Hypothesis Testing (Ch. 4)

- The Null Hypothesis: disproving a scientific theory is much easier than proving one.
- Thus,
  - "diabetic patients do not have raised blood pressure"
  - "oral contraceptives do not cause breast cancer"

Statistical Inference

- Hypothesis Testing: method of deciding whether the data are consistent with the null hypothesis.
- Given a study with a single outcome measure and a statistical test, hypothesis testing can be summarized in three steps:
    1. Choose a significance level, α, of the test.
    2. Conduct the study, observe the outcome, and compute the p-value.
    3. …

Statistical Inference

3. If the p-value <= α $\rightarrow$ data are not consistent with the null hypothesis.

   If p-value > α, do not reject the null hypothesis, and view it as "not yet disproven".

- Do not confuse the significance level and the p-value!
- If one rejects the null hypothesis when it is in fact true, then one makes a <u>Type I error</u>.
- The significance level α is the probability of making a Type I error. This is set <u>before</u> the test is carried out. The p-value is the result observed <u>after</u> the study is completed.

Hypothesis Testing (Ch. 4)

- Two Sample z-Test: We wish to test the null hypothesis that the means of two populations, estimated from two independent samples, are equal, when the samples are large.
- Sample 1: number of subjects n_1, mean $\overline{x}_1$, standard deviation s_1.
- Sample 2: number of subjects n_2, mean $\overline{x}_2$, standard deviation s_2.

Two Sample z-Test

- Assumptions:
    1. data are normally distributed
    2. data are independent
    3. samples are large (> ~30 for n_1, n_2)
- Calculate the <u>standard error of the difference between the means</u>:

$$SE(\overline{x}_1 - \overline{x}_2) = (s_1^2/n_1 + s_2^2/n_2)^{½}$$

$$z = (\overline{x}_1 - \overline{x}_2)/SE(\overline{x}_1 - \overline{x}_2)$$

Two Sample z-Test

- Under the null hypothesis, z is distributed approximately as a normal distribution with mean = 0 and standard deviation = 1.
- A 95% confidence interval for the difference is:

$$(\overline{x}_1 - \overline{x}_2) \pm 1.96\ SE(\overline{x}_1 - \overline{x}_2)$$

Two Sample z Test

Diastolic: during expansion of the heart

- Example of 2-sample z-test:
- Diabetics' diastolic blood pressure: n_1=100, $\overline{x}_1$=135mmHg, s_1=10mmHg
- Controls' diastolic blood pressure: n_2=90, $\overline{x}_2$=125mmHg, s_2=6mmHg

$$SE(\overline{x}_1 - \overline{x}_2) = 1.18, \ \ z = 8.47$$

- from Table of Normal distribution:

$$z = 3.09 \rightarrow p=0.002,$$

$$\text{thus } z=8.47 \rightarrow p<0.002$$

Two-Sample z-Test

- A 95% confidence interval for the difference in blood pressure between the groups is:

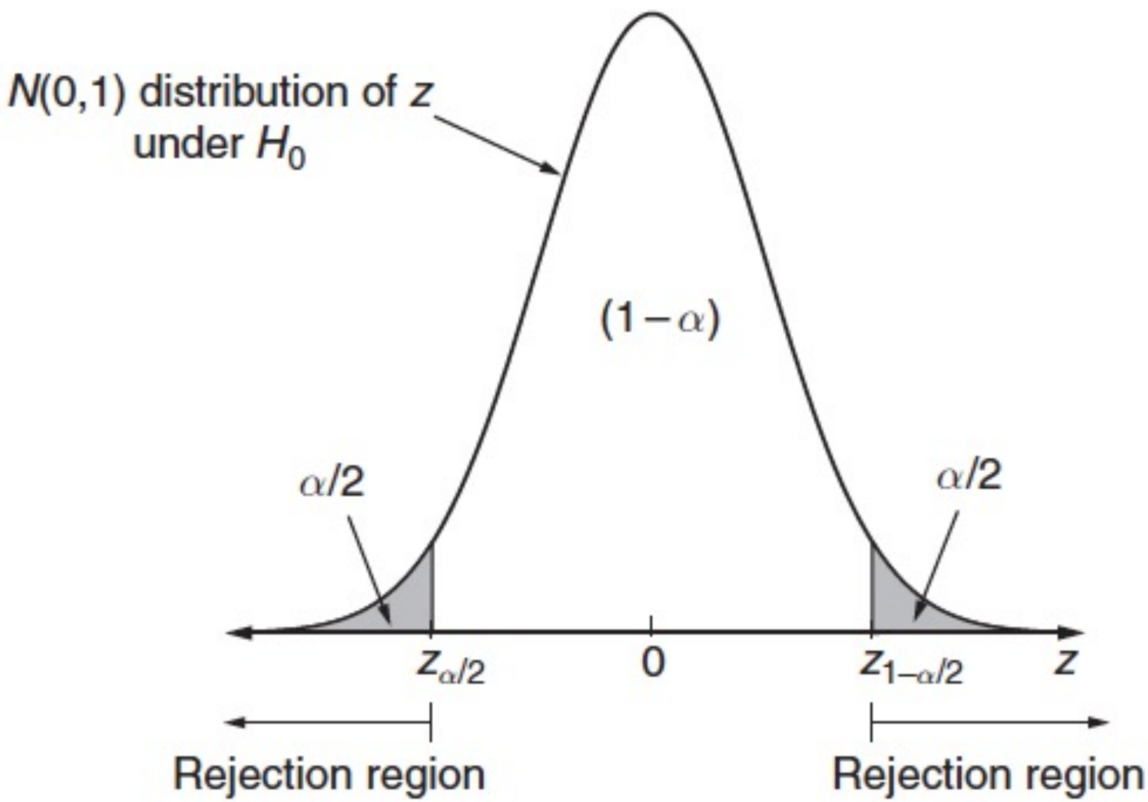
  $$10 \pm 1.96 \times 1.18$$

  $$7.7 - 12.3 \text{ mmHg}$$

- Does not intersect 0 → reject null hypothesis

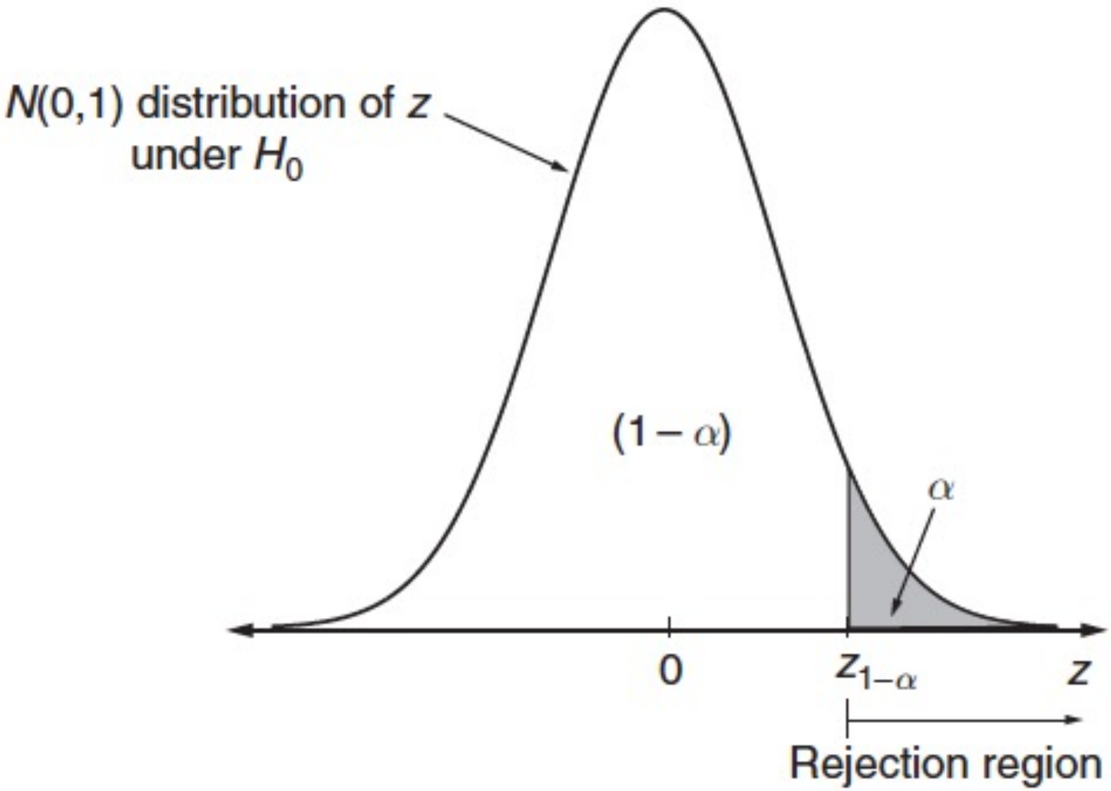
Two-sided test

Two-sided z test; z follows the $N(0,1)$ distribution curve when H_0 is true. The shaded areas are the rejection regions.



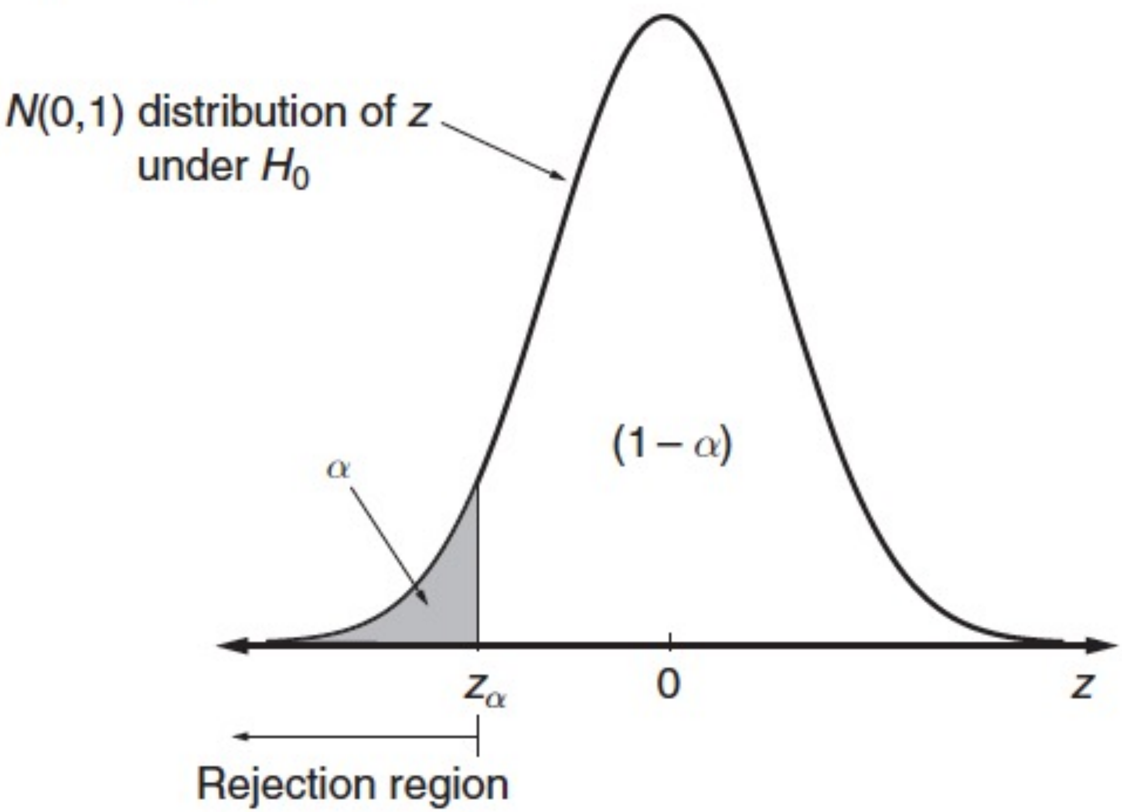
One-sided test

One-sided z test for an alternate hypothesis with positive directionality. The shaded area located in the right tail is the rejection region.



The "other" one-sided test

One-sided z test for an alternate hypothesis with negative directionality. The shaded area located in the left tail is the rejection region.



Two Sample z Test of Proportions

- Provided the sample sizes in the two groups are large, same method can be used on two proportions.
- Population mean $\mu_i \leftarrow\rightarrow$ sample mean x_i
- Population proportions of success: π_A, π_B

  ~ sample statistics: p_A, p_B
- Standard error for difference $p_1 - p_2$ given by:

$$SE(p_1 - p_2) = (p_1(1-p_1)/m + p_2(1-p_2)/n)^{1/2}$$

(Note difference from population means)

Two sample z-test of proportions

- Example: Clinical trial by Familiari et al., (1981) [Clin. Trial J., 18:383] comparing two drugs for treatment of peptic ulcers.

Drug	Healed	Not Healed	Total	% healed
A: Pirenzepine	23 (a)	7 (c)	30 (m)	76.67
B: Trithiozine	18 (b)	13 (d)	31 (n)	58.06
Total	41	20	61 (N)	

n_P=30, p_P = a/m = 0.7667

n_T=31, p_T = b/n = 0.5806

Two sample z test of proportions

$$\bar{d} = 0.1861, \quad SE(\bar{d}) = 0.1175$$

- The 95% confidence interval for δ is:

$$-0.0442 \text{ to } +0.4164$$

- Although there is an observed advantage of 0.1861 (19%) for pirenzepine over trithiozine, the 95% confidence interval includes the null hypothesis value of zero.

The p-value

- The choice of 95% for a confidence interval is quite arbitrary, although it has become conventional in the medical literature.
- A general $100(1 - \alpha)\%$ confidence interval can be calculated using:

$$\overline{d} \pm z_\alpha \times SE(\overline{d})$$

- z_α is the value along the Normal distribution table which leaves a total probability of a equally divided in the two tails.

The p-value

- If α=0.05, then 100(1 – α)% = 95%, z_α=1.96, and the 95% confidence interval is given as:

$$\bar{d} \pm 1.96 \times SE(d)$$

- In the comparison of the two treatments for peptic ulcer, a more general confidence interval for δ is:

$$0.1861 \pm (z_\alpha \times 0.1175)$$

The p-value

- Suppose that z_α is now chosen such that the lower limit equals zero. That is, the confidence interval just includes the null hypothesis value of $\delta = \pi_A - \pi_B = 0$.

$$0.1861 - (z_\alpha \times 0.1175) = 0$$

$$z_\alpha = 0.1861/0.1175 = 1.58$$

- From the Normal distribution table, α=0.11 and $100(1 - \alpha)\% = 89\%$.

The p-value

- The 89% confidence interval is:

  $0.1861 \pm (1.58 \times 0.1175)$

  or $0 – 0.37$.

- The calculated value of α is termed the p-value.
- The p-value is the probability of obtaining the observed difference (or other extreme) if the null hypothesis is true.