INM460 / IN3060
Computer Vision Mathematics Worksheet

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This worksheet contains a set of questions related to mathematics for computer vision.

Question 1

Let two 2D points be $\mathbf{p} = [1, 1]^T$ and $\mathbf{q} = [3, 4]^T$, and let $c = 3$ be a scalar value.

1.1

What is the distance d between $\mathbf{p}$ and $\mathbf{q}$?

1.2

Find a vector $\mathbf{t}$, that starts at $\mathbf{p}$ and ends at $\mathbf{q}$.

1.3

Find $\hat{\mathbf{t}}$, a normalised version of $\mathbf{t}$.

1.4

What is $c\hat{\mathbf{t}}$?

1.5

What is the element-wise product of $\mathbf{p}$ and $\mathbf{q}$?

1.6

What is the dot product of $\mathbf{p}$ and $\mathbf{q}$?

1.7

Let us homogenise $\mathbf{p}$ by adding a $w = 1$ value, i.e., $\mathbf{p} = [1, 1, 1]^T$. In homogeneous coordinates, is the 2D point $\mathbf{p} = [1, 1, 1]^T$ the same as a 2D homogeneous point $\mathbf{r} = [2, 2, 2]^T$?

Question 2

Let two 2D points in homogeneous coordinates be $\mathbf{p}_1 = [-1, 0, 1]^T$, $\mathbf{p}_2 = [0, -1, 1]^T$ and $\mathbf{p}_3 = [1, 0, 1]^T$. Let a matrix $\mathbf{A} = \begin{bmatrix} 2 & 0 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 1 \end{bmatrix}$ and a matrix $\mathbf{B} = \begin{bmatrix} 0 & 1 & 3 \\ 1 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$.

2.1

What is $\mathbf{A} - \mathbf{B}$?

2.2

What is the element-wise product $\mathbf{A}. * \mathbf{A}$?

2.3

What is $\mathbf{AB}$, the matrix multiplication of $\mathbf{A}$ with $\mathbf{B}$?

2.4

What is $\mathbf{A}\mathbf{p}_1$?

2.5

What is $\mathbf{A}\mathbf{p}_2$?

2.6

What is $\mathbf{Ap}_3$?

2.7

$\mathbf{A}$ is a special type of transformation matrix. On a 2D graph, draw points $\mathbf{p}_1$, $\mathbf{p}_2$, and $\mathbf{p}_3$, and draw lines between the points to form a triangle. On the same graph, draw $\mathbf{Ap}_1$, $\mathbf{Ap}_2$, and $\mathbf{Ap}_3$, and draw lines between the points to form a triangle. Based on the transformed shape, describe the effect the transformation $\mathbf{A}$ has on points.

Question 3

Let four $2D$ points be $\mathbf{p}_1 = [0, 0]^T$, $\mathbf{p}_2 = [2, 3]^T$, $\mathbf{p}_3 = [3, 2]^T$, $\mathbf{p}_4 = [5, 5]^T$.

3.1

Write four linear equations in the form $xm + b = y$ using the x and y values of the four points.

3.2

Express your four equations in matrix form, $\mathbf{Ac} = \mathbf{d}$, where $\mathbf{c} = [m, b]^T$ denote the slope and intercept for a line.

3.3

Use the pseudoinverse in Matlab to find the optimal $\mathbf{c}$ of the best fitting line through the points. Write Matlab code to display the points, and the line.

3.4

The question above is looking for a line $y = mx + b$ that passes through the points by finding the best m and b given the points. What if instead of a line, our task was to compute the best curve for the equation $y = a_1x + a_2x^2 + a_3x^3$? Can this be solved using linear least squares? If so, determine the solution as above.

Question 4

Let a single-variate function $f(x) = 4x^3 + 5x^2 + 2x + 7$, and a multi-variate function $g(x, y) = 3x^2 + xy - y^3 + 2$.

4.1

What is the derivative $\frac{df}{dx}$?

4.2

What is the partial derivative $\frac{\partial g}{\partial x}$?

4.3

What is the gradient of $g(x, y)$? Evaluate the gradient at a point $[4, 5]^T$.

4.4

What is the second partial derivative $\frac{\partial^2 g}{\partial y^2}$?

4.5

What is the Laplacian of $g(x, y)$?

4.6

Find the Taylor series expansion of $f(x)$ around a point $x = 0$.

4.7

What is the integral $\int f(x)dx$?