Lecture 2: 31/1-6/2

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2.1 Convex Functions

Read Chapter 3 from [BV04], focusing on the following sections:

- 3.1.1, 3.1.3 (not the proof), 3.1.4, 3.1.5, 3.1.6, 3.1.7, 3.1.8
- 3.2.1, 3.2.2, 3.2.3 (up to example 3.6), 3.2.4 (up to eq. (3.10) but including examples 3.12, 3.13), 3.2.5.

Read Section 1.6 to familiarise yourself with the notation.

2.2 Questions for practice

1. Which methods can we use to show that a function is convex?
2. Write the Taylor series expansion for the functions $f_1(x) : \mathbf{R} \to \mathbf{R}$ and $f_2(x) : \mathbf{R}^n \to \mathbf{R}$. Assume they are differentiable at all points.
3. Draw a function that does not satisfy the 1st-order condition (i.e., $f(y) \geq f(x) + \nabla f(x)^T (y - x)$ for all $x \in \mathbf{dom} f$).
4. What is the Sylvester's criterion; why do we need it in convex optimisation?
5. What is the difference between convexity and strict convexity? What does this mean in practice when we solve a problem? (hint: think about the optimal point(s) we get in each case).
6. Consider the function in quadratic form:

$$f(x) = \frac{1}{2} x^T P x + q^T x + r, \quad P \in \boldsymbol{S}^n,\ q \in \boldsymbol{R}^n,\ r \in \boldsymbol{R}$$

   and assume that $n = 2$, with $P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$, $q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$, and r. Then, do the algebra and write $f(x)$ without using the matrix notation (i.e., do the matrices multiplications), showing that indeed we get quadratic terms. Also, what properties the matrices P, q, and r should have in order function $f(x)$ to be convex?
7. Solve Exercise 3.1 from [BV04].
8. Solve Exercise 3.15(b) from [BV04].
9. Solve Exercise 3.16 from [BV04]; say only if the functions are convex or concave (ignore the quasi-convex or quasi-concave cases).
10. Solve Exercise 3.22 from [BV04].

2.3 Notes about Matlab Scripts

- Matlab scripts provide examples about convex functions.
- Prepare for programming convex problems by reading about CVX; more info here: http://cvxr.com/cvx/

References

[BV04] S. Boyd, L. Vandenberghe, "Convex Optimization", *Cambridge University Press*, 2004.