Example used in class Lecture notes ML part 2, slide 11

The simplest ML task: linear (linear) regression; one-dimensional data points X with target values t=Y Least squares: $E = \frac{1}{2} \sum_{i} (o_{i} - t_{i})^{2}$, where t is the target value and o is the output of the model for each example i. A machine learning task is to find a set of parameters w such that E is minimized. For example, under the assumption of a linear model, the task may be to find the values of w_{0} and w_{1} in the equation $o_{i} = w_{0} + w_{1}x_{i}$, where x is the set of examples, such that E is minimized.

To minimize E, we make the partial derivatives of E with respect to w_0 and w_1 equal to zero. In most ML tasks, this cannot be done analytically so that the computation needs to be approximated by selecting values for w_0 and w_1 according to the direction of the gradient and evaluating such choices incrementally; this can be efficient but it does not guarantee global optimization.

In the simple example given here, the values of w_0 and w_1 can be calculated directly by solving the following system of equations:

$$dE/dw_0 = \sum_i (w_0 + w_1 x_i - t_i) = 0$$

$$dE/dw_1 = \sum_i ((w_0 + w_1 x_i - t_i) \cdot x_i) = 0$$

Find the values of w_0 and w_1 . Plot these back onto the equation $o_i = w_0 + w_1 x_i$, which is your ML model.

Then, calculate the training set error by finding the values of o_i for each data point and calculating E.

Finally, find o_i for x=6, which is your prediction of 'Y=?' in the lecture notes.

Now, let (x=2.5, y=1.5), (x=3.5, y=2.5), (x=4.5, y=3.5) be your test set. Calculate your model's test set error. Please post your answers on Moodle.