Equazioni di Maxwell generali 1

Forma Locale	forma Globale
$\nabla \cdot \vec{\mathbf{D}} = \rho_{free}$ $\nabla \cdot \vec{\mathbf{B}} = 0$	$ \Phi(D) = \oint_{S} \vec{\mathbf{D}} \cdot d\Sigma = Q_{free} \Phi(B) = \oint_{S} \vec{\mathbf{B}} \cdot d\Sigma = 0 $
$\nabla \cdot \vec{\mathbf{B}} = 0$	$\Phi(B) = \oint_{S} \vec{\mathbf{B}} \cdot d\Sigma = 0$
$oldsymbol{ abla} imesec{\mathbf{E}}=-rac{\partialec{\mathbf{B}}}{\partial t}$	$\Gamma(E) = \oint_{\gamma} \vec{\mathbf{E}} \cdot dl = -\frac{\mathrm{d}\Phi(\vec{\mathbf{B}})}{\mathrm{d}t}$
$oldsymbol{ abla} imes oldsymbol{ec{H}} = oldsymbol{ec{J}} + rac{\partial oldsymbol{ec{D}}}{\partial t}$	$\Gamma(H) = \oint_{\gamma} \vec{\mathbf{H}} \cdot dl = \int_{S} \vec{\mathbf{J}} \cdot dS + \frac{\mathrm{d}\Phi(\vec{\mathbf{D}})}{\mathrm{d}t}$

$\mathbf{2}$ Relazioni tra i vari campi

2.1 Campi

$$\vec{\mathbf{D}} = \epsilon_0 \vec{\mathbf{E}} + \vec{\mathbf{P}} \tag{1}$$

$$\vec{\mathbf{P}} = \chi \epsilon_0 \vec{\mathbf{E}} \tag{2}$$

$$\vec{\mathbf{D}} = \epsilon_0 \epsilon_r \vec{\mathbf{E}}$$
 Nei materiali isotropi, omogenei e lineari (3)

$$\vec{\mathbf{H}} = \vec{\mathbf{B}}/\mu_0 - \vec{\mathbf{M}} \tag{4}$$

Relazione campi con sorgenti 2.2

$$\nabla \cdot \vec{\mathbf{P}} = -\rho_{bounded} \tag{5}$$

$$\vec{\mathbf{P}} \cdot \hat{\mathbf{n}} = \sigma_{bounded} \tag{6}$$

$$\nabla \cdot \vec{\mathbf{D}} = \rho_{free} \tag{7}$$

$$\vec{\mathbf{D}} \cdot \hat{\mathbf{n}} = \sigma_{free} \tag{8}$$

$$\nabla \times \vec{\mathbf{M}} = \vec{\mathbf{J}_{\mathbf{Amp},\mathbf{V}}} \tag{9}$$

$$\vec{\mathbf{M}} \times \hat{\mathbf{n}} = \vec{\mathbf{J}_{Amp,S}} \tag{10}$$

2.3 Potenziali

$$\vec{\mathbf{B}} = \nabla \times A \tag{11}$$

$$\nabla \times E = -\frac{\mathrm{d}}{\mathrm{d}t} \nabla \times A \tag{12}$$

$$-\nabla V = \vec{\mathbf{E}} + \frac{\partial A}{\partial t} \tag{13}$$

2.4 Onde

$$\vec{\mathbf{S}} = \frac{\vec{E} \times \vec{B}}{\mu_0 \mu_r}$$

$$I = \langle S \rangle = \frac{1}{2} \epsilon v E_0^2$$
(14)

$$I = \langle S \rangle = \frac{1}{2} \epsilon v E_0^2 \tag{15}$$

3 Relazioni notevoli (valgono in tutti i sistemi di riferimento)

$$\nabla \cdot \nabla f = \nabla \cdot (\nabla f) = \nabla^2 f$$
 Operatore di Laplace o Laplaciano (16)

$$\nabla \times \nabla f = \nabla \times (\nabla f) = 0 \tag{17}$$

$$\nabla \cdot \nabla \times \mathbf{F} = \nabla \cdot (\nabla \times \mathbf{F}) = 0 \tag{18}$$

$$\nabla \times \nabla \times \mathbf{F} = \nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla \cdot \mathbf{F}) - \nabla^2 \mathbf{F}$$
 (19)

$$\nabla^2 f g = f \nabla^2 g + 2 \nabla f \cdot \nabla g + g \nabla^2 f \tag{20}$$

Formula di Lagrange per il prodotto vettoriale:

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B}) \tag{21}$$

$$\nabla \cdot (f\mathbf{A}) = f\nabla \cdot \mathbf{A} + \mathbf{A} \cdot \nabla f \tag{22}$$

$$\nabla \times f \mathbf{A} = f \nabla \times \mathbf{A} - \mathbf{A} \times \nabla f \tag{23}$$

$$\nabla(\mathbf{A} \cdot \mathbf{B}) = (\mathbf{A} \cdot \nabla)\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{A} + \mathbf{A} \times (\nabla \times \mathbf{B}) + \mathbf{B} \times (\nabla \times \mathbf{A})$$
(24)

Table 1: Formule differenziali necessarie per esame

nome	Cartesiane	Cilindriche	Sferiche
$rac{1}{2}$	$\frac{\partial f}{\partial x}\vec{\mathbf{x}} + \frac{\partial f}{\partial y}\vec{\mathbf{y}} + \frac{\partial f}{\partial z}\vec{\mathbf{z}}$	$\frac{\partial f}{\partial \rho} \vec{\rho} + \frac{1}{\rho} \frac{\partial f}{\partial \phi} \vec{\phi} + \frac{\partial f}{\partial z} \vec{z}$	$\frac{\partial f}{\partial r}\vec{\mathbf{r}} + \frac{1}{r}\frac{\partial f}{\partial \theta}\vec{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\vec{\phi}$
$\mathbf{\nabla} \cdot F_{\downarrow}$		$\frac{1}{\rho} \frac{\partial \rho A_{\rho}}{\partial \rho} + \frac{1}{\rho} \frac{\partial A_{\phi}}{\partial \phi} + \frac{\partial A_{z}}{\partial z}$	$\frac{1}{r^2} \frac{\partial r^2 F_r}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial F_\theta \sin \theta}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial F_\phi}{\partial \phi}$
∨ ×	$\left(egin{array}{c} rac{\partial F_z}{\partial y} - rac{\partial F_y}{\partial z} ight) \hat{m{x}} \ rac{\partial F_x}{\partial z} - rac{\partial F_z}{\partial x} ight) \hat{m{y}} \ rac{\partial F_x}{\partial z} - rac{\partial F_z}{\partial x} ight) \hat{m{y}} \ angle$	$ \begin{pmatrix} \frac{1}{\rho} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \end{pmatrix} \hat{\boldsymbol{\rho}} \\ \begin{pmatrix} \frac{1}{\rho} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \\ \frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho} \end{pmatrix} \hat{\boldsymbol{\phi}} \\ \frac{1}{z} \begin{pmatrix} \frac{\partial (\rho A_\phi)}{\partial z} - \frac{\partial A_\rho}{\partial \rho} \end{pmatrix} \hat{\boldsymbol{z}} $	$\frac{1}{r \sin \theta} \left(\frac{\partial}{\partial \theta} (A_{\phi} \sin \theta) - \frac{\partial A_{\theta}}{\partial \phi} \right) \hat{r}$ $\frac{1}{r} \left(\frac{1}{\sin \theta} \frac{\partial A_{r}}{\partial \phi} - \frac{\partial}{\partial r} (rA_{\phi}) \right) \hat{\theta}$ $\frac{1}{r} \left(\frac{\partial}{\partial r} (rA_{\phi}) - \frac{\partial}{\partial r} A_{r} \right) \hat{\phi}$
$\nabla^2 f$	$\frac{\partial y}{\partial y^2} \int \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial y^2}$	$ \frac{1}{\rho} \left(\frac{\partial \rho}{\partial \rho} \left(\rho \frac{\partial f}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2} \right) $	$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial f}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial f}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 f}{\partial \phi^2}$
$ abla^2 {f A}$	$ abla^2 \mathbf{A} abla^2 A_x \hat{\mathbf{x}} + abla^2 A_y \hat{\mathbf{y}} + abla^2 A_z \hat{\mathbf{z}}$	$\left(abla^2 A_{ ho} - rac{A_{ ho}}{ ho^2} - rac{2}{ ho^2} rac{\partial A_{\phi}}{\partial \phi} ight) \hat{oldsymbol{ ho}}$ $\left(abla^2 A_{\phi} - rac{A_{\phi}}{2} + rac{2}{2} rac{\partial A_{\phi}}{2} ight) \hat{oldsymbol{\phi}}$	$\left(\nabla^2 A_r - \frac{2A_r}{r^2} - \frac{2}{r^2 \sin \theta} \frac{\partial (A_\theta \sin \theta)}{\partial \theta} - \frac{2}{r^2 \sin \theta} \frac{\partial A_\phi}{\partial \phi} \right) \hat{\boldsymbol{r}}$ $\left(\nabla^2 A_\theta - \frac{A_\theta}{r^2 \sin^2 \theta} + \frac{2}{r^2} \frac{\partial A_r}{\partial \theta} - \frac{2 \cos \theta}{r^2 \sin^2 \theta} \frac{\partial A_\phi}{\partial \phi} \right) \hat{\boldsymbol{\theta}}$
		$egin{array}{c} ho^2 & ho^2 \ 7^2 A_z) \hat{oldsymbol{z}} \end{array}$	$\left(\nabla^2 A_{\phi} - \frac{A_{\phi}}{r^2 \sin^2 \theta} + \frac{2}{r^2 \sin \theta} \frac{\partial A_r}{\partial \phi} + \frac{2 \cos \theta}{r^2 \sin^2 \theta} \frac{\partial A_{\theta}}{\partial \phi}\right) \hat{\phi}$