Dynamic Allocation of Reusable Resources to Strategic Agents under Long-Term Constraints

(NeurIPS'25; **Winner** of ACM Student Research @ SIGMETRICS'25)

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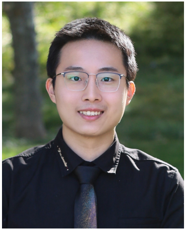
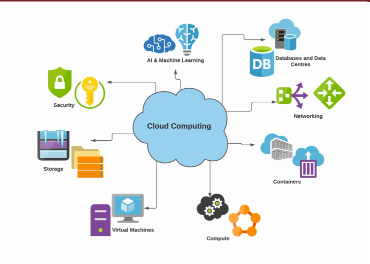
Massachusetts Institute of Technology



Today

Motivating Examples

GPU Allocation



- **Resource:** Reusable GPU
- **Agents:** Research groups
- **Constr:** Energy & budget

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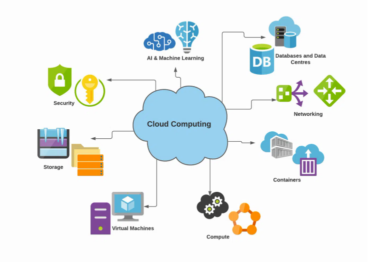

- **Resource:** Reusable GPU
- **Agents:** Research groups
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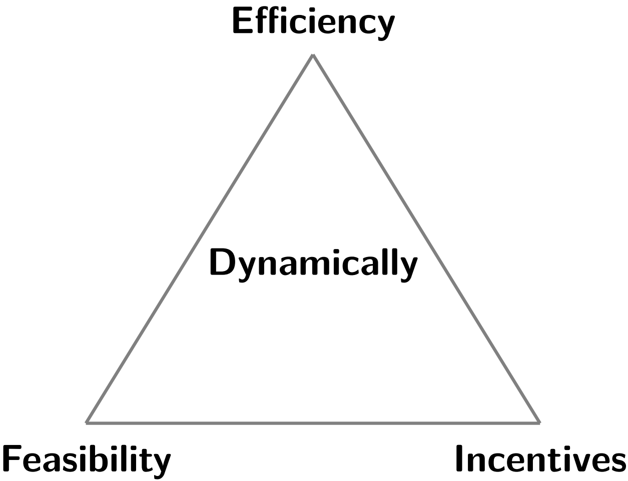
Mobile Health Unit

- **Resource:** MHU
- **Agents:** Remote regions
- **Constr:** Staffing & budget

Efficiency-Feasibility-Incentives Trilemma



Efficiency-Feasibility-Incentives Trilemma

Efficiency

- T rounds, K agents, value $v_{t,i} \sim$ **unknown** $\mathcal{V}_i$

$$\boxed{\text{Max value: } \sum_t v_{t,i_t}}$$

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Feasibility

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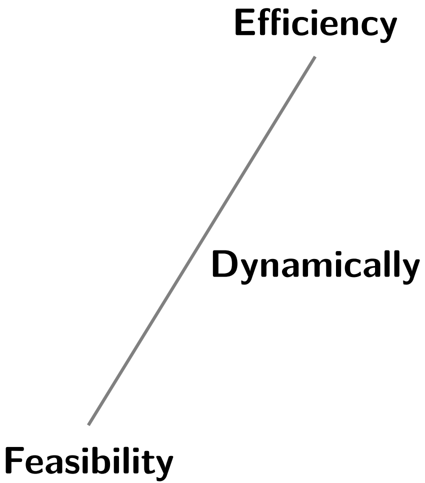

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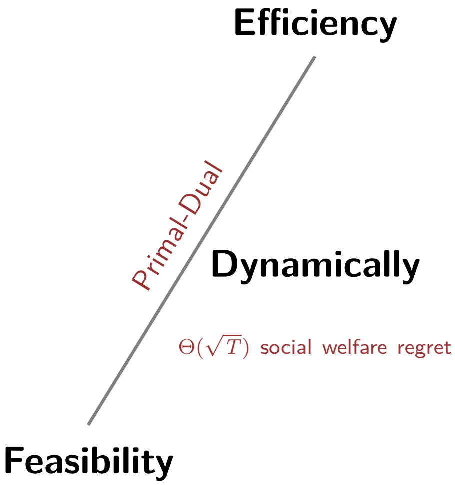

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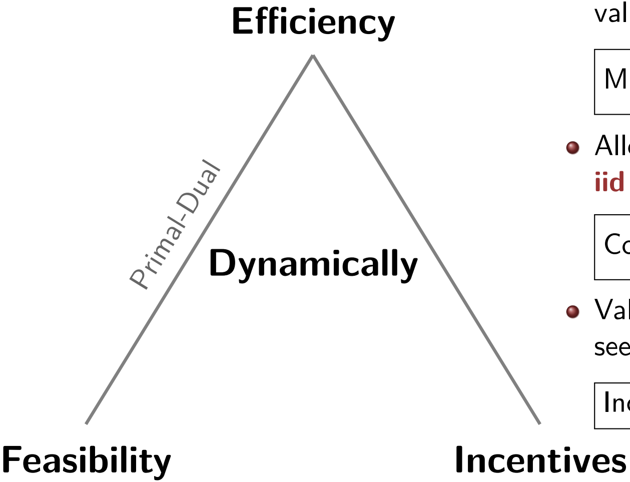

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Dynamically

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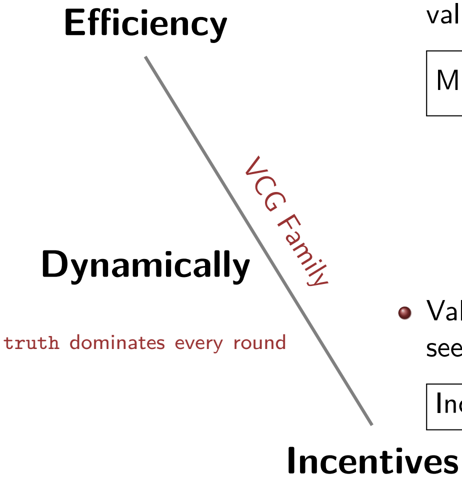

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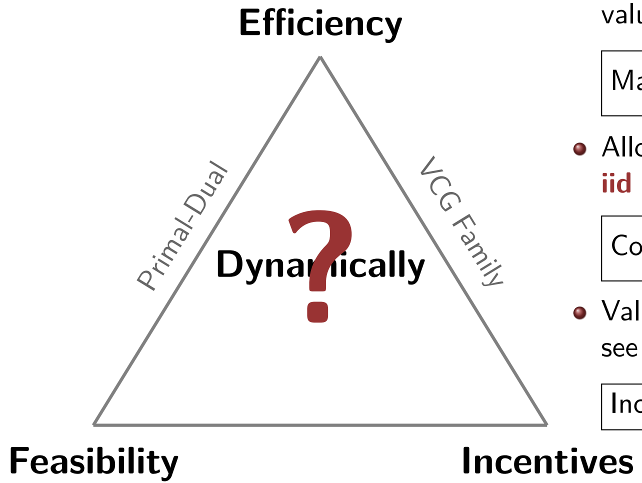

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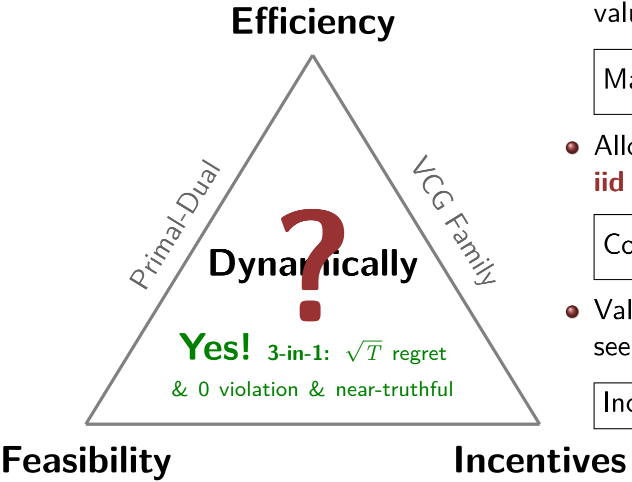

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Maximize social welfare:

$$\max \ \mathbb{E}\left[\sum_{t=1}^{T} v_{t,i_t}\right] \text{ s.t. } \frac{1}{T}\sum_{t=1}^{T} \boldsymbol{c}_{t,i_t} \le \boldsymbol{\rho}$$

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Regret. $\mathbb{E}[\sum_t (v_{t,i_t^*} - v_{t,i_t})]$ with $\{i_t^*\}_{t\in[T]}$ constrained offline opt

Standard Tool: Primal-Dual Framework (when Truthful)

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Primal (Good Allocations)

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- **Round-t primal.** Make good "dual-adjusted" allocations

  (based on $\tilde{v}_{t,i} := v_{t,i} - \boldsymbol{\lambda}_t^\top \boldsymbol{c}_{t,i}$; $v_{t,i}$ = true value & $\boldsymbol{c}_{t,i}$ = true cost)

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What's Wrong when Agents are Strategic?

Immediate feedback loop is target of **strategic manipulations!**

Standard Tool Fails: Vicious Cycle of Manipulation

What Happens With Strategic Agents?

$$\begin{tikzcd}
\text{Manipulate } v_{t,i} \arrow[d] \\
\text{Alter } i_t
\end{tikzcd}$$

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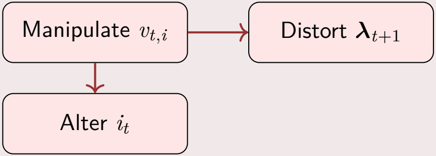

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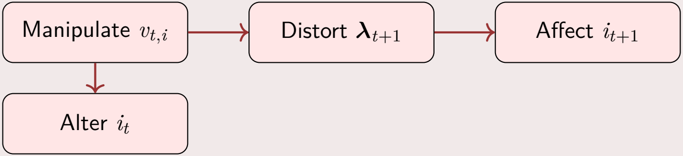

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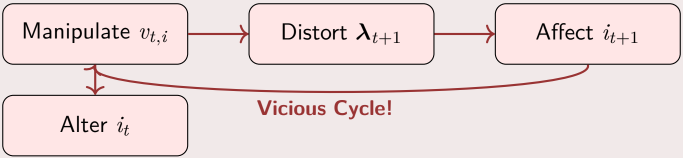

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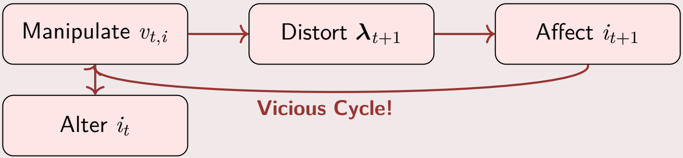
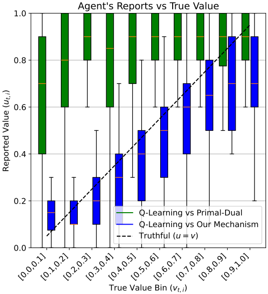
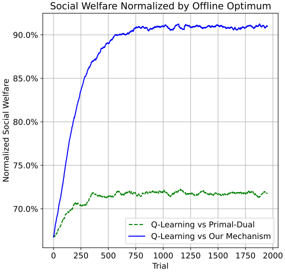


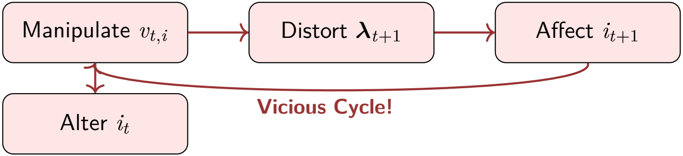




Today

1. Setup: A Trilemma in Online Mechanism Design
   - Efficiency-Incentive-Feasibility Trilemma
   - Formal Setup: Agents, Planner, Objectives
   - Vicious Cycle: Classical Primal-Dual Fails
2. Primal Side: Incentive-Aware Allocation
   - Pricing to Evict "Short-Term" Impact
   - Epoching to Mitigate "Long-Term" Impact
   - Exploration to Remove "Long-Term" Impact
3. Dual Side: Online Learning for Updates
   - Online Learning View of Dual Updates
   - Our Plan: Promise of Predictability
   - Beyond Optimistic FTRL: O-FTRL-FP

Pricing to Evict "Short-Term" Impact



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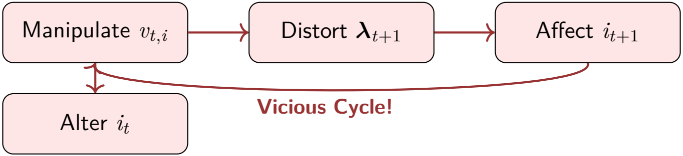

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- VCG-like auction on "dual-adjusted reports", defined as:

$$\tilde{u}_{t,i} := u_{t,i} - \boldsymbol{\lambda}_t^\mathsf{T} \boldsymbol{c}_{t,i}, \quad \forall i \in [K].$$

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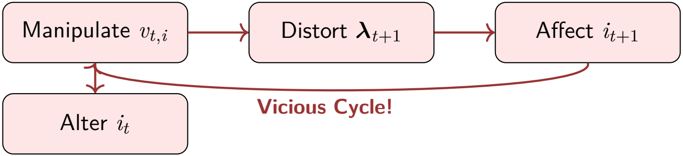

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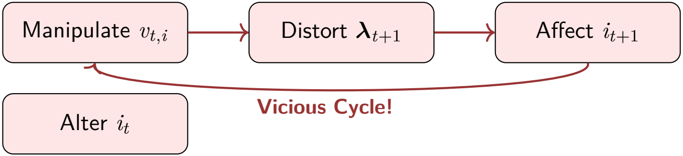

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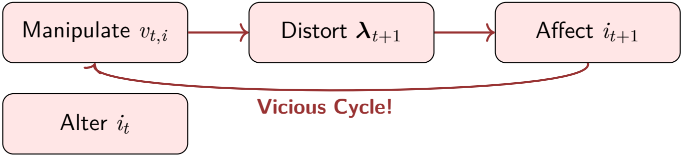
$\Longrightarrow$ `truth` **dominating** when maximizing round-t gain

Epoching to Mitigate "Long-Term" Impact



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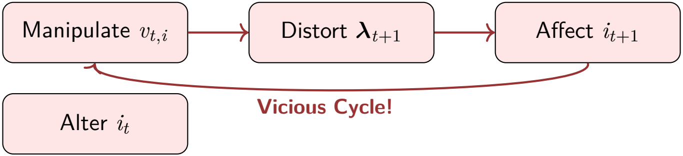

Ingredient 2: Epoch-Based Lazy Updates

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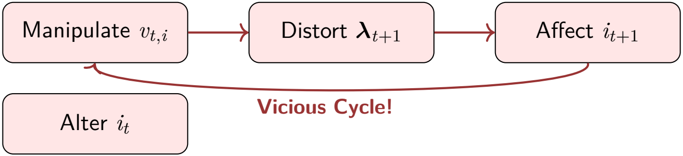

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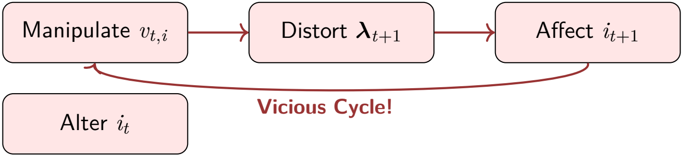

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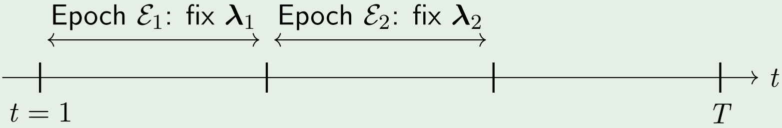

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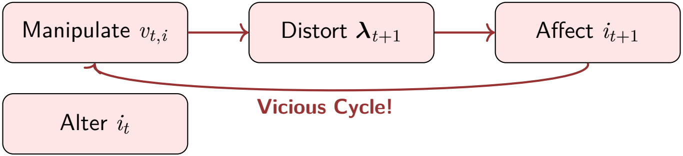

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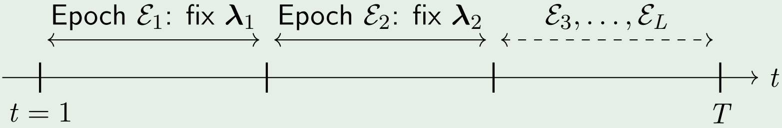

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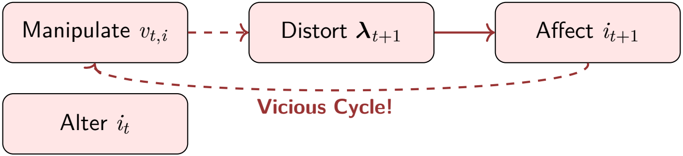
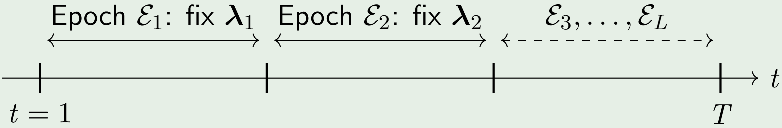


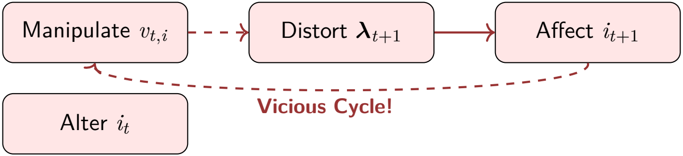
Ingredient 2: Epoch-Based Lazy Updates

Break $[T]$ into *epochs* $\mathcal{E}_1, \mathcal{E}_2, \ldots, \mathcal{E}_L$. Fix dual $\boldsymbol{\lambda}_t$ within $t \in \mathcal{E}_\ell$.



$\Longrightarrow$ **Make it hard** to affect dual updates

Exploration to Remove "Long-Term" Impact



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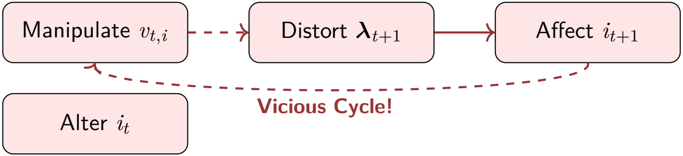

Ingredient 3: Randomized Exploration Rounds

With low probability ϵ, in each round $t \in \mathcal{E}_\ell$, we do...

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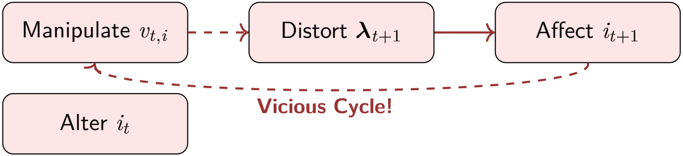

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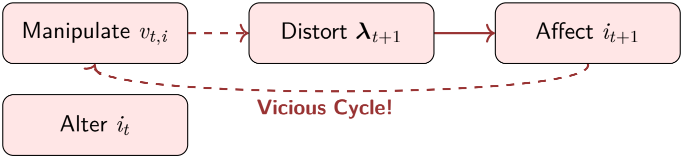

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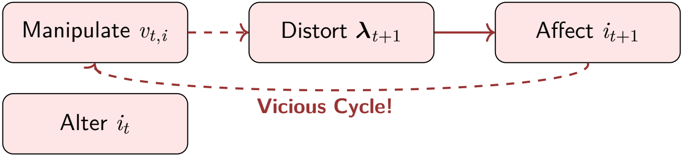

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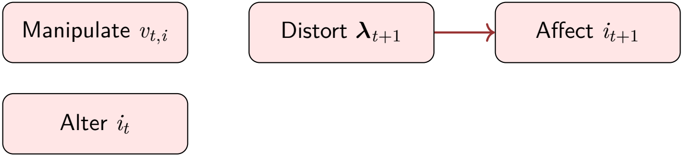

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$\Longrightarrow$ **Lying causes harm** $\ge \epsilon \frac{(u-v)^2}{2K}$ (outweighing benefits)

Primal Side Summary

1: **for** epoch $\ell = 1, 2, \dots, L$ **do**
2: Receive dual variable $\boldsymbol{\lambda}_\ell$ ▷ *Epoching*
3: **for** round t in epoch $\mathcal{E}_\ell$ **do**

3 Key Ingredients

1. Epoch-based lazy updates

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For each epoch $\mathcal{E}_\ell$, there exists a PBE where agents are **nearly truthful** ($\widetilde{\mathcal{O}}(1)$ misreports per epoch)

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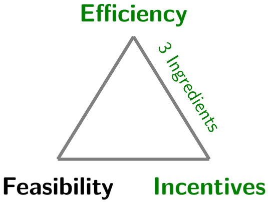
Today

Dual Updates



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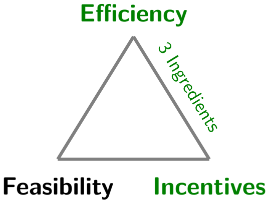
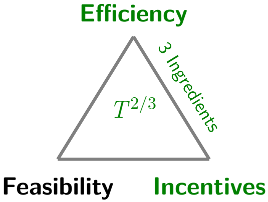


Tune duals $\boldsymbol{\lambda}_1, \boldsymbol{\lambda}_2, \ldots$ **before** knowing costs

Dual Updates: Online Learning gives $\widetilde{\mathcal{O}}(T^{2/3})$



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Theorem 1: Sublinear Regret ✓

3 ingredients (primal) + GD / FTRL (dual)
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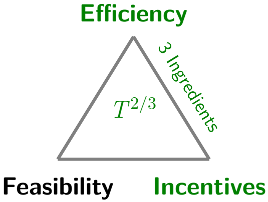

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Can we do better than $\Theta(T^{2/3})$?

Online Learning View of Dual Updates

Online Learning Formulation

1. **Setup.** L-round with actions $\boldsymbol{\lambda}_\ell \in \boldsymbol{\Lambda} \subseteq \mathbb{R}^d_{\geq 0}$ (a bounded set)

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3. **Objective.** Minimize online learning regret (derivation omitted)

$$\mathfrak{R}^{\boldsymbol{\lambda}} := \mathbb{E}\left[\max_{\boldsymbol{\lambda}^* \in \boldsymbol{\Lambda}} \sum_{\ell=1}^{L} (F_\ell(\boldsymbol{\lambda}_\ell) - F_\ell(\boldsymbol{\lambda}^*))\right]$$

Price of Incentives: An $\Omega(T^{2/3})$ Barrier

Dilemma of Lazy Updates

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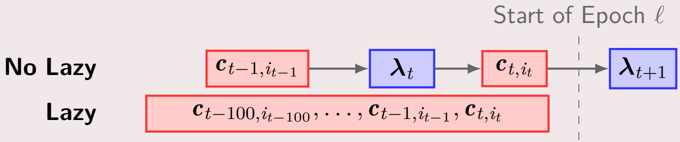

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Lazy $\boxed{\boldsymbol{c}_{t-100,i_{t-100}}, \ldots, \boldsymbol{c}_{t-1,i_{t-1}}, \boldsymbol{c}_{t,i_t}} \rightarrow \boxed{\boldsymbol{\lambda}_\ell}$

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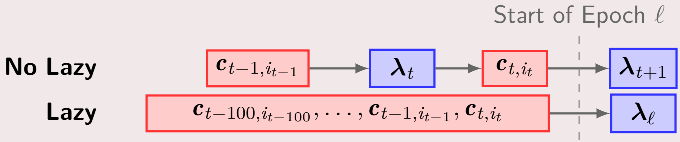

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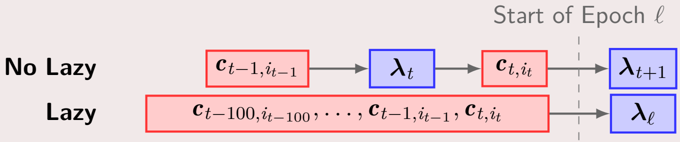

Theorem. "Low-switching online learning" has $\Omega(T/\sqrt{L})$ **regret**

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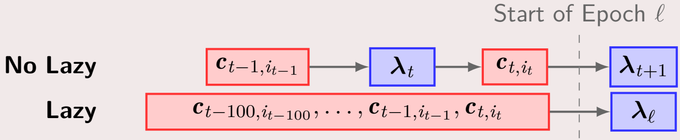

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Hardness (?) $\Omega(L)$ primal $+$ $\Omega(T/\sqrt{L})$ dual $=$ $\Omega(T^{2/3})$ **overall**

Our Plan: Promise of Predictability

Observation. $\Omega(T/\sqrt{L})$ is for fully adversarial loss functions

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2. **Truthful $\Longrightarrow$ reliable historical reports** to make predictions:

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Truthfulness not only boosts efficiency, but also helps dual!

Full View: Primal-Dual in Strategic Environments

Primal Side

Dual Side

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Primal Side

Incentive-aware $\{(i_t, p_{t,i_t})\}_{t \in \mathcal{E}_\ell}$ for a fixed dual price $\boldsymbol{\lambda}_\ell$

$\boldsymbol{\lambda}_\ell \in \mathbb{R}^d_{\geq 0}$

Dual Side

Allocation & Pricing

Given $\boldsymbol{\lambda}_\ell \in \mathbb{R}^d_{\geq 0}$ from dual:

- Allocate to an i_t maximizing $\tilde{v}_{t,i} \Longrightarrow$ **Efficiency**
- Incentivize truthfulness via $p_{t,i_t} \Longrightarrow$ **Incentives**

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$\boldsymbol{\lambda}_\ell \in \mathbb{R}^d_{\geq 0}$

$i_t \in [K]$

Dual Side
Learn next $\boldsymbol{\lambda}_{\ell+1}$
utilizing predictability

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Learning via Predictability

Given $\{i_t\}_{t\in\mathcal{E}_\ell}$ from primal:

- Learn a better $\boldsymbol{\lambda}_{\ell+1}$ for $\mathcal{E}_{\ell+1}$ $\Longrightarrow$ **Feasibility**
- Bypass $\Omega(T^{2/3})$ barrier via predictability $\Longrightarrow$ **Efficiency**

Exploiting Predictability with Optimism: O-FTRL

Optimistic FTRL Framework (O-FTRL)

O-FTRL considers *predicted loss* $\hat{F}_\ell \colon \boldsymbol{\Lambda} \to \mathbb{R}$ before deciding $\boldsymbol{\lambda}_\ell$
(where loss function $F_\ell(\boldsymbol{\lambda}) := \sum_{t \in \mathcal{E}_\ell} f_t(\boldsymbol{\lambda}) = \sum_{t \in \mathcal{E}_\ell} (\boldsymbol{\rho} - \boldsymbol{c}_{t,i_t})^\mathsf{T} \boldsymbol{\lambda}$):

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Exploiting Predictability with Optimism: O-FTRL

Start of Epoch ℓ

Lazy FTRL | Past costs → Action $\boldsymbol{\lambda}_\ell$ | $\Omega(T/\sqrt{L})$ **regret**

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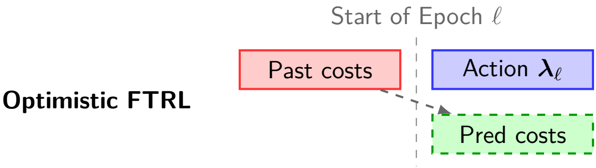

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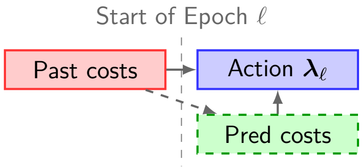
Exploiting Predictability with Optimism: O-FTRL

Optimistic FTRL



Optimistic FTRL Framework (O-FTRL)

O-FTRL considers *predicted loss* $\hat{F}_\ell \colon \boldsymbol{\Lambda} \to \mathbb{R}$ before deciding $\boldsymbol{\lambda}_\ell$ (where loss function $F_\ell(\boldsymbol{\lambda}) := \sum_{t \in \mathcal{E}_\ell} f_t(\boldsymbol{\lambda}) = \sum_{t \in \mathcal{E}_\ell} (\boldsymbol{\rho} - \boldsymbol{c}_{t,i_t})^\mathsf{T} \boldsymbol{\lambda}$):

$$\boldsymbol{\lambda}_\ell = \operatorname*{argmin}_{\boldsymbol{\lambda} \in \Lambda} \left(\sum_{\ell' < \ell} F_{\ell'}(\boldsymbol{\lambda}) + \hat{F}_\ell(\boldsymbol{\lambda}) + \frac{1}{\eta_\ell} \Psi(\boldsymbol{\lambda}) \right), \quad \forall \ell \in [L].$$

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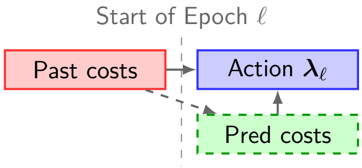

$\widetilde{\mathcal{O}}(T/L)$ **regret**

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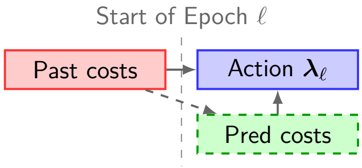
If $\hat{F}_\ell$'s are accurate, regret improves from $\widetilde{\mathcal{O}}(T/\sqrt{L})$ to $\widetilde{\mathcal{O}}(T/L)$.

Technical Challenge: Circular Dependency

Optimistic FTRL



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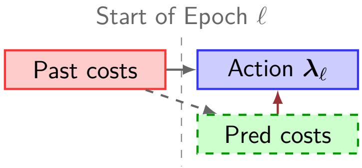

$\widetilde{\mathcal{O}}(T/L)$ **regret?**

Technical Challenge: Circular Dependency between $\boldsymbol{\lambda}_\ell$ and $\hat{F}_\ell$

- Yield $\boldsymbol{\lambda}_\ell$ **as-if true costs = pred costs**

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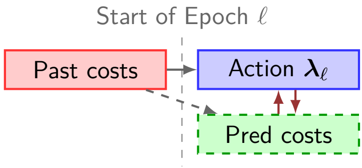

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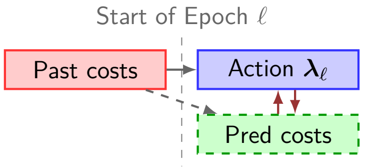

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Action $\boldsymbol{\lambda}_\ell$

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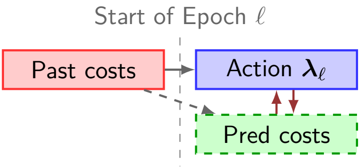

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Action $\boldsymbol{\lambda}_\ell \Longrightarrow$ Allocations $\{i_t\}_{t \in \mathcal{E}_\ell}$ $(i_t \approx \operatorname{argmax}_i (v_{t,i} - \boldsymbol{\lambda}_\ell^\top \boldsymbol{c}_{t,i}))$

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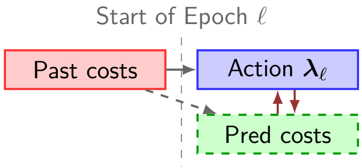

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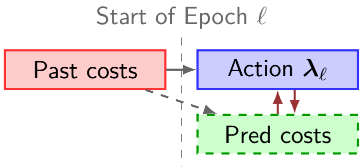

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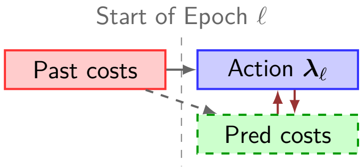

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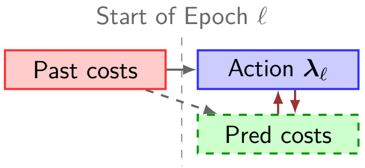

Circular issue!

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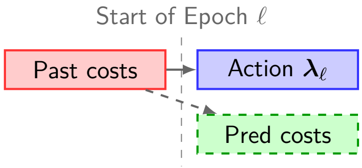
Our Solution: O-FTRL with Fixed Points (O-FTRL-FP)

O-FTRL-FP



Reformulating Update as Fixed-Point Problem

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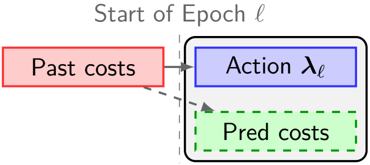

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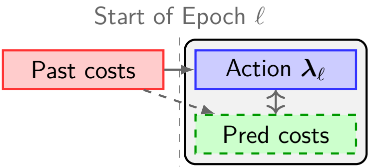

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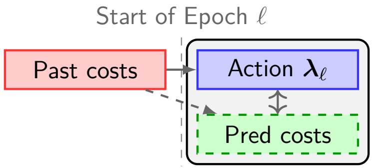

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Lemma. In our case, Eq. (*) has **approximate fixed points**

Dual Side Summary

Goal. Deciding duals despite only L switches & strategic agents.

Challenge: Lazy Updates $\Longrightarrow \Omega(T^{2/3})$ Barrier

$\Longrightarrow$ Primal framework + FTRL can only get $\widetilde{\mathcal{O}}(T^{2/3})$ **regret**

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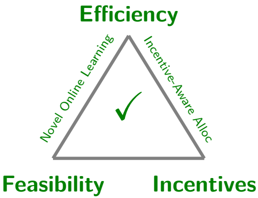
$\Longrightarrow$ Primal framework + our O-FTRL-FP = $\widetilde{\mathcal{O}}(\sqrt{T})$ **regret**

Main Results & Takeaway

Main Result

1st dynamic mechanism resolving **trilemma**:

- **Efficiency.** Optimal $\widetilde{\mathcal{O}}(\sqrt{T})$ regret
- **Feasibility.** Zero constraint violation
- **Incentives.** Robust to strategic agents



Key Techniques

- **Primal Side: Incentive-Aware Allocation.** Novel mixture of dual-adjusted pricing + epoching + random exploration
- **Dual Side: Online Learning for Updates.** Truthfulness ⇒ Predictability + novel O-FTRL-FP for circular dependencies

Questions are more than welcomed!

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