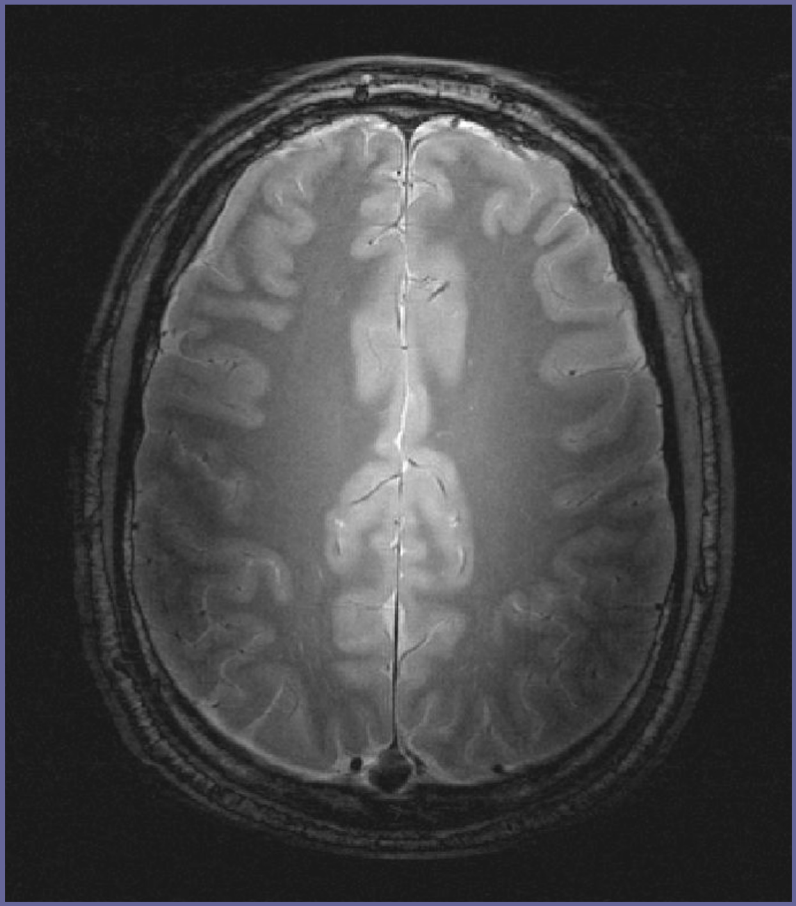
Image properties

Quantitative and Functional Imaging

BME 4420/7450

Fall 2022

How are these images different?



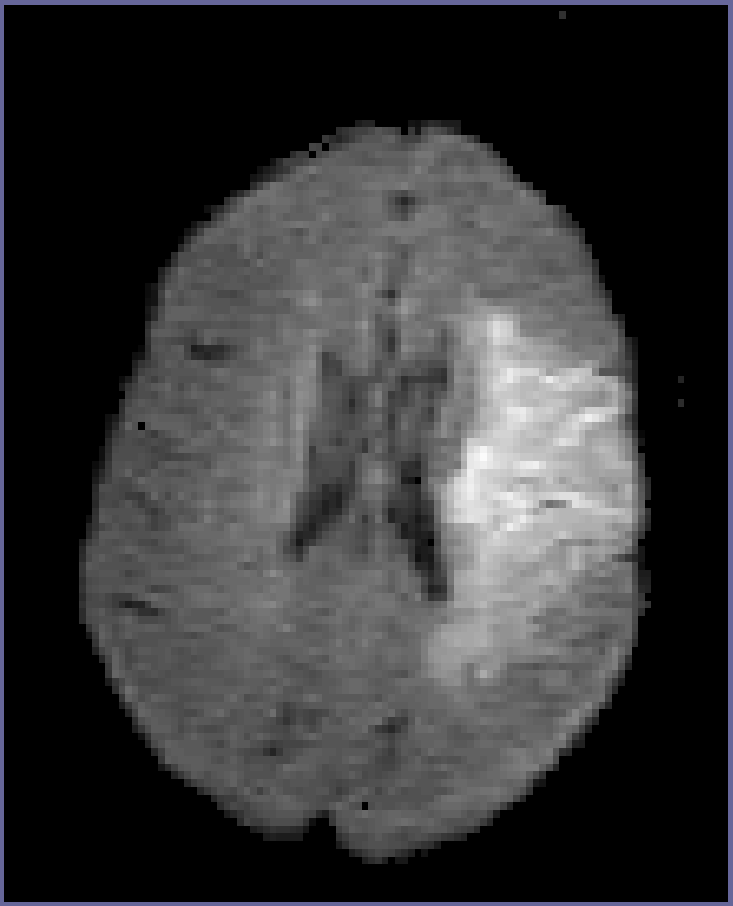


Image information

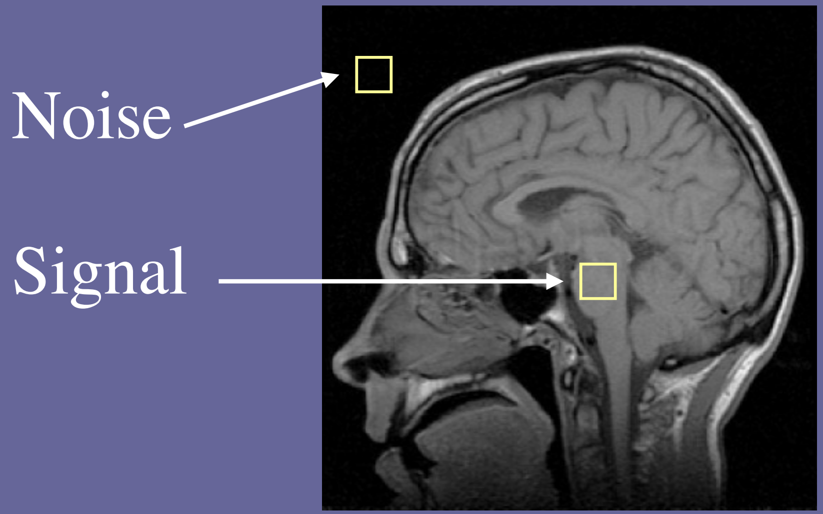
- An image represents the *spatial distribution of physical properties* in an object
  - Imperfect representation
- How do images differ from the true distributions they map?
- Let’s look at some examples…

What is the difference?

Signal to noise ratio (SNR)

- Measures the relative intensity of
  - Signal
  - Noise
- Independent of intensity scaling
- A standard measure of sensitivity

$$SNR = \frac{mean(Signal)}{std(Noise)}$$

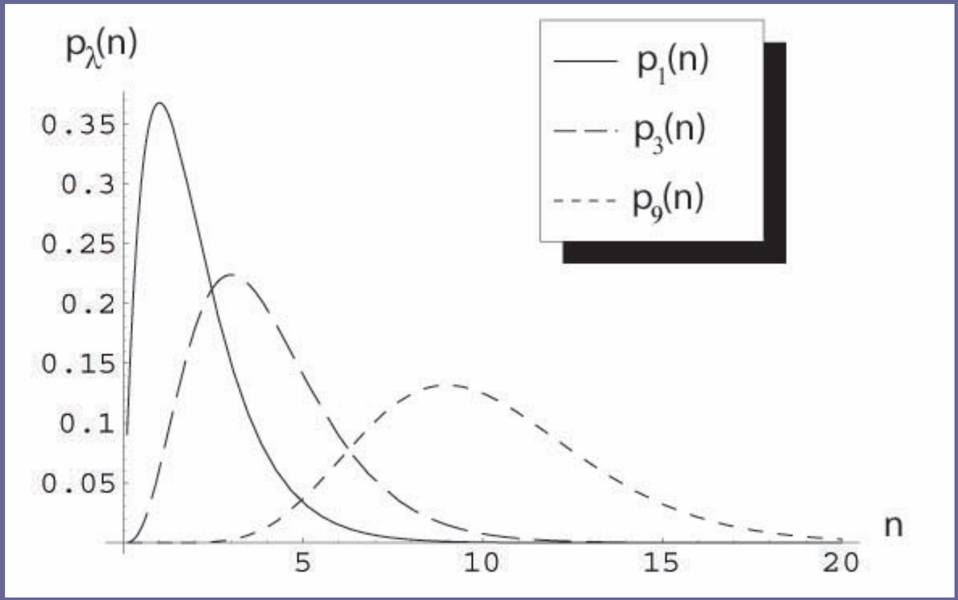


Models of noise

- Poisson noise
  - Counting independent events
  - CT, PET

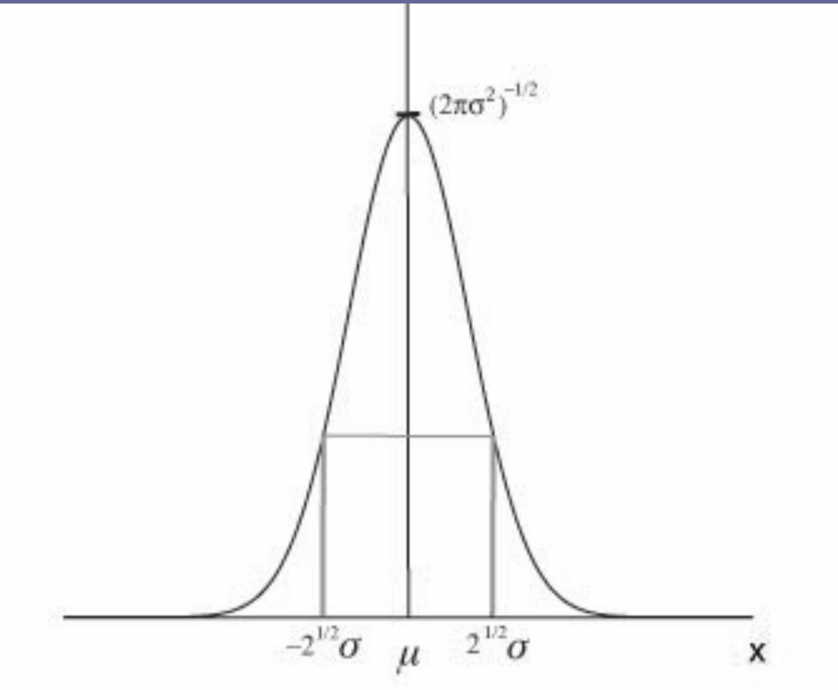
$$p_{\lambda}(n) = \frac{\lambda^{n}}{n!} e^{-\lambda}$$

λ = mean count



- Gaussian noise
  - ‘Normal’ distribution
  - Sum of many independent noise sources
  - MRI

$$\mathbf{p}(\mathbf{x}) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot \mathbf{e}^{-(\mathbf{x}-\mu)^2/2\sigma^2}$$



μ = mean value
σ = std deviation

What's the difference?

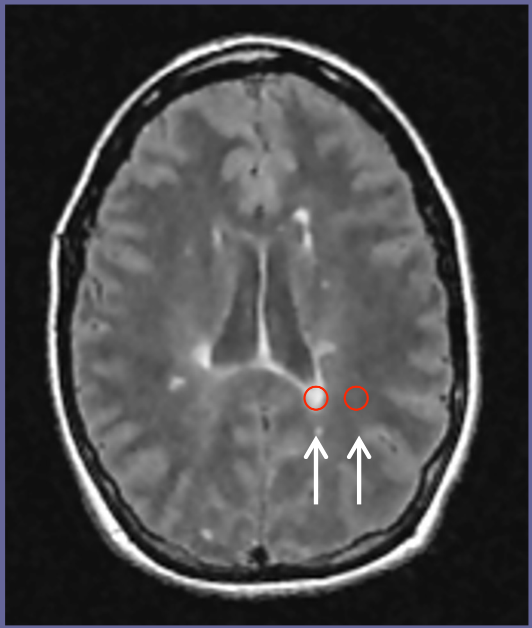
Contrast

- Image intensity difference (between regions)
- Allows regions to be distinguished
- Helps determine detection limits

$$\text{Contrast} = \Delta S$$
$$= S(\text{lesion}) - S(\text{white matter})$$

$$CNR = \frac{\Delta S}{\sigma}$$

Contrast to noise ratio



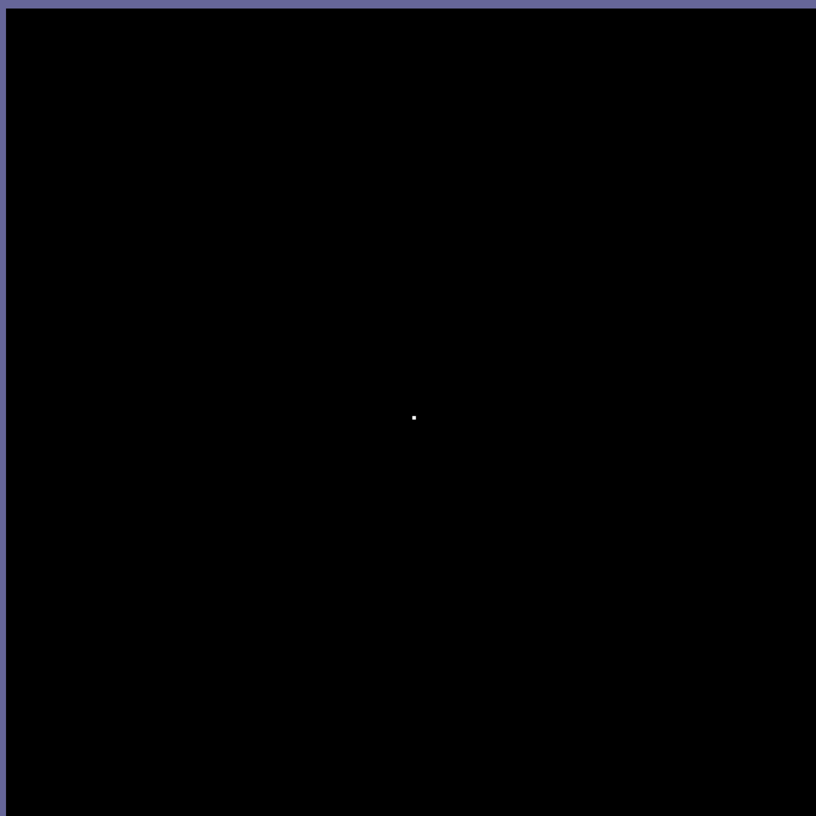
What is the difference?

Resolution

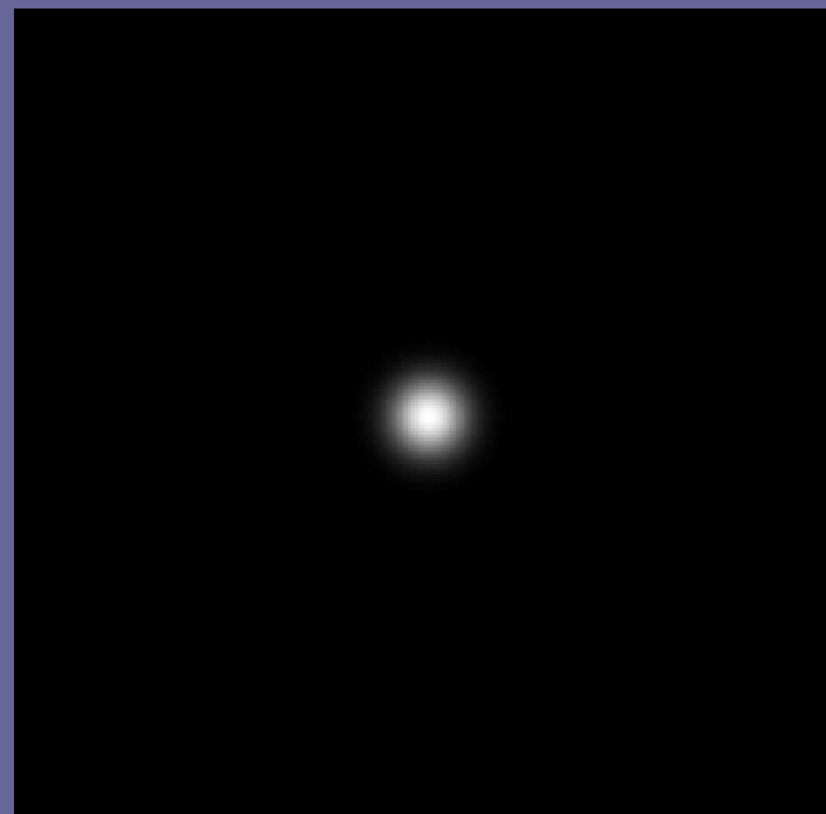
- Determines the smallest detectable feature in an image
- <u>Not</u> necessarily the pixel size
- May be different in x, y (and z)
- Characterized by the width of the *point spread function* (PSF)

Images are blurred representations

Ideal image



Blurred image



Convolution and image blurring

- The convolution of two functions, f(x) and h(x), is defined as

$$g(x) \equiv \int_{-\infty}^{\infty} f(x')\,h(x-x')\,dx'$$

$$= f(x) * h(x)$$

- Notice that the convolution is *commutative*: making the change of variables $x' = x - x''$,

$$\int_{-\infty}^{\infty} f(x')h(x-x')\,dx' = -\int_{\infty}^{-\infty} f(x-x'')h(x'')\,dx''$$

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$$\int_{-\infty}^{\infty} f(x')h(x-x')\,dx' = -\int_{\infty}^{-\infty} f(x-x'')h(x'')\,dx''$$

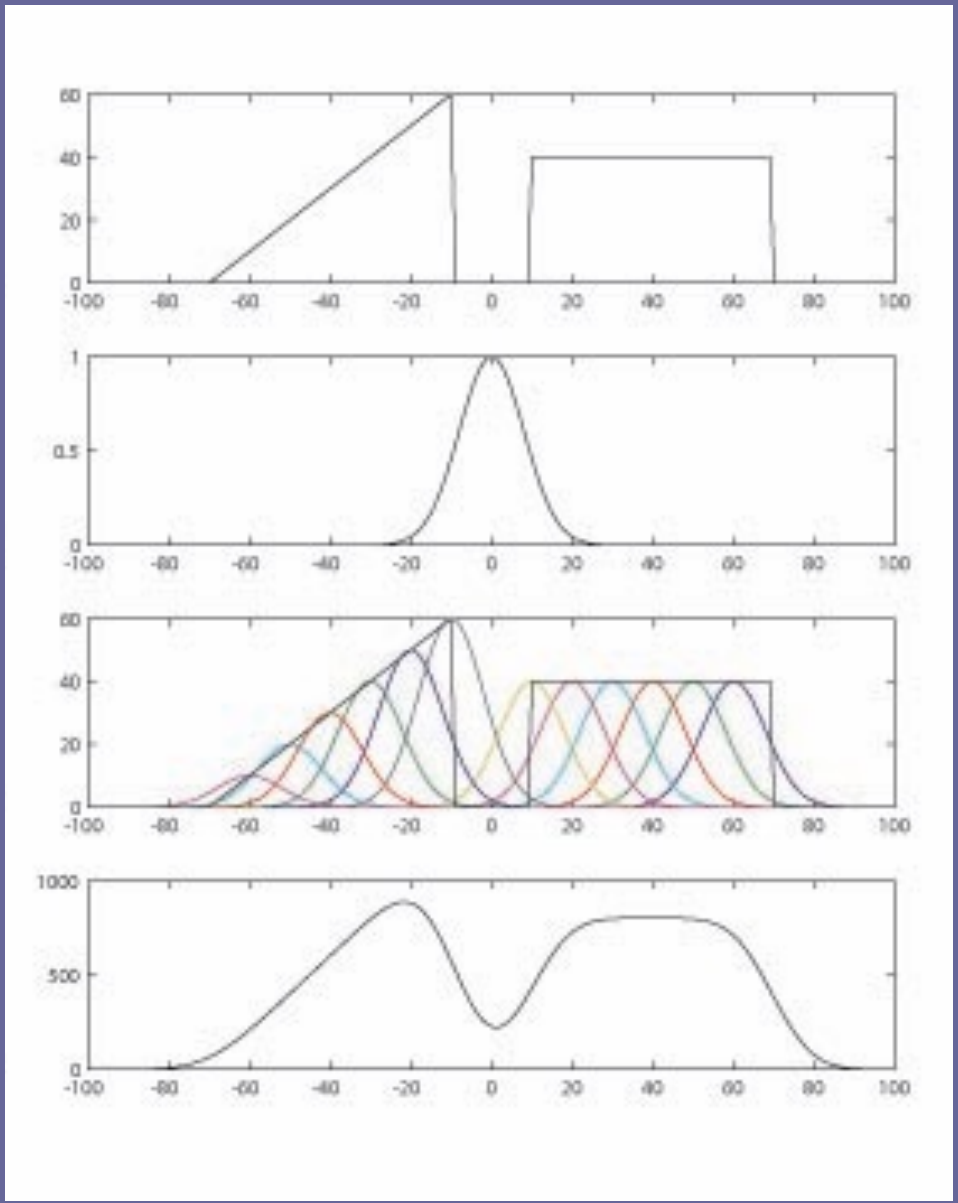
$$= \int_{-\infty}^{\infty} h(x'')f(x-x'')\,dx''$$

so

$$f(x)*h(x) = h(x)*f(x)$$

and the order doesn't matter.

- In words, the convolution is the sum of many weighted and shifted copies of h(x).



$$f(x)$$

$$h(x)$$

$$\int_{-\infty}^{\infty} f(x')\,h(x-x')\,dx'$$

$$= f(x) * h(x)$$

Describing a point source

- We can describe an idealized point source using the *Dirac delta* function, δ(x)
- The delta function is defined by

$$\delta\left(x-a\right)=\begin{cases}0, \text{ for } x\neq a\\ \infty, \text{ for } x=a\end{cases}$$

and

$$\int_{-\infty}^{\infty}\delta\left(x-a\right)dx=1$$

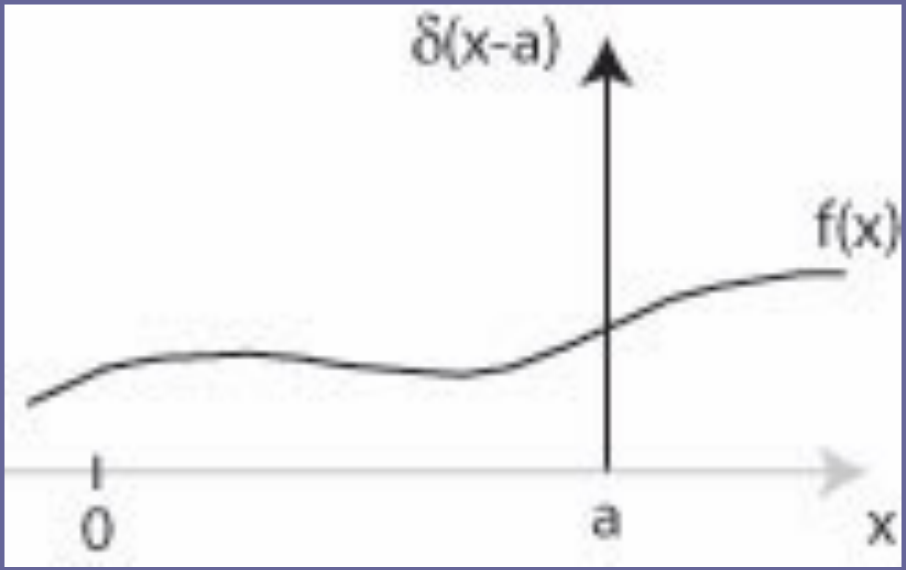
so the delta function has infinite height but unit area.

- A basic property of the delta function is

$$\int_{-\infty}^{\infty} f(x)\cdot\delta(x-a)\,dx = \int_{-\infty}^{\infty} f(a)\cdot\delta(x-a)\,dx$$

$$= f(a)\cdot\int_{-\infty}^{\infty}\delta(x-a)\,dx$$

$$= f(a)$$



- Convolving a function with δ(x) gives

$$f(x) * \delta(x) = \int_{-\infty}^{\infty} f(x')\delta(x - x')\,dx'$$
$$= f(x)$$

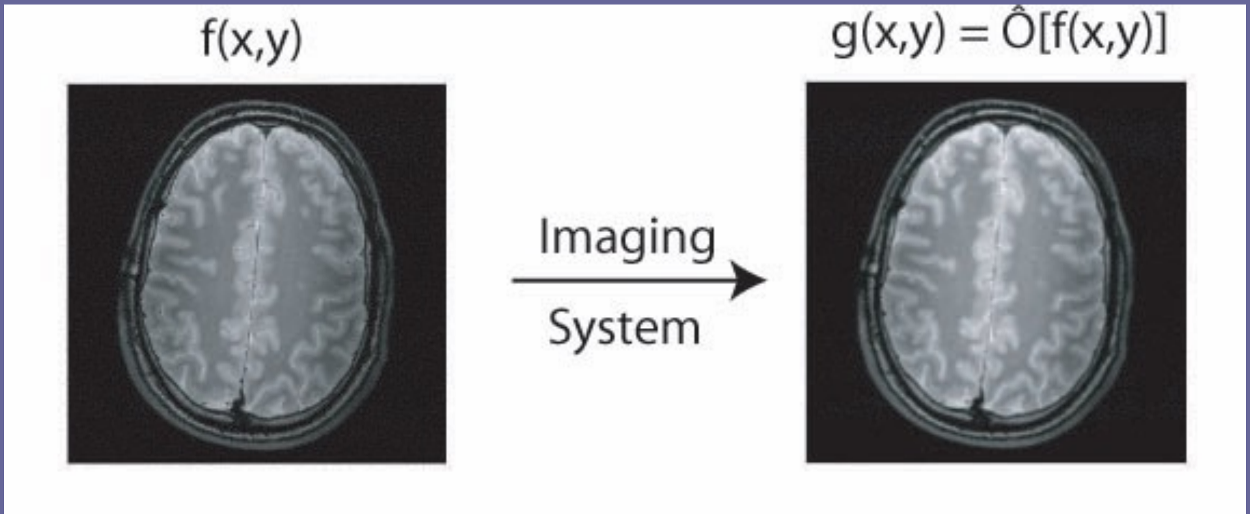
- The delta function can be considered the limit of a series of conventional (e.g., gaussian) functions:

$$\delta(x) = \lim_{\alpha \to \infty} \sqrt{\frac{\alpha}{\pi}} \cdot e^{-\alpha x^2}$$

In the limit, this has infinite amplitude (at x=0) and infinitesmal width.

Linear, shift invariant systems

- Imaging systems acquire signals from a spatial distribution of sources (an input) and produce an image (an output)



Let's denote the relation between input and output by the operator $\hat{O}$

- A system is *linear* if and only if

$$\hat{O}\left[\alpha f_1(x,y)+\beta f_2(x,y)\right]=$$
$$\alpha\cdot\hat{O}\left[f_1(x,y)\right]+\beta\cdot\hat{O}\left[f_2(x,y)\right]$$

- Suppose our system is linear and we use it to image an ideal point source

$$f(x,y)=\delta(x,y)=\delta(x)\cdot\delta(y)$$

- The image of a point source is called the *point spread function* (PSF)

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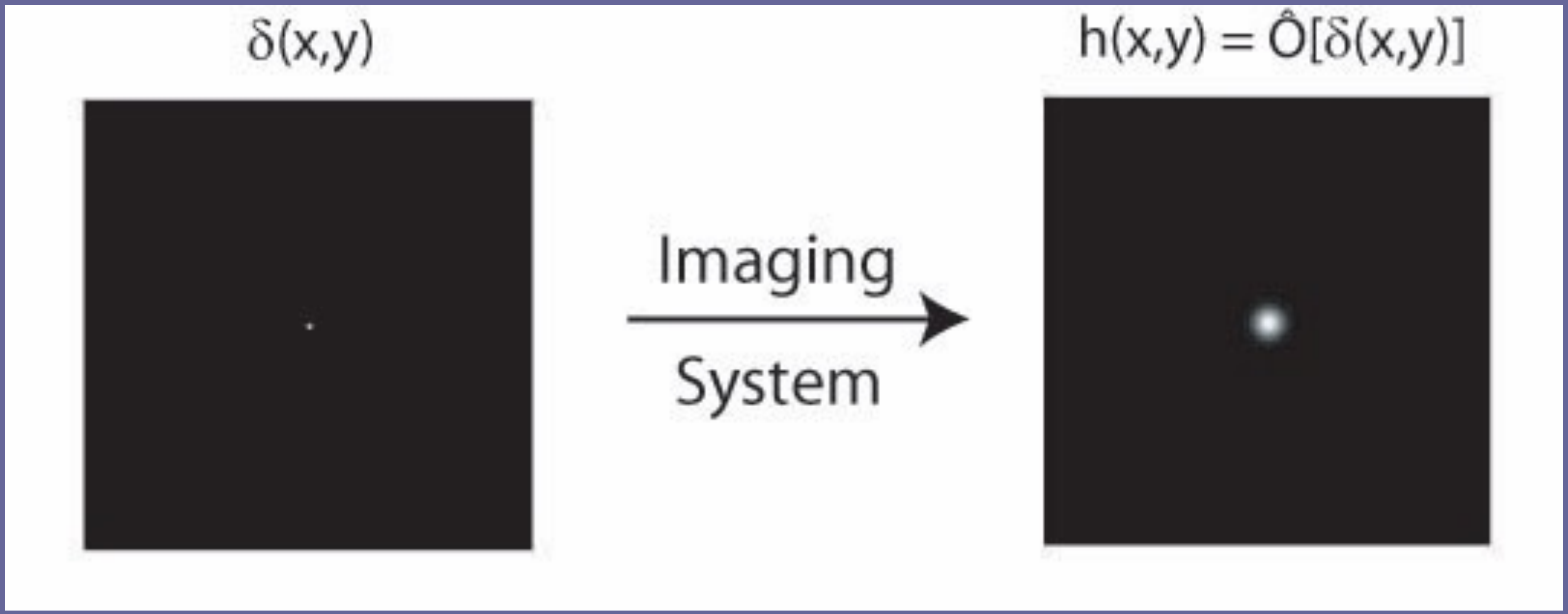
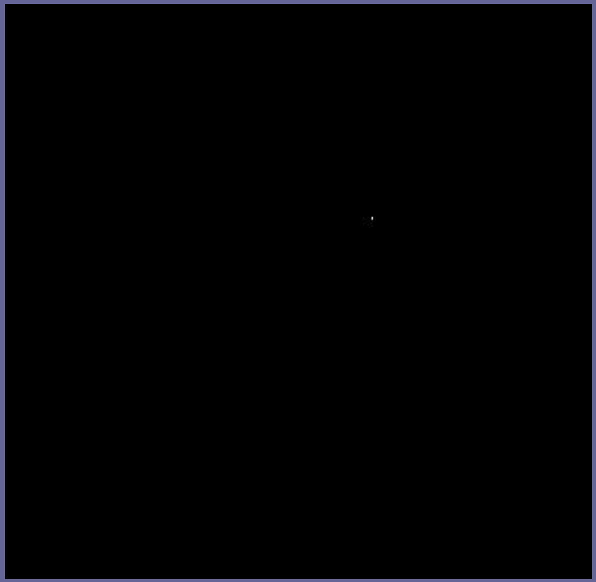


we'll write the PSF as h(x,y).

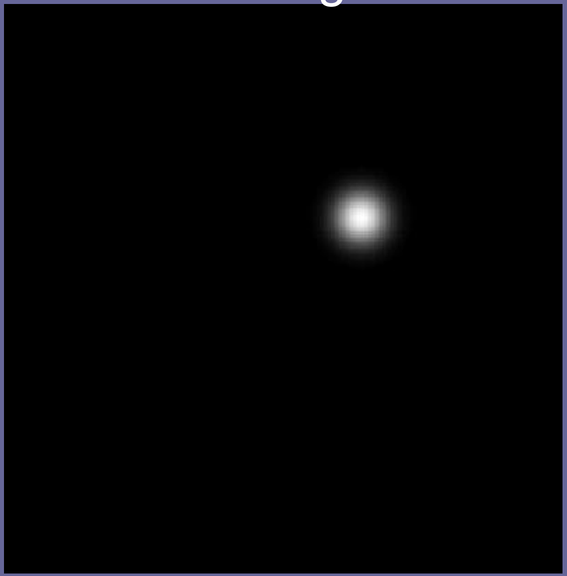
- Suppose that our system is not only linear, but also *shift invariant*
  - The only effect of shifting the input is to shift the output by the corresponding amount
  - For a delta function point source,

$$h(x-x_0, y-y_0) = \hat{O}\left[\delta(x-x_0, y-y_0)\right]$$

Point source



Image



- Suppose that our system is not only linear, but also *shift invariant*
  - The only effect of shifting the input is to shift the output by the corresponding amount
  - For a delta function point source,

$$h\left(x-x_0, y-y_0\right)=\hat{O}\left[\delta\left(x-x_0, y-y_0\right)\right]$$

- To save writing, we'll use the notation

input → *output*

so

$$\delta\left(x-x_0, y-y_0\right)\rightarrow h\left(x-x_0, y-y_0\right)$$

- Given an array of point sources with different magnitudes, linearity and shift invariance imply

$$\sum_n f(x_n, y_n) \cdot \delta(x - x_n, y - y_n) \rightarrow$$

$$\sum_n f(x_n, y_n) \cdot h(x - x_n, y - y_n)$$

- In the limit that δ function spacing goes to zero, the sums are replaced by integrals:

$$\int f(x', y') \cdot \delta(x - x', y - y') dx'dy' \rightarrow$$

$$\int f(x', y') \cdot h(x - x', y - y') dx'dy'$$

- In the limit that δ function spacing goes to zero, the sums are replaced by integrals:

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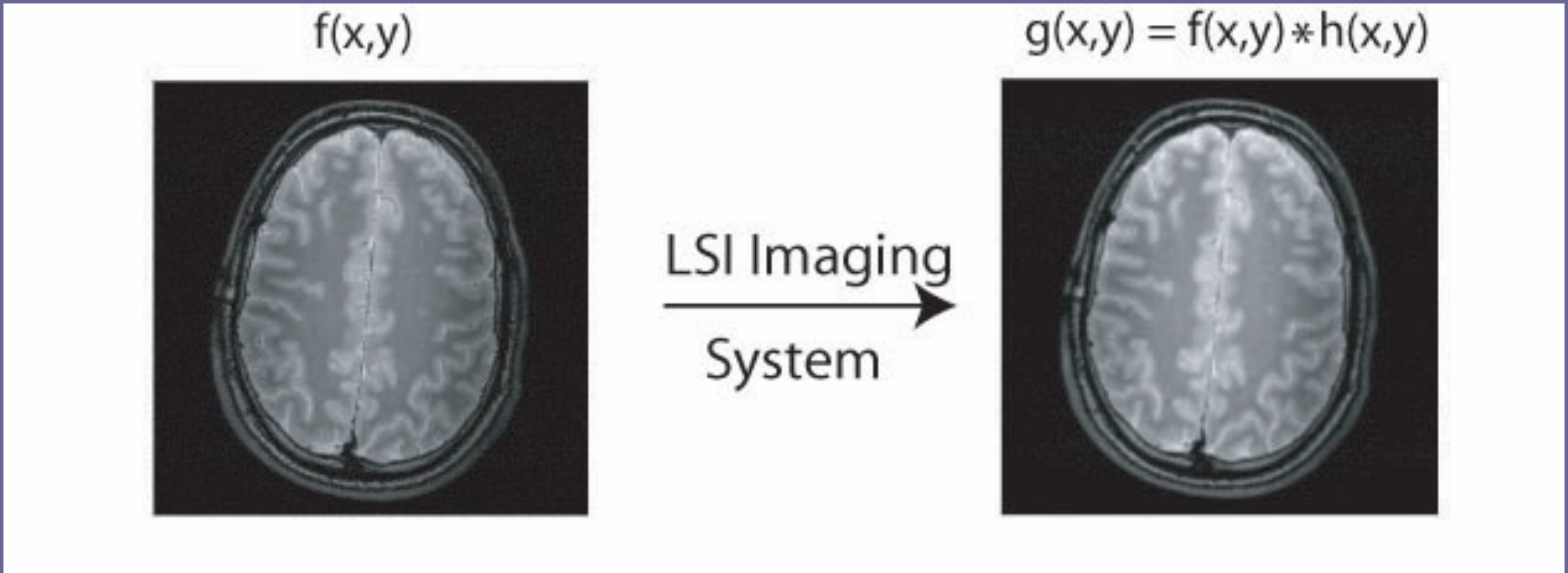
$$\int f(x',y')\cdot h(x-x',y-y')dx'dy'$$

or

$$f(x,y) \rightarrow f(x,y) * h(x,y)$$

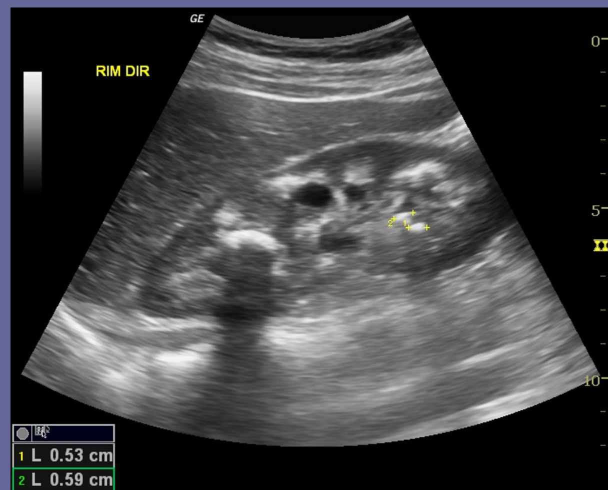
The output is equal to the true distribution, convolved (i.e., blurred) by the PSF.



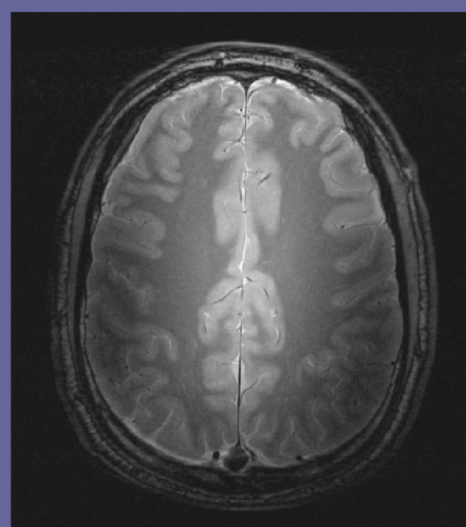
- Equivalent statements:
  - An imaging system is LSI
  - The image is equal to a convolution of the input with a PSF

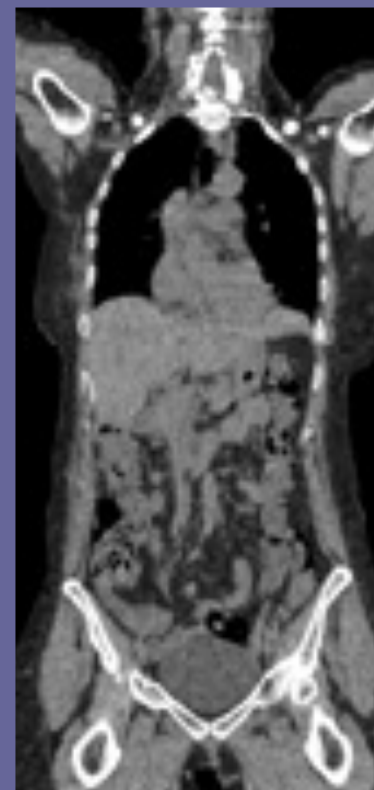
Which imaging modality made these?

Sureshbabu, J Nuc Med Tech (2005)



Case courtesy of Dr Bruno Di Muzio, Radiopaedia.org

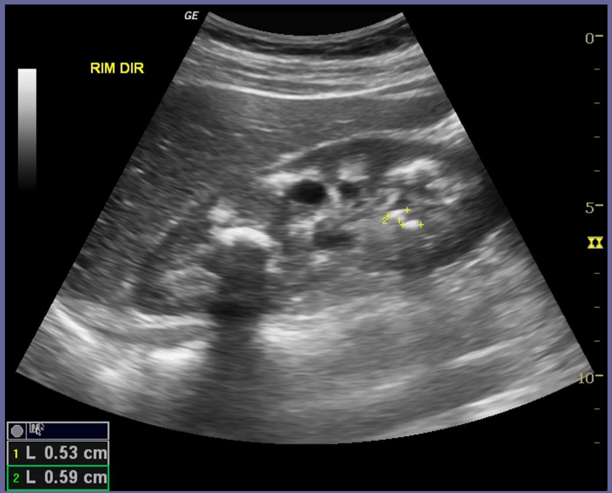




Kinahan (AAPM)

In-class exercise:
Choose an imaging modality—is it LSI?

Sureshbabu, J Nuc Med Tech (2005)



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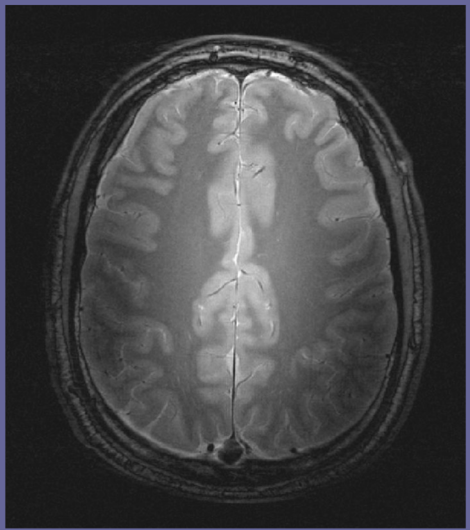
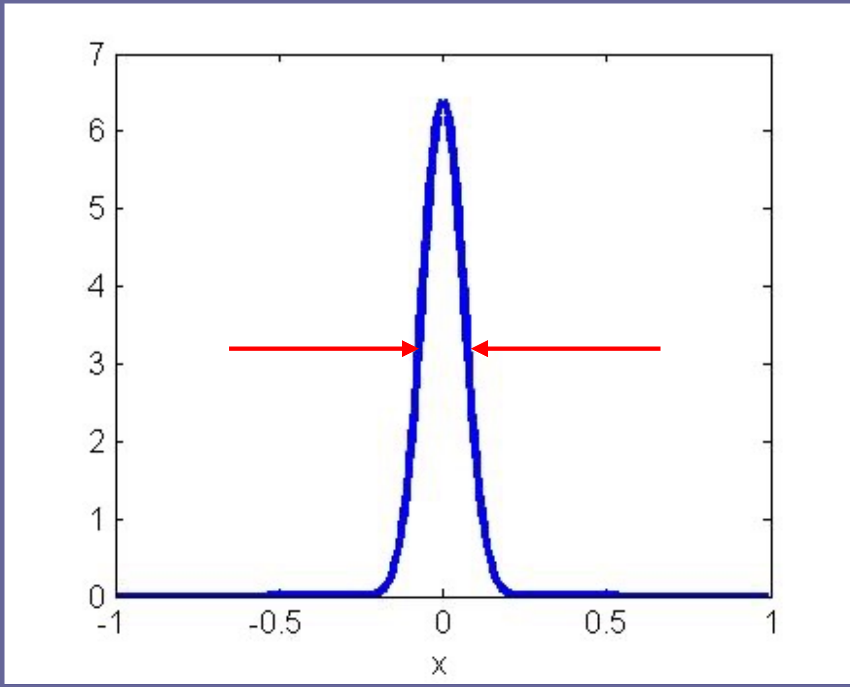


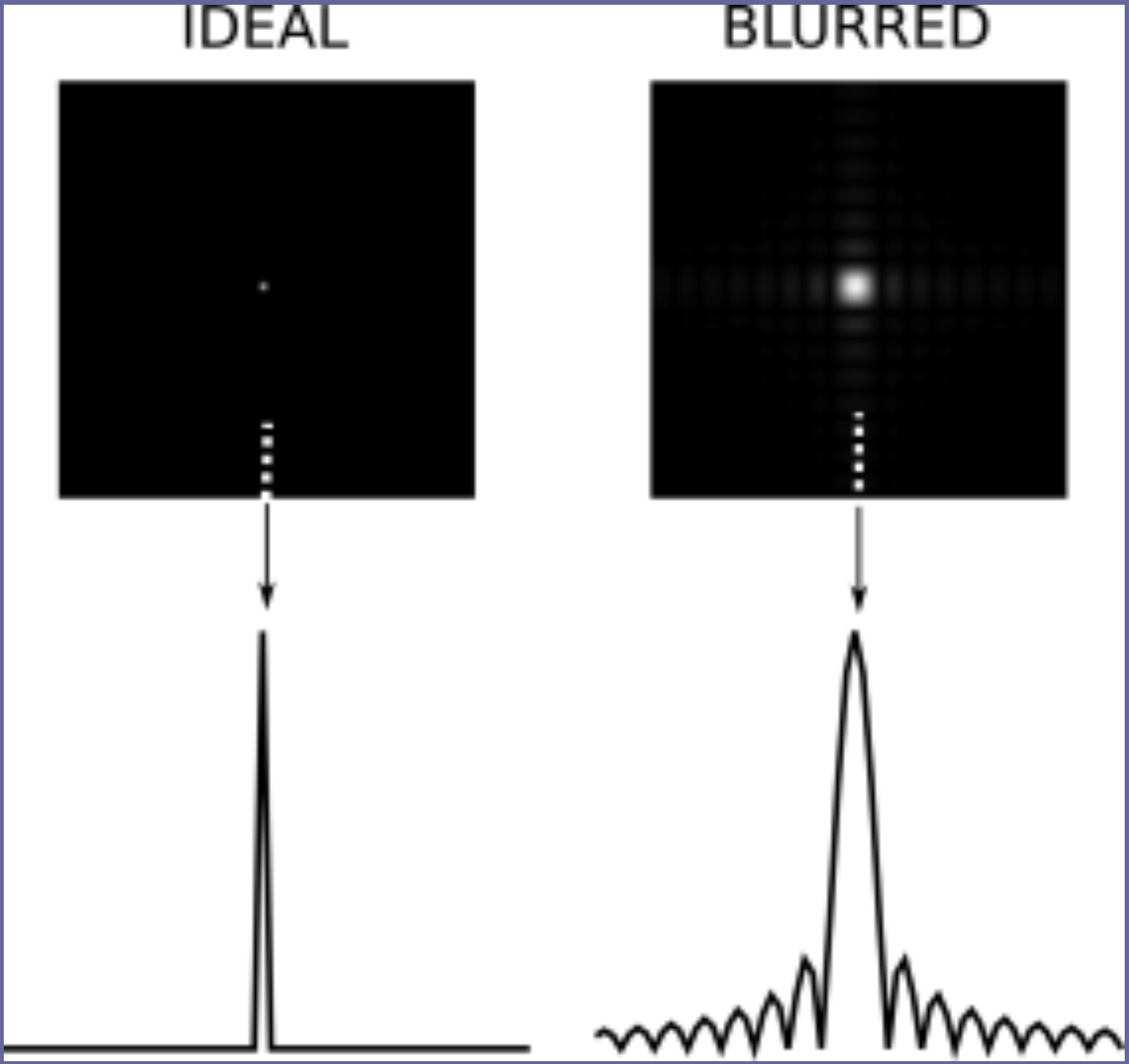
Kinahan (AAPM)

The PSF determines resolution

- A common definition of resolution is the full width at half maximum (FWHM) of the PSF



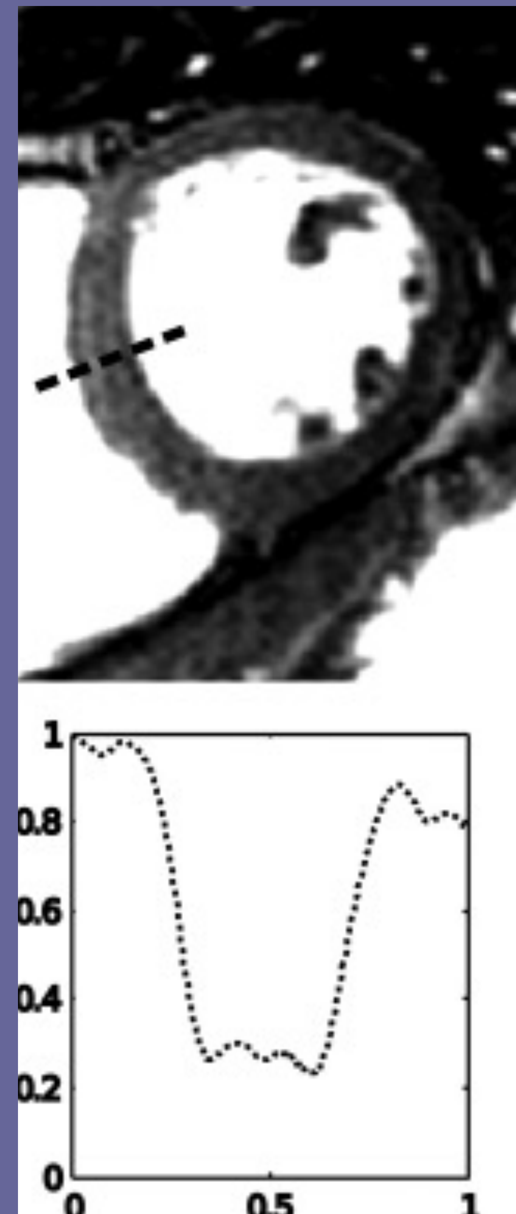
The Point Spread Function describes blurring



Journal of Magnetic Resonance Imaging
25 JUN 2014 DOI: 10.1002/jmri.24687

Effects of the PSF

- PSF width causes blurring in the image
- Oscillation in the PSF causes ringing in the image
  - Edge artifacts that can be mistaken for real structure
  - Most obvious near strong contrast boundaries
- Solutions?



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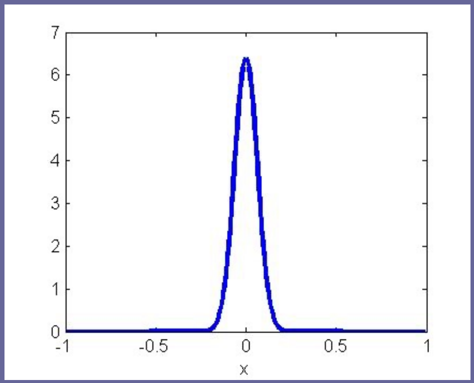
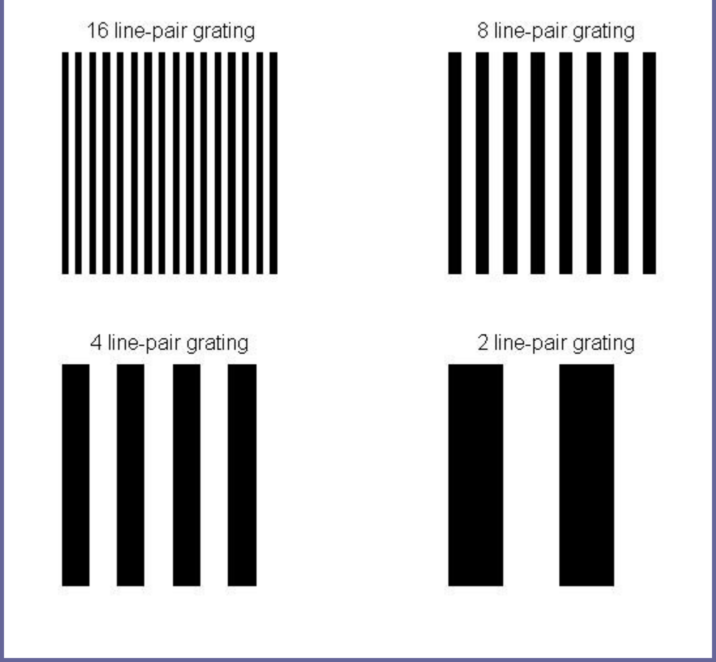
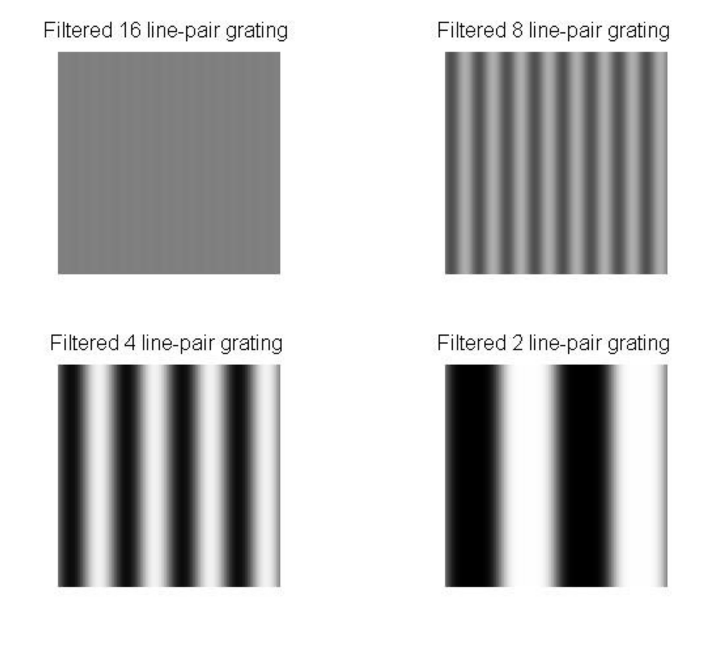
Resolution affects contrast



PSF





Partial volume averaging

- Each volume element (*voxel*) of the image represents a volume of tissue
  - Voxels are not cubes—they are broadened by the PSF
- Tissue properties may not be uniform in the voxel
- Measurements from the image data usually don't reflect true properties of heterogeneous voxels
- Measurements from the data may not give good estimates of volume-averaged properties

Artifacts

- Image reconstruction errors
  - Missing signal
  - Extra signal
  - Displaced signal
- Violation of assumptions of reconstruction
  - Motion
  - Model relating signal to tissue properties is too simple…
- Artifacts interfere with analysis (usually don't average out)

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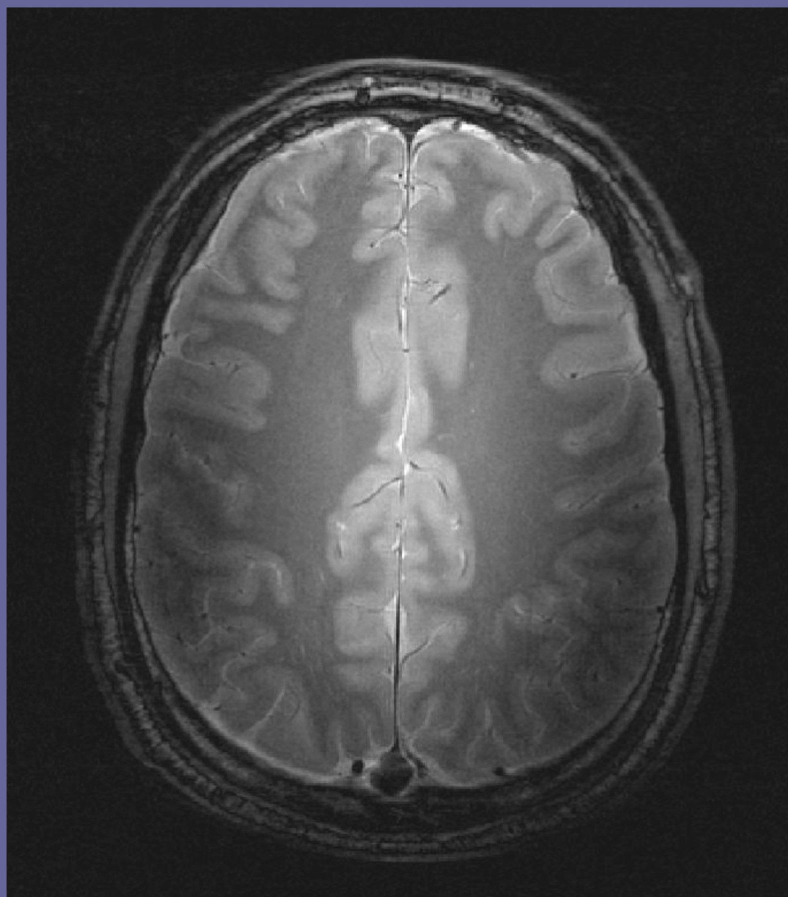
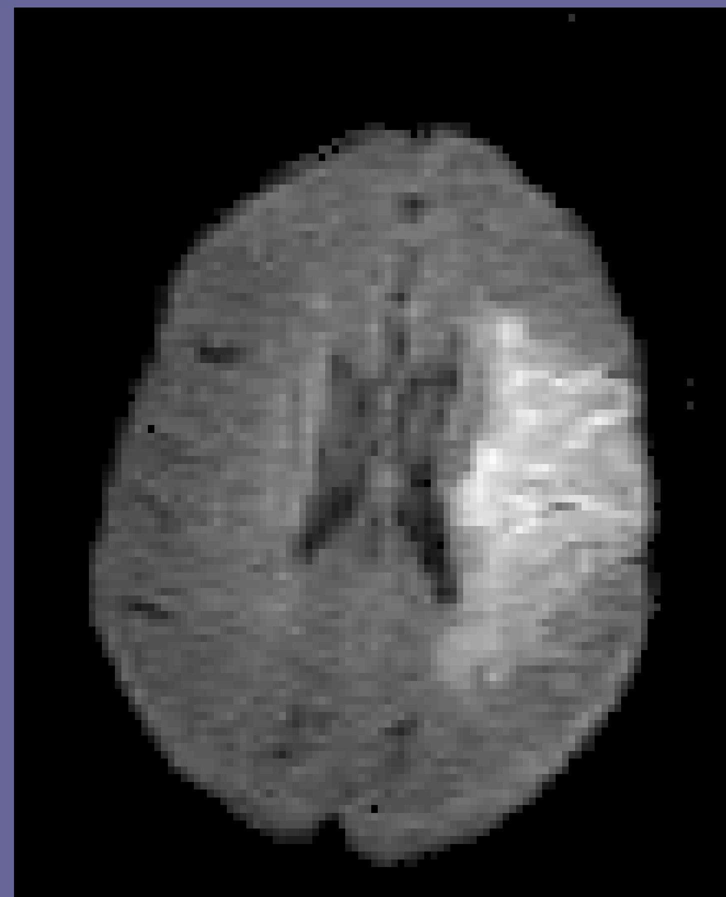
How are these images different?





Summary

- Information in an image depends on
  - SNR
  - Contrast
  - Resolution
  - Artifacts
- These are the most important aspects of image quality

References

- Hansen MS, Kellman P. Image reconstruction: An overview for clinicians. Journal of Magnetic Resonance Imaging. 25 JUN 2014 DOI: 10.1002/jmri.24687
- Suetens P. Fundamentals of Medical Imaging (Cambridge, 2002).