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$$\frac{1}{\sigma_b^2} \min_{\underset{\sim}{x}} \underset{\sim}{r}^T \begin{pmatrix} m_1 & & & 0 \\ & m_2 & & \\ & & \ddots & \\ 0 & & & m_n \end{pmatrix} \underset{\sim}{r},$$

or the i^{th} point is counted m_i times. The result for $\underset{\sim}{x}$ would be the same (on average) as if you had used all of the points individually!

!

OK, what is the formula for $\underset{\sim}{x}$?

Just as before we take the gradient:

$$\underset{\sim}{\nabla}_x \left\{ \underset{\sim}{r}^T \left(\underset{\approx}{\Sigma}_b^2 \right)^{-1} \underset{\sim}{r} \right\} = 0$$

$\underset{\sim}{r} = \underset{\sim}{b} - \underset{\approx}{A}\underset{\sim}{x}$

which yields:

$$\underset{\approx}{A}^T \left(\underset{\approx}{\Sigma}_b^2 \right)^{-1} \underset{\approx}{A}\, \underset{\sim}{x} = \underset{\approx}{A}^T \left(\underset{\approx}{\Sigma}_b^2 \right)^{-1} \underset{\sim}{b}$$

or $\underset{\sim}{x} = \underset{\approx}{K}_w \underset{\sim}{b} \equiv \left\{ \underset{\approx}{A}^T \left(\underset{\approx}{\Sigma}_b^2 \right)^{-1} \underset{\approx}{A} \right\}^{-1} \underset{\approx}{A}^T \left(\underset{\approx}{\Sigma}_b^2 \right)^{-1} \underset{\sim}{b}$

with error:

$$\underset{\approx}{\Sigma}_x^2 = \underset{\approx}{K}_w \underset{\approx}{\Sigma}_b^2 \underset{\approx}{K}_w^T$$