

# The p value

- Example: The mean change in blood pressure after treatment in 36 patients with hypertension is 5.0mmHg with standard deviation = 15mmHg. Such data may arise when the subject serves as his or her own control. (e.g., blood pressure may be recorded before commencement of treatment, and then after 1 week of treatment.)
- Null hypothesis: there is no effect of treatment on blood pressure.

# Paired differences

- If the change in B.P.  $d$  is calculated for each patient and the null hypothesis is true, then the 36  $d$ 's should be close to zero.
- The  $d$ 's are termed the paired differences.  
 $\bar{d} = \Sigma d/n = 5.0$      $SD = (\Sigma (d - \bar{d})^2 / (n - 1))^{1/2} = 15.0$
- Thus,  $SE(\bar{d}) = SD(d)/\text{sqrt}(n) = 2.5$   
and  $z = 5.0/2.5 = 2.0$
- $z$  determines  $p$  as 0.046 (from normal dist'n table)
- A statistical significance test considers this  $p$ -value.

# Statistical Inference

- Hypothesis Testing: method of deciding whether the data are consistent with the null hypothesis.
- Given a study with a single outcome measure and a statistical test, hypothesis testing can be summarized in three steps:
  1. Choose a significance level,  $\alpha$ , of the test.
  2. Conduct the study, observe the outcome, and compute the p-value.
  3. ...

# Statistical Inference

3. If the p-value  $\leq \alpha \rightarrow$  data are not consistent with the null hypothesis.

If p-value  $> \alpha$ , do not reject the null hypothesis, and view it as “not yet disproven”.

- Do not confuse the significance level and the p-value!
- If one rejects the null hypothesis when it is in fact true, then one makes a Type I error.
- The significance level  $\alpha$  is the probability of making a Type I error. This is set before the test is carried out. The p-value is the result observed after the study is completed.

# Small samples of continuous data

- Student's t-distribution: If samples are small,  $\bar{x}$  and  $s$  will not be close to  $\mu$  and  $\sigma$ .
- (In one way sample size is already taken into account through the calculation of the standard deviation of the mean,  $SE(\bar{x})$  when dividing by  $\sqrt{n}$ . )
- In small samples, however, values of  $s$  far from  $\sigma$  are not uncommon.

# Small samples of continuous data

- → although  $\bar{d}$  will still have a Normal distribution, it can no longer be assumed the ratio  $\bar{d}/SD(\bar{d})$  will.
- Our previous analyses were based on the assumption that  $s$  (sample) was close to  $\sigma$  (population).
- We must modify the calculation of both the p-value and a confidence interval.

# Small samples of continuous data

- First, relabel the ratio as **t** instead of **Z** to avoid confusion.
- The confidence interval  $Z_{\alpha}$  is replaced by  $t_{\alpha}$ , to yield

$$\bar{d} \pm t_{\alpha} \times SE(\bar{d})$$

- The ratio  $\bar{d}/SE(\bar{d})$  is then known as the Student's t-statistic and under the null hypothesis is assumed to be distributed as the Student's t-distribution.

# Small samples of continuous data

- $t_{\alpha}$  depends not only on  $\alpha$  but also on the number of degrees of freedom, df, on which  $\sigma$  is estimated.
- The number of degrees of freedom depends on two factors:
  1. The number of groups we wish to compare;
  2. The number of parameters we need to estimate to calculate the standard deviation of the contrast of interest.
- (calculation of df depends on the particular problem...)



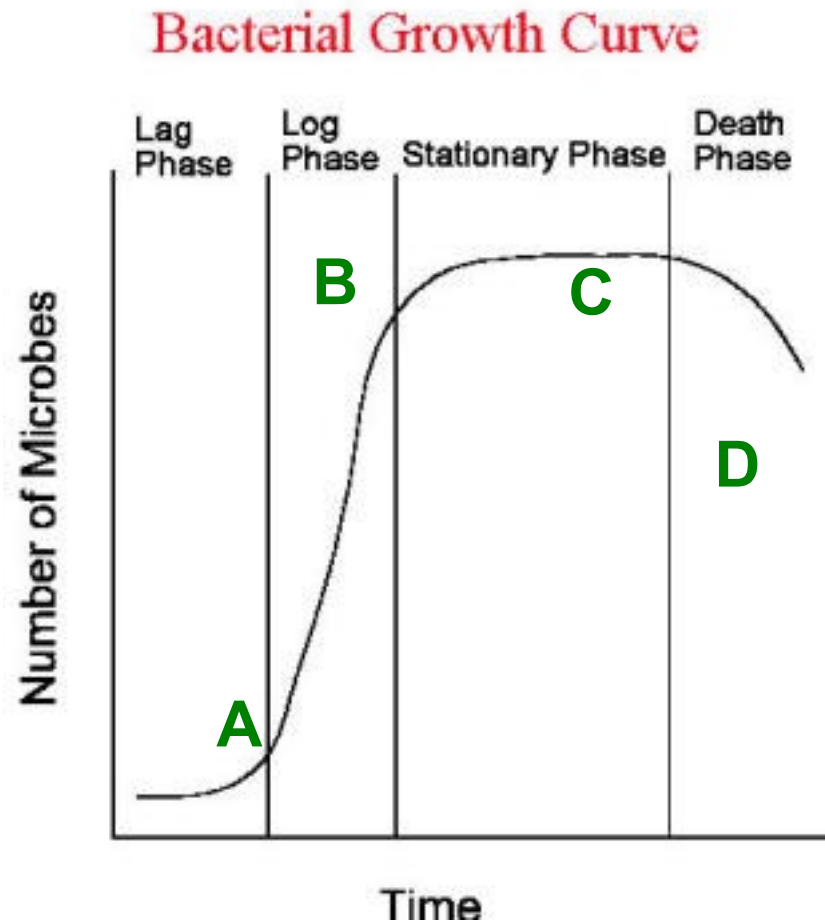
# The t-statistic

- Given  $df$  and  $\alpha$ ,  $t_{\alpha}$  is obtained from the Student's t-distribution table.
- For  $df = \infty$ ,  $t_{\alpha}$  is equal to  $Z_{\alpha}$  as read off of the Normal distribution table.
- $t_{\alpha} \downarrow$  as  $df \uparrow$ , reflecting the increasing uncertainty concerning the estimate of  $s$  as sample sizes get smaller.
- There are two main types of t-tests: Paired and Unpaired.

# Two types of clinical trial

- Cross-over trial: subjects receive both the test and control treatments in a randomized order. Useful in stable, chronic diseases such as diabetes or arthritis, where the purpose of treatment is palliation and not cure.
- Matched case-control study: identifies group of persons with the disease, and control group without the disease, and examines the relationship of a risk factor to the disease. Control group is picked to resemble diseased group w.r.t. certain risk factor (e.g., % of smokers).

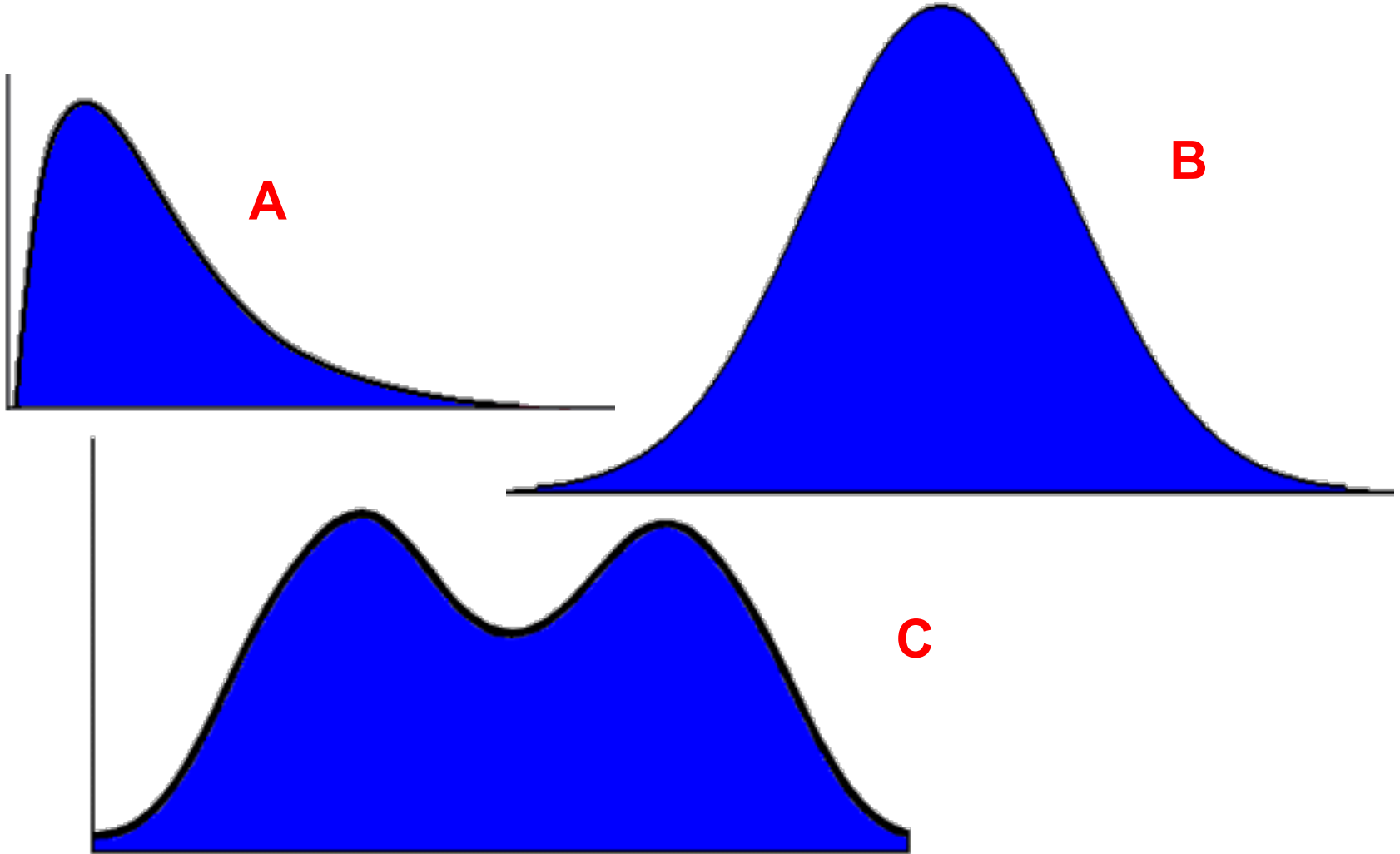
**Q1: If an adaptive step size integration is performed, which region of the function will permit the largest step size?**



**Q2: Which of the following is NOT a “null hypothesis”?**

- A. Diabetic patients do not have raised blood pressure.**
- B. Oral contraceptives do not cause breast cancer.**
- C. Smoking cigarettes increases the risk of heart disease.**

**Q3: Which of the following is an appropriate population for a two sample z-test?**



# Experiment

- Develop hypotheses
- Collect sample/Conduct experiment
- Calculate test statistic
- Compare test statistic with what is expected when  $H_0$  is true
  - Reference distribution
  - Assumptions about distribution of outcome variable

# Example: Hypertension/Cholesterol

- Mean cholesterol hypertensive men
- Mean cholesterol in male general population (20-74 years old)
- In the 20-74 year old male population the **mean** serum cholesterol is 211 mg/ml with a **standard deviation** of 46 mg/ml

# Cholesterol Hypotheses

- $H_0: \mu_1 = \mu_2$
- $H_0: \mu = 211 \text{ mg/ml}$ 
  - $\mu = \text{population}$  mean serum cholesterol for male hypertensives
  - Mean cholesterol for hypertensive men = mean for general male population
- $H_A: \mu_1 \neq \mu_2$
- $H_A: \mu \neq 211 \text{ mg/ml}$



# Cholesterol Sample Data

- 25 hypertensive men
- Mean serum cholesterol level is 220mg/ml  
( $\bar{X} = 220$  mg/ml)
  - Point estimate of the mean
- Sample standard deviation:  $s = 38.6$  mg/ml
  - Point estimate of the variance =  $s^2$

# Experiment

- Develop hypotheses
- Collect sample/Conduct experiment
- Calculate test statistic
- Compare test statistic with what is expected when  $H_0$  is true
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# Basic Research Terminology

- **Retrospective:** (historic) look back at events that have already taken place
- **Prospective:** Watch for future outcomes
- **Case Control Study:** Persons w/ disease & those w/out are compared
- **Cohort Study:** Persons w/ and/or w/out disease are followed over time

# Terminology (Cont.)

- **Cross-sectional Study:** Presence or absence of exposure to possible risk factor measured at one point in time. Prevalence obtained.
- **Prevalence:** The # of new cases and existing cases during specified time period.
- **Incidence:** The # of NEW cases per unit of a population at risk for disease occurring during stated time period.

# **Historical Minute**

## **First “Clinical Trials”**

### **Historical Highlights of Drug Trials**

- **1909: Paul Ehrlich – Arsphenamine (syphilis)**
- **1929: Alexander Fleming – Penicillin (1<sup>st</sup> bacterial)**
- **1935: Gerhard Domagk – Sulfonamide (bacterial)**
- **1944: Schatz/Bugie/Waksman – Streptomycin (TB)**
- **By 1950, the British Medical Res. Council developed a systematic methodology for studying & evaluating therapeutic interventions**

# **Core Components of Clinical Trials**

- **Involve human subjects**
- **Move forward in time**
- **Most have a comparison CONTROL group**
- **Must have method to measure intervention**
- **Focus on unknowns: effect of medication**
- **Must be done before medication is part of standard of care**
- **Conducted early in the development of therapies**

# **Core Components of Clinical Trials**

- **Must review existing scientific data & build on that knowledge**
- **Test a certain hypothesis**
- **Study protocol must be built on sound & ethical science**
- **Control for any potential biases**
- **Most study medications, procedures, and/or other interventions**

# Simplified

- **Randomized:** Schemes used to assign participant to one group
  - Ex: Every 3 gets higher dose
- **Nonrandomized:** All with Hep. C = cases; others = controls
- **Protocol:** Study design - instructions
- **Blinded:** Participants do not know if in experimental or control group
- **Double Blinded:** Participants **AND** staff do not know group assignment
- **Placebo:** Inactive pill w/ no therapeutic value



# Components of Clinical Trial Protocols

- Investigating two or more conditions so have two(+) groups
  - Ex: drug vs. placebo; medicine vs. surgery; low dose vs. high dose
- Specific inclusion/exclusion criteria
- Sample size & power calculations
- Plan re: potential biases
- Plan re: handling of attrition/loss to follow up

# Study Participant Recruitment

- Identify eligible participants
- Explain study
- Provide informed consent
- Reassess eligibility
- Assign to one group

## Participants should be told:

- May have side effects (adverse effects)
- Time commitment
- Benefits & risks
- May withdraw at any time
- Enrollment 100% voluntary

# Phases of Clinical Trials

- **Phase I:** Small group [20-80] for 1st time to evaluate safety, determine safe dosage range & identify SE
- **Phase II:** Rx/tx given to larger group [100-300] to confirm effectiveness, monitor SE, & further evaluate safety

# Phases of Clinical Trials (cont.)

- **Phase III:** Rx/tx given to even larger group [1,000-3,000] to fulfill all of Phase II objectives & compare it to other commonly used txs & collect data that will allow it to be used safely
- **Phase IV:** Done after rx/tx has been marketed - studies continue to test rx/tx to collect data about effects in various populations & SE from long term use.

# Summary of Phases I-III

	<b># Subs.</b>	<b>Length</b>	<b>Purpose</b>	<b>% Drugs Successfully Tested</b>
<b>Phase I</b>	20 – 100	Several months	Mainly Safety	70%
<b>Phase II</b>	Up to several 100	Several months-2 yrs.	Short term safety; mainly effectiveness	33%
<b>Phase III</b>	100s – several 1000	1-4 yrs.	Safety, dosage & effectiveness	25-30%

# **Issues in Clinical Trials: Use of Placebo Trials**

**On international realm, 1999 “Declaration of Helsinki” revised to address use of placebos:**

- Placebos not ethical in virtually all studies that involve diseases with PROVEN tx**
- Remain ethical in trials where no proven tx**
- Revisions due to controversy over use of placebos in attempting to find easy/cheap way to reduce HIV perinatal transmission**
  - 1998 study in Ivory Coast, Uganda, & Thailand: HIV+ pregnant women given either placebo or shorter course of AZT**

# t-test example

- Hindmarsh and Brook (1987) give the heights of 16 children before treatment and one year after treatment with a growth hormone.
- Results were standardized for age (observed height minus predicted height for age divided by SD of height).

# t-test example

Subject	Baseline	At 1 year	Difference
1	-0.7	4.1	4.8
2	0.0	3.4	3.4
3	-0.7	3.1	3.8
4	-0.7	3.0	3.7
5	0.5	2.8	2.3
6	-0.7	2.7	3.4
7	-0.6	2.5	3.1
8	-0.3	2.3	2.6
9	-0.7	2.2	2.9
10	-0.7	2.0	2.7
11	-0.5	1.8	2.3
12	-0.7	1.6	2.3
13	-0.5	1.3	1.8
14	-0.7	0.9	1.6
15	-0.4	0.8	1.2
16	-0.3	0.3	0.6
Mean:	-0.48	2.18	2.66
Median:	-0.65	2.25	2.65
SD:	0.33	1.03	1.06



# t-test example

- The distribution is skewed, as demonstrated by the coefficient

$$\begin{aligned} sk &= 3(\text{mean} - \text{median})/\text{SD} \\ &= 1.55 \end{aligned}$$

- Which suggests that the data are not normally distributed.
- Note that just because the basic observations appear not to have normal distributions, this does not necessarily mean that our methods do not apply!

# t-test example

- In this case it is the differences before-after, that have to be checked.
- In fact the differences appear to be symmetric ( $sk = 0.03$ ).
- $SD(d) = 1.06$ , and  $SE(\bar{d}) = 1.06/\sqrt{16} = 0.265$
- This is a small study so we need to use the confidence interval with  $t_{\alpha}$ , not  $Z_{\alpha}$ .
- In this example the data are paired and the degrees of freedom are  **$df = n - 1 = 15$**

# t-test example

- 95% confidence interval for the mean difference is:

$$2.66 \pm (t_{0.05} \times 0.265)$$

- From Student's t-distribution table, with  $df=15$ ,  $t_{0.05} = 2.131$ .
- $\rightarrow 2.10 - 3.22$  cm.
- We can calculate the Student's t-statistic as  $t=2.66/0.265=10.0$ .

# t-test example

- From the table, the largest value with 15 degrees of freedom is 4.073, corresponding to  $\alpha=0.001$ .
- Thus,  $p<0.001$ .
- We conclude that the growth hormone was effective in increasing the stature of these children.