Homework 7, Math 189, Fall 2023

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In each problem, please show both the method of computation and a numerical answer. In some cases I may advise you that the number is huge and you do not need to compute it explicitly (but you always need to show how to compute it).

1. Levin, section 1.2 problem 6:

   How many 10-bit strings contain 6 or more 1's?

   The hint is to break it into five cases, and this should be reflected in the computation you show me.

$\binom{10}{6} + \binom{10}{7} + \binom{10}{8} + \binom{10}{9} + \binom{10}{10} = 386$

or very cunningly

$512 - \frac{1}{2}\binom{10}{5}$ can you figure out why this works?

2. Based on problems 1,2 in section 1.2: Answer the following questions about the set $\{1, 2, 3, 4, 5, 6\}$.

   (a) How many subsets of size 3 does it have? How many of these contain 3 as an element?

   (b) How many subsets does it have which contain only even numbers?

   (c) How many subsets does it have which have an even number of elements?

20

10

8

32

$\binom{6}{3} = 20$ subsets of size 3

$\binom{5}{2} = 10$ " " " containing 3

$2^3 = 8$ subsets containing only evens

$\binom{6}{0} + \binom{6}{2} + \binom{6}{4} \binom{6}{6}$ $1 + 15 + 15 + 1 = 32$ subsets with even number of members

3. Write out row 13 of Pascal's triangle without computing the row above it. Show calculations using only multiplication and division (not punching combination buttons on your calculator).

$1 \times \frac{13}{1} = 13 \times \frac{12}{2} = 78 \times \frac{11}{3} = 286 \times \frac{10}{4} = 715 \times \frac{9}{5} = 1287 \times \frac{8}{6}$

$= 1716 \times \frac{7}{7} = 1716$

I accepted fancy calculators with factorials, but this is what I wanted.

1, 13, 78, 286, 715, 1287, 1716, 1716, 1287, 715, 286, 78, 13, 1

4. Consider the sets $A = \{1, 2, 3, 4\}$ and $B = \{a, b\}$. $2^4 = 16$

   (a) How many functions are there from A to B? How many functions are there from B to A? $4^2 = 16$

   (b) How many injections (one-to-one) functions are there from A to B? From B to A? (Hint: one of these is trivial to answer with no computation). $A \to B$ none $B \to A$ $4 \cdot 3 = 12$

   (c) How many surjections are there from A to B? (Hint: how many functions from A to B are not surjections?)

There are two functions $\begin{pmatrix} 1 & 2 & 3 & 4 \\ a & a & a & a \end{pmatrix}$ $\begin{pmatrix} 1 & 2 & 3 & 4 \\ b & b & b & b \end{pmatrix}$

which are not surjective,

so $16 - 2 = 14$

5. A committee with 20 members, 13 of whom are men and 7 are women, wants to appoint a subcommittee with four members.

   (a) How many ways are there to choose the subcommittee, with just that information?

   (b) How many ways are there to choose the committee if in addition a chair and a secretary have to be designated from the subcommittee?

   (c) How many ways are there to choose the committee if it has to be gender balanced (equal numbers of men and women?)

a $\binom{20}{4} = 4845$

b $4845 \cdot 4 \cdot 3 = 58140$

alt. $20 \cdot 19 \cdot \binom{18}{2} = 58140$

c. $\binom{13}{2}\binom{7}{2} = 1638$

6. How many ways are there to rearrange the letters in the word REARRANGE?

$$\frac{9!}{3!\,2!\,2!} = 15120$$

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$$\text{also. } \binom{9}{3}\cdot\binom{6}{2}\cdot\binom{4}{2}\cdot\binom{2}{1}\cdot\binom{1}{1} = 15120$$

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