Ranks

- Practical differences between parametric and NP are that NP methods use the ranks of values rather than the actual values
- E.g.
  1,2,3,4,5,7,13,22,38,45 - actual
  1,2,3,4,5,6, 7, 8, 9,10 - rank

Median

- The median is the value above and below which 50% of the data lie.
- If the data is ranked in order, it is the middle value
- In symmetric distributions the mean and median are the same
- In skewed distributions, median more appropriate

Median

- BPs:
  135, 138, 140, 140, 141, 142, 143
  Median=140

- No. of cigarettes smoked:
  0, 1, 2, 2, 2, 3, 5, 5, 8, 10
  Median=2.5

Wilcoxon Signed Rank Test

- NP test relating to the median as measure of central tendency
- The ranks of the absolute differences between the data and the hypothesised median calculated
- The ranks for the negative and the positive differences are then summed separately (W_- and W_+ resp.)
- The minimum of these is the test statistic, W

Wilcoxon Signed Rank Test: Example

The median heart rate for an 18 year old girl is supposed to be 82bpm. A student takes the pulse rates of 8 female students (all aged 18):

83, 90, 96, 82, 85, 80, 81, 87

Do these results suggest that the median might not be 82?

Wilcoxon Signed Rank Test: Example

H_0: median=82

H_1: median≠82

Two-tailed test

Because one result equals 82 this cannot be used in the analysis

Wilcoxon Signed Rank Test: Example

Result	Above or below median	Absolute difference from median=82	Rank of difference
83	+	1	1.5
90	+	8	6
96	+	14	7
85	+	3	4
80	-	2	3
81	-	1	1.5
87	+	5	5

W_+= 1.5+6+7+4+5=23.5 W_-= 3+1.5=4.5 So, W=4.5

n=7, so the value of W > tabulated value of 2, so p>0.05

Wilcoxon table: note that for significance, our W should be smaller

	alpha values						
n	0.001	0.005	0.01	0.025	0.05	0.10	0.20
5	--	--	--	--	--	0	2
6	--	--	--	--	0	2	3
7	--	--	--	0	2	3	5
8	--	--	0	2	3	5	8
9	--	0	1	3	5	8	10
10	--	1	3	5	8	10	14
11	0	3	5	8	10	13	17
12	1	5	7	10	13	17	21
13	2	7	9	13	17	21	26
14	4	9	12	17	21	25	31
15	6	12	15	20	25	30	36
16	8	15	19	25	29	35	42
17	11	19	23	29	34	41	48
18	14	23	27	34	40	47	55
19	18	27	32	39	46	53	62
20	21	32	37	45	52	60	69
21	25	37	42	51	58	67	77
22	30	42	48	57	65	75	86
23	35	48	54	64	73	83	94
24	40	54	61	72	81	91	104
25	45	60	68	79	89	100	113
26	51	67	75	87	98	110	124
27	57	74	83	96	107	119	134

	alpha values						
n	0.001	0.005	0.01	0.025	0.05	0.10	0.20
28	64	82	91	105	116	130	145
29	71	90	100	114	126	140	157
30	78	98	109	124	137	151	169
31	86	107	118	134	147	163	181
32	94	116	128	144	159	175	194
33	102	126	138	155	170	187	207
34	111	136	148	167	182	200	221
35	120	146	159	178	195	213	235
36	130	157	171	191	208	227	250
37	140	168	182	203	221	241	265
38	150	180	194	216	235	256	281
39	161	192	207	230	249	271	297
40	172	204	220	244	264	286	313
41	183	217	233	258	279	302	330
42	195	230	247	273	294	319	348
43	207	244	261	288	310	336	365
44	220	258	276	303	327	353	384
45	233	272	291	319	343	371	402
46	246	287	307	336	361	389	422
47	260	302	322	353	378	407	441
48	274	318	339	370	396	426	462
49	289	334	355	388	415	446	482
50	304	350	373	406	434	466	503

Wilcoxon Signed Rank Test: Example

Therefore, the student should conclude that these results could have come from a population which had a median of 82 as the result is not significantly different to the null hypothesis value.

Wilcoxon Signed Rank Test Normal Approximation

- As the number of ranks (n) becomes larger, the distribution of W becomes approximately Normal
- Generally, if n>20
- Mean W=n(n+1)/4
- Variance W=n(n+1)(2n+1)/24
- Z=(W-mean W)/SD(W)

Wilcoxon Signed Rank Test Assumptions

- Population should be approximately symmetrical but need not be Normal
- Results must be classified as either being greater than or less than the median ie exclude results=median
- Can be used for small or large samples

Wilcoxon signed rank test

- To test difference between paired data

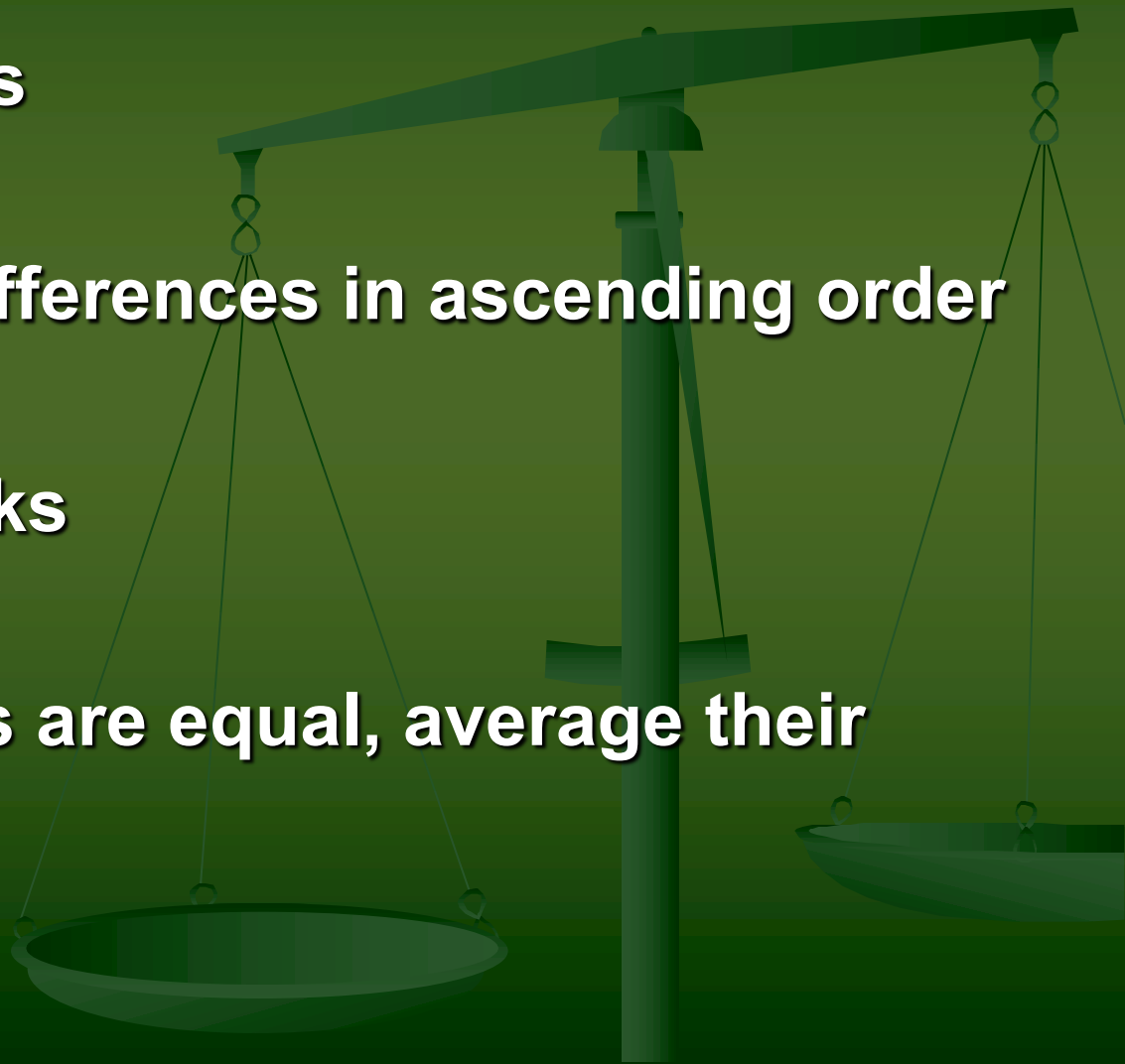
EXAMPLE

Patient	Hours of sleep	
	Drug	Placebo
1	6.1	5.2
2	7.0	7.9
3	8.2	3.9
4	7.6	4.7
5	6.5	5.3
6	8.4	5.4
7	6.9	4.2
8	6.7	6.1
9	7.4	3.8
10	5.8	6.3

Null Hypothesis: Hours of sleep are the same using placebo & the drug

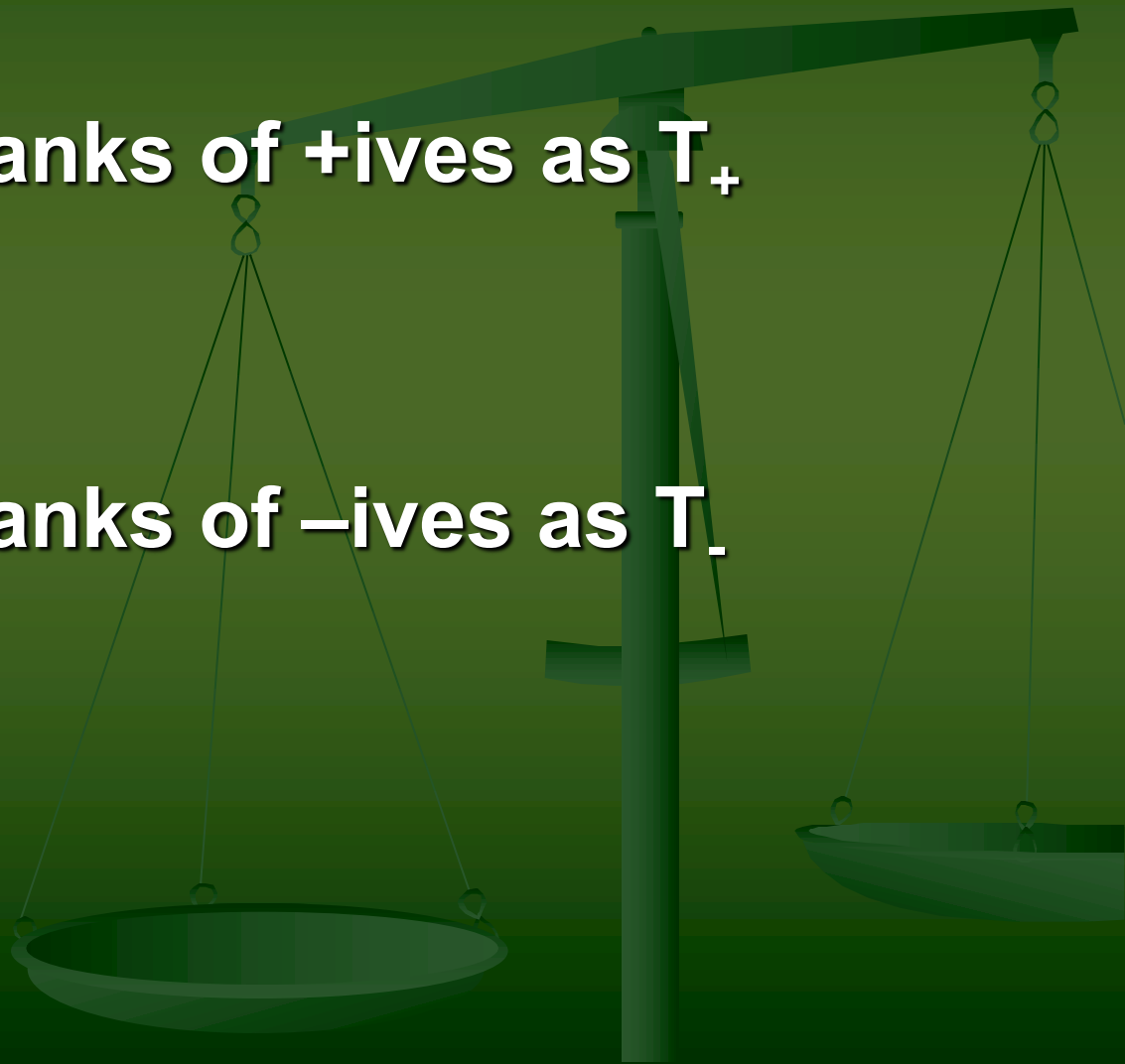
STEP 1

- Exclude any differences which are zero
- Ignore their signs
- Put the rest of differences in ascending order
- Assign them ranks
- If any differences are equal, average their ranks



STEP 2

- Count up the ranks of +ives as T_+
- Count up the ranks of –ives as T_-



STEP 3

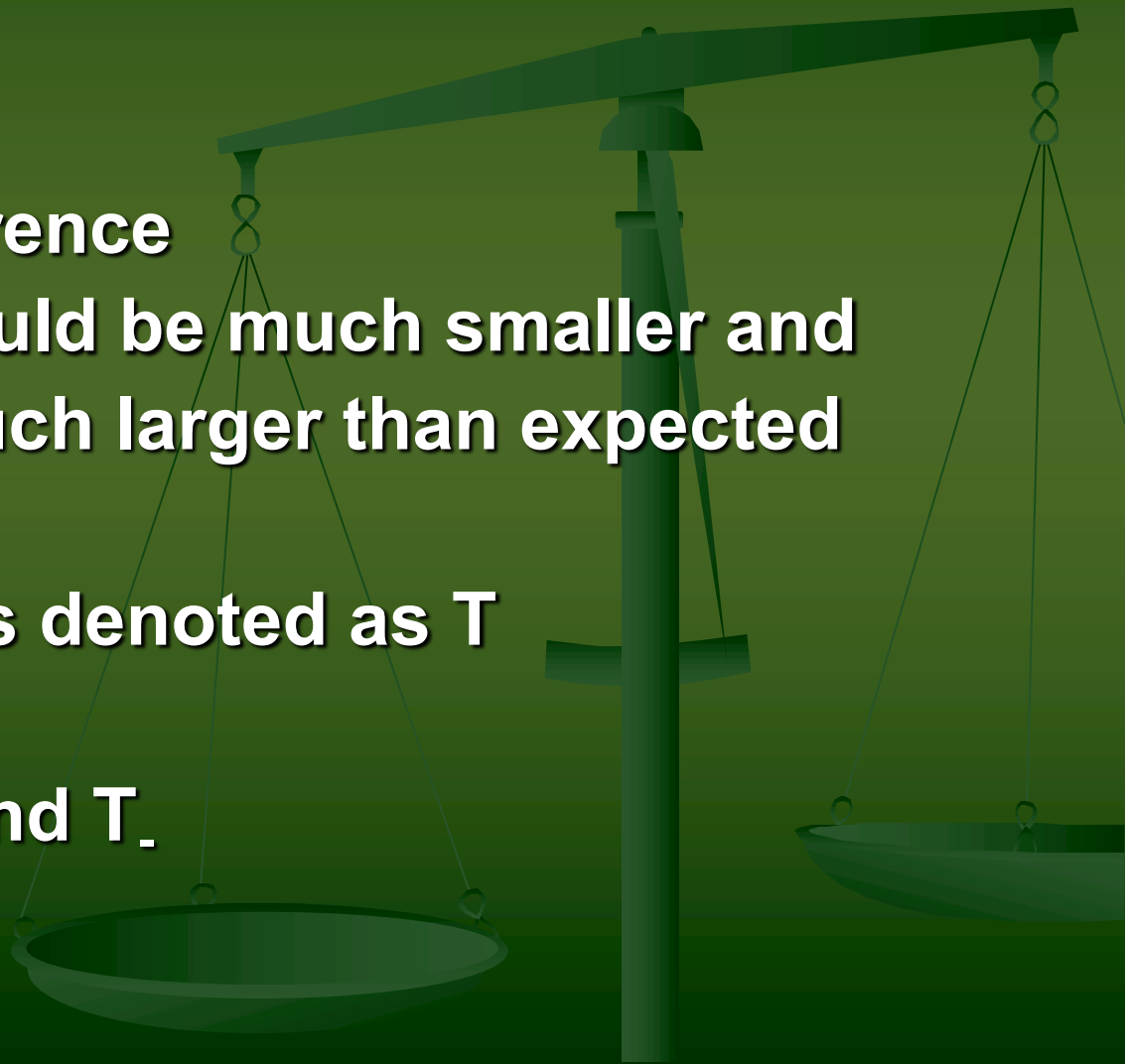
- If there is no difference between drug (T_+) and placebo (T_-), then T_+ & T_- would be similar

- If there is a difference

  one sum would be much smaller and the other much larger than expected

- The larger sum is denoted as T

- T = larger of T_+ and T_-



STEP 4

- Compare the value obtained with the critical values (5%, 2% and 1%) in table
- N is the number of differences that were ranked (**not the total number of differences**)
- So the zero differences are excluded

Patient	Hours of sleep		Difference	Rank Ignoring sign
	Drug	Placebo		
1	6.1	5.2	0.9	3.5*
2	7.0	7.9	-0.9	3.5*
3	8.2	3.9	4.3	10
4	7.6	4.7	2.9	7
5	6.5	5.3	1.2	5
6	8.4	5.4	3.0	8
7	6.9	4.2	2.7	6
8	6.7	6.1	0.6	2
9	7.4	3.8	3.6	9
10	5.8	6.3	-0.5	1

3^{rd} & 4^{th} ranks are tied hence averaged; T= larger of T_+ (50.5) and T_- (4.5)

Here, calculated value of T= 50.5; tabulated value of T= 47 (at 5%)

significant at 5% level indicating that the drug (hypnotic) is more effective than placebo

In Matlab: signrank.m

signrank(x)

or

signrank(x,y)