g., other molecules neighboring H_2O
  - Basis of contrast in routine diagnostic imaging
  - Varies with physiological and pathological state
- Quantifying relaxation times helps in optimizing
  - Contrast
    - Differentiate healthy tissues/organs
    - Improve detection of diseased tissue
  - Accuracy of calculated quantities (T_2 example)

MRI measures magnetization (M)

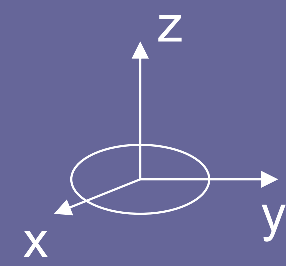
- M is usually in an equilibrium state between
  - Polarizing influence of external field (B_0 || z axis)
  - Randomizing influence of thermal energy
- When the spin system is disturbed, it takes some time to return to equilibrium
  - How is equilibrium restored?
  - How long until equilibrium is restored?
- In equilibrium, $M_z = M_0$, $M_x=M_y=0$.



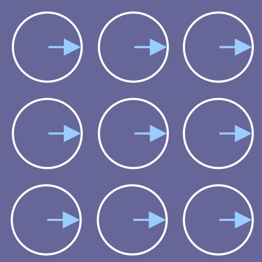
Transverse Relaxation

Transverse relaxation

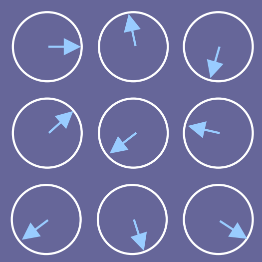
- No net M_x or M_y in equilibrium state



- After spins are put in the xy plane, large transverse magnetization:



- Over time, spin orientations are randomized:



- Occurs at rate $R_2 = 1/T_2$

Quantitative description of transverse relaxation

- Magnitude of magnetization perpendicular to B_0 is M_{xy}
- Bloch's phenomenological equation for M_{xy}:

$$\frac{dM_{xy}}{dt} = -R_2 \cdot M_{xy}$$

Quantitative description of transverse relaxation

- Magnitude of magnetization perpendicular to B_0 is M_{xy}
- Bloch's phenomenological equation for M_{xy}:

$$\frac{dM_{xy}}{dt} = -R_2 \cdot M_{xy}$$

$$\frac{dM_{xy}}{M_{xy}} = -R_2 \cdot dt$$

Quantitative description of transverse relaxation

- Magnitude of magnetization perpendicular to B_0 is M_{xy}
- Bloch's phenomenological equation for M_{xy}:

$$\frac{dM_{xy}}{dt} = -R_2 \cdot M_{xy}$$

$$\frac{dM_{xy}}{M_{xy}} = -R_2 \cdot dt$$

$$ln\left(\frac{M_{xy}(t)}{M_{xy}(0)}\right) = -R_2 \cdot t$$

Quantitative description of transverse relaxation

- Magnitude of magnetization perpendicular to B_0 is M_{xy}
- Bloch's phenomenological equation for M_{xy}:

$$\frac{dM_{xy}}{dt} = -R_2 \cdot M_{xy}$$

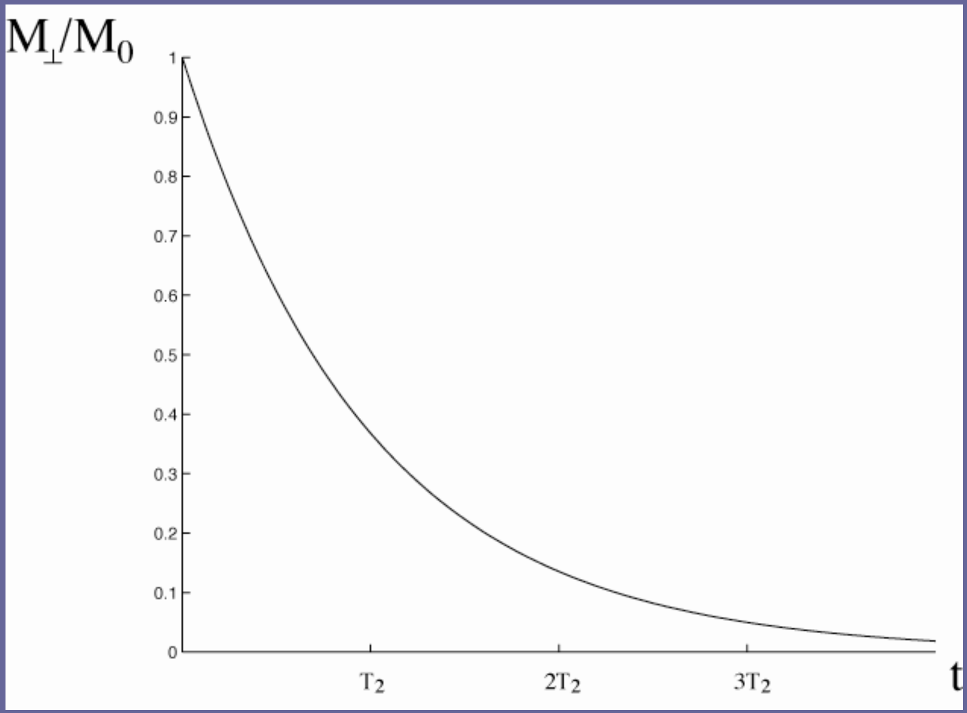
$$\frac{dM_{xy}}{M_{xy}} = -R_2 \cdot dt$$

$$ln\left(\frac{M_{xy}(t)}{M_{xy}(0)}\right) = -R_2 \cdot t$$

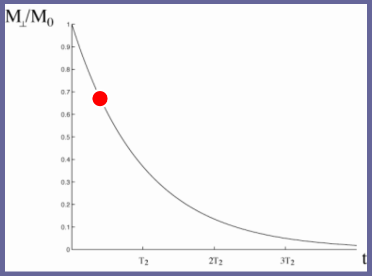
$$\boxed{M_{xy}(t) = M_{xy}(0) \cdot e^{-R_2 t}}$$

Multiple echo time measurements of R_2

- Acquire a series of images, each at a different time after the tipping B_1 pulse
- Signal intensity decays exponentially with rate constant R_2.

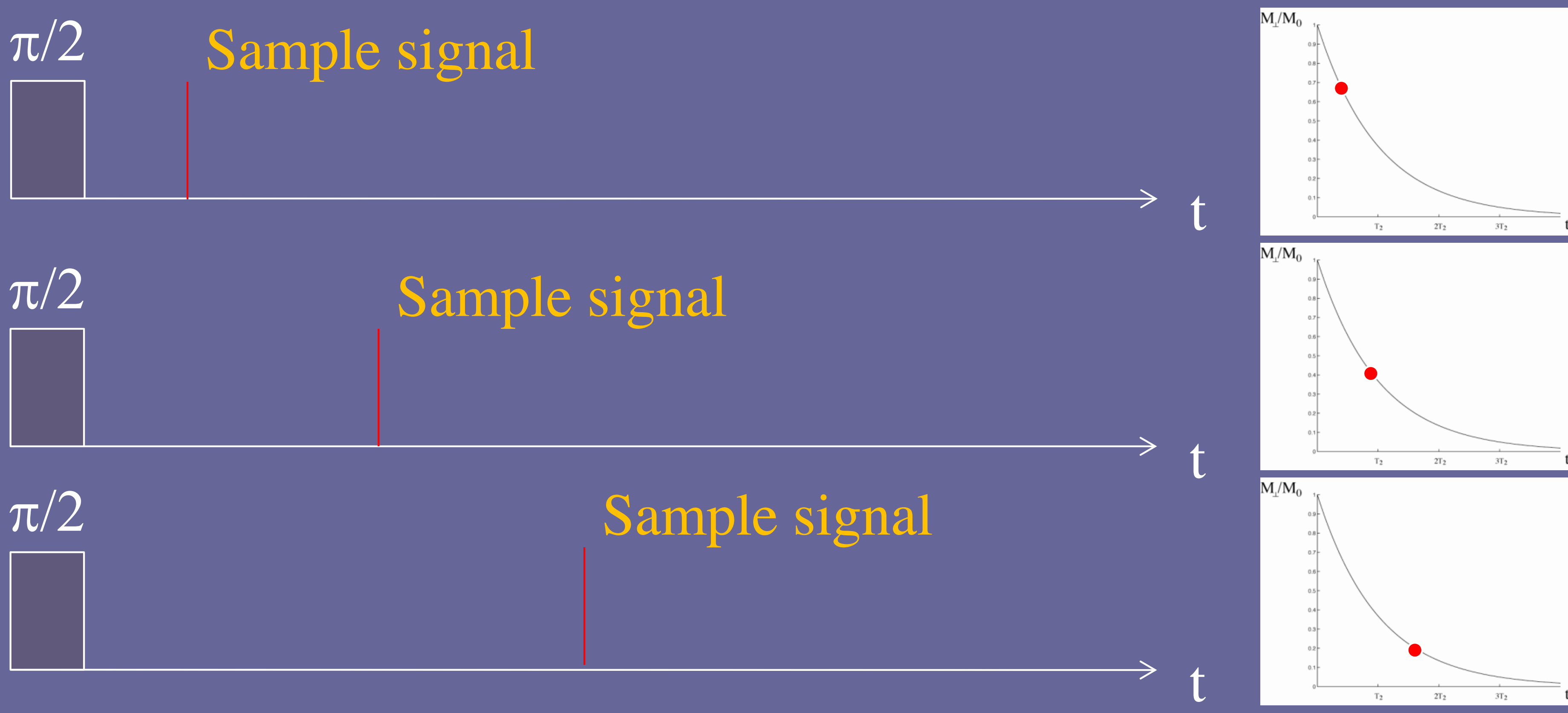


T_2 measurement

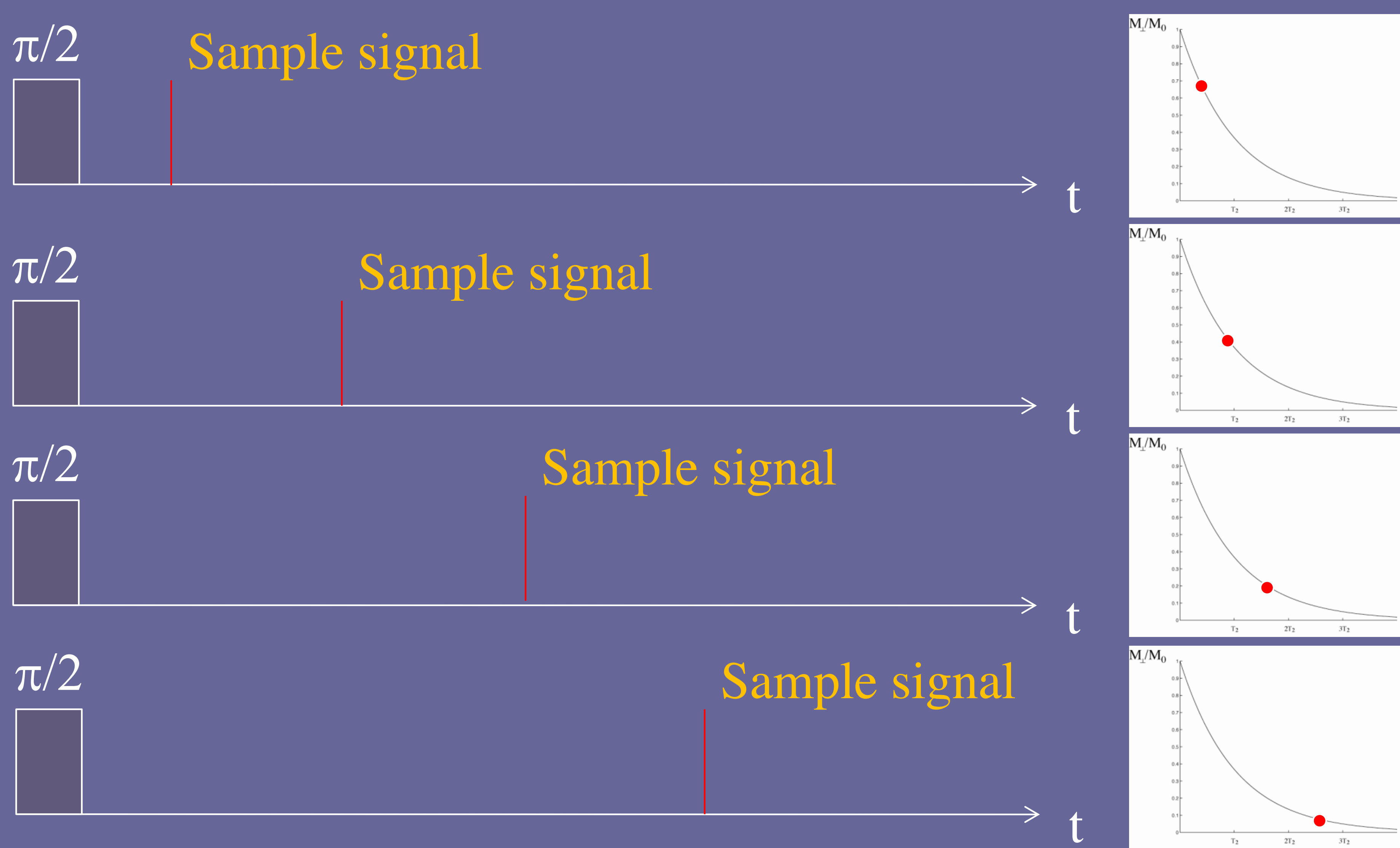


T_2 measurement

T_2 measurement



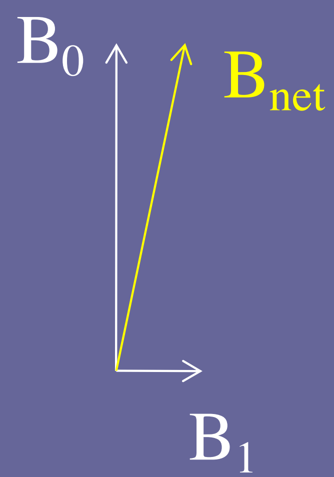
T_2 measurement



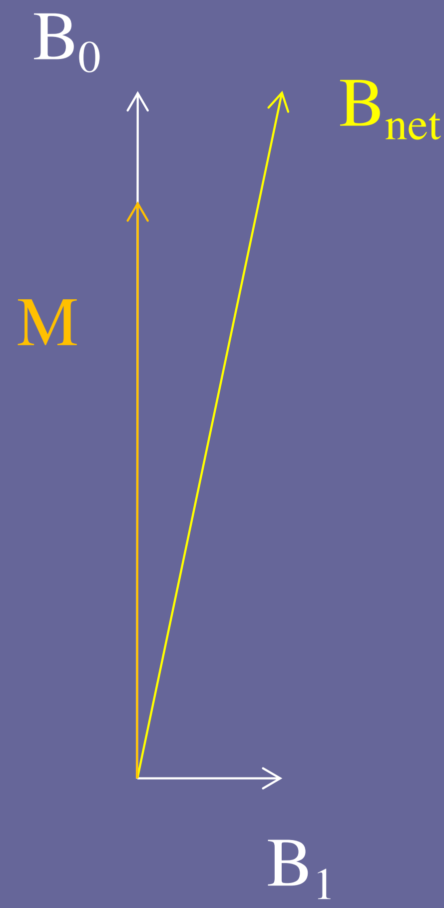
Recall the effects of a transverse field

- B_1 field should be $\perp$ B_0 to tip spins into the transverse plane
- A time-independent B_1 just tilts the polarizing field
  - Does not create large transverse magnetization, $M_\perp$

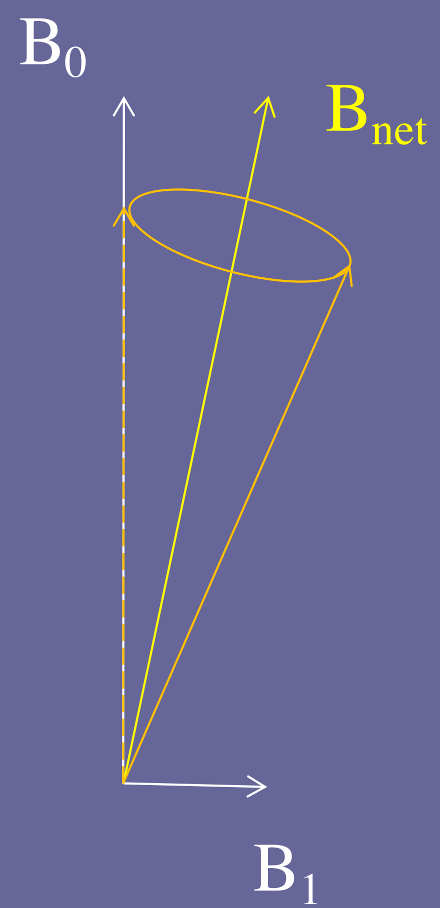
– A time-varying B_1 field at the precession frequency can create large $M_\perp$



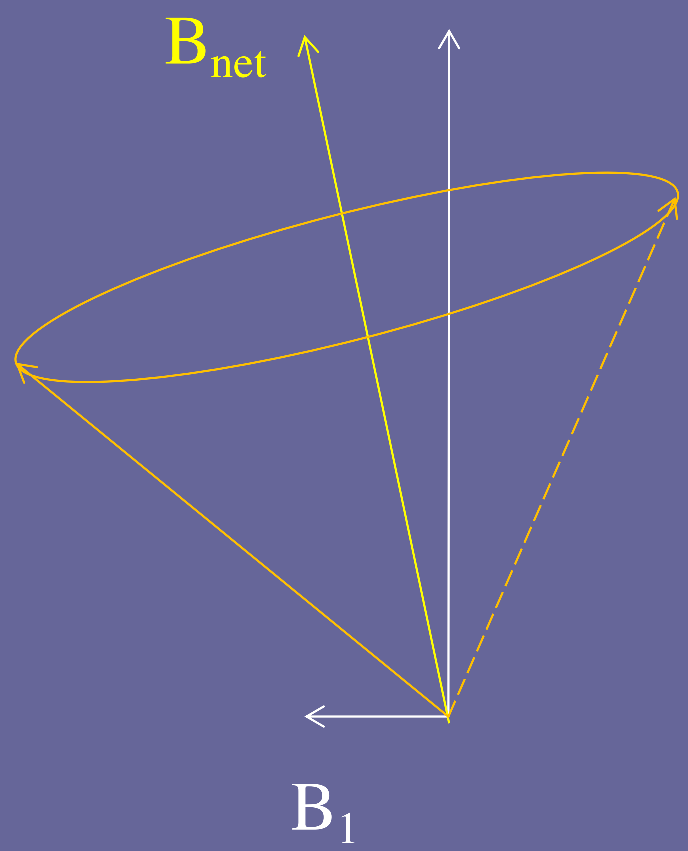
At time = 0



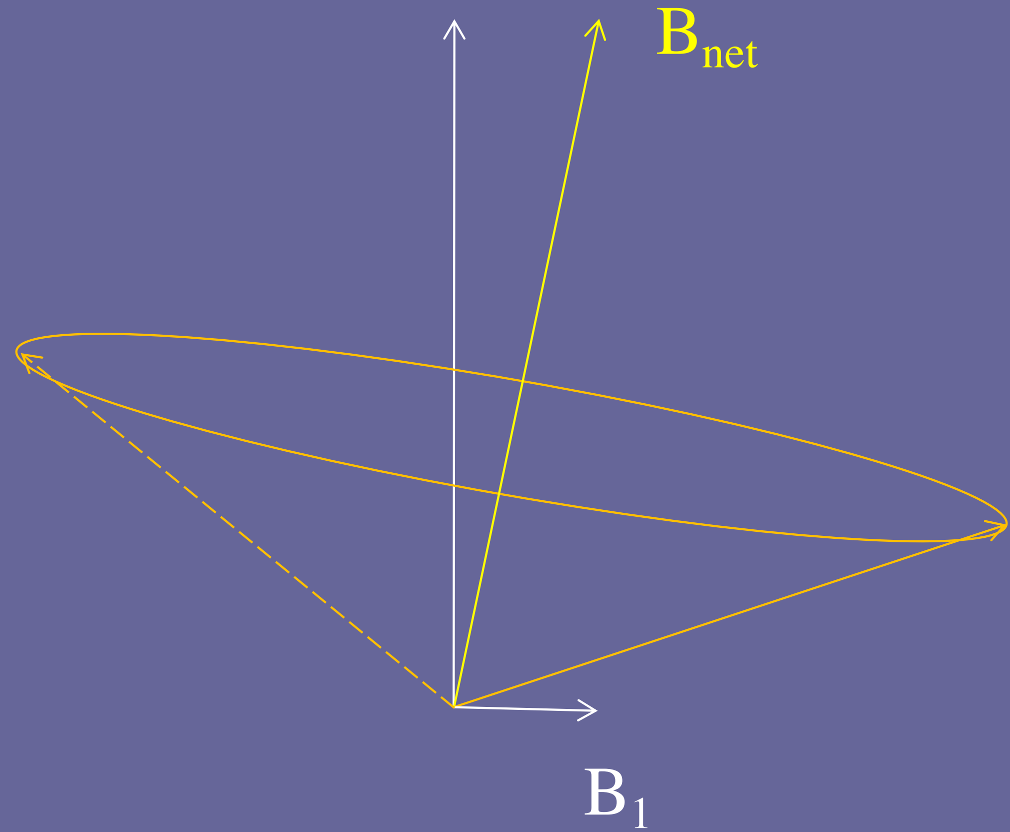
At time = $\tau/2$



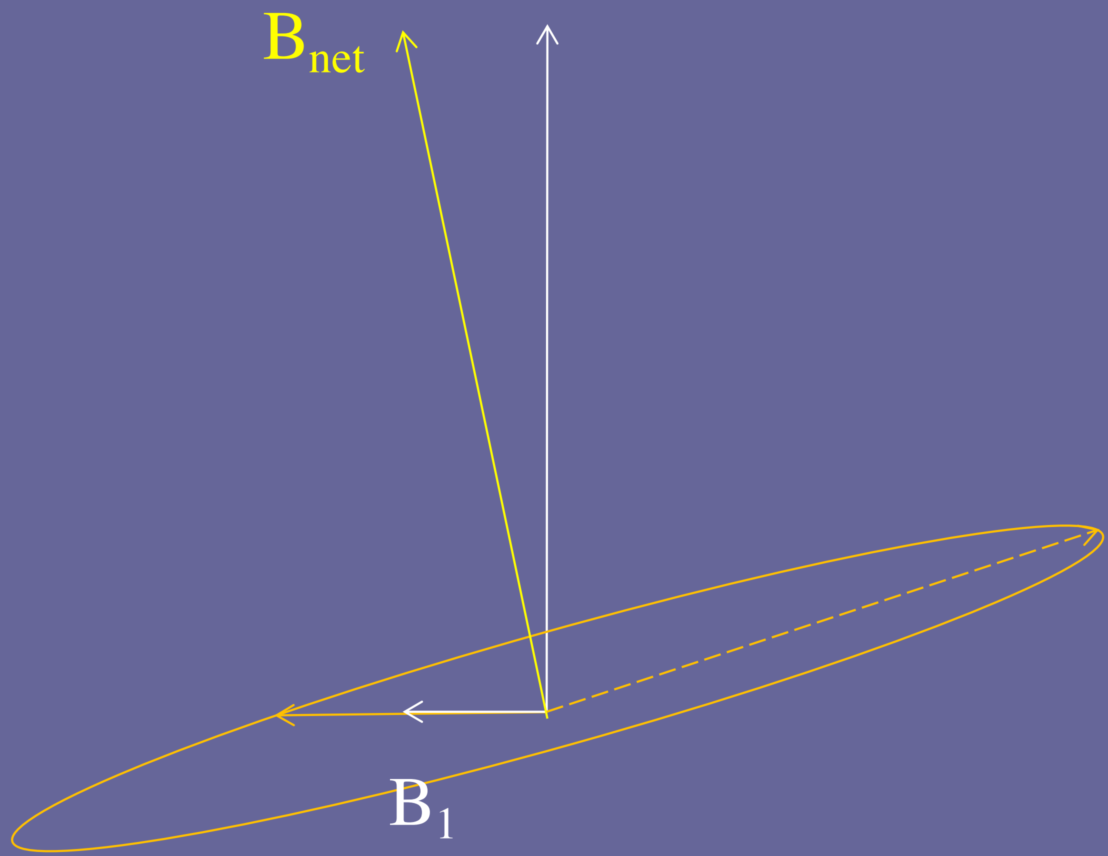
At time = τ



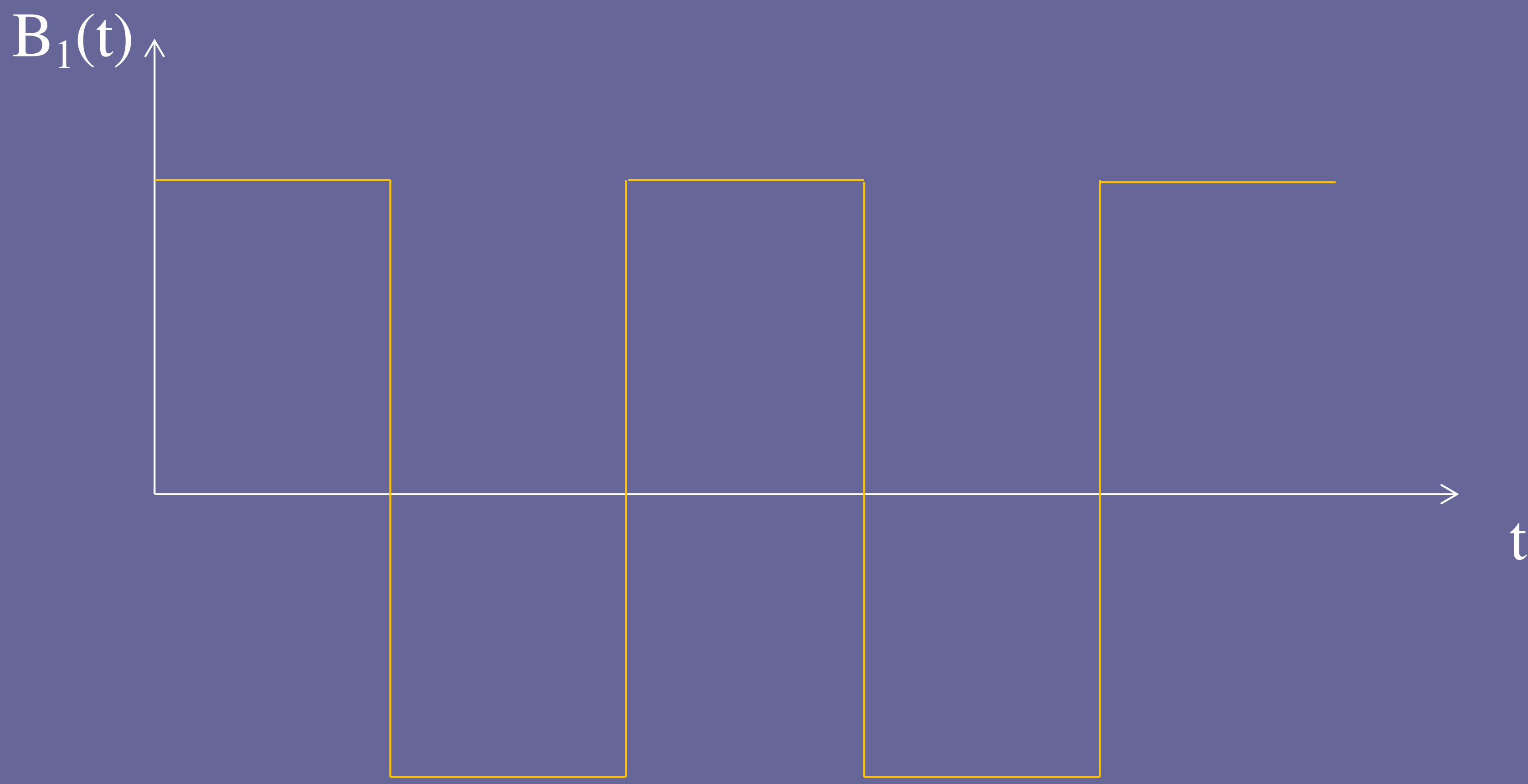
At time = $3\tau/2$



Tip angle = 90°



Time dependence of B_1 field



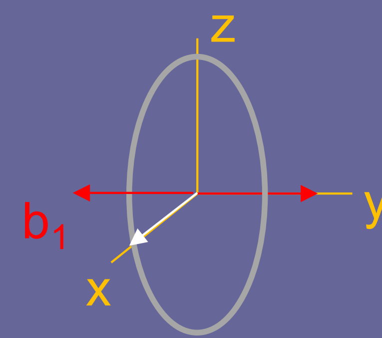
Frequency = $\omega = \gamma B_0$

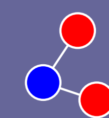
Effect of a transverse field

- A small, time-varying B_1 field can create large $M_\perp$
  - If the B_1 field precesses with the spins, slow tipping can accumulate over time
- Frequency of B_1 field must match spin precession frequency to have a significant effect
  - B_1 must be resonant with spin precession

What causes relaxation?

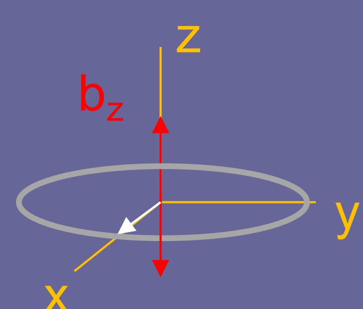
- Spins are influenced by fields generated by neighboring spins
  - Larger magnetic moments have greater effect
- Tumbling molecules produce time-varying magnetic fields, $b_1(t)$, in their neighborhoods
- A transverse magnetic field at frequency $\sim\omega_0$ changes μ_z
- Frequency components of $b_1(t)$ at $\sim\omega_0$ drive magnetization towards equilibrium
  - Equilibrium between randomizing effect of thermal energy and ordering effect of the external (B_0) magnetic field



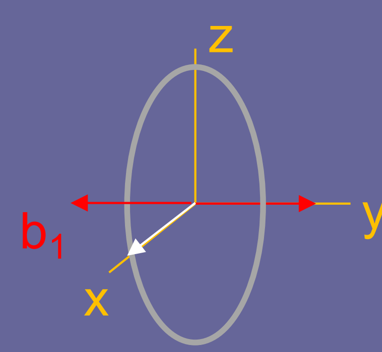


Determinants of R_2

- Neighboring spins drive transverse relaxation
- $b_1(t)$ = transverse magnetic field from neighbors
  - randomizes spin orientation in xy plane
- b_z = longitudinal magnetic field from neighbors
  - randomizes spin orientation in xy plane
- b_1 and b_z contribute to R_2



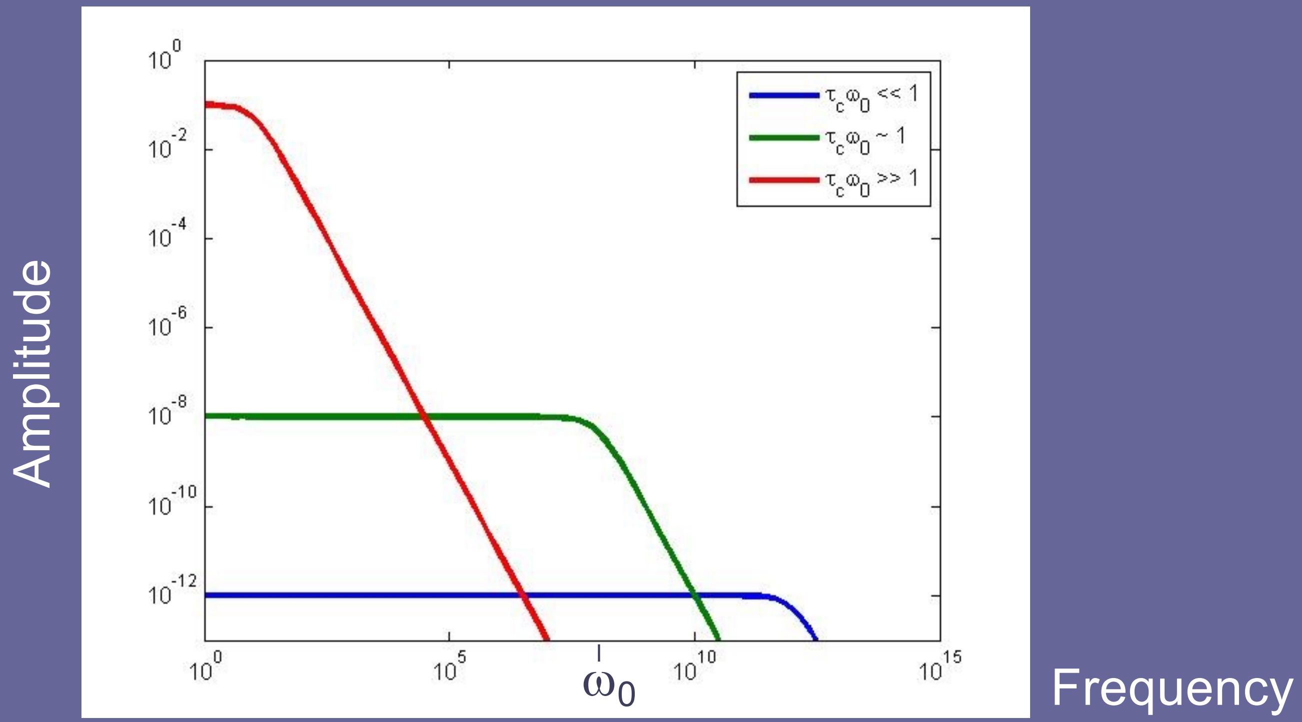
b_z drives precession in the xy plane



b_1 drives precession between z and the xy plane

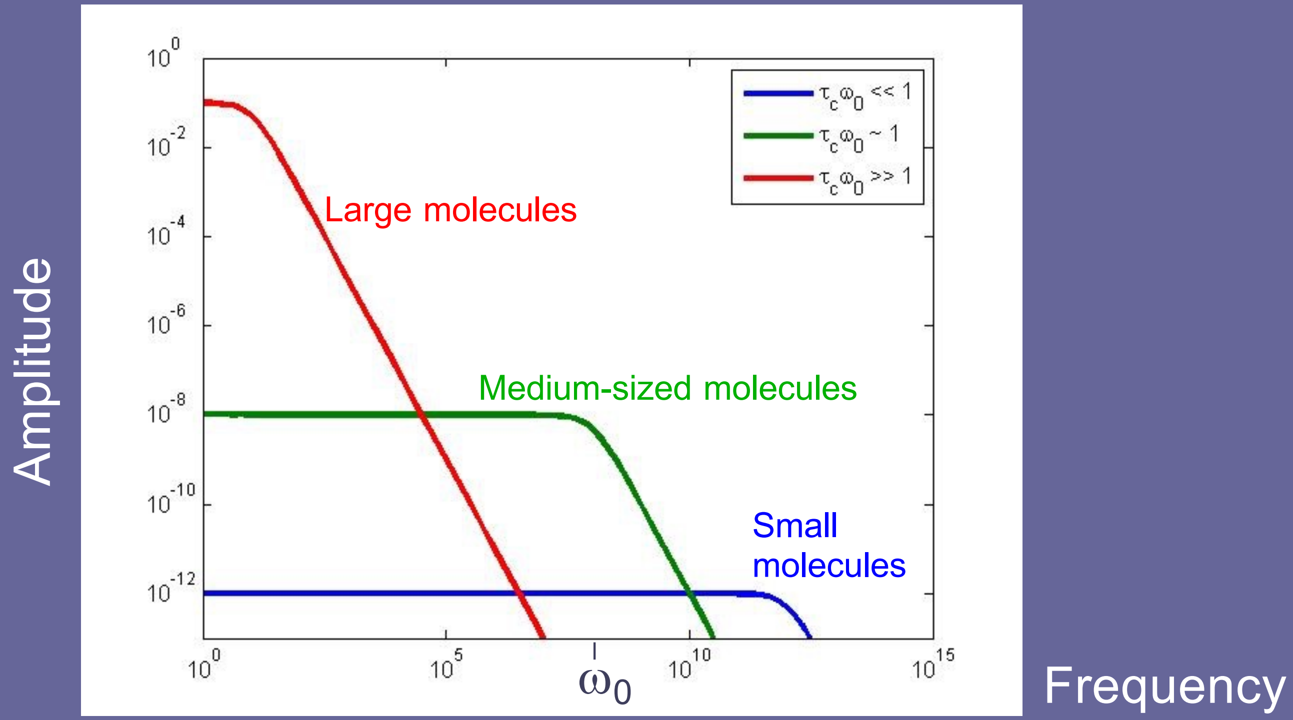
Molecular motion

- The *correlation time*, τ_c, is the interval required for a molecule to change orientation or position appreciably
- The frequency spectrum of $b_1(t)$ depends on τ_c:



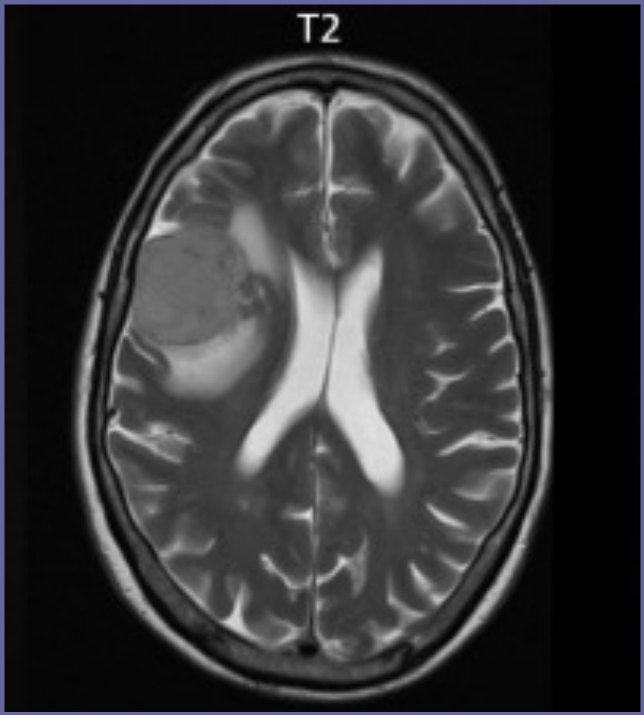
Determinants of R_2

- Magnetic moments of neighbors
- Tumbling motion of neighbors (τ_c)
  - Time-varying $b_1(t)$ at $\sim\omega_0$
  - Slow b_z component at ~0 Hz



In-class exercise

- Which part of the image has the smallest R_2?
- Which part has the largest R_2?



$$M_{xy}(t) = M_{xy}(0) \cdot e^{-R_2 t}$$

Longitudinal Relaxation

Quantitative description of longitudinal relaxation

- The z axis is defined by B_0
- Magnetic moment per unit volume is magnetization, M
- The component of magnetization along B_0 is M_z
- Bloch's phenomenological equation for M_z:

$$\frac{d\left(M_0 - M_z\right)}{dt} = -R_1 \cdot \left(M_0 - M_z\right)$$

$$\frac{d\left(M_0 - M_z\right)}{dt} = -R_1 \cdot \left(M_0 - M_z\right)$$

$$\frac{d\left(M_0 - M_z\right)}{dt} = -R_1 \cdot \left(M_0 - M_z\right)$$

$$\frac{d\left(M_0 - M_z\right)}{\left(M_0 - M_z\right)} = -R_1 \cdot dt$$

$$\frac{d\left(M_0 - M_z\right)}{dt} = -R_1 \cdot \left(M_0 - M_z\right)$$

$$\frac{d\left(M_0 - M_z\right)}{\left(M_0 - M_z\right)} = -R_1 \cdot dt$$

$$\int_0^t \frac{d\left(M_0 - M_z\right)}{\left(M_0 - M_z\right)} = -R_1 \cdot \int_0^t dt$$

$$\frac{d\left(M_0 - M_z\right)}{dt} = -R_1 \cdot \left(M_0 - M_z\right)$$

$$\frac{d\left(M_0 - M_z\right)}{\left(M_0 - M_z\right)} = -R_1 \cdot dt$$

$$\int_0^t \frac{d\left(M_0 - M_z\right)}{\left(M_0 - M_z\right)} = -R_1 \cdot \int_0^t dt$$

$$ln\left(M_0 - M_z\left(t\right)\right) - ln\left(M_0 - M_z\left(0\right)\right) = -R_1 \cdot \left(t - 0\right)$$

$$\frac{d\left(M_0 - M_z\right)}{dt} = -R_1 \cdot \left(M_0 - M_z\right)$$

$$\frac{d\left(M_0 - M_z\right)}{\left(M_0 - M_z\right)} = -R_1 \cdot dt$$

$$\int_0^t \frac{d\left(M_0 - M_z\right)}{\left(M_0 - M_z\right)} = -R_1 \cdot \int_0^t dt$$

$$ln\left(M_0 - M_z\left(t\right)\right) - ln\left(M_0 - M_z\left(0\right)\right) = -R_1 \cdot \left(t - 0\right)$$

$$ln\,\frac{M_0 - M_z\left(t\right)}{M_0 - M_z\left(0\right)} = -R_1 t$$

$$\frac{d\left(M_0 - M_z\right)}{dt} = -R_1 \cdot \left(M_0 - M_z\right)$$

$$\frac{d\left(M_0 - M_z\right)}{\left(M_0 - M_z\right)} = -R_1 \cdot dt$$

$$\int_0^t \frac{d\left(M_0 - M_z\right)}{\left(M_0 - M_z\right)} = -R_1 \cdot \int_0^t dt$$

$$ln\left(M_0 - M_z\left(t\right)\right) - ln\left(M_0 - M_z\left(0\right)\right) = -R_1 \cdot \left(t - 0\right)$$

$$ln\frac{M_0 - M_z\left(t\right)}{M_0 - M_z\left(0\right)} = -R_1 t$$

$$\frac{M_0 - M_z\left(t\right)}{M_0 - M_z\left(0\right)} = e^{-R_1 t}$$

$$\frac{d\left(M_0 - M_z\right)}{dt} = -R_1 \cdot \left(M_0 - M_z\right)$$

$$\frac{d\left(M_0 - M_z\right)}{\left(M_0 - M_z\right)} = -R_1 \cdot dt$$

$$\int_0^t \frac{d\left(M_0 - M_z\right)}{\left(M_0 - M_z\right)} = -R_1 \cdot \int_0^t dt$$

$$ln\left(M_0 - M_z(t)\right) - ln\left(M_0 - M_z(0)\right) = -R_1 \cdot (t - 0)$$

$$ln\frac{M_0 - M_z(t)}{M_0 - M_z(0)} = -R_1 t$$

$$\frac{M_0 - M_z(t)}{M_0 - M_z(0)} = e^{-R_1 t}$$

$$M_z(t) = M_0 - \left[M_0 - M_z(0)\right] e^{-R_1 t}$$

$$M_z(t) = M_0 - \left[M_0 - M_z(0) \right] e^{-R_1 t}$$

$$M_z(t) = M_0 - \left[M_0 - M_z(0)\right]e^{-R_1 t}$$

- At time t=0 the magnetization is inverted,

$$M_z(0) = -M_0$$

$$M_z(t) = M_0 - \left[M_0 - M_z(0)\right]e^{-R_1 t}$$

- At time t=0 the magnetization is inverted,

$$M_z(0) = -M_0$$

so

$$M_z(t) = M_0 - [2M_0]e^{-R_1 t}$$

$$M_z(t) = M_0 - \left[M_0 - M_z(0)\right]e^{-R_1 t}$$

- At time t=0 the magnetization is inverted,

$$M_z(0) = -M_0$$

so

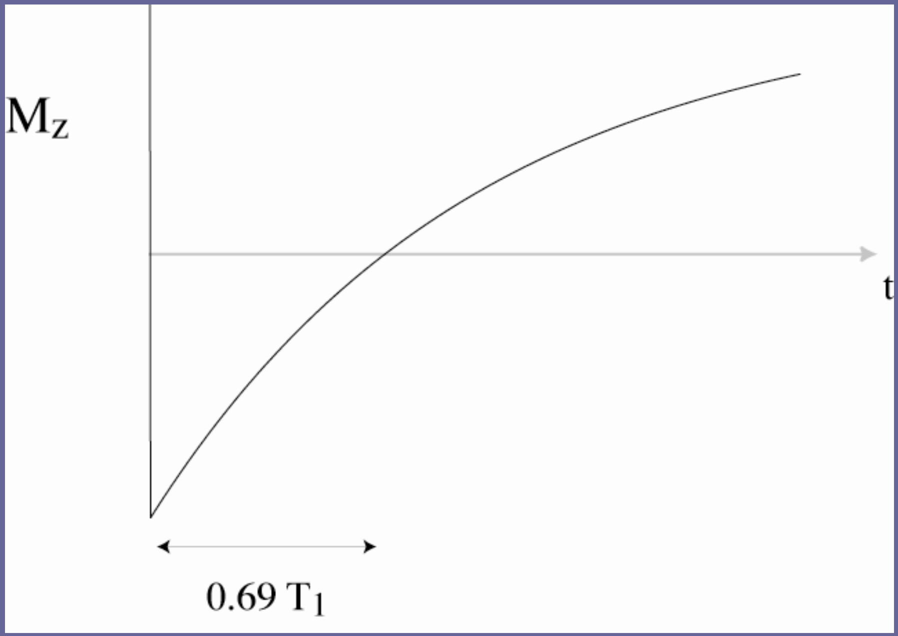
$$M_z(t) = M_0 - \left[2M_0\right]e^{-R_1 t}$$

$$\boxed{M_z(t) = M_0 \cdot \left(1 - 2e^{-R_1 t}\right)}$$

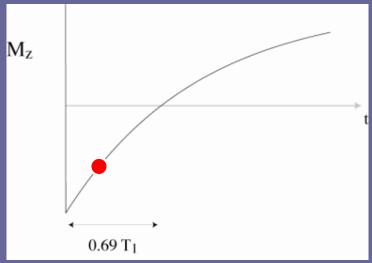
This gives the time dependence of the longitudinal magnetization in terms of the relaxation rate $R_1 = 1/T_1$.

Inversion recovery measurements of R_1

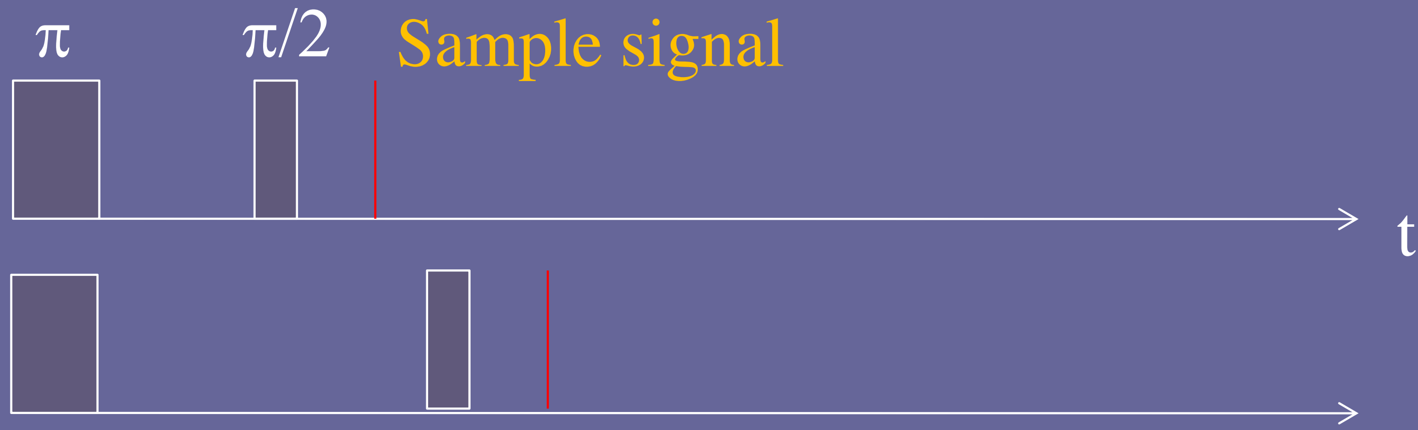
- Apply an 'inversion' B_1 pulse to orient M along –Z
- Wait some delay (inversion) time, TI, during which M_z recovers toward M_0
- Tip spins into the transverse plane and measure magnetization immediately

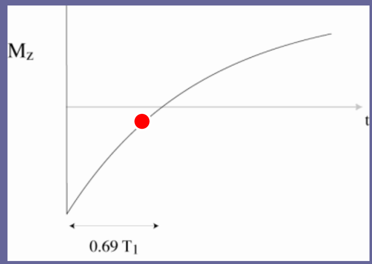


$B_1(t)$ for Inversion Recovery

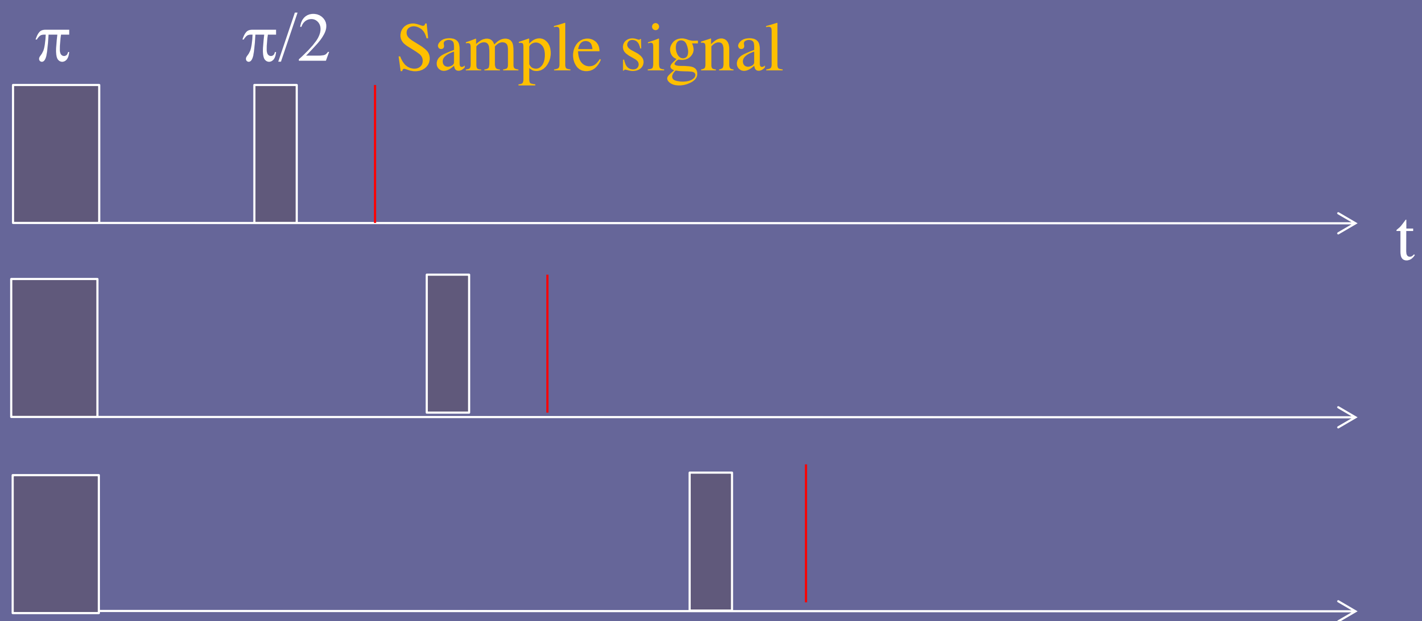


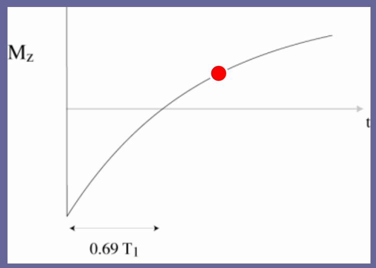
$B_1(t)$ for Inversion Recovery



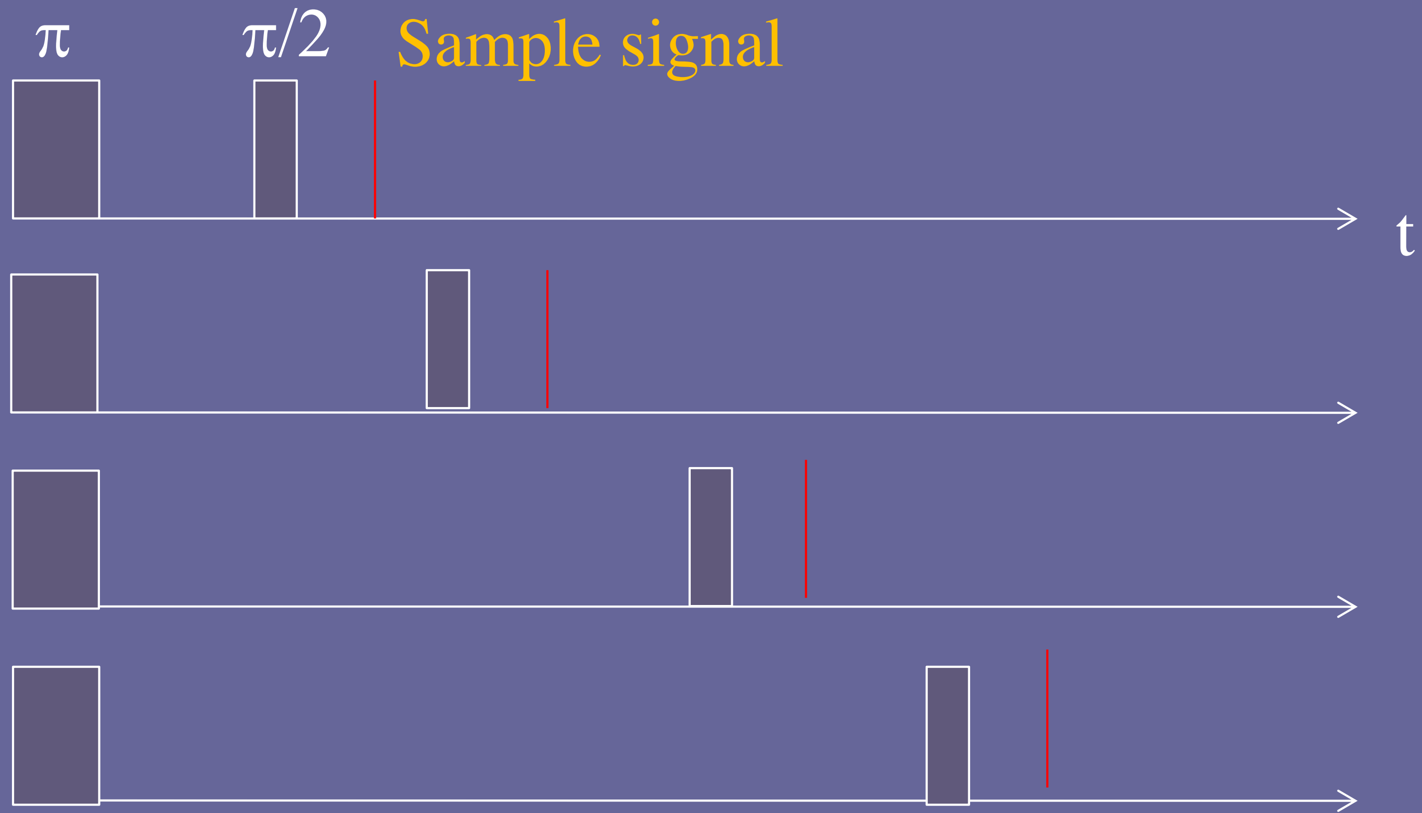


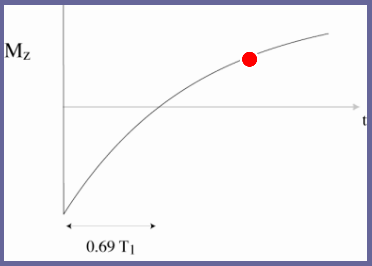
$B_1(t)$ for Inversion Recovery



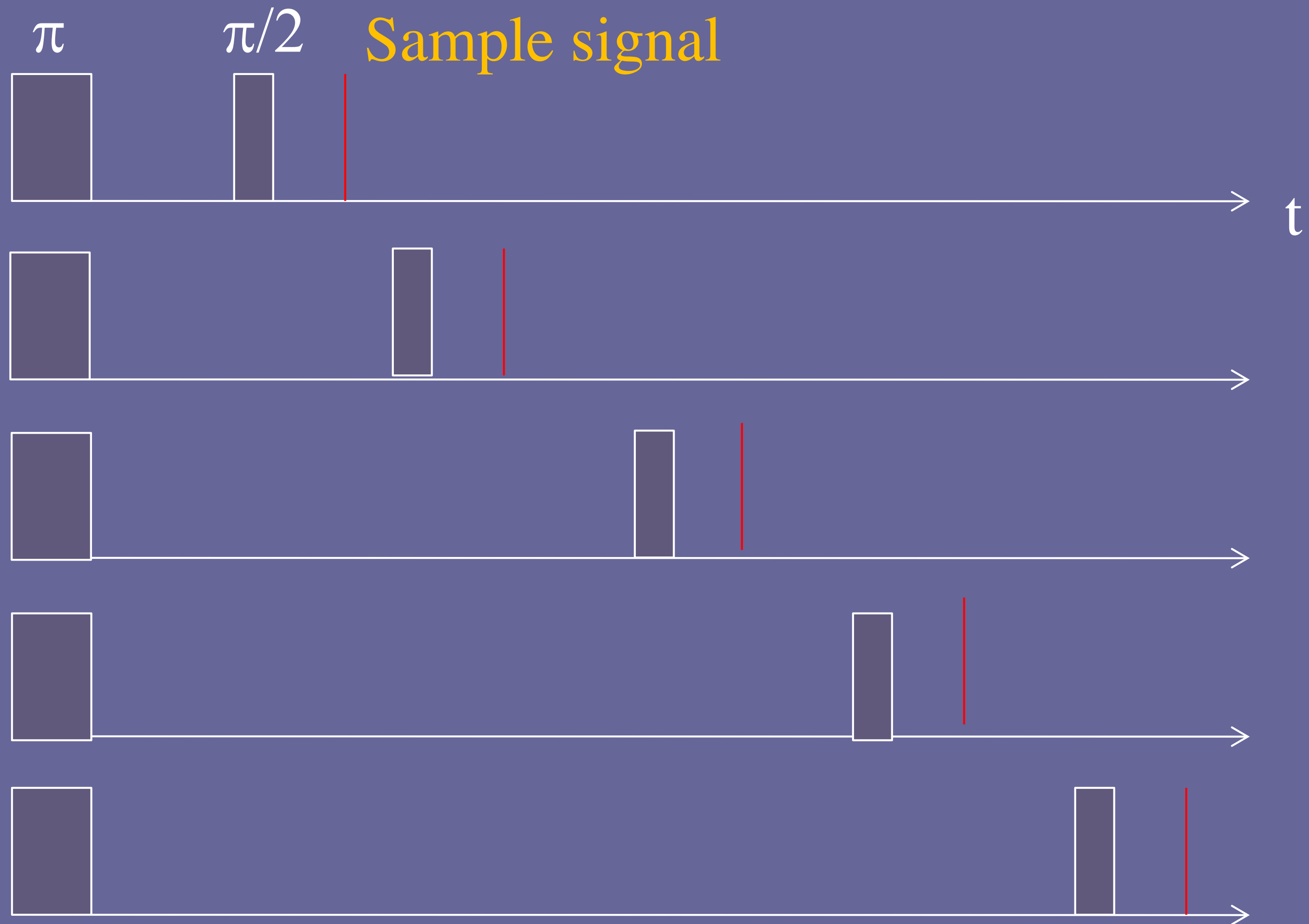


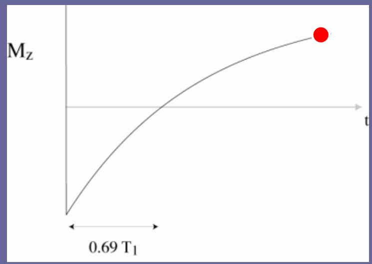
$B_1(t)$ for Inversion Recovery





$B_1(t)$ for Inversion Recovery

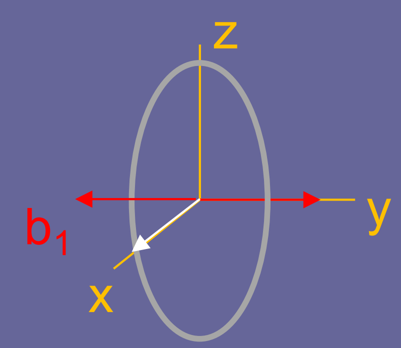




Determinants of R_1

- Neighboring spins drive transverse relaxation
- $b_1(t)$ = transverse magnetic field from neighbors
  - Drives precession between z and the xy plane (and –z)
  - Randomizes spin orientation in xy plane
- b_z = longitudinal magnetic field from neighbors
  - Randomizes spin orientation in xy plane
- b_1 and b_z contribute to R_2
- b_1 (only) contributes to R_1

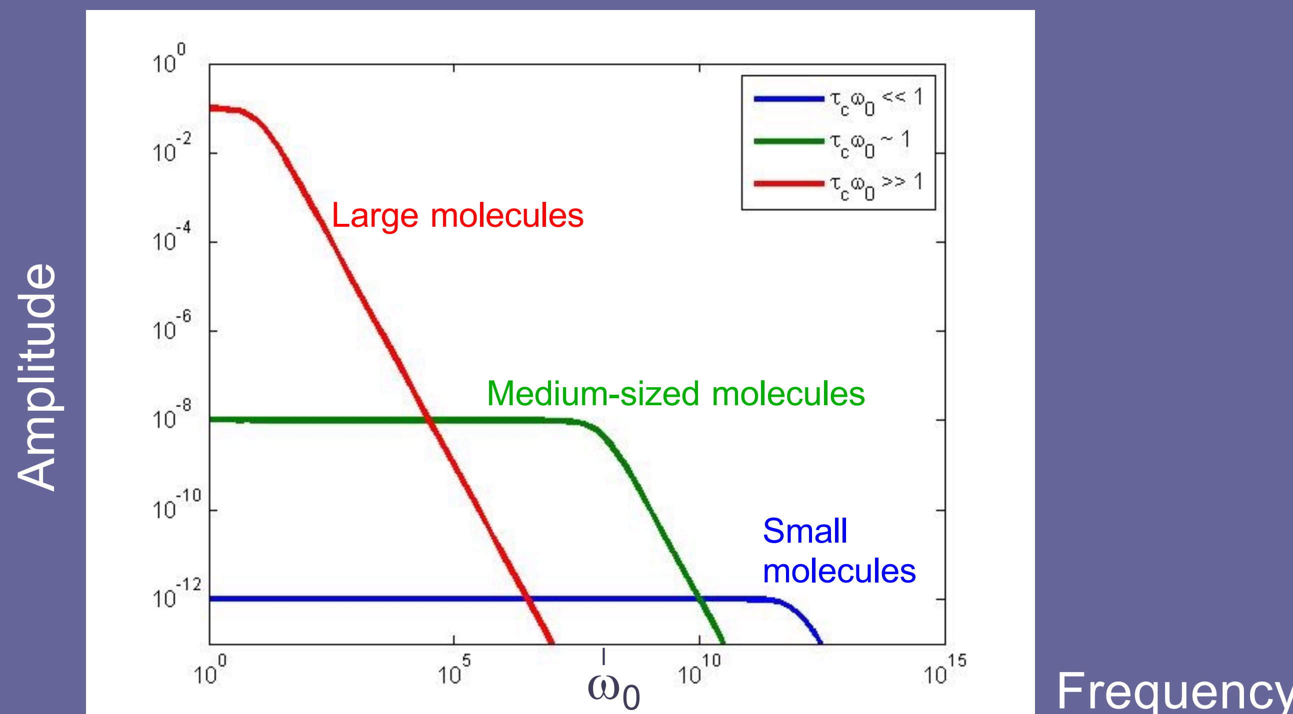
$$R_2 \geq R_1$$



b_1 drives precession between z and -z

Molecular motion

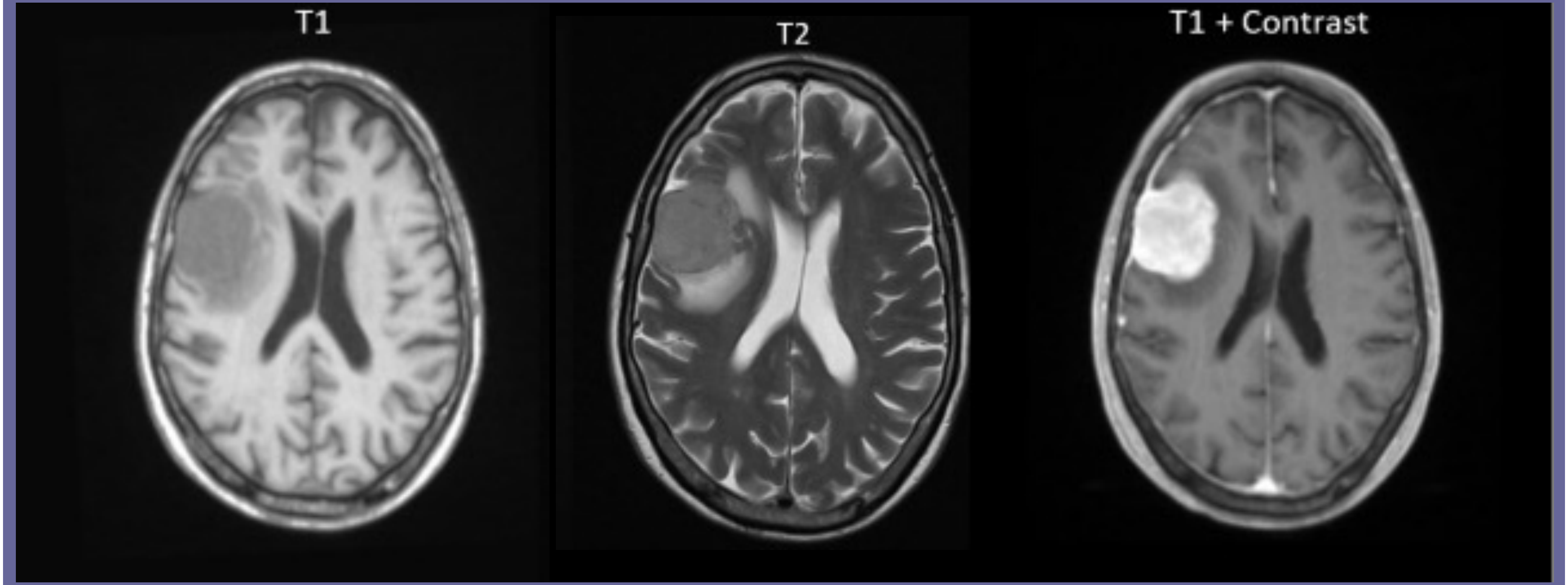
- The *correlation time*, τ_c, is the interval required for a molecule to change orientation or position appreciably
- The frequency spectrum of $b_1(t)$ depends on τ_c:



Determinants of R_1 and R_2 in tissues

- Water content
- Concentration of macromolecules
- State of water (free vs. bound to protein)
- Exchange of water between bound and free pools
- Ion concentration
- Paramagnetic ion concentration

What makes these different?



Geekymedics.com

Summary

- NMR relaxation times reveal changes in the *molecular environment* of tissue water
- T_2 can be measured from images acquired at different echo (measurement) times
- $$M_{xy}(t) = M_{xy}(0) \cdot e^{-R_2 t}$$
- T_1 can be measured from images acquired at different inversion delay times
- $$M_z(t) = M_0 \cdot (1 - 2e^{-R_1 t})$$
- Coming up next:
  - Image analysis in MATLAB (on Brightspace)
  - Project 1—mapping NMR relaxation times