QUESTION NO. 3 : INTEGER PROGRAMMING (30%)

A new restaurant chain aims at capturing market share in Beijing. The marketing team of the chain has divided the city into 7 districts (A, B, ..., G) which it would like to cover. A district is considered "covered" if a restaurant is build within the same district or within a district that is within a 5km radius (i.e., ≤ 5km). The distances (in km) between the districts are given in the following table:

Distance (km)	A	B	C	D	E	F	G
District A	-	6	2	1	8	3	4
District B	6	-	1	1	6	9	2
District C	2	1	-	7	3	3	8
District D	1	1	7	-	9	6	3
District E	8	6	3	9	-	1	2
District F	3	9	3	6	1	-	7
District G	4	2	8	3	2	7	-

Sub-question (a): The manager would like to decide in which districts to open a restaurant such that all districts are covered. Given the high costs to open a restaurant, the manager would like to minimize the number of necessary restaurants. Write the corresponding integer linear programming model by well defining the decision variables, the objective function and the required constraints.

Sub-question (b): After a more thorough study of the rental costs in the different districts in Beiing, the manager concludes that opening a restaurant in districts A, B or E would cost 12 million CNY (Chinese Yuan), opening a restaurant in districts C or F would cost 8 million CNY and opening a restaurant in districts D or G would cost 14 million CNY. Write the new objective function that now minimizes the total opening costs.

Sub-question (c): The manager realizes an important detail in her planning. Specifically, she notices that if a restaurant is opened in district A, then district D requires a restaurant as well. If there is no restaurant in district A, then a restaurant in district D is optional. Write the corresponding constraint implementing such a requirement.

Sub-question (d): The marketing team would like to add further requirements. Specifically, if both districts B and C have a restaurant, then district E cannot have a restaurant. Write the corresponding constraint implementing such a requirement.

Attention: Only write the linear programming models. Do NOT solve them.