1 Equazioni di Maxwell generali

Forma Locale	forma Globale
$\nabla \cdot \vec{\mathbf{D}} = \rho_{free}$	$\Phi(D) = \oint_S \vec{\mathbf{D}} \cdot d\Sigma = Q_{free}$
$\nabla \cdot \vec{\mathbf{B}} = 0$	$\Phi(B) = \oint_S \vec{\mathbf{B}} \cdot d\Sigma = 0$
$\nabla \times \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{B}}}{\partial t}$	$\Gamma(E) = \oint_\gamma \vec{\mathbf{E}} \cdot dl = -\frac{\mathrm{d}\Phi(\vec{\mathbf{B}})}{\mathrm{d}t}$
$\nabla \times \vec{\mathbf{H}} = \vec{\mathbf{J}} + \frac{\partial \vec{\mathbf{D}}}{\partial t}$	$\Gamma(H) = \oint_\gamma \vec{\mathbf{H}} \cdot dl = \int_S \vec{\mathbf{J}} \cdot dS + \frac{\mathrm{d}\Phi(\vec{\mathbf{D}})}{\mathrm{d}t}$

2 Relazioni tra i vari campi

2.1 Campi

$$\vec{\mathbf{D}} = \epsilon_0 \vec{\mathbf{E}} + \vec{\mathbf{P}} \tag{1}$$

$$\vec{\mathbf{P}} = \chi \epsilon_0 \vec{\mathbf{E}} \tag{2}$$

$$\vec{\mathbf{D}} = \epsilon_0 \epsilon_r \vec{\mathbf{E}} \quad \text{Nei materiali isotropi, omogenei e lineari} \tag{3}$$

$$\vec{\mathbf{H}} = \vec{\mathbf{B}}/\mu_0 - \vec{\mathbf{M}} \tag{4}$$

2.2 Relazione campi con sorgenti

$$\begin{array}{rl} \nabla \cdot \vec{\mathbf{P}} = -\rho_{bounded} & \vec{\mathbf{P}} \cdot \hat{\mathbf{n}} = \sigma_{bounded} \\ \nabla \cdot \vec{\mathbf{D}} = \rho_{free} & \vec{\mathbf{D}} \cdot \hat{\mathbf{n}} = \sigma_{free} \quad [cc] \\ \nabla \times \vec{\mathbf{M}} = \vec{\mathbf{J}}_{\mathbf{Amp,V}} & \vec{\mathbf{M}} \times \hat{\mathbf{n}} = \vec{\mathbf{J}}_{\mathbf{Amp,S}} \end{array}$$

2.3 Potenziali

$$\vec{\mathbf{B}} = \nabla \times A \tag{5}$$

$$\nabla \times E = -\frac{\mathrm{d}}{\mathrm{d}t} \nabla \times A \tag{6}$$

$$-\nabla V = \vec{\mathbf{E}} + \frac{\partial A}{\partial t} \tag{7}$$

2.4 Formule Particolari

Legge di Biot Savart (Campo Magnetico $\vec{B}$ generato da un circuito $\mathcal{C}$ percorso da una corrente i):

$$\frac{\mu_0 i}{4\pi} \oint_{\mathcal{C}} \frac{dl \times r}{r^3} \tag{8}$$

Forza di Lorentz:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \tag{9}$$

Autoinduttanza:

$$\Phi = Li \tag{10}$$

Mutua Induttanza:

$$\Phi_{k,i} = M_{k,i} \cdot i_k \tag{11}$$

2.5 Onde

$$\vec{\mathbf{S}} = \frac{\vec{E} \times \vec{B}}{\mu_0 \mu_r} \tag{12}$$

$$I =< S >= \frac{1}{2} \epsilon v E_0^2 \tag{13}$$

Formula per assorbimento energia:

$$\mathcal{E} = \epsilon_r \epsilon_0 E_0^2 \qquad \text{materiali assorbenti} \tag{14}$$

$$\mathcal{E} = 2\epsilon_r \epsilon_0 E_0^2 \qquad \text{materiali riflettenti} \tag{15}$$

3 Relazioni notevoli (valgono in tutti i sistemi di riferimento)

$$\boldsymbol{\nabla} \cdot \boldsymbol{\nabla} f = \nabla \cdot (\nabla f) = \nabla^2 f \quad \text{Operatore di Laplace o Laplaciano} \tag{16}$$

$$\boldsymbol{\nabla} \times \boldsymbol{\nabla} f = \nabla \times (\nabla f) = 0 \tag{17}$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{\nabla} \times \mathbf{F} = \nabla \cdot (\nabla \times \mathbf{F}) = 0 \tag{18}$$

$$\boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \mathbf{F} = \nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla \cdot \mathbf{F}) - \nabla^2 \mathbf{F} \tag{19}$$

$$\nabla^2 fg = f\nabla^2 g + 2\nabla f \cdot \nabla g + g\nabla^2 f \tag{20}$$

Formula di Lagrange per il prodotto vettoriale:

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B}) \tag{21}$$

$$\nabla \cdot (f\mathbf{A}) = f\nabla \cdot \mathbf{A} + \mathbf{A} \cdot \nabla f \tag{22}$$

$$\nabla \times f\mathbf{A} = f\nabla \times \mathbf{A} - \mathbf{A} \times \nabla f \tag{23}$$

$$\nabla(\mathbf{A} \cdot \mathbf{B}) = (\mathbf{A} \cdot \nabla)\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{A} + \mathbf{A} \times (\nabla \times \mathbf{B}) + \mathbf{B} \times (\nabla \times \mathbf{A}) \tag{24}$$

Tabella 1: Formule differenziali necessarie per esame

nome	Cartesiane	Cilindriche	Sferiche
$\boldsymbol{\nabla} f$	$\frac{\partial f}{\partial x}\vec{\mathbf{x}}+\frac{\partial f}{\partial y}\vec{\mathbf{y}}+\frac{\partial f}{\partial z}\vec{\mathbf{z}}$	$\frac{\partial f}{\partial \rho}\vec{\rho}+\frac{1}{\rho}\frac{\partial f}{\partial \phi}\vec{\phi}+\frac{\partial f}{\partial z}\vec{\mathbf{z}}$	$\frac{\partial f}{\partial r}\vec{\mathbf{r}}+\frac{1}{r}\frac{\partial f}{\partial \theta}\vec{\theta}+\frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\vec{\phi}$
$\boldsymbol{\nabla}\cdot\vec{F}$	$\frac{\partial f}{\partial x}+\frac{\partial f}{\partial y}+\frac{\partial f}{\partial z}$	$\frac{1}{\rho}\frac{\partial \rho A_\rho}{\partial \rho}+\frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi}+\frac{\partial A_z}{\partial z}$	$\frac{1}{r^2}\frac{\partial r^2 F_r}{\partial r}+\frac{1}{r\sin\theta}\frac{\partial F_\theta \sin\theta}{\partial \theta}+\frac{1}{r\sin\theta}\frac{\partial F_\phi}{\partial \phi}$
$\boldsymbol{\nabla}\times\vec{F}$	$\left(\frac{\partial F_z}{\partial y}-\frac{\partial F_y}{\partial z}\right)\hat{\boldsymbol{x}}$ $\left(\frac{\partial F_x}{\partial z}-\frac{\partial F_z}{\partial x}\right)\hat{\boldsymbol{y}}$ $\left(\frac{\partial F_y}{\partial x}-\frac{\partial F_x}{\partial y}\right)\hat{\boldsymbol{z}}$	$\left(\frac{1}{\rho}\frac{\partial A_z}{\partial \phi}-\partial A_\phi\frac{\partial}{\partial z}\right)\hat{\boldsymbol{\rho}}$ $\left(\partial A_\rho \partial z-\partial A_z \partial \rho\right)\hat{\boldsymbol{\phi}}$ $1\rho\left(\partial(\rho A_\phi)\partial\rho-\partial A_\rho\partial\phi\right)\hat{\boldsymbol{z}}$	$\frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}(A_\phi\sin\theta)-\frac{\partial A_\theta}{\partial\phi}\right)\hat{\boldsymbol{r}}$ $1r\left(1\sin\theta\partial A_r\partial\phi-\frac{\partial}{\partial r}(rA_\phi)\right)\hat{\boldsymbol{\theta}}$ $\frac{1}{r}\left(\frac{\partial}{\partial r}(rA_\theta)-\frac{\partial A_r}{\partial\theta}\right)\hat{\boldsymbol{\phi}}$
$\nabla^2 f$	$\frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 f}{\partial y^2}+\frac{\partial^2 f}{\partial z^2}$	$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial f}{\partial\rho}\right)+\frac{1}{\rho^2}\frac{\partial^2 f}{\partial\phi^2}+\frac{\partial^2 f}{\partial z^2}$	$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\phi^2}$
$\nabla^2\mathbf{A}$	$\nabla^2 A_x\hat{\mathbf{x}}+\nabla^2 A_y\hat{\mathbf{y}}+\nabla^2 A_z\hat{\mathbf{z}}$	$\left(\nabla^2 A_\rho-\frac{A_\rho}{\rho^2}-\frac{2}{\rho^2}\frac{\partial A_\phi}{\partial\phi}\right)\hat{\boldsymbol{\rho}}$ $\left(\nabla^2 A_\phi-\frac{A_\phi}{\rho^2}+\frac{2}{\rho^2}\frac{\partial A_\rho}{\partial\phi}\right)\hat{\boldsymbol{\phi}}$ $(\nabla^2 A_z)\hat{\boldsymbol{z}}$	$\left(\nabla^2 A_r-\frac{2A_r}{r^2}-\frac{2}{r^2\sin\theta}\frac{\partial(A_\theta\sin\theta)}{\partial\theta}-\frac{2}{r^2\sin\theta}\frac{\partial A_\phi}{\partial\phi}\right)\hat{\boldsymbol{r}}$ $\left(\nabla^2 A_\theta-\frac{A_\theta}{r^2\sin^2\theta}+\frac{2}{r^2}\frac{\partial A_r}{\partial\theta}-\frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial A_\phi}{\partial\phi}\right)\hat{\boldsymbol{\theta}}$ $\left(\nabla^2 A_\phi-\frac{A_\phi}{r^2\sin^2\theta}+\frac{2}{r^2\sin\theta}\frac{\partial A_r}{\partial\phi}+\frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial A_\theta}{\partial\phi}\right)\hat{\boldsymbol{\phi}}$