LECTURE 4 → SLIDE 7 & 8.

1. What is P(A)?

Using sum rule,

$$P(A) = [P(A|B) * P(B)] + (P(A|\sim B) * P(\sim B)] \rightarrow (1)$$

In the above equation, B is not known to us, so if we assume Bayes theorem,

$$P(B|C) = \frac{P(C|B) * P(B)}{P(C)} \rightarrow (2)$$

Let's bring P(B) to left hand side

$$P(B) = \frac{P(B|C) * P(C)}{P(C|B)}$$ → unknown,

or a much simpler way is to use sume rule again for P(B)

$$P(B) = [P(B|C) * P(C)] + (P(B|\sim C) * P(\sim C)]$$
$$= [0.8 * 0.6] + [0.5 * 0.4]$$
$$= 0.68 \simeq \boxed{0.7}$$

$$\therefore P(\sim B) = 1 - P(B)$$
$$= 1 - 0.7 \simeq \boxed{0.3}$$

Now that we have got~~t~~ $P(B)$,

$$P(A) = (0.1 * 0.7) + (1 * 0.3)$$

$$= 0.07 + 0.3$$

$$= 0.37$$

$$\simeq \boxed{P(A) = 0.37}$$

For Exercise 1.

1) $P(A) = 0.38$ → Correct answer

2) $$P(C|A) = \frac{P(C) \cdot P(a|c)}{P(a)} \quad — \quad 1$$

to find $p(a|c)$

$$p(a,c) = p(a|c) \cdot p(c) \quad — \quad 2$$

$$p(a,c) = p(a,b,c) + p(a,\sim b,c)$$

$$= 0.048 + [p(a|\sim b) \cdot p(\sim b|c) \cdot p(c)]$$

$=$

So $p(\sim b|c) = 1 - p(b|c)$

$= 1 - 0.8$

$= 0.2$

$\therefore p(a,c) = 0.048 + [1 * 0.2 * 0.6]$

$= 0.048 + [0.12]$

$$\boxed{p(a,c) = 0.168}$$

Now from Equation 2,

$$P(a|c) = \frac{p(a,c)}{p(c)} = \frac{0.168}{0.6} = \boxed{0.28}$$

Putting this value back, in Eqn 1

$$P(C|A) = \frac{0.6 * 0.28}{0.38}$$

$$\therefore \boxed{P(C|A) = 0.44}$$ ✓

3) Most likely explanation for Mike not answering his phone,

We need to find out all possible combinations,

$P(B|A)$
$P(\sim B|A)$
$P(C|A)$
$P(\sim C|A)$
$P(B, C|A)$
$P(\sim B, C|A)$
$P(\sim B, \sim C|A)$
$P(B, \sim C|A)$

Any of these events could be the reason ~~we~~ Mike is not answering his phone

After calculation, $\boxed{P(\sim B/A) = 0.789}$ has the highest value which suggests that " Mike is not answering the phone because he is not at home"

a) $P(B|A) = \dfrac{P(A|B) * P(B)}{P(A)}$

$= \dfrac{0.1 * 0.7}{0.38}$

$= \boxed{0.184}$

b) $P(\sim B|A) = \dfrac{P(A|\sim B)\ P(\sim B)}{P(A)}$

$= \dfrac{1 * 0.3}{0.38}$

$= \boxed{0.789} \simeq 0.79.$

c) $P(C|A) = 0.44$

d) $P(\sim C|A) = \dfrac{P(A|\sim C) * P(\sim C)}{P(A)}$

for $P(A|\sim C) =$

$P(A, \sim C) = P(A|\sim C) \cdot P(\sim C)$

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$$P(A, \sim C) = P(A, B, \sim C) + P(A, \sim B, \sim C)$$

$$= [P(A|B)\,P(B|\sim C)\,P(\sim C)]$$

$$+ [P(A|\sim B) \cdot P(\sim b|\sim C) \cdot P(\sim C)]$$

$$= [0.1 * 0.5 * 0.4] + [1 * P(\sim b|\sim C) \cdot 0.4]$$

$$= [0.02] + [0.4 * \underbrace{P(\sim b|\sim C)}_{1 - P(b|\sim C)}]$$

$$= [0.02] + [0.4 * 0.5]$$

$$= [0.02] + [0.2]$$

$$= 0.22$$

$$\therefore P(A|\sim C) = \frac{P(A, \sim C)}{P(\sim C)}$$

$$= \frac{0.22}{0.4}$$

$$= \boxed{0.55}$$

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Final answer

$$\therefore P(\sim c \mid A) = \frac{P(A \mid \sim c) \cdot P(\sim c)}{P(A)}$$

$$= \frac{0.55 * 0.4}{0.38}$$

$$= \boxed{0.578}$$

e) $$P(B, c \mid A) = \frac{P(A, B, c)}{P(A)}$$

$$= \frac{0.048}{0.38}$$

$$= \boxed{0.126}$$

f) $$P(B, \sim c \mid A) = \frac{P(A, B, \sim c)}{P(A)}$$

$$= \frac{P(A \mid B) \cdot P(B \mid \sim c) \cdot P(\sim c)}{P(A)}$$

$$= \frac{0.1 * 0.5 * 0.4}{0.38} = \boxed{0.052}$$

g) $P(\sim B, \sim C | A) = \frac{P(A, \sim B, \sim C)}{P(A)}$

$= \frac{P(A|\sim B) \cdot P(\sim B|\sim C) \cdot P(\sim C)}{P(A)}$

$= \frac{1 * 0.5 * 0.4}{0.38}$

$= \boxed{0.526}$

h) $P(\sim B, C, A) = \frac{P(A, \sim B, C)}{P(A)}$

$= \frac{P(A|\sim B) . P(\sim B|C) . P(C)}{P(A)}$

$= \frac{1 * (1 - P(B|C)) \cdot P(C)}{0.38}$

$= \frac{1 * 0.2 * 0.6}{0.38} = \boxed{0.315}$