Use of Operational Amplifier as an Integrator and Differentiator

Stefano Pilosio

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1 Abstract

The objective was to measure the frequency response of two similar circuits. Even thought the circuits use the same components, they show different behaviors, not only in the operation applied to the input signal to produce the output signal, but also in the Frequency Response.

2 Introduction

2.1 Operational Amplifier

The circuits are made of a resistor (R), a capacitor (C) and an operational amplifier. To analyze the circuits' behavior it is required to know the feature of an ideal operational amplifier, herein referred as "OPAMP".

The ideal OPAMP is a voltage-controlled voltage source, with a voltage gain $E = \frac{v_{OUT}}{v_{IN}}$, where v_{OUT} is the output voltage and V_{IN} is the input voltage. In this case v_{OUT} is referenced to ground and v_{IN} is the voltage difference between two poles named inverting (v^+) and not inverting (v^-) inputs, so $v_{IN} = v^+ - v^-$. In the ideal OPAMP $E \to \infty$.

To run the circuit in the desired configuration, the circuit has to have a negative feedback loop, so the conditions to use *virtual short circuit principle* are respected. From this principle is obtained the stability of negative feedback, $E \to \infty$ and $v^+ - v^- = 0$. The principle is also named 'Virtual Ground': as in this case the not-inverting input is at ground potential, so $v^+ = 0 = v^-$.

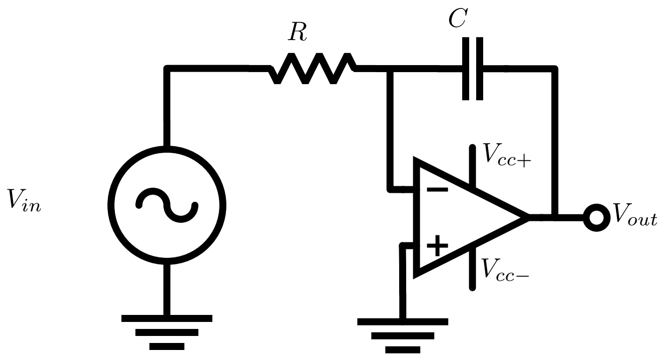


Figure 1: Schematic of an Integrator Circuit

2.2 Integrator Circuit

Referring to Figure 1, in the negative feedback loop there is a capacitor and a resistor. Using Ohm's Law, Capacitor Law ($q = Cv \implies \frac{\mathrm{d}q}{\mathrm{d}t} = i = C\frac{\mathrm{d}v}{\mathrm{d}t}$) and KCL (Kirchhoff's current law) it is obtained:

$$\begin{cases} v_{IN} - R \cdot i_R = v^- \\ i_R = i_C \\ v^+ - v^- = 0 \\ v^+ = 0 \\ i_C = C\frac{\mathrm{d}v^- - v_{OUT}}{\mathrm{d}t} \end{cases}$$

$$i = \frac{v_{IN}}{R} = -C\frac{\mathrm{d}v_{OUT}}{\mathrm{d}t}$$

From last equation, resolving for v_{OUT}:

$$v_{OUT} = -\frac{1}{CR}\int_0^t v_{IN}\, dt + v(0) \qquad (1)$$

It is visible from (1), that this circuit in time domain produces the integral function of the input signal.

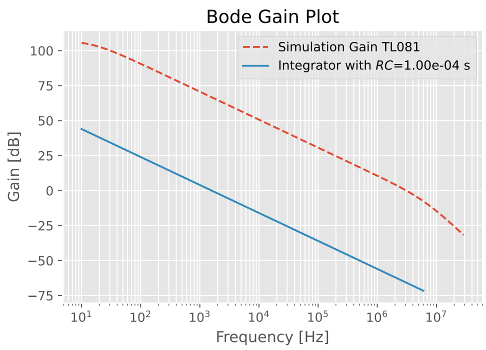
The frequency response is defined as

$$H = \frac{v_{OUT}}{v_{IN}}(f)$$

. To obtain $H(f)$, it is applied Fourier Transform (FT) over (1):

$$V_{OUT} = -\frac{1}{CR(j2\pi f)}V_{IN} \tag{2}$$

$$H = \frac{V_{OUT}}{V_{IN}} = \frac{1}{RC(j2\pi f)} \tag{3}$$



The Bode Gain Diagram in Figure 2.2shows a constant negative monotony of $-20\,\mathrm{dB/Dec}$, it means the output attenuation has a direct proportionality with the frequency.

In the diagram is also shown the open-loop gain of the OPAMP, that shows the behavior of the OPAMP used without a feedback loop. The OPAMP has much higher possible gain than what permits the frequency response of the circuit, so the OPAMP is never saturated, so there will be not a distortion in the output. This is due to the negative feedback loop that limits the circuit gain in a dynamical way compared to a resistive voltage divider which statically limits the gain.

2.3 Differentiator Circuit

As shown in Figure 2, the circuit is similar to the previous case, but in this case the capacitor and the resistor are swapped. This change

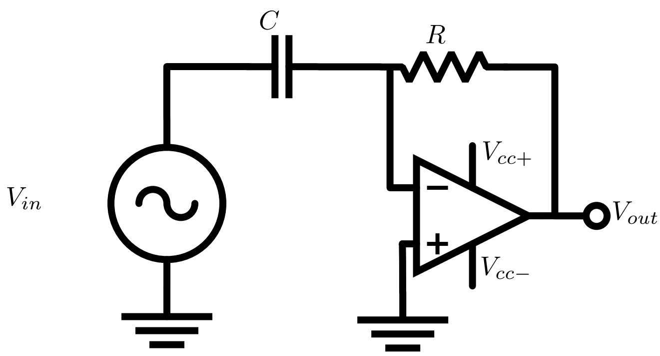


Figure 2: Schematic of a Differentiator Circuit

produces a completely different response to the input waveform, it can be understood writing KCL, the Ohm's Law and the capacitor law.

$$\begin{cases} i_C = C\frac{\mathrm{d}v_{IN}-v^-}{\mathrm{d}t} \\ i_R = \frac{v^- - v_{OUT}}{R} \\ i_R = i_C \\ v^+ = v^- \\ v^- = 0 \end{cases} \qquad C\frac{\mathrm{d}v_{IN}}{\mathrm{d}t} = \frac{-v_{OUT}}{R}$$

As in the precedent case resolving for v_{OUT}

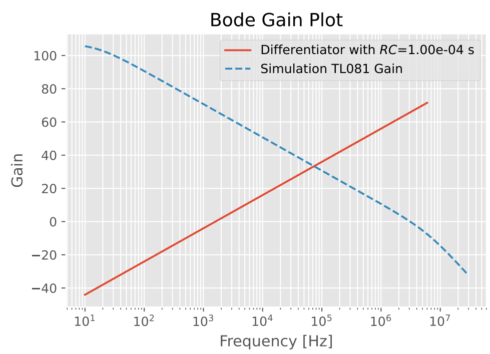
$$v_{OUT} = RC\frac{\mathrm{d}v_{IN}}{\mathrm{d}t} \tag{4}$$

(4) shows that v_{OUT} is a waveform proportional to the derivative of the input waveform.

Calculating H produces:

$$V_{OUT} = RC \cdot (j2\pi f)V_{IN} \tag{5}$$

$$H = RC \cdot (j2\pi f) \tag{6}$$



In this case the circuit has a constant positive monotony of 20 dB/Dec, so output signal amplification is directly proportional to the frequency. In this case we can expect a peak at the frequency in the point where the open loop OPAMP gain intersects the circuit frequency response, then the behavior of the circuit will be similar to the open loop OPAMP due to the limitation of a real OPAMP.

Also in this circuit we can see a dynamic change in the output gain of the circuit but with the opposite behavior of the precedent cases, but it is limited to what can supply the OPAMP. From this we can expect a distortion at high frequency if the theoretical output has a higher amplitude than the supplied voltage to the OPAMP, this situation can be avoided lowering the input voltage.

A second conclusion is that with a more complex feedback loop it is possible to amplify only some chosen frequency, or set of frequencies, with the versatility in gain of an OPAMP. Maybe with the right condition it is possible to produce periodical signal without an input signal.

3 Measurement Methods

3.1 Materials

3.1.1 Instruments

- Oscilloscope "Tektronix TDS 1012B";
- Function Generator "Agilent 3322A";
- Power Supply.

3.1.2 Equipment

- OPAMP "LT081";
- Breadboard;
- Wires;
- Resistor (value : $10\,\text{k}\Omega$, tolerance : 1%);
- Capacitor (value : $10\,\text{nF}$);

3.2 Procedure

The two circuits are built as shown in the previous Figures 2 and 1, the Oscilloscope probe was connected in the point named V_{OUT}, grounded to ground. The function generator was connected to V_{IN} and ground. The OPAMP was powered by a dual power supply, supplying the necessary V_{CC+} and V_{CC-} respectively of $15\,\text{V}$ and $-15\,\text{V}$.

The measure procedure was:

1. Choosing a frequency for the sine wave on the function generator;
2. Using the oscilloscope it was measured the voltage peak to peak of the input and the output signal;
3. Then was used the cursor mode on oscilloscope to measure the shift in the peak of the input and the output to obtain the phase, for this the cursor was set on time domain and moved in correspondence of a peak of the input waveform and the nearest peak of the output waveform, at the measure was assigned a positive sign if the output is to the right of input, negative if the output is to the left of input.

The frequencies have been chosen to use an approximate log scale (like 1, 2, 3, 5, 8, 10).

4 Analysis

4.1 Computation

During the measuring process data were saved on a comma separated value text file (CSV). Data have been analyzed using a Python environment with "*NumPy, Pandas and Matplotlib*" libraries. This environment was included in a Jupyter Notebook to visualize the analysis results in real time and to use the advantages of an interpreted language like Python to modify the code in real time. The code computed and plotted from the data the Bode Diagram and the Nyquist Diagram. Using another software, "*NGSpice*", it was simulated the expected behavior in frequency, in the simulation is added a $50\,\Omega$ resistor to mimic the function generator impedance. At last data from experiment and simulation are compared in the Python environment.

4.2 Integrator

As shown in Figure 3 the frequency response is very similar to the expected one. There is a difference in the curl at high frequency, this is probably due to non-ideal aspect of OPAMP amplification frequency response due to the fact that is present also in the simulation, even so it is not to exclude some breadboard parasitic capacitance, that could have caused some deviation from simulation in the experimental data. The breadboard adds some capacitance, in the order of pF, and some resistance, that influences the response of the circuit. There is an intrinsic difficulty in the manual measure of a low voltage at high frequency, due to signal stability and external electromagnetic effects (Radio waves). The phase graph reinforces the suspect of a parasitic capacitance, in fact the phase change starts at a lower frequency than expected, that is compatible to a higher capacitance.

4.3 Differentiator

As shown in Figure 4 we have a resonance peak at around 75 kHz, also in this case the experimental trend resembles the simulated very closely. The lack of points around the resonance frequency is due to the used frequency sequence.

In the phase Diagram and also in Nyquist plot is clearly visible when the behavior of the circuit changes from the Differentiator to a classical OPAMP. Even in the case of the Differentiator there is a difficulty in measuring the change in phase due to signal stability.

5 Conclusion

At the end we have two interesting circuits that can calculate in real time two complex linear operations such as the derivative and integral of a function, it is interesting also to opposite behavior in respect to the frequency of the circuits and its possible application.

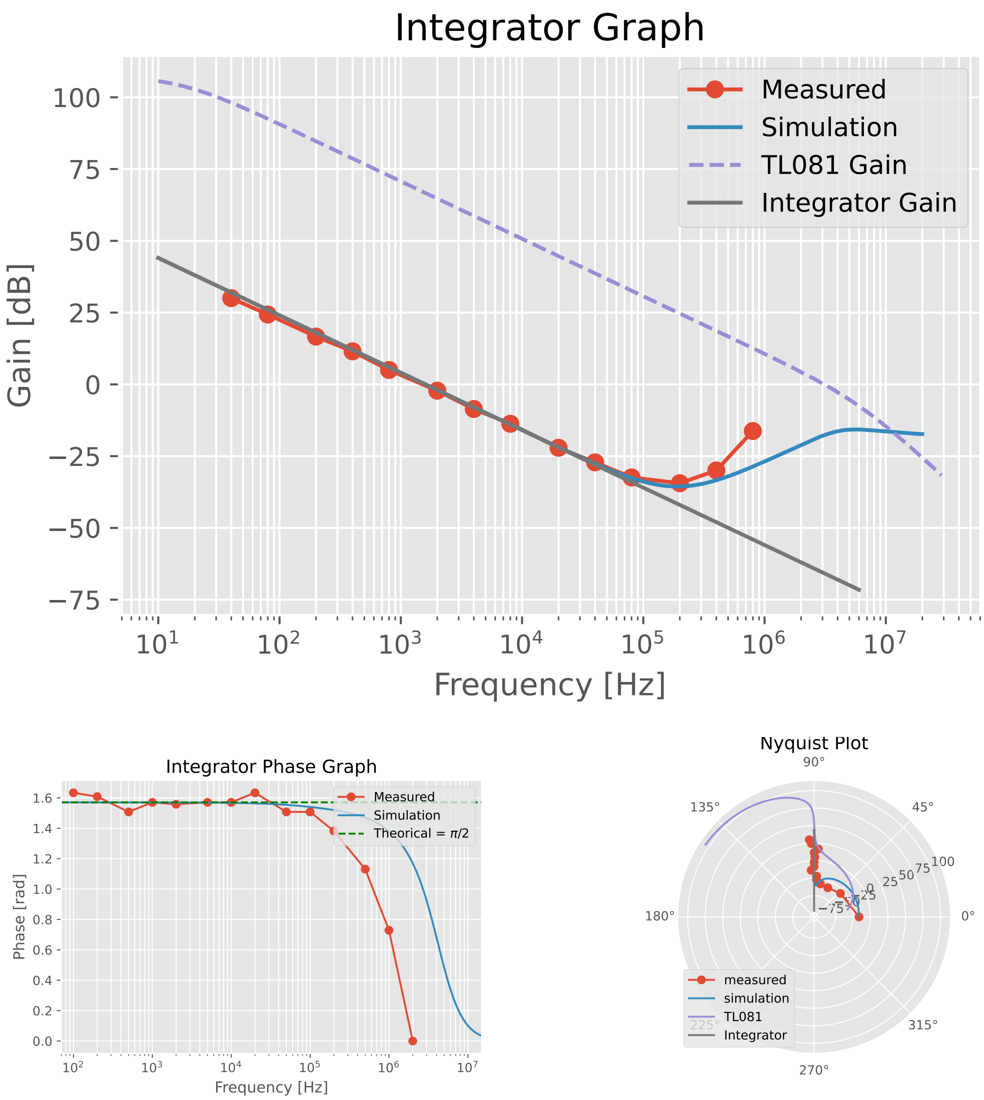


Figure 3: Resulting plot after experiment and simulation, it is notable that the frequency response resemble closely the simulated behavior and the expected behavior from section 2.2.

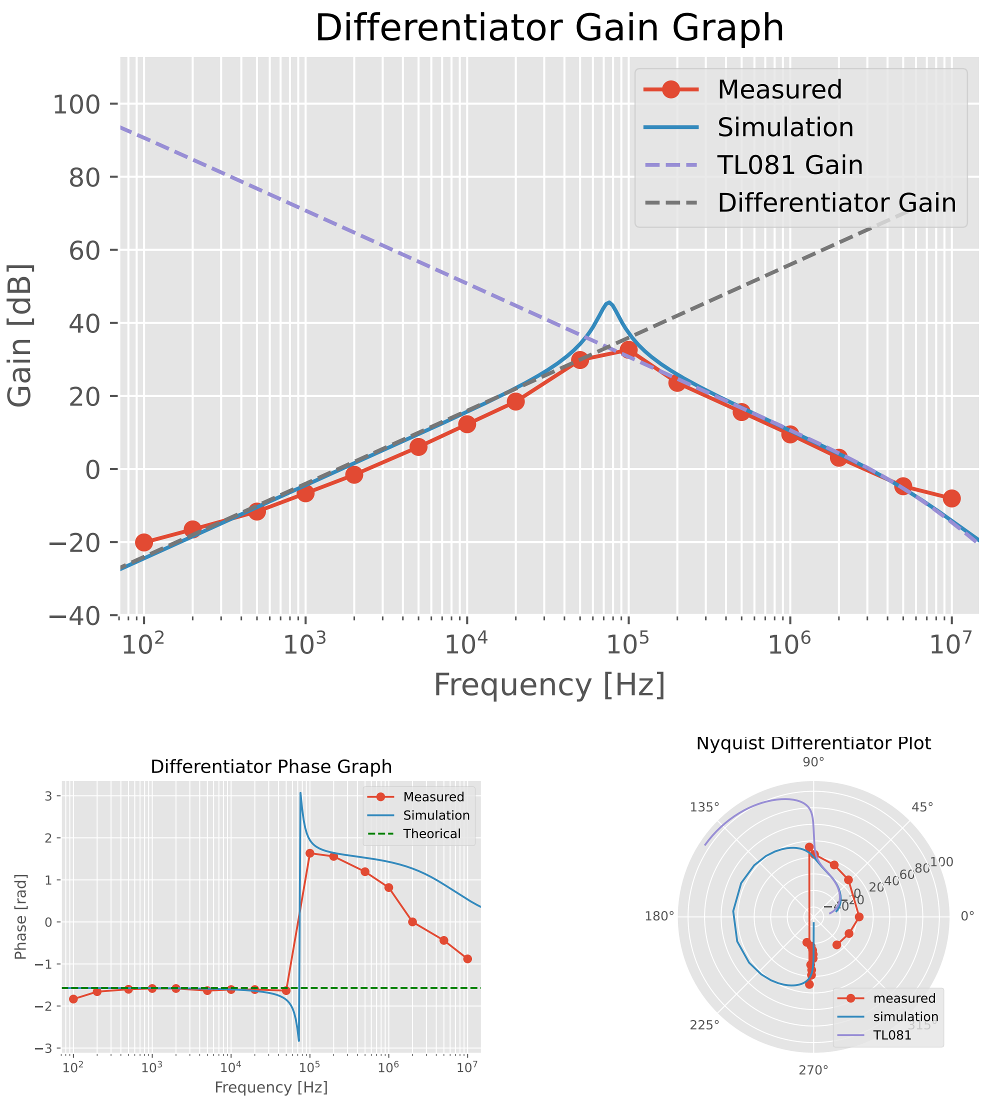


Figure 4: Resulting plot after experiment and simulation, it is notable that the frequency response resemble closely what expected in section 2.3.