We define the Oneka-type regional flow model by the following discharge potential:

$$\Phi(x, y) = Ax^2 + By^2 + Cxy + Dx + Ey + F \tag{1}$$

Equation (1) is a quadratic form in real variables. We define z and $\bar{z}$ as

$$z = x + iy \qquad\qquad \bar{z} = x - iy \tag{2}$$

Inverting yields

$$x = \frac{z+\bar{z}}{2} \qquad\qquad y = \frac{z-\bar{z}}{2i} \tag{3}$$

Substituting (3) into (1) yields

$$\Phi(z, \bar{z}) = A\left(\frac{z+\bar{z}}{2}\right)^2 + B\left(\frac{z-\bar{z}}{2i}\right)^2 + C\left(\frac{z+\bar{z}}{2}\right)\left(\frac{z-\bar{z}}{2i}\right) + D\left(\frac{z+\bar{z}}{2}\right) + E\left(\frac{z-\bar{z}}{2i}\right) + F \tag{4}$$

Rearranging (4) yields

$$\Phi(z, \bar{z}) = az^2 + b\bar{z}^2 + cz\bar{z} + dz + e\bar{z} + f \tag{5}$$

where

$$a = \frac{A - B - iC}{4} \tag{6}$$
$$b = \frac{A - B + iC}{4} \tag{7}$$
$$c = \frac{A + B}{2} \tag{8}$$
$$d = \frac{D - iE}{2} \tag{9}$$
$$e = \frac{D + iE}{2} \tag{10}$$
$$f = F \tag{11}$$

Alternatively,

$$\frac{1}{4}\begin{bmatrix} 1 & -1 & -1i & 0 & 0 & 0 \\ 1 & -1 & 1i & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & -2i & 0 \\ 0 & 0 & 0 & 2 & 2i & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 \end{bmatrix}\begin{bmatrix} A \\ B \\ C \\ D \\ E \\ F \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix} \tag{12}$$

Inverting yields

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ -1 & -1 & 1 & 0 & 0 & 0 \\ 2i & -2i & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1i & -1i & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \\ D \\ E \\ F \end{bmatrix} \tag{13}$$

Note that $b = \bar{a}$ and $e = \bar{d}$, so (5) simplifies to

$$\Phi(z, \bar{z}) = az^2 + \bar{a}\bar{z}^2 + cz\bar{z} + dz + \bar{d}\bar{z} + f \tag{14}$$
$$= \mathrm{real}\left\{az^2 + dz + f\right\} + cz\bar{z} \tag{15}$$

where a and d are complex, but c and f are real.