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1. $F(\frac{1}{80}) = 480 \sin(\frac{\pi}{80}) + 160(1 - \frac{1}{80}) - 480$ (+10)

iter	a	b	m	f(a)	f(b)	f(m)
0	1	75	38	-312.5	+76.98	-58.26
1	38	75	56.5	-58	+77	+30
2	38	56.5	47.25	-58	+30	-9.5
3	47.25	56.5	51.875	-9.5	+30	+11.5

m = 49.56 (+1)

← (+1) each

2.

$$V_z = \frac{C_1}{z} + C_2 z^2 + C_3 z + C_4$$

(+10)
$$\underline{\hat{A}} = \begin{pmatrix} \frac{1}{z_1} & z_1^2 & z_1 & 1 \\ \frac{1}{z_2} & z_2^2 & z_2 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ \frac{1}{z_n} & z_n^2 & z_n & 1 \end{pmatrix}$$

(+2)
$$\underline{\hat{V}}_z = \begin{pmatrix} V_z^{(1)} \\ V_z^{(2)} \\ \vdots \\ V_z^{(n)} \end{pmatrix}$$

(+9)
$$\underline{\hat{C}} = \underbrace{[\underline{\hat{A}}^T \underline{\hat{A}}]^{-1} \underline{\hat{A}}^T}_{\underline{\hat{K}}} \underline{\hat{V}}_z = \underline{\hat{K}} \underline{\hat{V}}_z$$

(+5)
$$\sigma_v^2 = S_v^2 = \frac{1}{n-4} (\underline{\hat{A}} \underline{\hat{C}} - \underline{\hat{V}}_z)^T (\underline{\hat{A}} \underline{\hat{C}} - \underline{\hat{V}}_z)$$

(+5)
$$\sum_c^2 = \sigma_v^2 \underline{\hat{K}} \underline{\hat{K}}^T$$

(+5)
$$\sum_{V_z^{(m)}}^2 = \underline{\hat{A}}^{(m)} \sum_c^2 \underline{\hat{A}}^{(m)T}$$

where

(+2)
$$\underline{\hat{A}}^{(m)} = \begin{pmatrix} \frac{1}{z_1} & z_1^2 & z_1 & 1 \\ \frac{1}{z_2} & z_2^2 & z_2 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ \frac{1}{z_n} & z_n^2 & z_n & 1 \end{pmatrix}$$

 ↑ at z points where we want a prediction

(+2)
$$\sigma_{V_z^{(m)}}^2 = \text{diag} \left\{ \sum_{V_z^{(m)}}^2 \right\}$$

$$3. S_d^2 = \left(\frac{\partial f}{\partial V} \bigg|_{\bar{V}} \right)^2 S_V^2 \quad (+4)$$

(15)

$$d = f(V) = \left(\frac{6V}{\pi} \right)^{1/3}$$

$$\frac{\partial f}{\partial V} = \frac{1}{3} \frac{6}{\pi} \left(\frac{6V}{\pi} \right)^{-2/3} \quad (+4) \quad @ \bar{V} \leadsto \frac{2}{\pi} \left(\frac{6\bar{V}}{\pi} \right)^{-2/3}$$

$$\leadsto S_d^2 = \cancel{\frac{1}{9} \frac{36}{\pi^2}} \frac{4}{\pi^2} \left(\frac{6\bar{V}}{\pi} \right)^{-4/3} S_V^2$$

$$\bar{d} = \left(\frac{6 \cdot 623.6}{\pi} \right)^{1/3} = 10.6 \mu m$$

~~$\frac{\partial f}{\partial V}$~~

$$S_d^2 = 3.21 \times 10^{-5} \cdot (289.1)^2 = 2.68$$

$$S_d = 1.64 \mu m \quad (+4)$$

$$\leadsto \bar{d} = 10.6 \pm 1.6 \text{ (s.d.) } \mu m$$