Module IN3031 / INM378 Digital Signal Processing and Audio Programming

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Digital Filtering (moving from theory to applications)

RECAP: Convolution

- **Convolution** combines two signals, **similarly** to **cross-correlation**
  - it's the correlation with a reversed signal

$$conv(s1, s2)[t1] = \sum_{t=0}^{N2-1} s1[t1-t]\, s2[t]$$

N2 is the length of s2, s1[i] = 0 assumed where i<0 or i>=N

- Often **written** as **s1 * s2**

RECAP: Convolution Theorem

- The **most important property** of the convolution is given by the convolution theorem:
  A convolution in the time domain
  is equivalent to
- ***a multiplication in the frequency domain***:

$$x * y \circ\!\!-\!\!\bullet X \cdot Y$$

$$meaning:\ FT(conv(x, y)) = FT(x) \cdot FT(y)$$

Digital Filters

- Sound **spectra** are **changed** by **filters**
- **SFFT** manipulation and resynthesis - a form of **filtering in the frequency domain**
- **Most filtering** happens in the **time domain** by **convolution**

Linear Filters

- Linear filters **sum scaled and delayed copies** of the signal to itself (**convolution** with the **scaling factors**)
- **2 types**, depending on where they take the signal from
  - **Finite Impulse Response** (**FIR**) filters (use input signal)
  - **Infinite Impulse Response** (**IIR**) filters (use input & output signal)

The Order of Filters

- An **FIR** filter ***f*** of **order *k*** has this **structure**
  f(x[n]) = b_0x[n] + b_1 x[n-1] + b_2 x[n-2] + ... + b_k x[n-k]

- An **IIR** filter *g* of **order *k*** has this **recursive** structure
  g(x[n]) = + b_0x[n] + b_1 x[n-1] + b_2 x[n-2] + ... + b_k x[n-k]
  - a_1g(x[n-1]) - a_2g(x[n-2]) - ... - a_kg(x[n-k])

- or as a **difference equation**

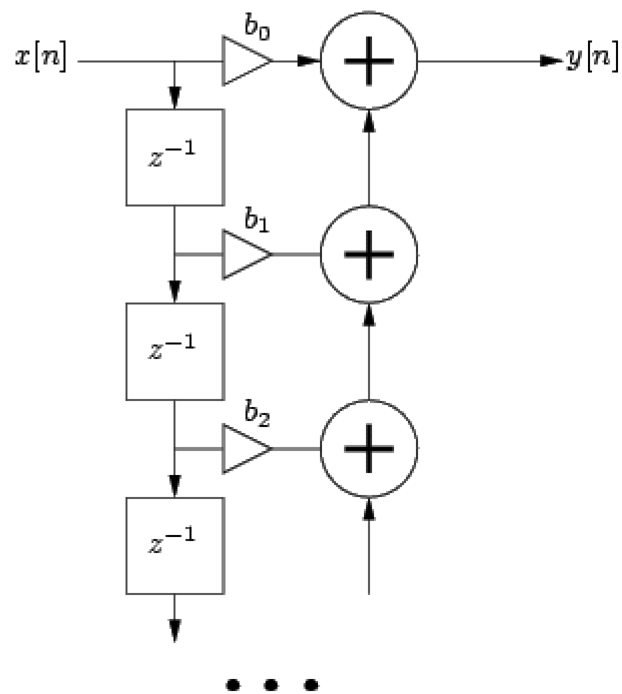
$$y[n] = -\sum_{i=1}^{k} a_i\, y[n-i] + \sum_{i=0}^{k} b_i\, x[n-i]$$

- $\mathbf{a_n}$ and $\mathbf{b_n}$ are called **filter coefficients**

An FIR Filter

- Am **FIR** filter *f* of order *k* has this **structure**
  $f(x[n]) = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] + \ldots + b_k x[n-k]$
  with **coefficients** $b = [b_0, b_1, b_2, \ldots, b_k]$

- **Graphically**:



▷ : multiplication with a scalar

z^{-1}: delay by 1 sample

Uses of Digital Filters

- Digital filters are **used for**
  - **anti-aliasing** (before downsampling)
  - **equalisation** (removing frequency imbalances of microphones, room acoustics etc)
  - user **sound modification** (adjust to personal taste)
  - sound **analysis** (select the frequency range to analyse)
  - sound **synthesis** (shape the timbre of a synthetic sound)

Types of Digital Filters

- There are four **common types** of filters:
  - **low pass** (anti-aliasing, synthesis, HF noise removal)
  - **high pass** (remove rumbling, protect speakers)
  - **band pass** (sound analysis)
  - **band stop** (removing unwanted signal, e.g. from power supply)
- **Other** types of filters:
  - **comb** filters (usually the result of short delays)
  - **all pass** filters (modify only the phase)

Properties of Digital Filters

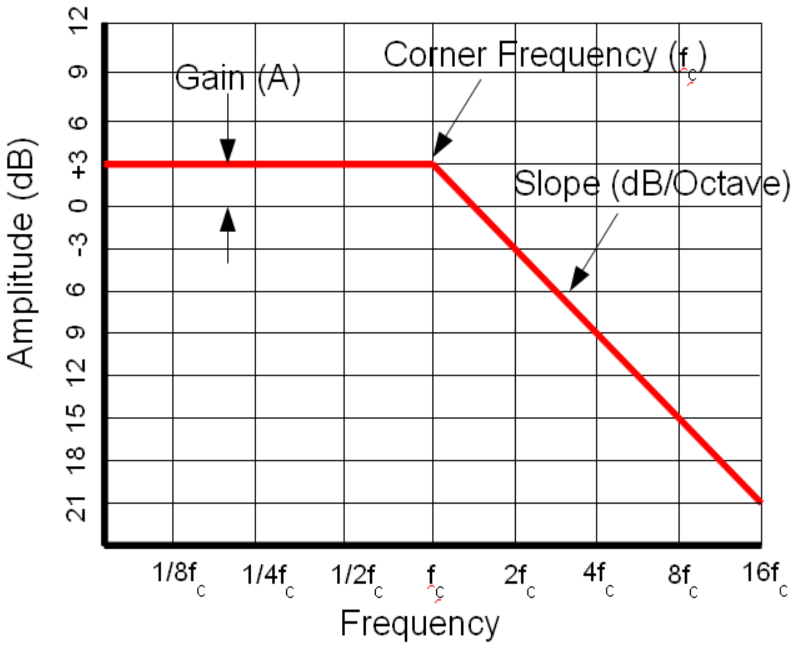
- Filter **architecture** (FIR or IIR)
- Filter **order** (#sample delays = #coefficients-1)
- Filter **coefficients** (the values defining the filter operation)

resulting from these

- **Frequency response** (mainly magnitude)
- **Impulse response** (sometimes step response)
- **Time behaviour** (phase response, group delay)

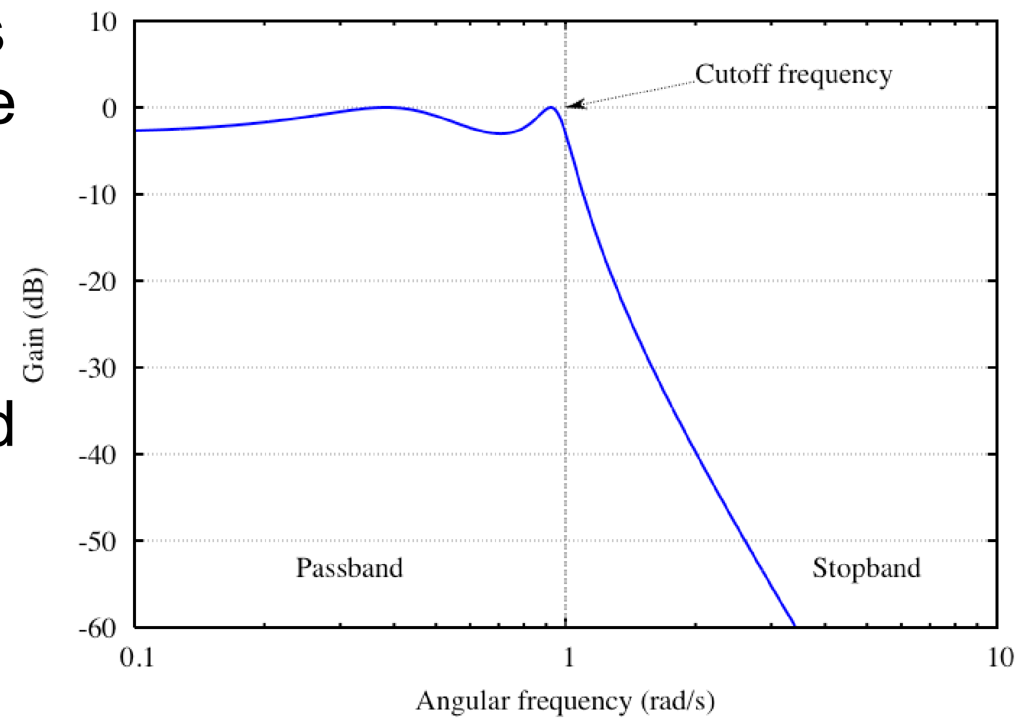
Filter Parameters

- The ***pass band*** is a range of frequencies, that should pass the filter unattenuated.
- The ***stop band*** is a range of frequencies, that should not pass the filter.
- The pass band ends at the ***corner frequency*** (usually at -3dB).
- The **slope** of the freq. resp. is measured in dB/octave (somtimes decade)



More Filter Parameters

- ***Ripple*** is the unevenness of the frequency response
- ***Resonance*** is a peak in frequency response near cut-off frequency
- ***Stability***: The filter should (usually) not oscillate by itself

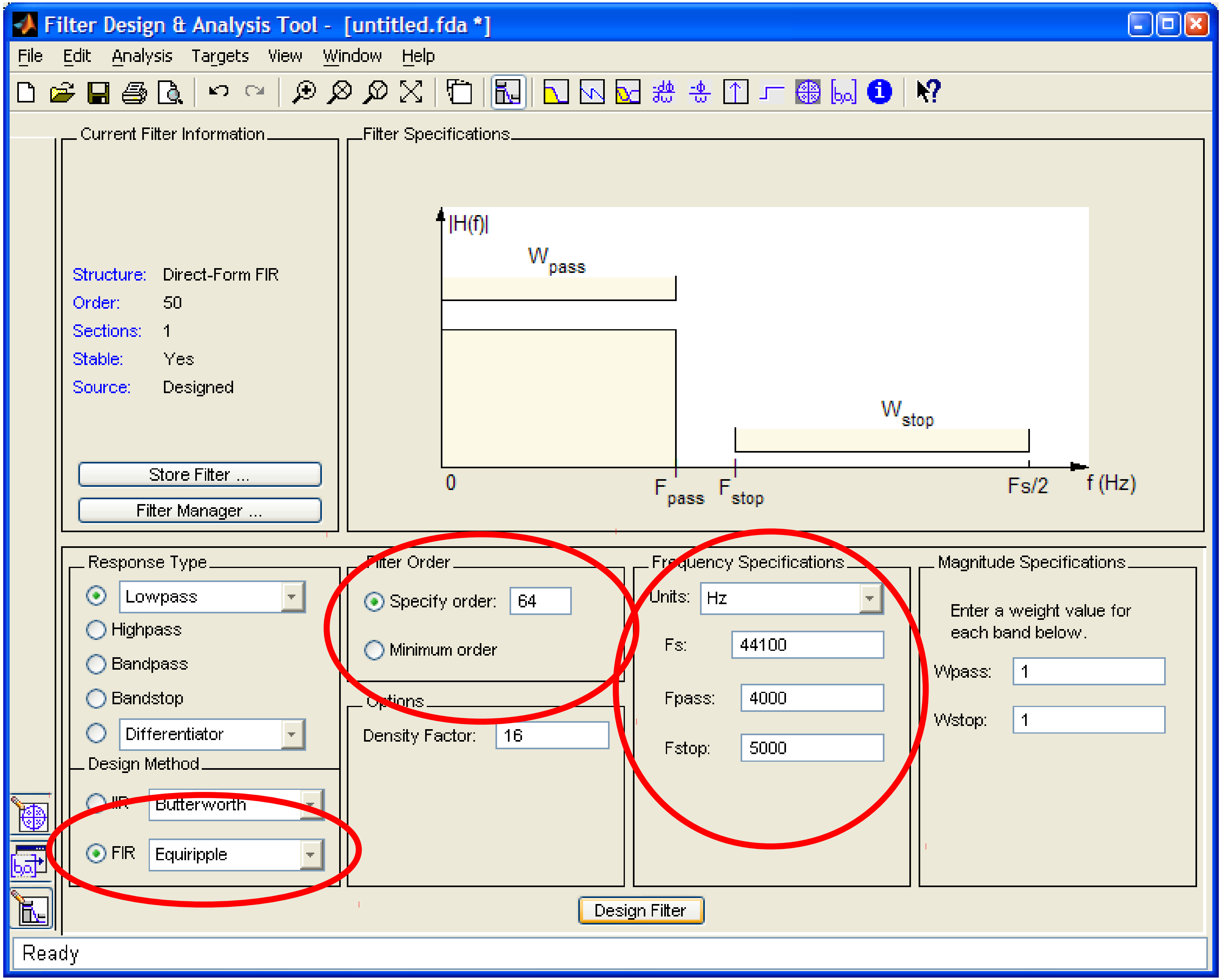


FIR Filter Design

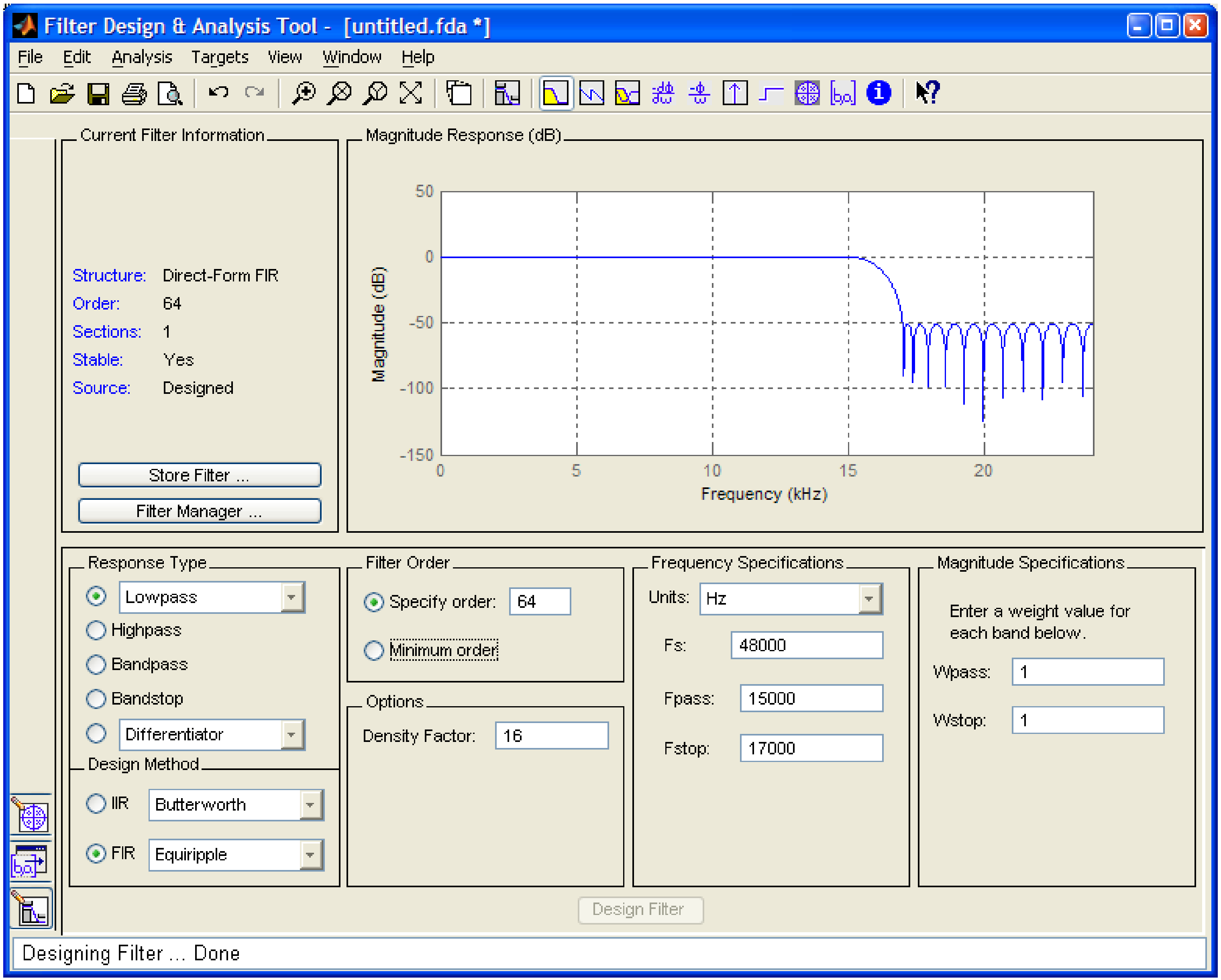
FIR:

- Approach: coefficients as **iFFT** of frequency response
- **Pro**:
  - **FIR** filters are **always stable**
  - Good phase behaviour
- **Cons**:
  - for steep slope in the transition band, we need high **number of coefficients** (and thus computation time)

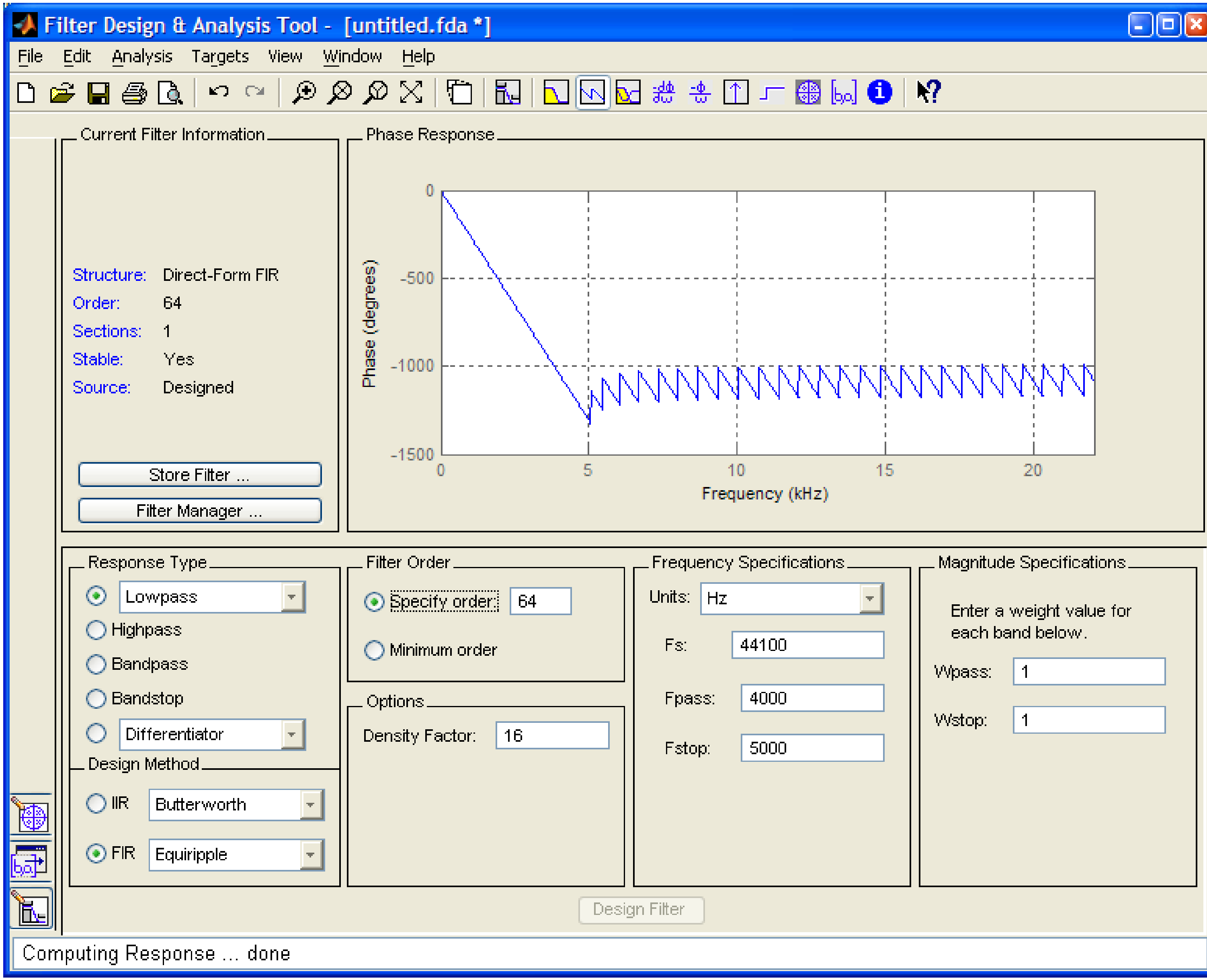
FIR Filter Design in Matlab



Frequency Response



Phase Response



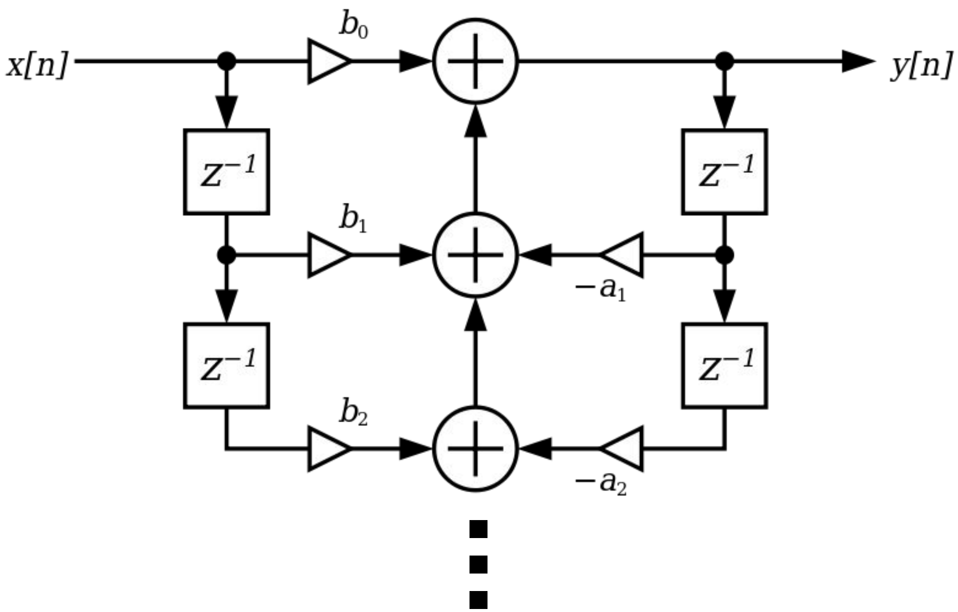
IIR Filter Design

IIR:

- Approach: Define **numerical methods** for finding appropriate filter coefficients (mathematically demanding).
- Pro:
  - IIR filters can be **very efficient**.
- Cons:
  - Uncontrolled **phase** behaviour
  - **May be unstable**
  - Quantisation noise may multiply through recursion

IIR Filter Diagram

$$y[n] = -\sum_{i=1}^{k} a_i\, y[n-i] + \sum_{i=0}^{k} b_i\, x[n-i]$$



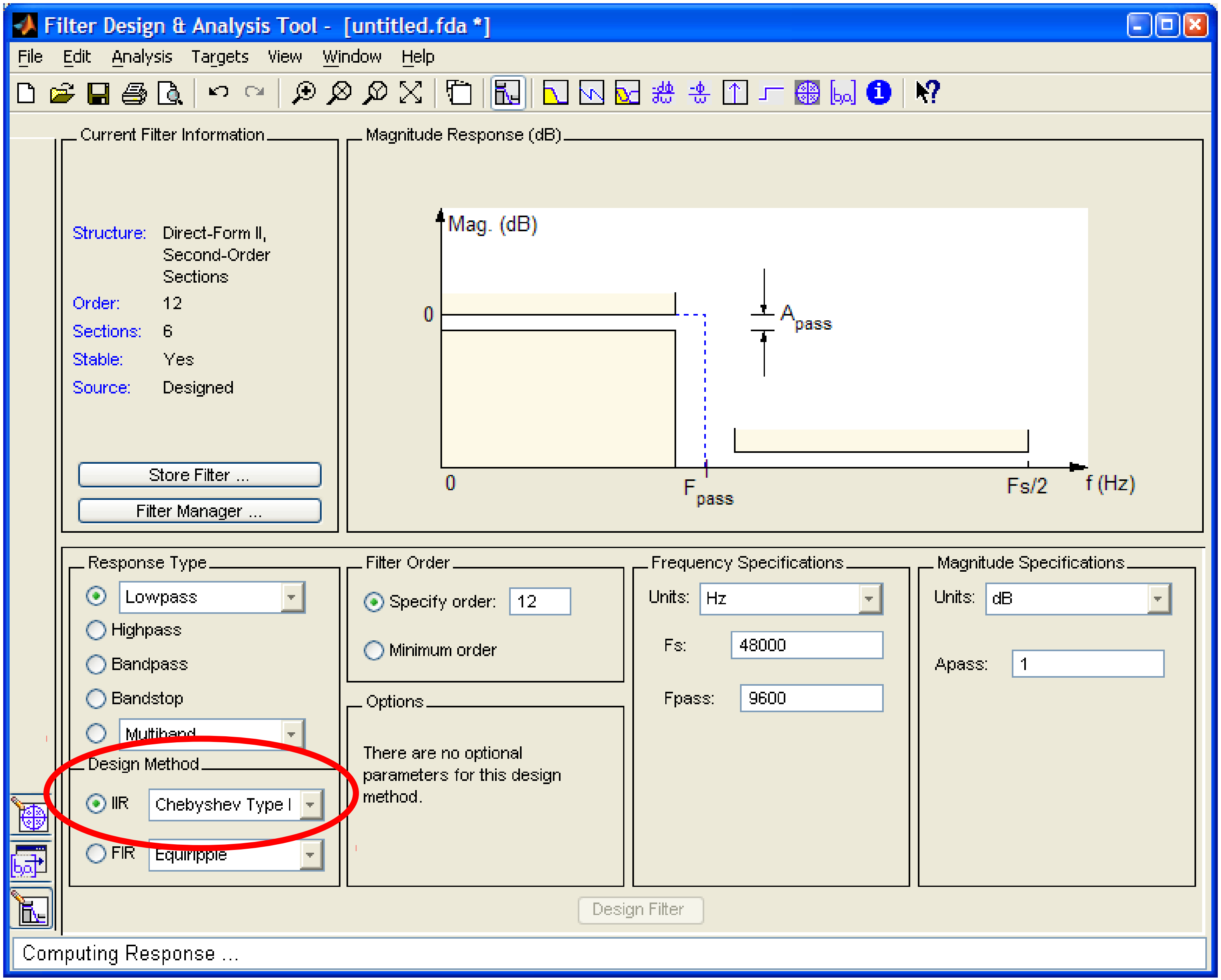
z^{-1} represents a delay by one sample

This structure is called *Direct Form 1*

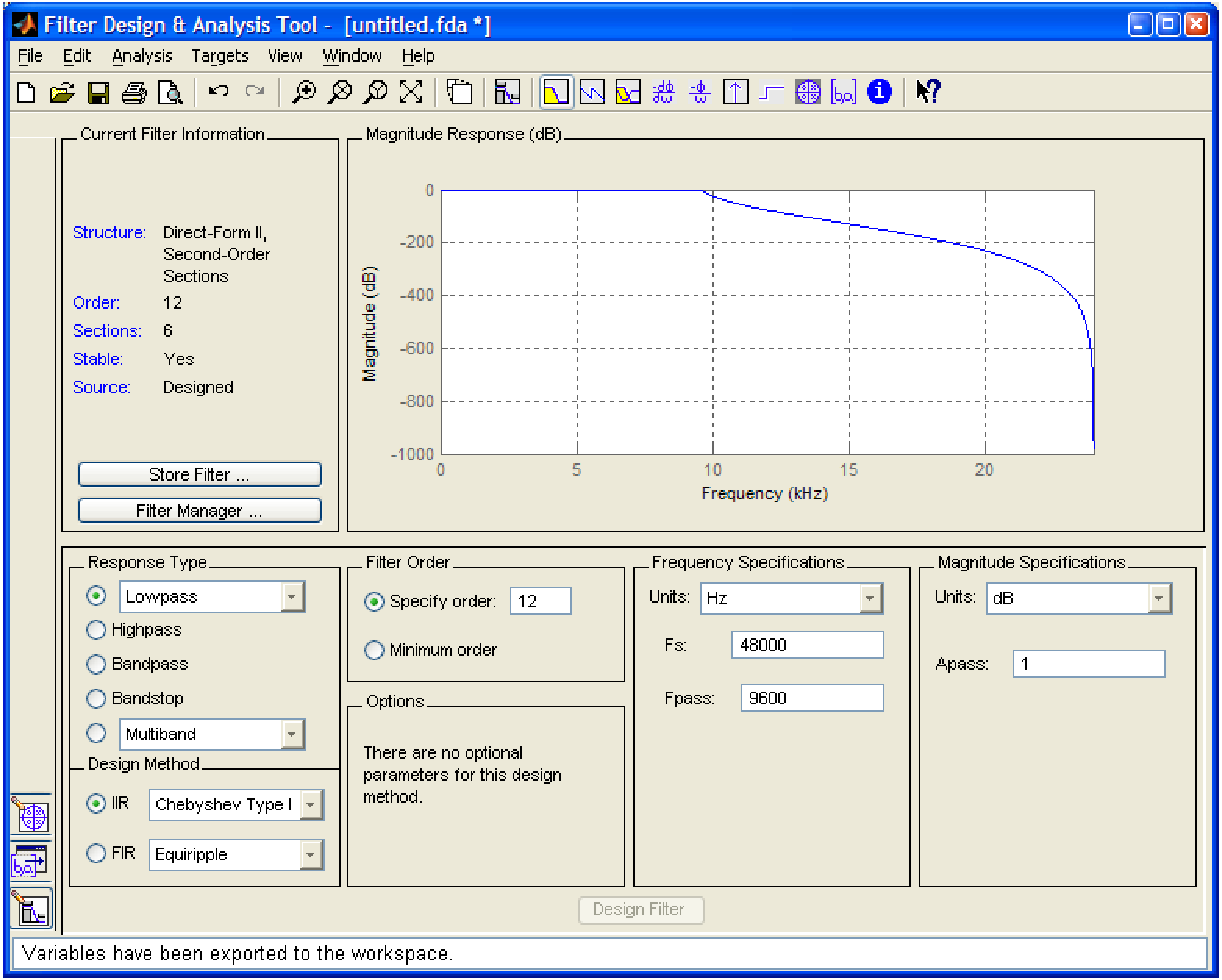
The Impulse Response of an IIR Filter

- An IIR filter g
  $$g(x[n]) = - a_1 g(x[n-1]) - a_2 g(x[n-2]) - \dots - a_k g(x[n-k]) + b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] + \dots + b_k x[n-k]$$
- The impulse response of g has to be computed recursively and may be infinitely long
- IIR filters
  - allow very effective filtering with few coefficients
  - may oscillate by themselves
  - frequency response is hard to compute

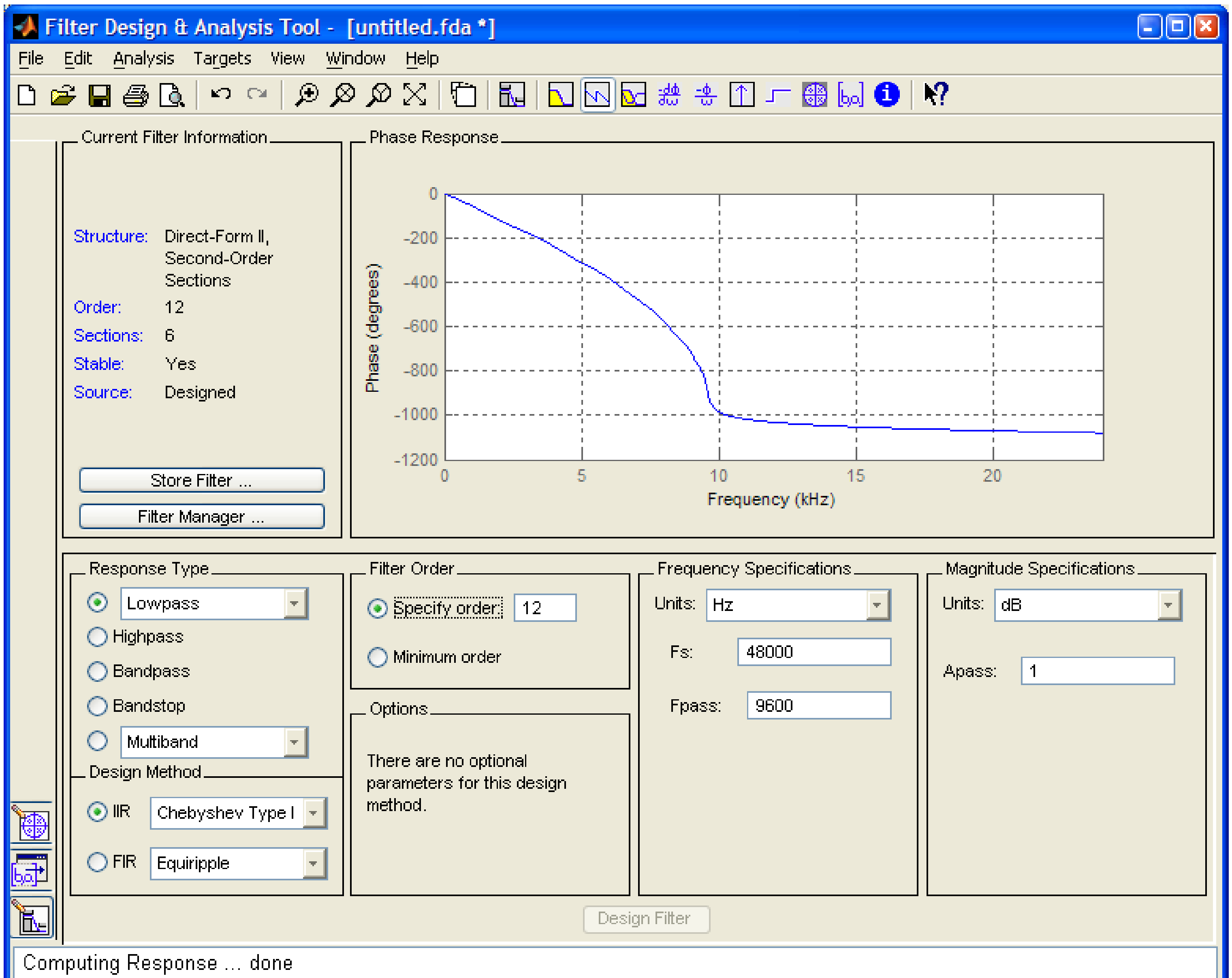
IIR Filter Design in Matlab



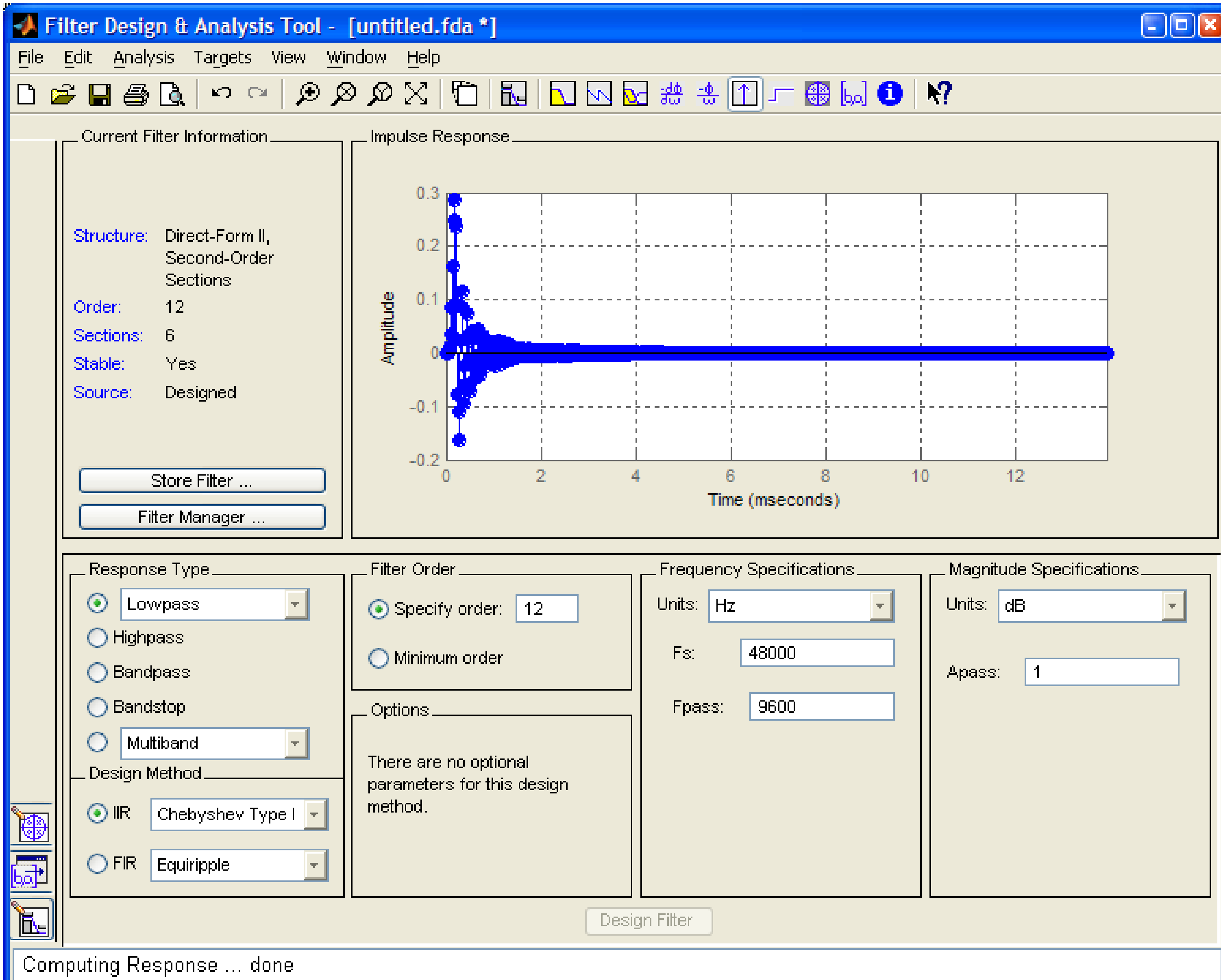
Frequency Response



Phase Response

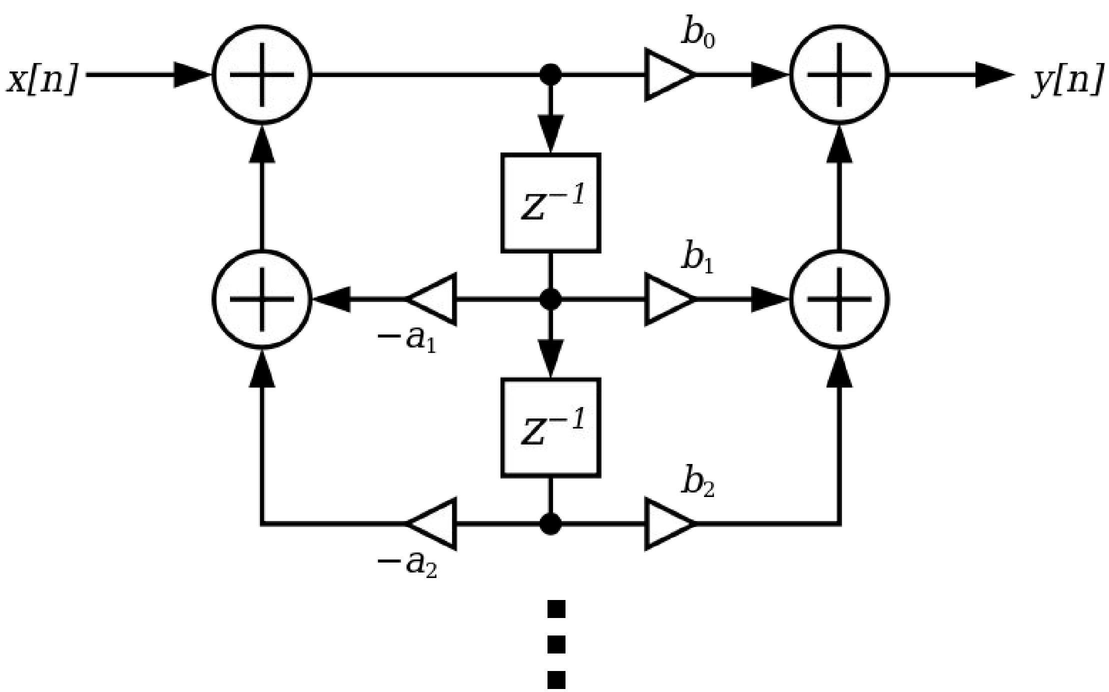


Impulse Response



IRR Filters in Practice

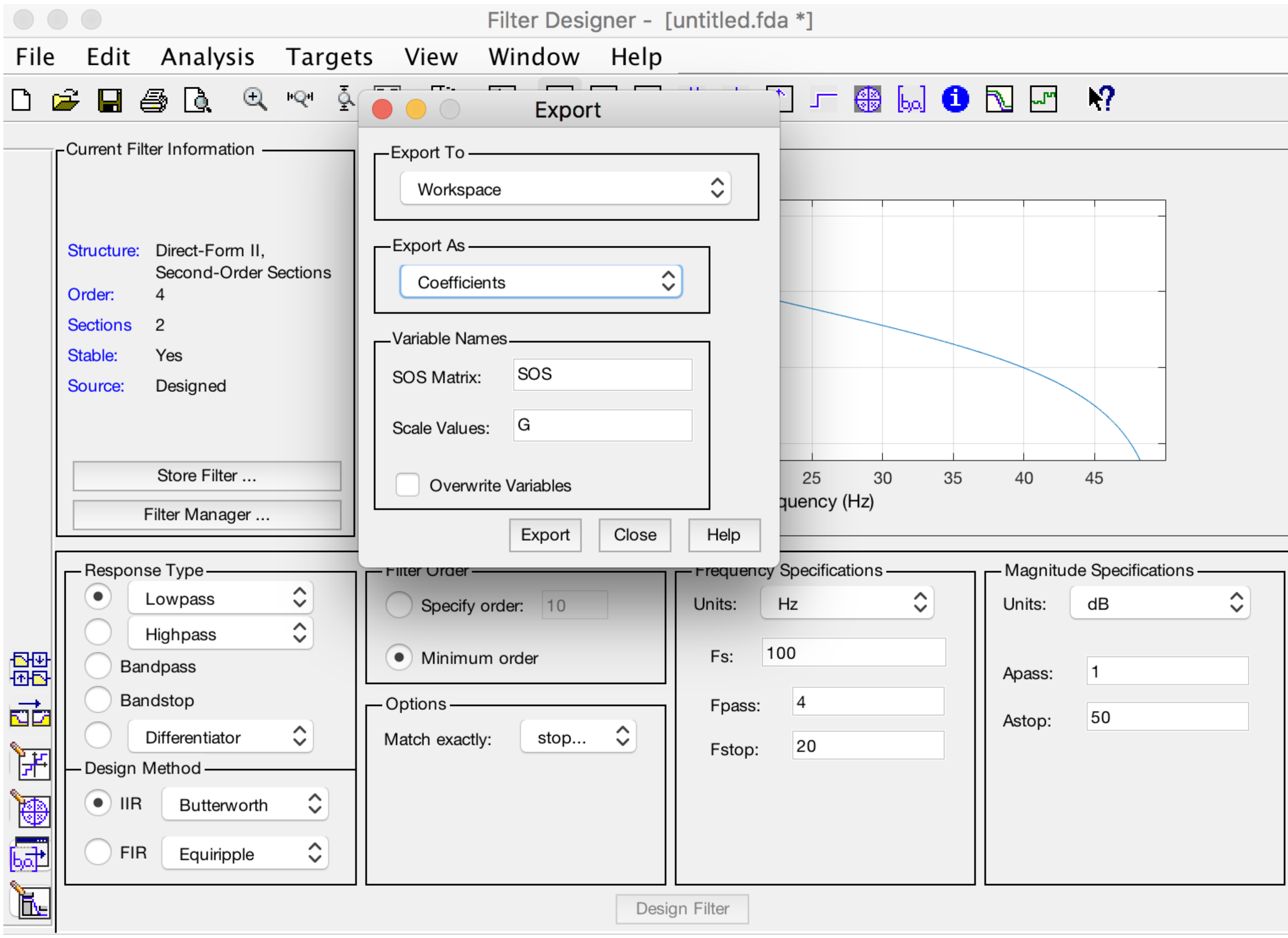
- IRR filters are used in *Direct Form 2*
- Equivalent to Direct Form 1, but more efficient



IRR Filters in Practice (2)

- Many architectures exist
  - equivalent possibilities (linear systems …)
  - numerical and computational trade-offs
- Standard DF2 uses SOS (internal) and G (scaling) coefficients
- Can be transformed to A and B coefficients in `[B,A]=sos2tf(SOS,G)`
- Apply A and B coefficients with `filter(A,B,sig)`

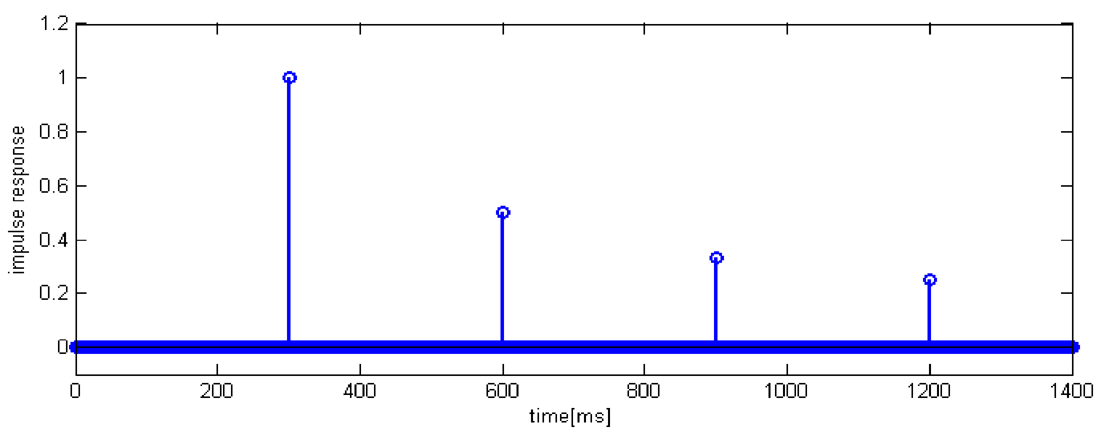
Exporting an IRR filter to the Matlab workspace



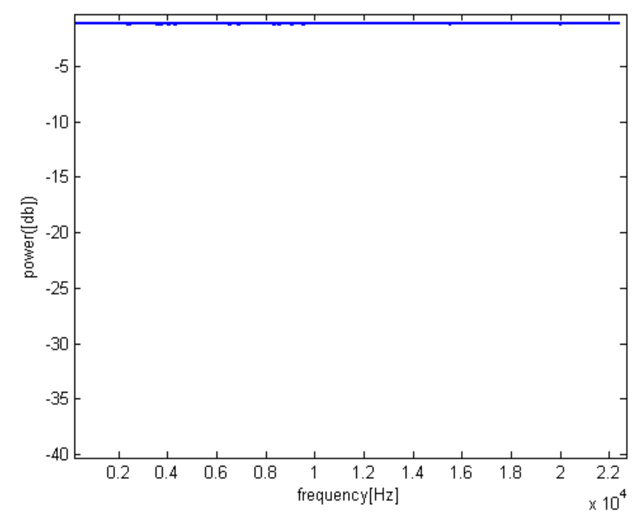
Impulse Responses and Audio Effects

Echo

- Impulse response: Few filter coefficients **span over seconds**
- Frequency response is **flat**



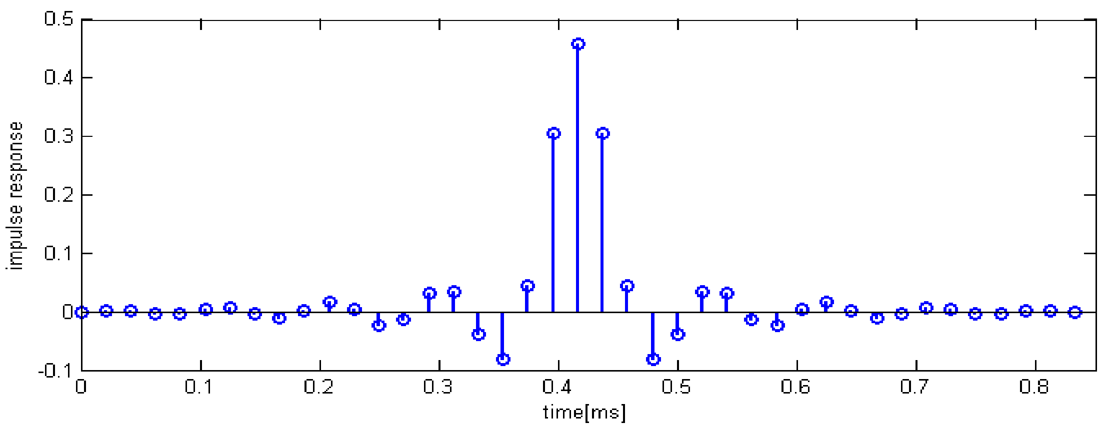
Impulse Response



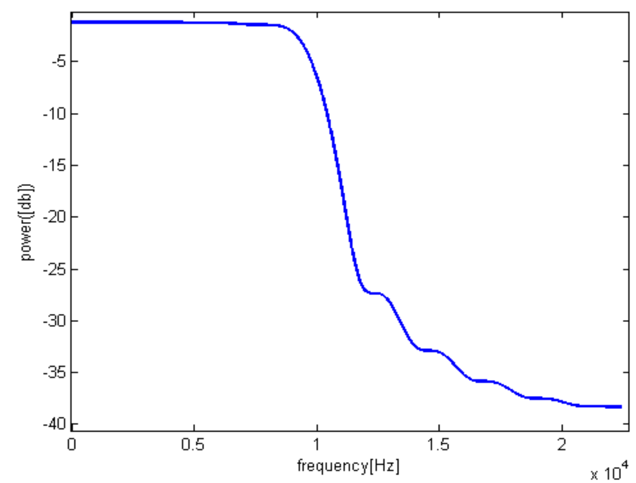
Frequency Domain

Low-Pass

- Impulse response: Many filter coefficients in the **first few milliseconds**
- **Approximates a rectangular window** in frequency domain



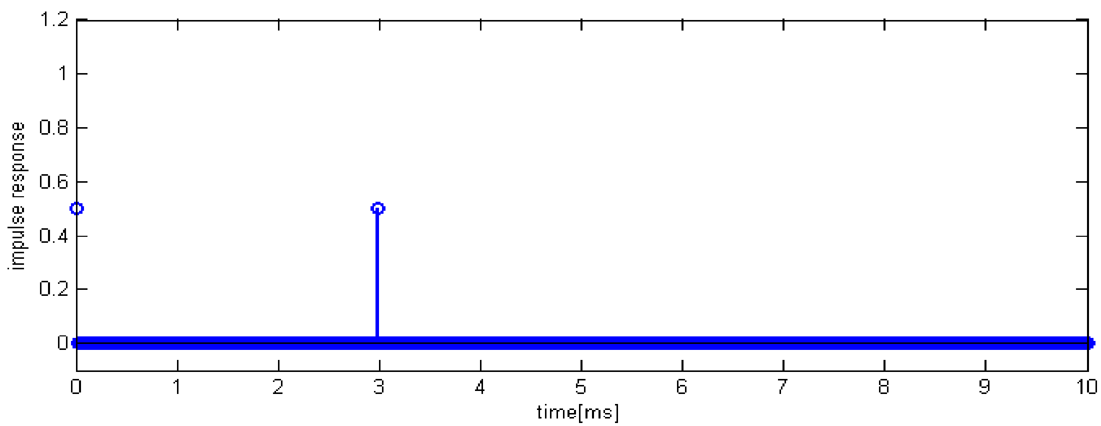
Impulse Response



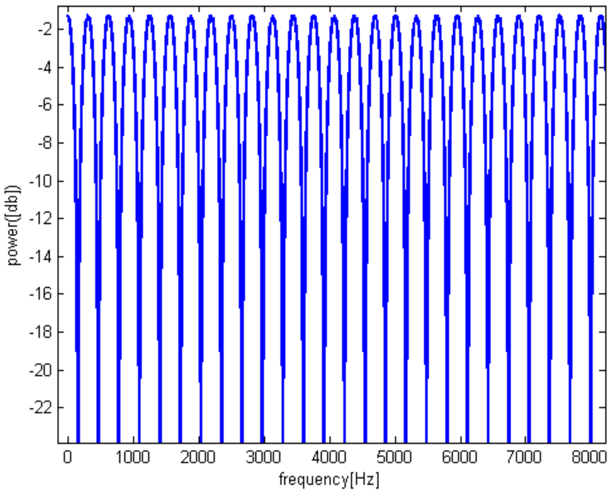
Frequency Domain

Flanger as Filter

- Impulse response: 2 filter coefficients in the **first few milliseconds (identity + delay)**
- **Comb filter shape** in frequency domain
- IR changes over time



Impulse Response

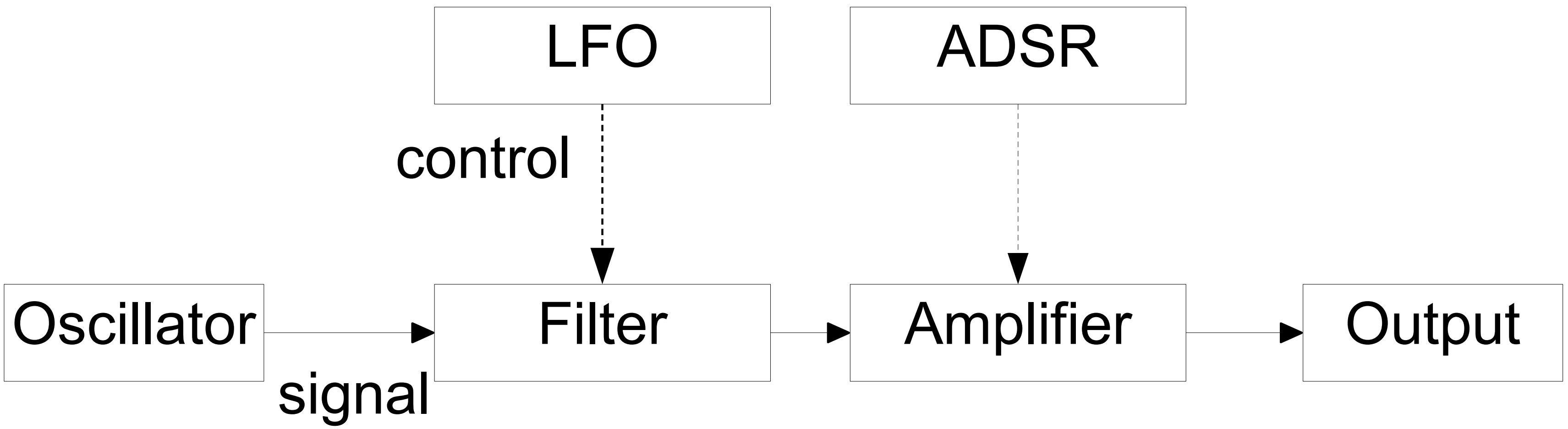


Frequency Domain

Using Filters for Subtractive Synthesis

Subtractive Sound Synthesis

- Most common form of analogue synthesis
- Generates a sound, filters and amplifies (attenuates) it
- Exemplary set-up:



“Virtual” Synthesizer

Take-Home Messages

- FIR and IIR filters are often used to remove frequency bands (Hi-Pass, Lo-Pass, …)
- Filters need delay-lines, i.e. memory buffers
- Filters can be time-variant (flanger)
- Can be used in subtractive synthesis
- Games need real-time programming
- Complexity often hidden by building blocks (FMOD)

Reading:
FMOD Studio API Docs
Smith, DSP Guide, chpt 15