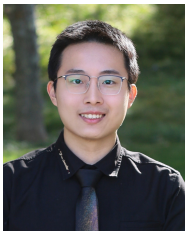


Dynamic Allocation of Reusable Resources to Strategic Agents under Long-Term Constraints

(NeurIPS'25; **Winner** of ACM Student Research @ SIGMETRICS'25)

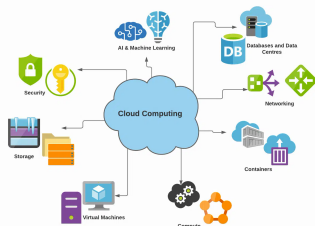
Yan Dai Negin Golrezaei Patrick Jaillet

Massachusetts Institute of Technology



Resource Allocation under Incentives & Constraints

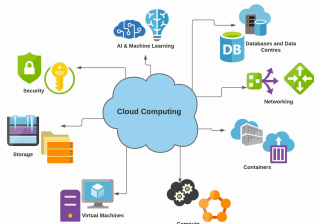
GPU Allocation



- **Resource:** Reusable GPU
- **Agents:** Research groups
- **Constr:** Energy & budget

Resource Allocation under Incentives & Constraints

GPU Allocation



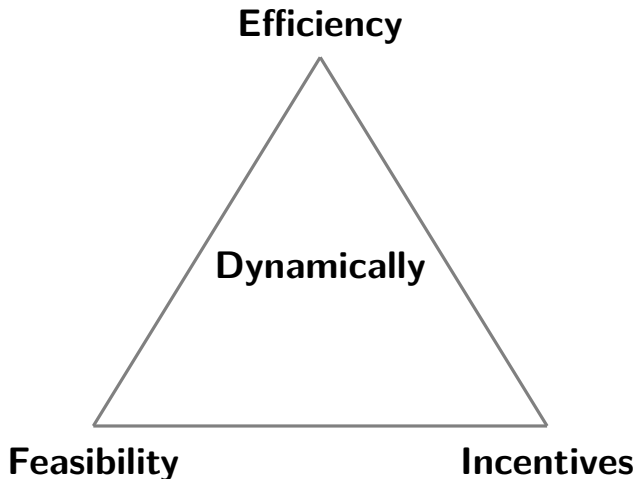
- **Resource:** Reusable GPU
- **Agents:** Research groups
- **Constr:** Energy & budget

Mobile Health Unit



- **Resource:** MHU
- **Agents:** Remote regions
- **Constr:** Staffing & budget

Efficiency-Feasibility-Incentives Trilemma



Efficiency-Feasibility-Incentives Trilemma

Efficiency

- T rounds, K agents,
value $v_{t,i} \sim$ **unknown** \mathcal{V}_i

Max value: $\sum_t v_{t,i_t}$

Efficiency-Feasibility-Incentives Trilemma

Efficiency

- T rounds, K agents,
value $v_{t,i} \sim$ **unknown** \mathcal{V}_i

$$\text{Max value: } \sum_t v_{t,i_t}$$

- Alloc cost $c_{t,i}$ (d -dim),
iid \sim **unknown** \mathcal{C}_i

$$\text{Constr: } \sum_t c_{t,i_t} \leq T\rho$$

Feasibility

Efficiency-Feasibility-Incentives Trilemma

Efficiency

Dynamically

Feasibility

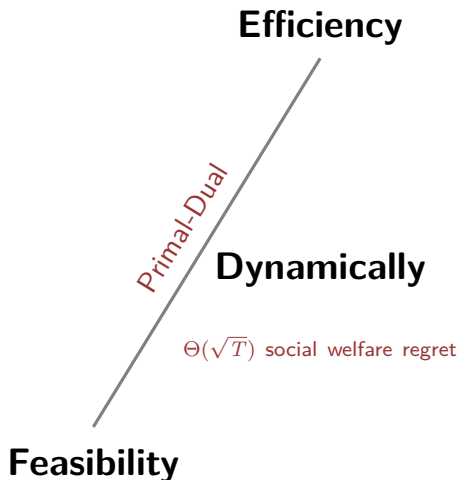
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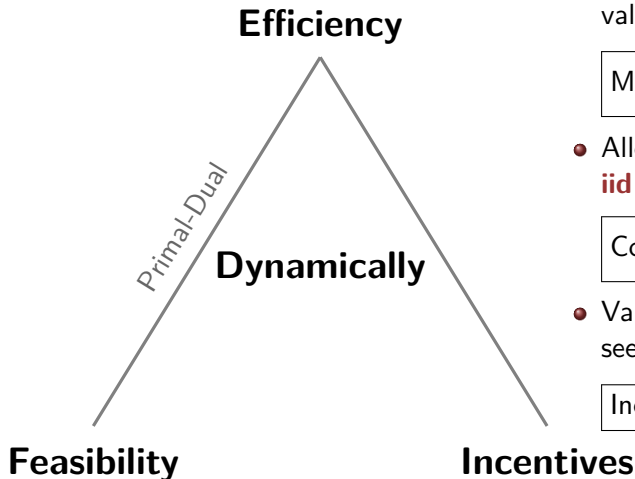
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- Value $v_{t,i}$ private; only see **strategic report** $u_{t,i}$

$$\text{Incentivize } u_{t,i} \approx v_{t,i}$$

Efficiency-Feasibility-Incentives Trilemma

Efficiency

- T rounds, K agents, value $v_{t,i} \sim$ **unknown** \mathcal{V}_i

$$\text{Max value: } \sum_t v_{t,i_t}$$

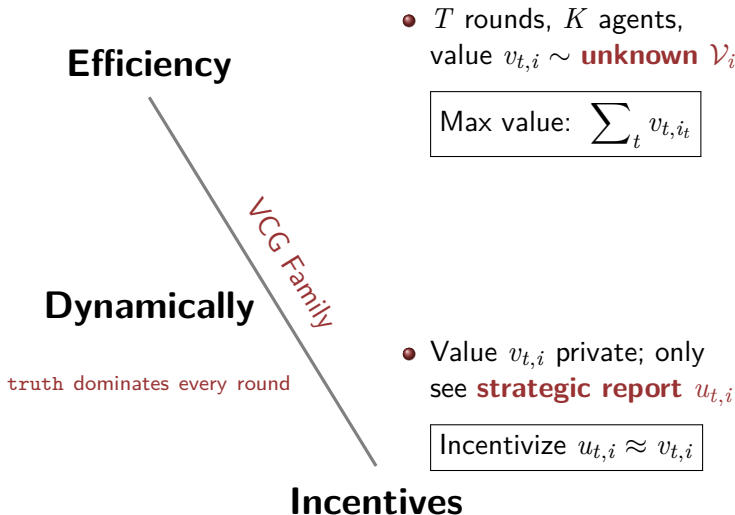
Dynamically

- Value $v_{t,i}$ private; only see **strategic report** $u_{t,i}$

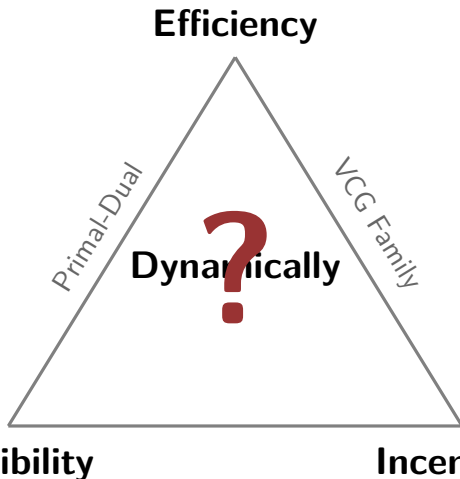
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Incentives

Efficiency-Feasibility-Incentives Trilemma



Efficiency-Feasibility-Incentives Trilemma



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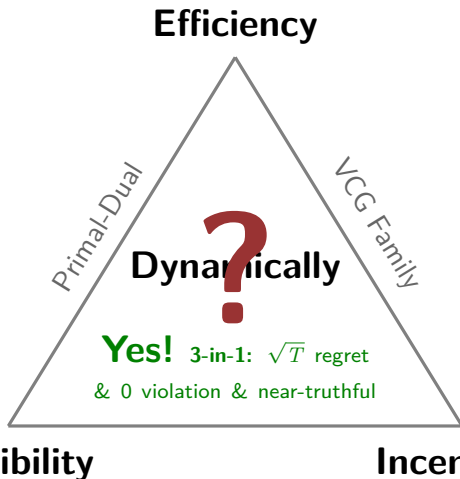
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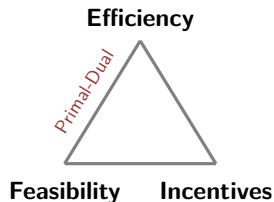
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Classical Primal-Dual Fails with Strategic Agents



Primal (Good Allocations)

Dual (Track Constraints)

Figure: Classical Primal-Dual in Round t

Classical Primal-Dual Fails with Strategic Agents

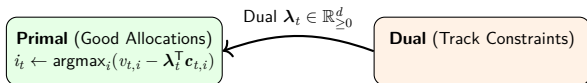
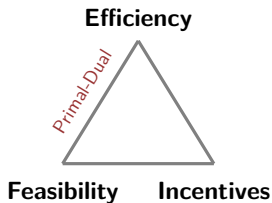


Figure: Classical Primal-Dual in Round t

Classical Primal-Dual Fails with Strategic Agents

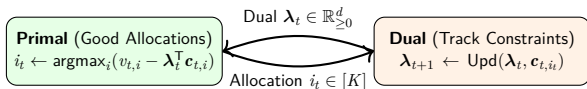
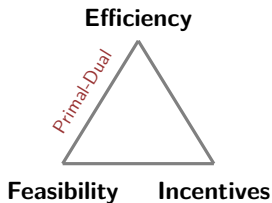


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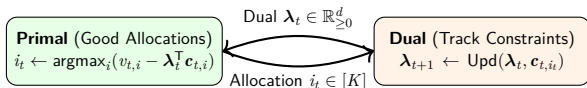
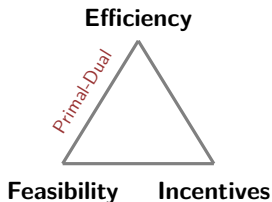


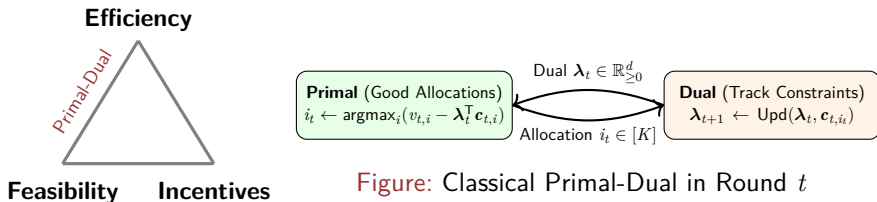
Figure: Classical Primal-Dual in Round t

What Happens With Strategic Agents?

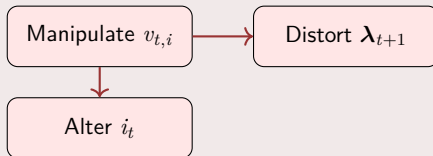
Manipulate $v_{t,i}$

Alter i_t

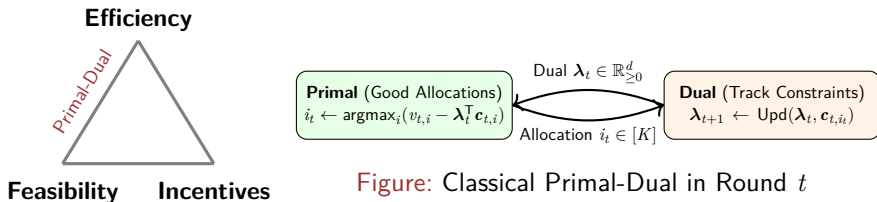
Classical Primal-Dual Fails with Strategic Agents



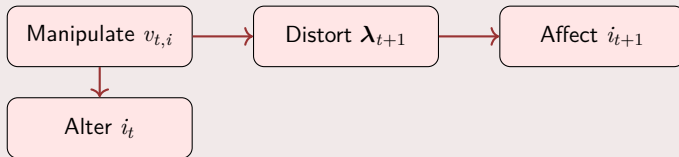
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What Happens With Strategic Agents?



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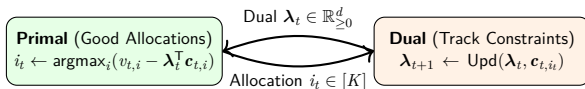
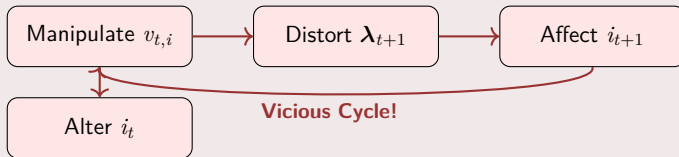


Figure: Classical Primal-Dual in Round t

What Happens With Strategic Agents?



Classical Primal-Dual Fails with Strategic Agents

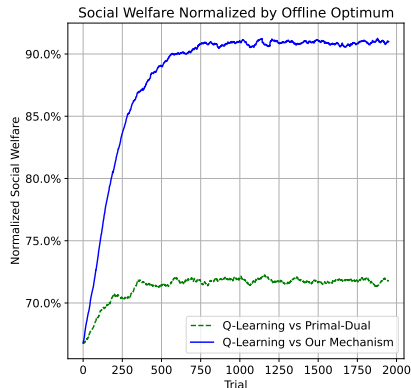
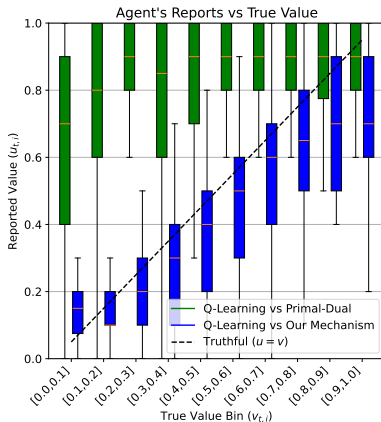


Figure: Vanilla Primal-Dual vs Our Mechanism

Classical Primal-Dual Fails with Strategic Agents

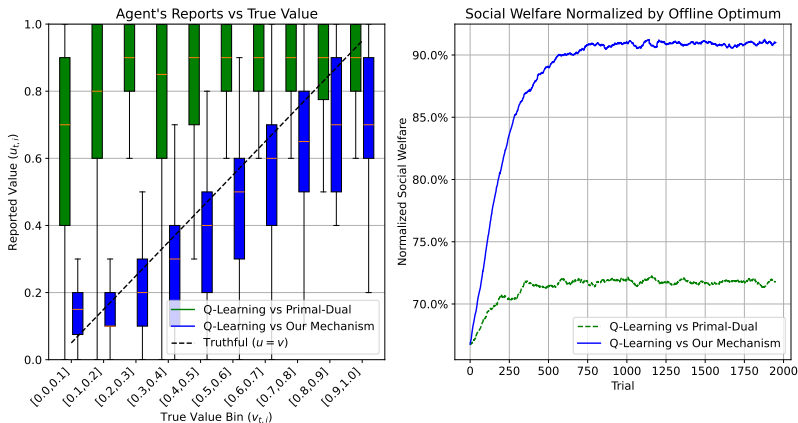


Figure: Vanilla Primal-Dual vs Our Mechanism

misreport vs truthful

Classical Primal-Dual Fails with Strategic Agents

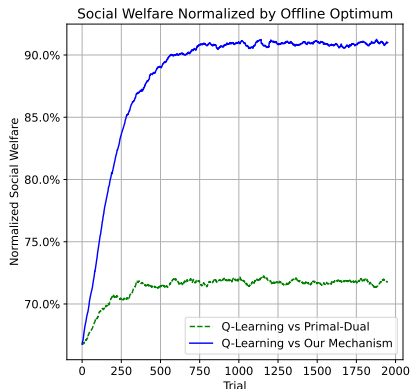
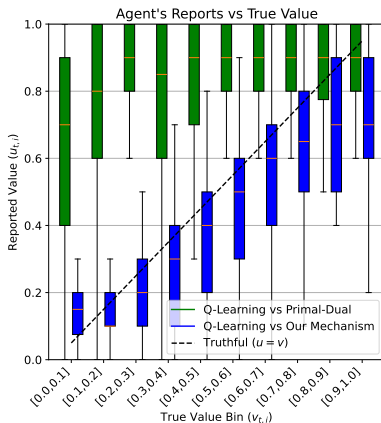
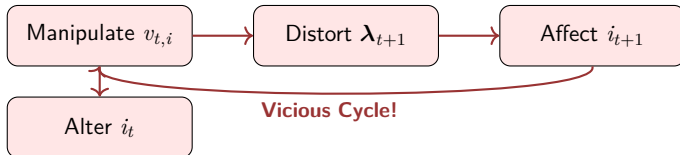


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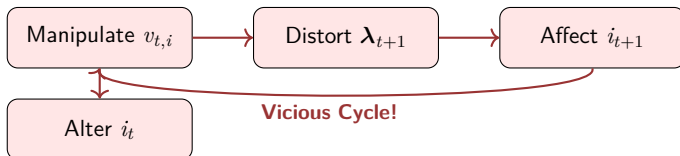
misreport vs truthful

low vs high efficiency

Primal Allocations

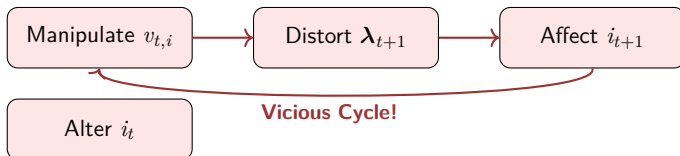


Primal Allocations: Pricing



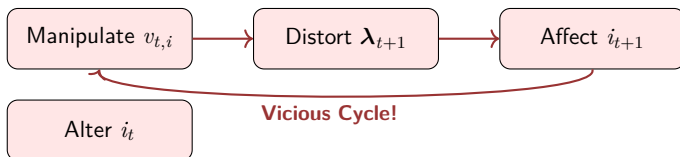
- 1 **Dual-Adjusted Pricing.** VCG-like rule (adapted for λ)

Primal Allocations: Pricing



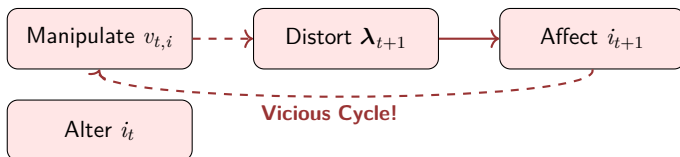
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 \implies truth dominates (for static setups)

Primal Allocations: Pricing + Epoching



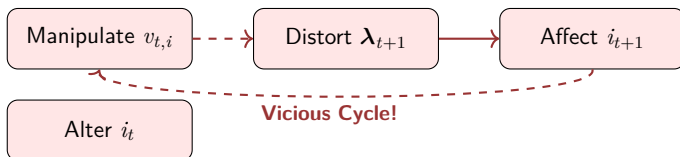
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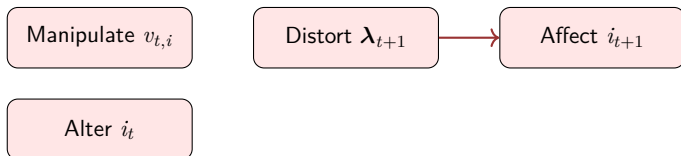
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Primal Allocations: Pricing + Epoching + Exploration



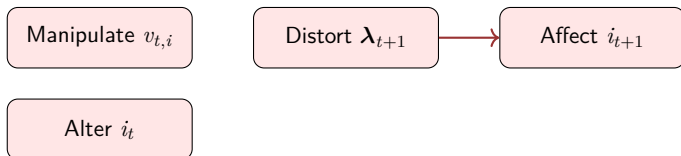
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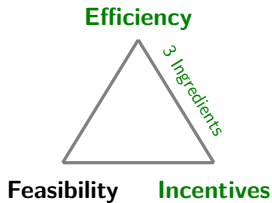
Primal Allocations: Pricing + Epoching + Exploration



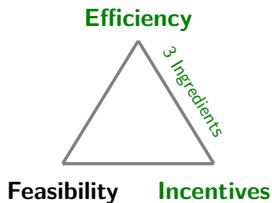
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Theorem. $\tilde{O}(1)$ misreports & $\tilde{O}(1)$ misallocations per epoch

Dual Updates

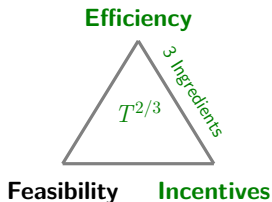


Dual Updates



Dynamically tune $\lambda_1, \lambda_2, \dots$ according to costs

Dual Updates: Online Learning gives $\tilde{O}(T^{2/3})$

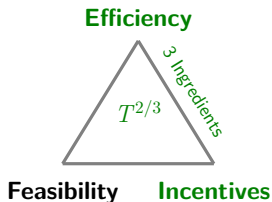


Dynamically tune $\lambda_1, \lambda_2, \dots$ according to costs

Theorem 1: Sublinear Regret ✓

3 ingredients (primal) + GD / FTRL (dual)
 $\Rightarrow \tilde{O}(T^{2/3})$ **regret** ("no-regret" guarantee)

Dual Updates: Online Learning gives $\tilde{O}(T^{2/3})$



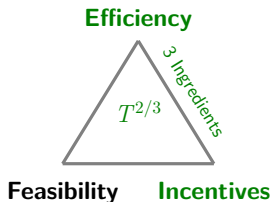
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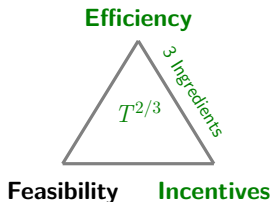
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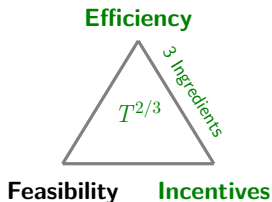
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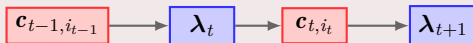
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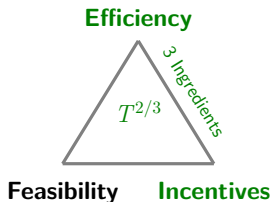
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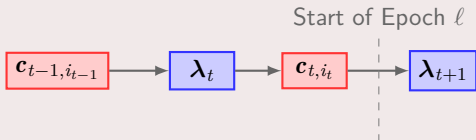
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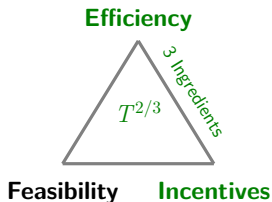
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Lazy



Dual Updates: Online Learning gives $\tilde{O}(T^{2/3})$



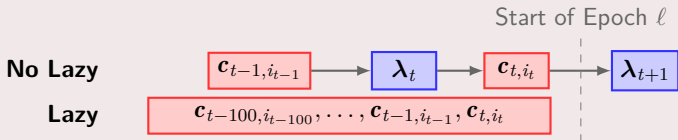
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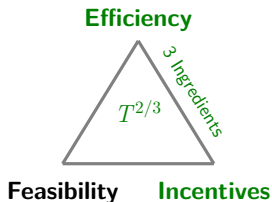
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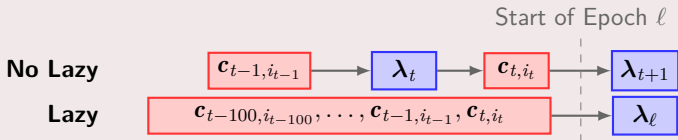
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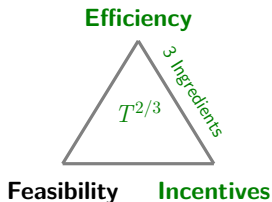
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Dual Updates: Online Learning gives $\tilde{O}(T^{2/3})$



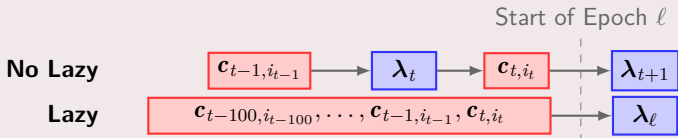
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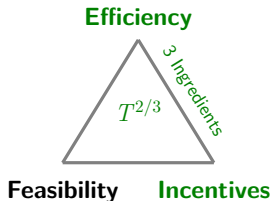
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Dual Updates: Online Learning gives $\tilde{O}(T^{2/3})$



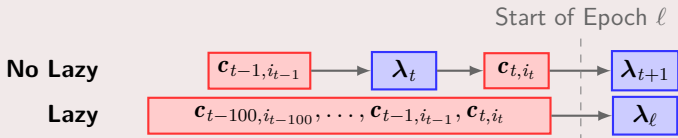
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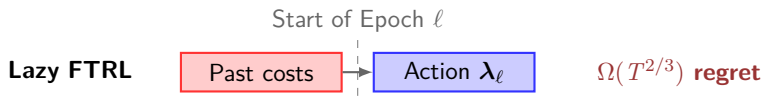
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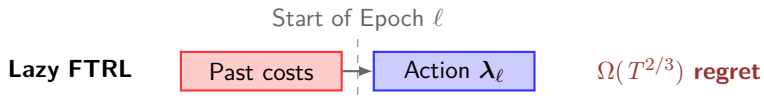


Theorem. “Low-switching online learning” has $\Omega(T^{2/3})$ **regret**

Dual Updates: Even Better via Predictability

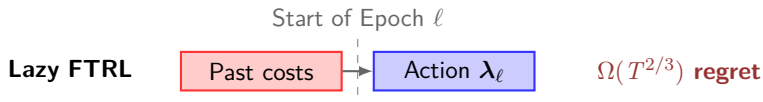


Dual Updates: Even Better via Predictability



Key Insight: (Almost-)Truthfulness \implies Predictability

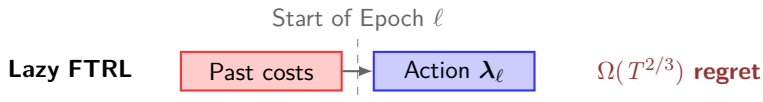
Dual Updates: Even Better via Predictability



Key Insight: (Almost-)Truthfulness \implies Predictability

- 1 Truthful \implies iid future cost \mathbf{c}_{t,i_t} ($i_t \approx \arg\max_i (v_{t,i} - \lambda_\ell^\top \mathbf{c}_{t,i})$)

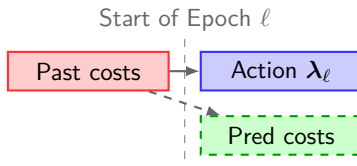
Dual Updates: Even Better via Predictability



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- 1 Truthful \implies **iid future cost** \mathbf{c}_{t,i_t} ($i_t \approx \arg\max_i (v_{t,i} - \lambda_\ell^\top \mathbf{c}_{t,i})$)
- 2 Truthful \implies **reliable history** (for distributions \mathcal{V}_i and \mathcal{C}_i)

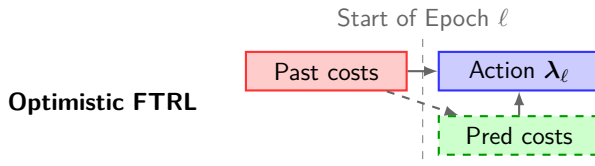
Dual Updates: Even Better via Predictability



Key Insight: (Almost-)Truthfulness \implies Predictability

- ① Truthful \implies **iid future cost** \mathbf{c}_{t,i_t} ($i_t \approx \arg\max_i (v_{t,i} - \boldsymbol{\lambda}_\ell^\top \mathbf{c}_{t,i})$)
 - ② Truthful \implies **reliable history** (for distributions \mathcal{V}_i and \mathcal{C}_i)
- \implies **Predict new costs**

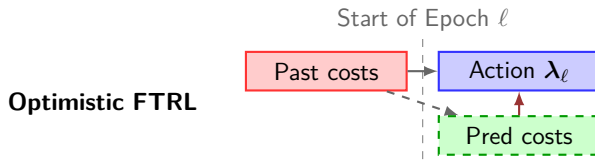
Dual Updates: Even Better via Predictability



Key Insight: (Almost-)Truthfulness \implies Predictability

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 - ② Truthful \implies **reliable history** (for distributions \mathcal{V}_i and \mathcal{C}_i)
- \implies **Predict new costs** for better action λ_ℓ

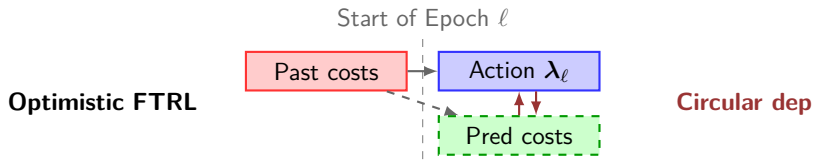
Dual Updates: Even Better via Predictability



Issue: Circular Dependency

- Yield λ_ℓ **as-if true costs = pred costs**

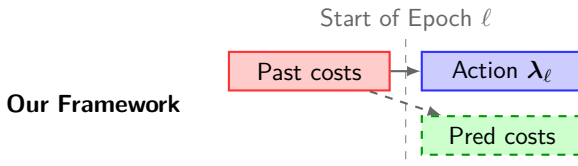
Dual Updates: Even Better via Predictability



Issue: Circular Dependency

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Dual Updates: Even Better via Predictability

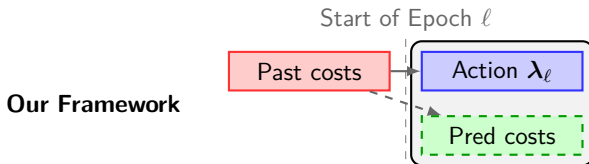


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Novel online learning

Dual Updates: Even Better via Predictability

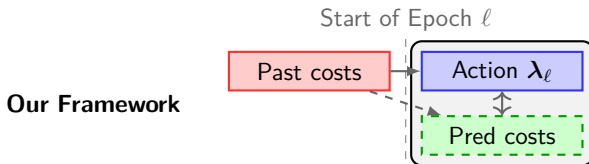


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Novel online learning

Dual Updates: Even Better via Predictability

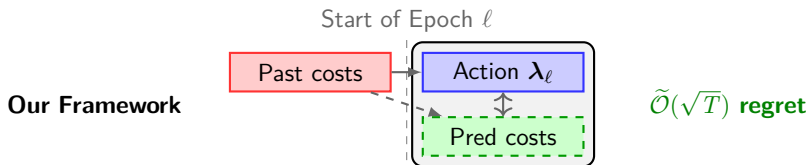


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Novel online learning (decide action λ_ℓ & pred costs simultaneously via fixed-point subroutine; named **O-FTRL-FP**)

Dual Updates: Even Better via Predictability

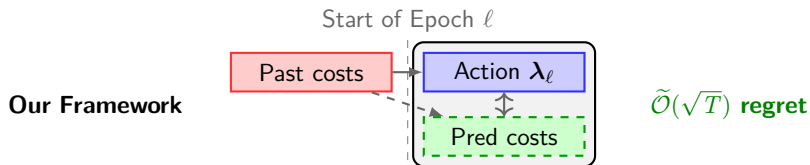


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Novel online learning (decide action λ_ℓ & pred costs simultaneously via fixed-point subroutine; named **O-FTRL-FP**) $\Rightarrow \tilde{O}(\sqrt{T})$ regret

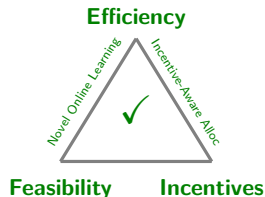
Recall. Non-strategic lower bound = $\Omega(\sqrt{T})$ regret

Main Results & Takeaway

Main Result

1st dynamic mechanism resolving **trilemma**:

- **Efficiency.** Optimal $\tilde{O}(\sqrt{T})$ regret
- **Feasibility.** Zero constraint violation
- **Incentives.** Robust to strategic agents



Key Techniques

- **Primal Side: Incentive-Aware Allocation.** Novel mixture of dual-adjusted pricing + lazy updates + random exploration
- **Dual Side: Online Learning for Updates.** Truthfulness \Rightarrow Predictability + novel framework for circular dependencies

Questions are more than welcomed!

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