BURGLARY EXAMPLE

We use marginalisation as a way to find $P(a)$

How can we find $p(a)$?

$$p(a) = p(a, b) + p(a, \sim b)$$

Further:

$$p(a, b) = p(a, b, e) + p(a, b, \sim e)$$

$$p(a, b, e) = p(a, b, e, d) + p(a, b, e, \sim d)$$

and

$$p(a, b, \sim e) = p(a, b, \sim e, d) + p(a, b, \sim e, \sim d)$$

$$p(a, \sim b) = p(a, \sim b, e) + p(a, \sim b, \sim e)$$

$$p(a, \sim b, e) = p(a, \sim b, e, d) + p(a, \sim b, e, \sim d)$$

and

$$p(a, \sim b, \sim e) = p(a, \sim b, \sim e, d) + p(a, \sim b, \sim e, \sim d)$$

These are all of the values we need to find:

Let us start with the basic equation

$$p(a) = p(a, b) + p(a, \sim b) \quad \text{— } ①$$

Here we need to find $p(a, b)$ and $p(a, \sim b)$ which in turn we need to find $p(a, b, e)$ and $p(a, b, \sim e)$

So,

$p(a,b,e) = p(a,b,e,d) + p(a,b,e,\sim d)$ — ②

$p(a,b,e,d) = p(a|b,e) \cdot p(b) \cdot p(e) \cdot p(d|e)$

$= 0.95 * 0.09 * 0.0001 * 0.7$

$= \boxed{0.000005985}$

And,

$p(a,b,e,\sim d) = p(a|b,e) \cdot p(b) \cdot p(e) \cdot p(\sim d|e)$

$= 0.95 * 0.09 * 0.0001 * [1 - p(d|e)]$

$= 0.95 * 0.09 * 0.0001 * 0.3$

$= \boxed{0.0000002565}$

Plugging these 2 values into Equation ②,

$\boxed{p(a,b,e) = 0.00000855}$

Next step is to find $p(a,b,\sim e)$

$p(a,b,\sim e) = p(a,b,\sim e,d) + p(a,b,\sim e,d)$ — ③

First

$p(a, b, \sim e, d) = p(a|b, \sim e) \cdot p(b) \cdot p(\sim e) \cdot p(d|\sim e)$

$= 0.94 * 0.09 * 0.9999 * 0.01$

$= \boxed{0.00084592}$ — 4

And,

$p(a, b, \sim e, \sim d) = p(a|b, \sim e) \cdot p(b) \cdot p(\sim e) \cdot p(\sim d|\sim e)$

$= 0.94 * 0.09 * 0.9999 * [1 - p(d|\sim e)]$

$= 0.94 * 0.09 * 0.9999 * 0.99$

$= \boxed{0.0837}$ — 5

From equation 1 we now have $p(a \cap b)$

$p(a, b) = p(a, b, e) + p(a, b, \sim e)$

$= 0.00000855 + [\,4 + 5\,]$

$= 0.00000855 + 0.0845$

$\boxed{p(a, b) = 0.0845}$

Using the same approach above, we need to find $p(a, \sim b)$ for Eqn 1

$p(a, \sim b) = p(a, \sim b, e) + p(a, \sim b, \sim e)$ — 6

From (6),

$$p(a, \sim b, e) = p(a, \sim b, e, d) + p(a, \sim b, e, \sim d) \quad \text{— (7)}$$

To find the 2 parts of equation (7)

$$p(a, \sim b, e, d) = p(a|\sim b, e) \cdot p(\sim b) \cdot p(e) \cdot p(d|e)$$

$$= 0.29 * 0.91 * 0.0001 * 0.7$$

$$= \boxed{0.000018473}$$

And,

$$p(a, \sim b, e, \sim d) = p(a|\sim b, e) \cdot p(\sim b) \cdot p(e) \cdot p(\sim d|e)$$

$$= 0.29 * 0.91 * 0.0001 * 0.3$$

$$= \boxed{0.00000792}$$

Plugging these values into (7),

$$p(a, \sim b, e) = 0.000018473 + 0.00000792$$

$$= \boxed{0.00002639}$$

Now From Eqn 6, we still have to find $P(a, \sim b, \sim e)$

So,

$$p(a,\sim b,\sim e) = p(a,\sim b,\sim e,d) + p(a,\sim b,\sim e,\sim d) \quad — (8)$$

$$p(a,\sim b,\sim e,d) = p(a|\sim b,\sim e) * p(\sim b) \cdot p(\sim e) \cdot p(d|\sim e)$$

$$= 0.001 * 0.91 * 0.9999 * 0.01$$

$$= \boxed{0.000091}$$

And,

$$p(a,\sim b,\sim e,\sim d) = p(a|\sim b,\sim e) \cdot p(\sim b) \cdot p(\sim e) \cdot p(\sim d|\sim e)$$

$$= 0.001 * 0.91 * 0.9999 * 0.99$$

$$= \boxed{0.00090081}$$

Finally,

$$p(a,\sim b,\sim e) = 0.000091 + 0.00090081$$

$$= \boxed{0.00099181}$$

Back to our original Equation 1

$$P(a) = p(a,b) + p(a,\sim b)$$

$$= 0.0845 + [p(a,\sim b,e) + p(a,\sim b,\sim e)]$$

$$= 0.0845 + [0.00002639 + 0.00099181]$$

giving

$$\boxed{P(a) = 0.0855}$$

Going back to the Lecture Questions:-

1) $P(B|A) = \dfrac{P(A|B) \cdot P(B)}{P(A)}$

$P(A|B)$ is unknown so,

$$P(A|B) = \frac{P(A,B)}{P(B)} = \frac{0.0845}{0.09} = \boxed{0.93}$$

$$\therefore P(B|A) = \frac{0.93 * 0.09}{0.0855}$$

$$\boxed{P(B|A) = 0.978} //$$

2) Find $P(E|A)$

$$P(E|A) = \frac{P(A|E) \cdot P(E)}{P(A)}$$

We do not have $P(A|E)$ but can find it using

$$P(A,E) = P(A|E) \cdot P(E)$$

Using Marginalisation,

$$P(a,e) = p(a,b,e) + p(a,\sim b,e)$$

We already have both these values

$$\therefore p(a,e) = 0.00000855 + 0.00002639$$

$$= \boxed{0.00003494}$$

Using this value,

$$p(a,e) = p(a|e) \cdot p(e)$$

$$\therefore p(a|e) = \frac{p(a,e)}{p(e)} = \frac{0.00003494}{0.0001}$$

$$= \boxed{0.3494}$$

Going back to our initial question of finding $P(e|a)$

$$P(e|a) = \frac{p(a|e) \cdot p(e)}{p(a)}$$

$$= \frac{0.3494 * 0.0001}{0.0855}$$

$$\boxed{p(e|a) = 0.000408}$$

3) Find $P(b,e|a)$.

$$\begin{tikzcd} \text{Burglary} \arrow[dr] & & \text{Earthquake} \arrow[dl] \\ & \text{Alarm} & \end{tikzcd}$$

Here burglary and earthquake are independent.

$$\therefore p(b,e|a) = \frac{p(a,b,e)}{p(a)} \rightarrow \text{[Bishop Ch-8, Pg 376]}$$

$$= \frac{0.00000855}{0.0855}$$

$$= \boxed{0.0001}$$

Note

B and E are intercausal (i.e 2 events influences a 3rd event). In our case given $p(b|a)$ and $p(e|a)$, the chances of $p(b,e|a)$ happening is much lower.