SIR Model

As the first step in the modelling process, we identify the independent variables and the dependent variables. The independent variable is time **t** (measured in days).

We consider two related sets of dependent variables –
The first set of dependent variables count people in each group as a function of time –

S = S(t)

Susceptible group

I = I(t)

Infected group

R = R(t)

Recovered group

At t = 0,

$S(0) = S_0$

Susceptible group

$I(0) = I_0$

Infected group

$R(0) = 0$

Recovered group

The second set of dependent variables represent the fraction of total population in each of the three categories. So, if N is the total population, we have –

Assumptions –

- o We ignore births and immigration. The only way a person leaves the susceptible group is by becoming infected. The rate of change of **S(t)** depends on the number already susceptible, the number of individuals already infected and the amount of contact between the susceptible and the infected. We assume that each infected individual has a fixed number of contacts **beta** per day. However, not all these contacts are with susceptible individuals. Therefore, we assume a homogenous mixture of the total population. A fraction of these contacts that are with the susceptible is $\frac{S}{N}$. Thus, on an average, each infected individual generates **beta** $* \frac{S}{N}$ new infected individuals per day.
- o We can also assume that a fixed fraction **gamma** of the infected will recover any given day. A person who dies is also a part of the recovered group.

$$\frac{ds}{dt} = -beta * \frac{S}{N} * I$$

Susceptible Equation

$$\frac{di}{dt} = beta * \frac{S}{N} * I - gamma * I$$

Infected Equation

$$\frac{dr}{dt} = gamma * I$$

Recovered Equation

* The rate of change of the susceptible group is negative because the susceptible group is getting smaller as they transition into the infected group with respect to time. Similarly, individuals in the infected group gradually recover and transition into the recovered group.