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# Problem Set 1

**BME 7450**

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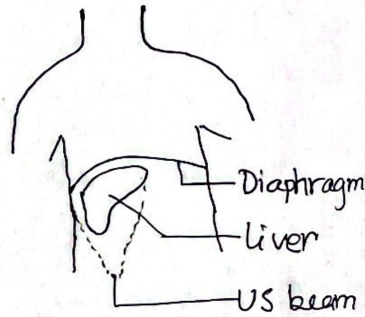
## Problem No -01

(A)

I don't think it is an abnormal diaphragm. It is just an image artifact.

(B)

This kind of discontinuous diaphragm appears because of speed displacement artifact or also known as propagation velocity artifact.




The way an US system measures depth is that, it measures the time ~~to~~ between releasing ~~#~~ a wave and receiving the echo, multiplies it with 1540 (assuming US propagation velocity for human tissue is 1540 m/s) and then halved it.

So, if the propagation velocity of US varies, there will be error in determining depth. Hence, if there is a differential variation in tissue composition under the same US beam, then, different return times to the transducer will be processed as different depths of tissue resulting ~~a~~ in a discontinuity in the displayed US image.

In case of hepatic steatosis, fat accumulates in liver which slows down the US beam ~~fat~~. which may result in discontinuous diaphragm.

Scanned with CamScanner

Problem NO-02

(A)

Since the bright object is causing streaking effect, it must be a solid, hard material. In this instance, I think it is a metal object.

(B)

When photons pass through a metal, it absorps the relatively low energy photons and attenuate the others. This phenomenon is called beam hardening.

(C)

Metal artifacts are due to a combination of beam hardening, scatter, non-linear volume effect, and noise

Beam hardening: metal absorbs the low-energy photons and hardens it. And Harder beam is less attenuated. All beams passing through a particular pixel follow different paths & therefore, experience a different degree of beam hardening. Hence, they percieve different attenuation value in that pixel causing streaks that connect objects with strong attenuation.

Scatter: Not all photons arrive to the detector follow a straight path; about 30% of photons received are due to scattering. Because of scattered photons, attenuation of a particular beam is underestimated which yields streaks in image.

Non-linear partial volume effect: Because of the finite beam width, every measurement ~~repreml~~ represents an intensity averaged over this beam width which is then log converted to calculate the integrated linear attenuation. However, there's an underestimation which gets bigger with more attenuation and can cause streaks tangent to edges.

Noise: The image reconstruction algorithm transforms measured signal noise into structured image noise. In the presence of a metal object, this results in alternating dark and bright thin streaks radiating from the metal object.

Problem 8 No- 03

We know that MRI violates shift invariance. MRI measures the net magnetization of a selected slice and reconstructs an image relative to tissue property. Since it is a relative representation, shifting an object in the ~~p~~ input causes variance in the output image.

To ~~correct this behavior, I think the slice selection is crucial. The slice should be thin~~

To understand this, if a patient's position is shifted in the scanner, the resulting image would be different, hence we can say that MRI violates shift invariance. To tackle this, I think we should change the selection of slice and select a new slice on the new position of the patient. Also, we can use image registration methods (linear and non-linear) to normalize the image into it's previous position.

Problem No-4

A

One of the methods to increase SNR is to take multiple images and average them out. This method works ~~bef~~ because noise in different images may vary and doesn't correlate.

If we acquire N images and average them, we can write,

$$S = \frac{1}{N} \sum_{i=1}^{N} S_i$$

$$\Rightarrow S = f(S_1, S_2, \dots S_N) = \frac{1}{N} (S_1 + S_2 + \dots + S_N) \quad \text{--- (1)}$$

From propagation of error formula, we can write.

$$\sigma_S^2 = \sigma_{S_1}^2 \left(\frac{\partial S}{\partial S_1}\right)^2 + \sigma_{S_2}^2 \left(\frac{\partial S}{\partial S_2}\right)^2 + \dots + \sigma_{S_N}^2 \left(\frac{\partial S}{\partial S_N}\right)^2$$

assuming noise in different images uncorrelated. We also assume, noise variance is same in each image, i.e.,

$$\sigma_{S_1}^2 = \sigma_{S_2}^2 = \dots = \sigma_{S_N}^2 = \sigma^2$$

Here, $\frac{\partial S}{\partial S_1} = \frac{1}{N}(1 + 0 + \dots + 0) = \frac{1}{N}$

Similarly, $\frac{\partial S}{\partial S_2} = \frac{1}{N}$ and $\frac{\partial S}{\partial S_N} = \frac{1}{N}$

So, from (1), we can write,

$$\sigma_S^2 = \sigma^2\left(\frac{1}{N^2} + \frac{1}{N^2} + \dots + \frac{1}{N^2}\right) = \frac{\sigma^2 N}{N^2} = \frac{\sigma^2}{N}$$

$$\Rightarrow \sigma_S = \frac{\sigma}{\sqrt{N}}$$

We know that, $SNR = \dfrac{\text{mean (Signal)}}{\text{std (Noise)}}$

$$\therefore \quad SNR(N) = \frac{S}{\sigma_S} = \frac{S\sqrt{N}}{\sigma} \quad \text{---} \; (2)$$

(B)

From (2), we can say that to double the SNR, we need to take four times the number of image compared to previously, i.e., $\boxed{N_2 = 4N_1}$

And to increase SNR by a factor of 100,

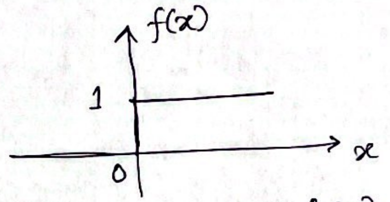
$$\boxed{N_2 = 10000\,N_1}$$

That's is 10,000 times more images are needed.

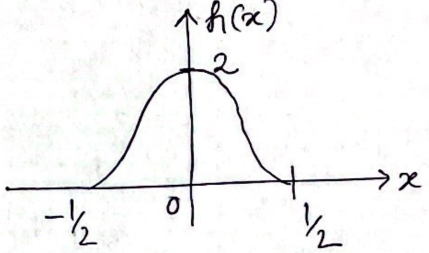
This leads to the practical limitation of this method. To improve SNR, the number of images needed increases exponentially which is practically not effiecient.

## Problem NO-05

(A) Here, $f(x) = \begin{cases} 1, & x \geq 0 \\ 0, & x < 0 \end{cases}$



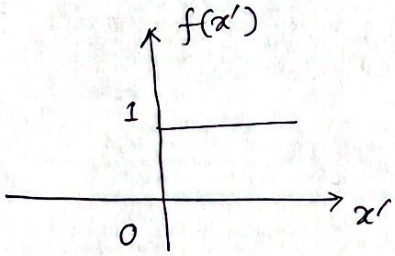

$h(x) = \begin{cases} 1 + \cos 2\pi x, & |x| \leq \frac{1}{2} \\ 0, & \text{otherwise} \end{cases}$

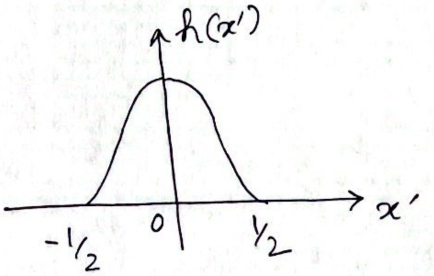


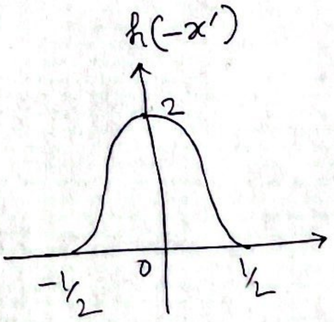

$g(x) = f(x) * h(x)$

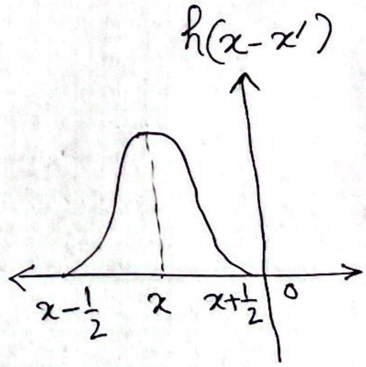
$= \int_{-\infty}^{\infty} f(x')\, h(x - x')\, dx'$

### Graphical Representation:



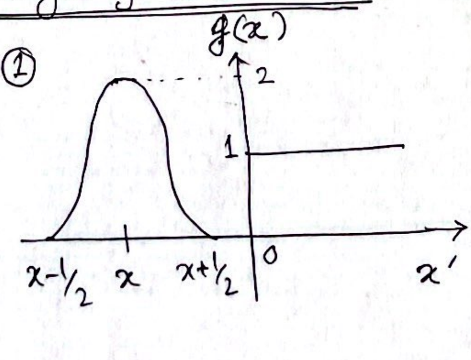



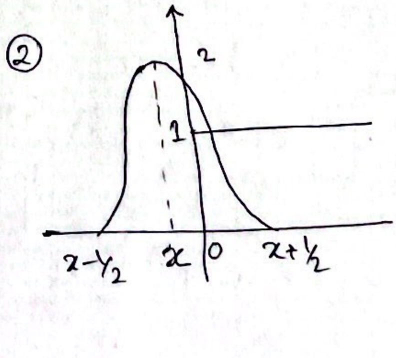




### Stages of convolution:

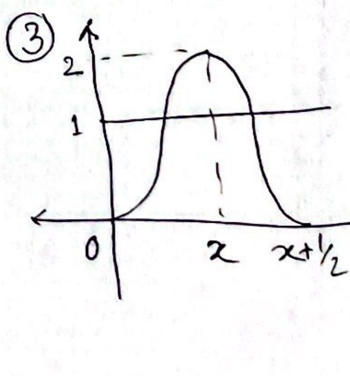
(1)




(2)




(3)



① when $x+\frac{1}{2}<0 \Rightarrow x<-\frac{1}{2}$

$g(x)=0$ because there is no overlap.

② when $0 \leq x+\frac{1}{2} \leq 1 \Rightarrow -\frac{1}{2} \leq x \leq \frac{1}{2}$

$$g(x) = \int_0^{x+1/2} f(x')\, g\, h(x-x')\, dx'$$

$$= \int_0^{x+1/2} 1 \, . \, 1+\cos 2\pi(x-x') \, . \, dx'$$

$$= \left[ x' + \frac{\sin 2\pi(x-x')}{-2\pi} \right]_0^{x+1/2}$$

$$= \left[ x+\frac{1}{2} - \frac{\sin(-\pi)}{2\pi} \right] - \left[ 0 - \frac{\sin 2\pi x}{2\pi} \right]$$

$$= x+\frac{1}{2} + \cancel{\sin}\frac{2\pi x}{2\pi}$$

③ when $x-\frac{1}{2}>0$ or $x>\frac{1}{2}$

$$g(x) = \int_{x-1/2}^{x+1/2} 1 \, . \, 1+\cos 2\pi(x-x') \, . \, dx'$$

$$= \left[ x' + \frac{\sin 2\pi(x-x')}{-2\pi} \right]_{x-1/2}^{x+1/2}$$

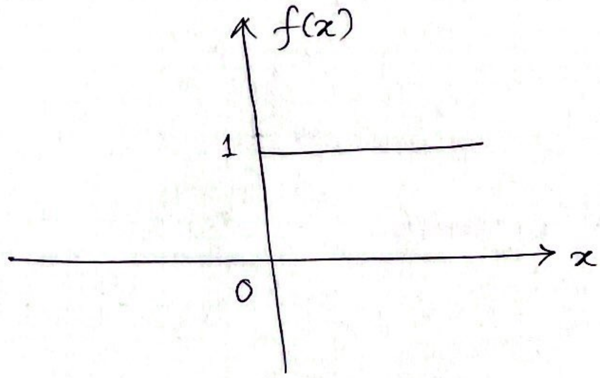
$$= \left[ x+\frac{1}{2} - \frac{\sin(-\pi)}{2\pi} \right] - \left[ x-\frac{1}{2} - \frac{\sin \pi}{2\pi} \right]$$

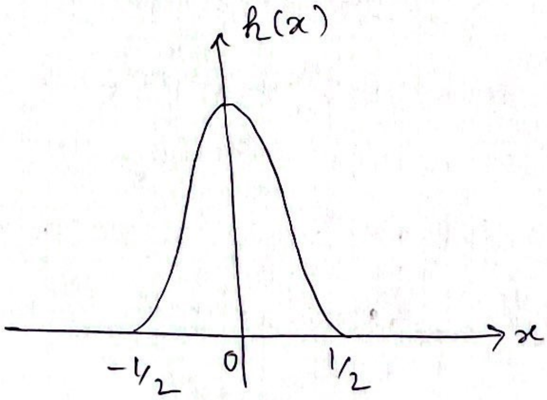
$$= x+\frac{1}{2} - x + \frac{1}{2}$$

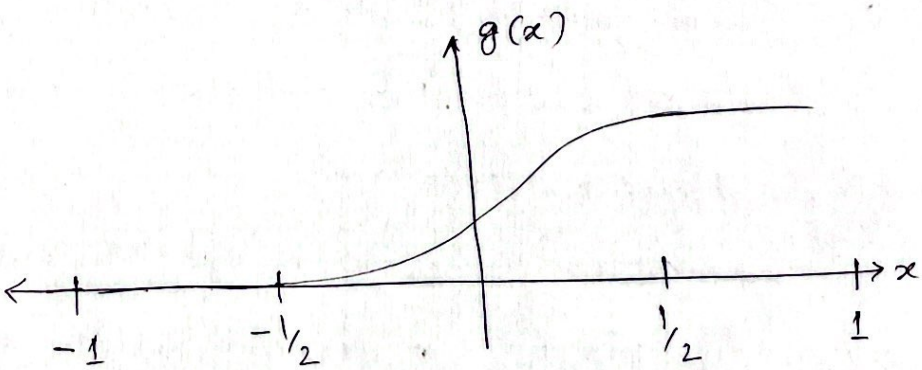
$$= 1$$

So, $g(x) = \begin{cases} 0 & , \quad x < -\frac{1}{2} \\ x + \frac{1}{2} + \frac{\sin 2\pi x}{2\pi} & , \quad -\frac{1}{2} \leq x \leq \frac{1}{2} \\ 1 & , \quad x > \frac{1}{2} \end{cases}$

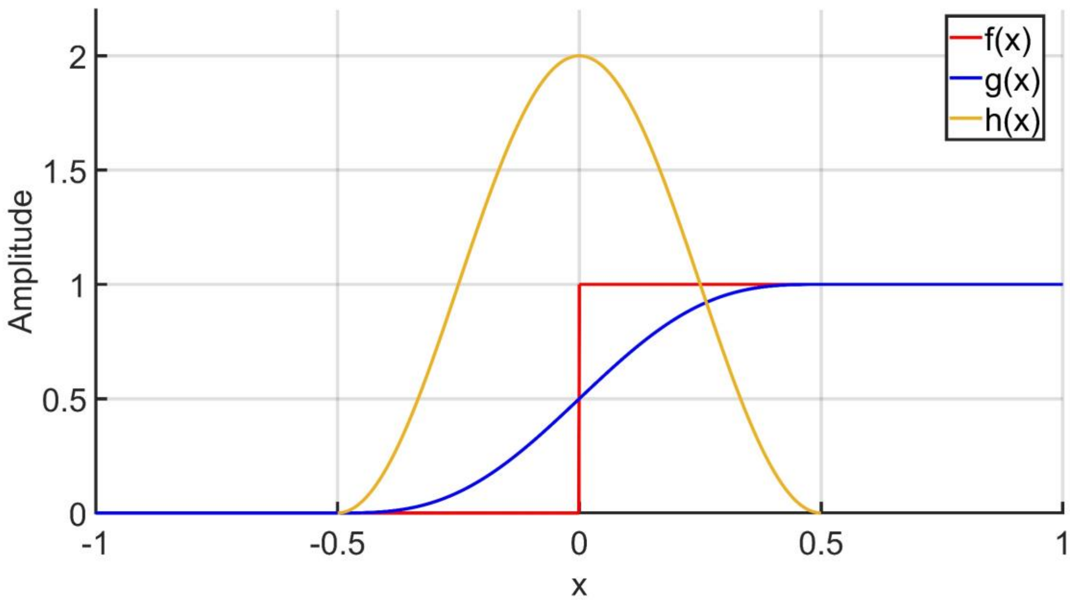
(B)




From the plot of $g(x)$, we can say that the PSF blurrs the sharp edge at $x = 0$ and makes the intensity going gradually up.



f(x)
g(x)
h(x)
Amplitude
2
1.5
1
0.5
0
-1
-0.5
0
0.5
1
x