Effects of Noise on Image Data

Quantitative and Functional Imaging

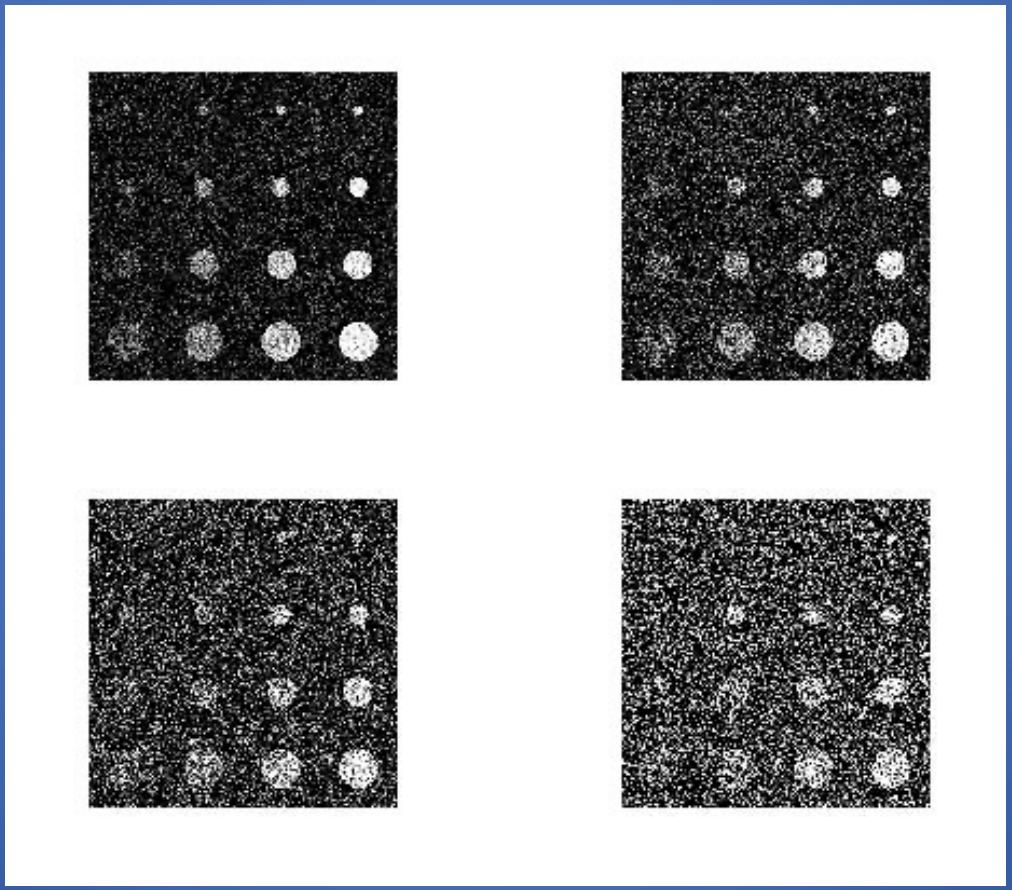
BME 4420/7450

Fall 2022

Effects of noise

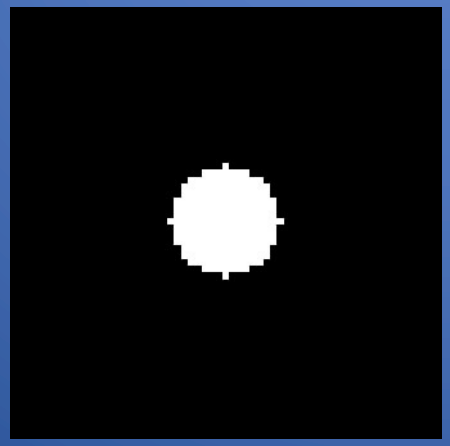
- Random noise in images
  - Interferes with feature detection
  - Causes errors in calculated parameters

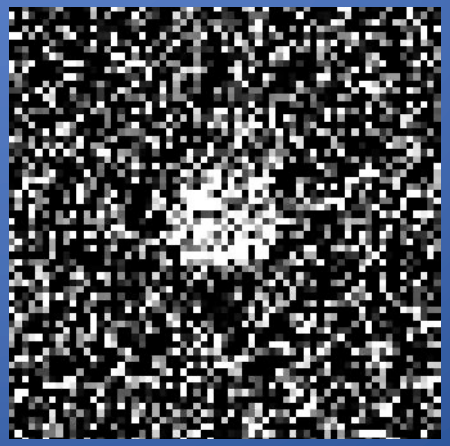
Detection threshold depends on contrast, noise, and resolution



Feature detection

- Example: threshold detection of an object in a noisy image
  - Is the object present?
  - Simple rule: compare intensity to a threshold value





Binary decisions

- Correct answers
  - True positive
  - True negative
- Incorrect answers
  - False positive
  - False negative
- Always possible to get true positives
  - At the cost of false positives
- Get best compromise between
  - True positives
  - False positives

	Object reported to be absent	**Object reported to be present**
Object absent	True negative (TN)	False positive (FP)
Object present	False negative (FN)	True positive (TP)

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Rating

Binary decisions

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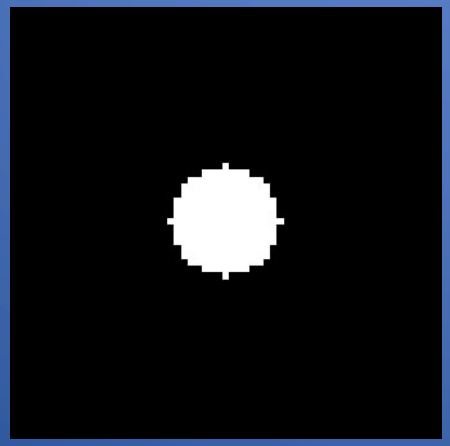
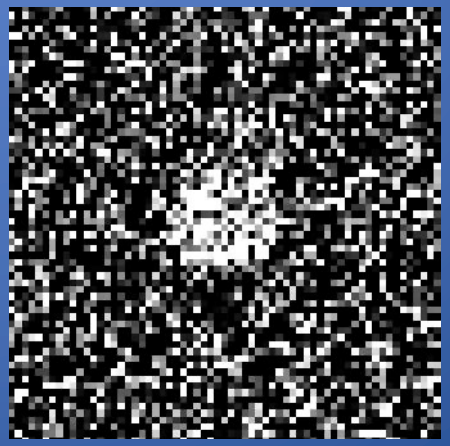
Correct?

Optimizing feature detection

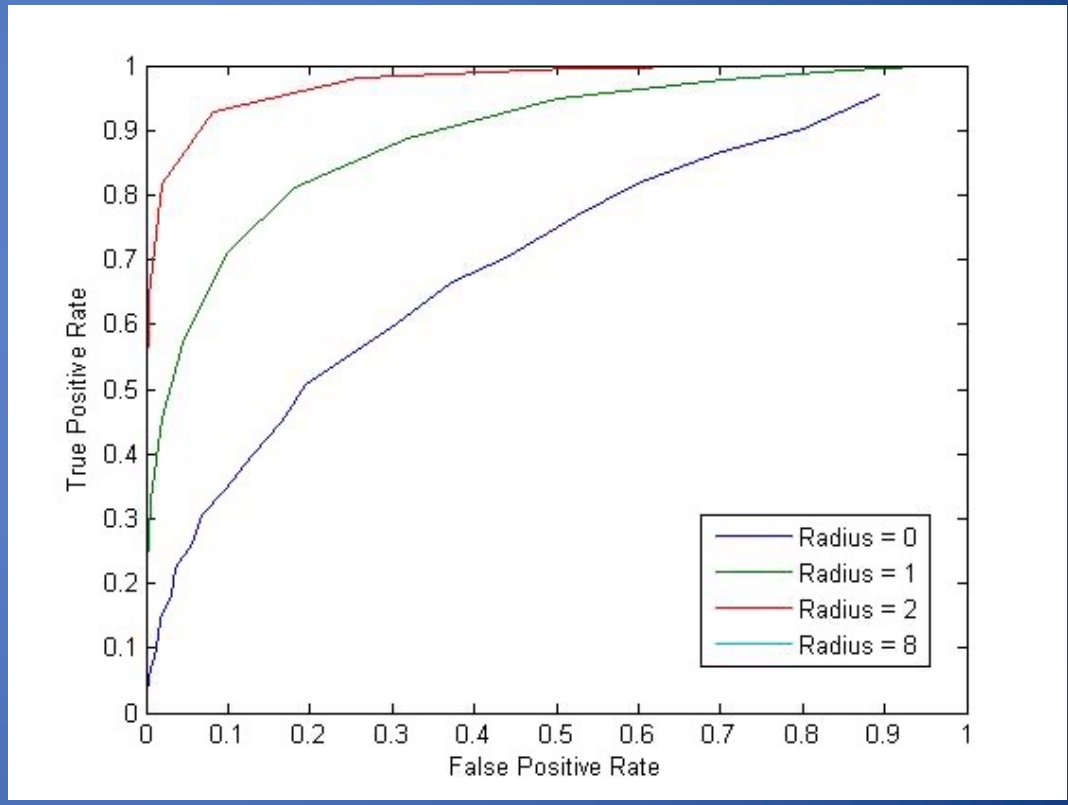
- Example: threshold detection of an object in a noisy image
  - Is the object present?
  - Simple rule: compare intensity to a threshold value
- Optimizing detection:
  - Algorithm
  - Parameters of the algorithm
- How can we quantify the effect of different choices?





Receiver Operating Characteristic (ROC) curves

- Fraction of true positives plotted as a function of false positives
  - Points on curve correspond to values of continuous parameter
  - Different curves correspond to values of discrete parameters or algorithms
- Perfect detector : curve passes through the upper-left corner of the plot
- Example: comparison to a threshold value
- The ROC curve quantifies **detector performance**

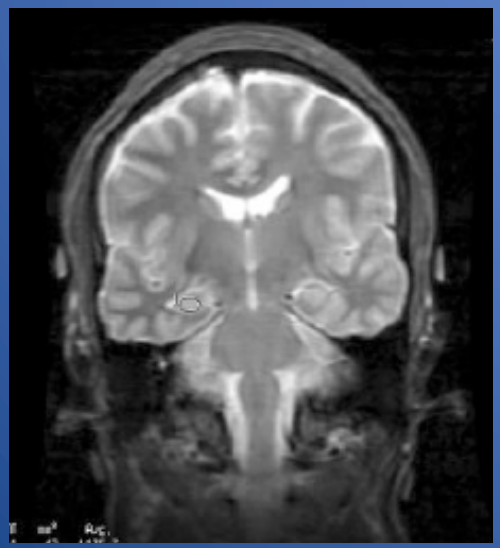


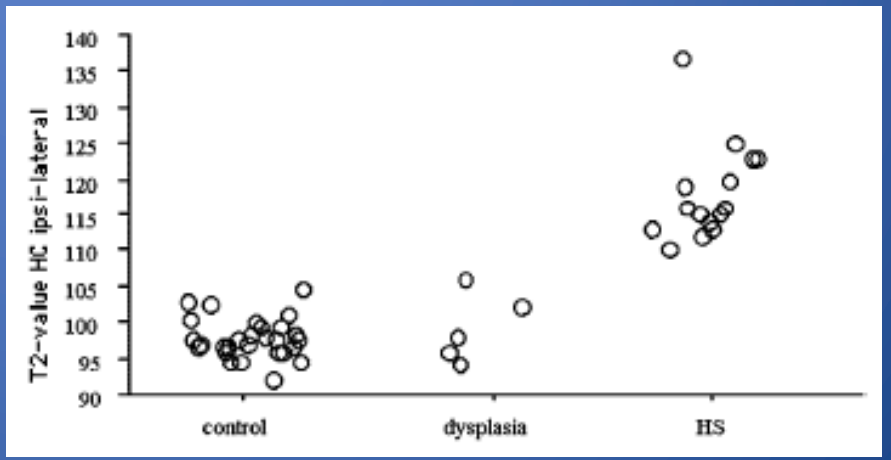
Effects of noise

- Random noise in images
  - Interferes with feature detection
  - Also causes errors in calculated parameters
- How can we assess these?
  - How does noise affect calculated quantities?
- Let's look at an example…

Hippocampal sclerosis

- Major cause of temporal lobe epilepsy
- Refractory cases may require surgery
- Where is the seizure focus?
- Localize gliosis based on T_2



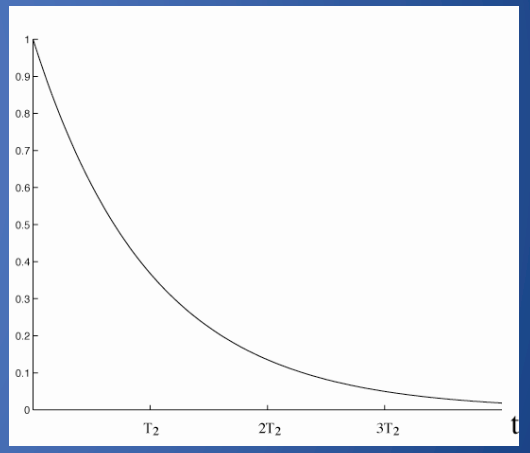


Briellmann et al, AJNR (2004)

Imaging protocol design task

- To identify the seizure focus, you want to make an accurate measurement of hippocampal T_2
- Image intensity is given by

$$S = S_0 \cdot \exp\left(-T_E / T_2\right)$$



- Acquire 2 images
  - Very short echo time (T_E=0): $S(0) = S_0$
  - Longer echo time (T_E>0): $S = S_0 \cdot \exp\left(-T_E / T_2\right)$
- What is the uncertainty in the estimated T_2?

Protocol design task

- The uncertainty in the measured T_2 is

$$\frac{\sigma_{T_2}}{T_2} = \left(\frac{\sigma}{S_0}\right) \cdot \frac{T_2}{T_E} \cdot \sqrt{1 + \exp\left(2T_E / T_2\right)}$$

- What value of T_E minimizes the uncertainty?
- What is the fractional uncertainty in the T_2 estimate at the optimal T_E?

Protocol design task

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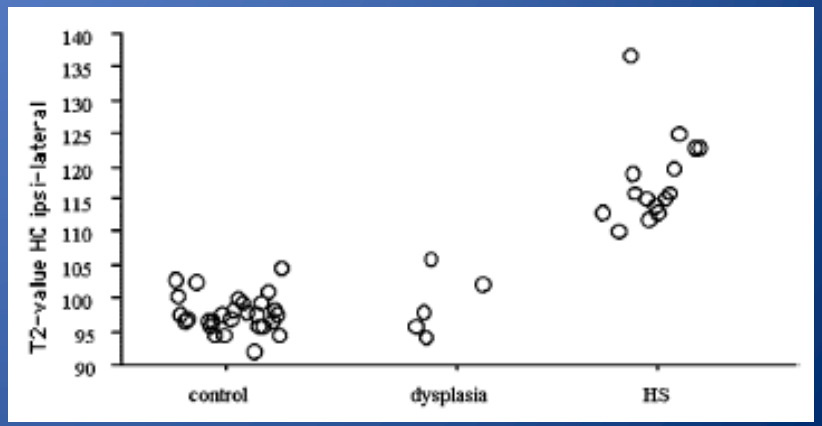
- What value of T_E minimizes the uncertainty?
- What is the fractional uncertainty in the T_2 estimate at the optimal T_E?
- What is the minimum SNR to detect hippocampal sclerosis?



Protocol design task

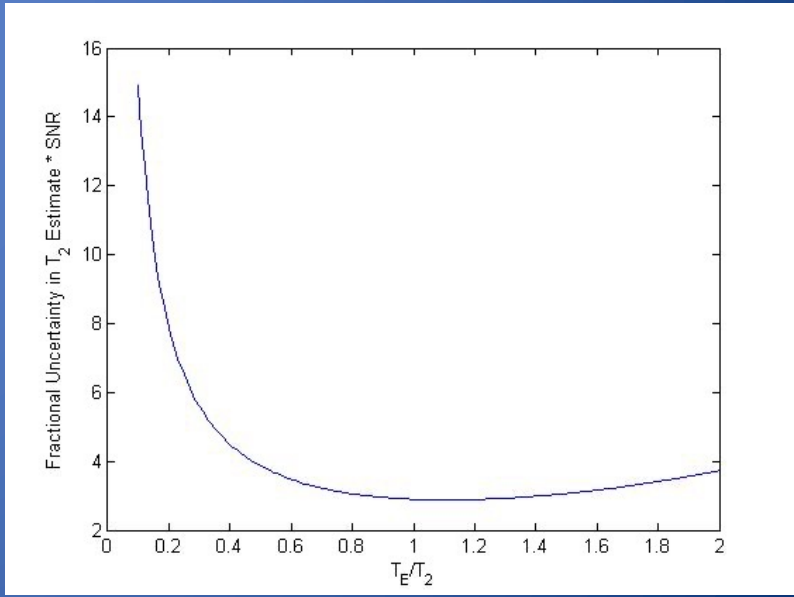
- The optimal echo time is

$$T_E \approx 1.11 \cdot T_2$$

- The fractional uncertainty at this T_E is

$$\frac{\sigma_{T_2}}{T_2} = \left(\frac{1}{SNR}\right) \cdot \frac{1}{1.11} \cdot \sqrt{1+\exp(2.22)}$$

$$= \frac{2.88}{SNR}$$



- How can SNR be improved?

In-class exercise: Improving SNR through averaging

- Calculate the average of two images:

$$\bar{S} = (S_1 + S_2)/2$$

- Suppose the noise variance, $\sigma^2(S_1) = \sigma^2(S_2) = \sigma^2$ and the noise is independent in the two images
- What is the noise variance $\sigma^2(\bar{S})$in the $\bar{S}$ image?
- Is the SNR of the average image higher?

Image accuracy

- Images can be inaccurate in many ways
  - Geometry
    - Blurring (resolution)
    - Distortion (shift variance)
    - Misplaced signal (reconstruction artifact)
  - Sensitivity
    - Undetectable features
    - Errors due to noise
    - Spatial dependence (shift variance)
    - Temporal drift

Summary

- Noise affects quantities (like T_2) calculated from image data
- Propagation of errors is a convenient way to find the uncertainty in calculated quantities
  - And to find the experimental parameters that minimize uncertainty
- Image SNR can be improved by averaging multiple images
  - Assumes stationary signal