Solutions to Homework 5

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Solutions to Homework 5, last minute test review. I may use these to mark Homework 5; I may actually check off Homework 5 as turned in, depending how much energy I have.

Little of this is actually on the exam (some is, the review sheet should tell you what is relevant).

5 different Pythagorean triples: You get a primitive Pythagorean triple from each pair of relatively prime odd numbers $s > t$ as st, $\frac{s^2-t^2}{2}$, $\frac{s^2+t^2}{2}$.

$s = 1, t = 3$: 3,4,5

$s = 5, t = 1$: 5,12,13

$s = 5, t = 3$: 15,8,17

$s = 7, t = 1$: 7.24.25

$s = 7, t = 3$: 21,20,29

I set up a little spreadsheet to produce these. You should include the calculations on this problem or on the test question if this appears in the test.

problem 14 p. 34 Crisman: Try $s = 23, t = 15$: this gives 345, 152, 377

Pick large s, t fairly far from each other.

problem 18: Suppose $x^2 + y^2 = z^2$ and that y is even but not divisible by 4. So $y = 4n + 2$ for some integer n. z will be $2m + 1$ for some integer m because it is odd. $z^2 = x^2 + y^2 = (2m + 1)^2 + (4n + 2)^2 = 4m^2+4m+1+16n^2+16n+4$. $4m^2+4m$ is divisible by 8 because m^2+m is even; so is $16n^2$. It follows that the remainder of z^2 on division by 8 is 5. But no square is equal to 5 mod 8: in fact any square of an odd number, $2n+1 = 4n^2+4n+1$ has remainder 1 on position by 8.

problem 19: In mod 3 arithmetic, $0^2 = 0, 1^2 = 1, 2^2 = 1$. Two squares adding to a square have to be $0+0=0$ (not primitive) or $0+1=1$ or $1+0=0$. They do not say this, but z cannot be divisible by 3, and only one of x, y can be, if the triple is to be primitive.

patterns in PPTs re dvisibility by five: In mod 5 arithmetic

$0^2 = 0; 1^2 = 1; 2^2 = 4; 3^2 = 4; 4^2 = 1$

The ways to get Pythagorean triples are $0+0=0$, not primitive, and $1+0=1$, $4+0=4$, $1+4=0$ so again you can see as in the case of 3 that exactly one of the three will be divisible by 5, and it may in this case be z.

Crisman p. 75 problem 5: Without the FTA, we have by the Bezout identity u, v such that $au+bv=1$, so $c = auc+bvc$. ab goes into auc because a is explicitly tiven as a factor and b goes into c; ab goes into bvc because b is explicitly given as a factor and a goes into c.

Using the FTA, if a and b are relatively prime, they share no prime factors. x goes into y iff the exponent of each prime in the prime factorization of y is greater than or equal to its exponent in the factorization of x. Now the exponent of each prime in the prime factorization of ab is less than or equal to its exponent in a, because a and b share no common factors, and similarly less than or equal to the exponent of that prime in the prime factorization of b, and so every exponent of a prime in the factorization of ab is less than or equal to its exponent in the factorizations of both a and $b4$, and both of the exponents are less than the exponent of the given prime in the factorization of c, because $a|c$ and $b|c$, so $ab|c$ as well.

problem 10: Suppose $a^3|b^2$. This means that for each prime, three times the exponent of that prime in the factorization of a is less than or equal to two times the exponent of that prime in the factorization of b, which implies that the exponent of that prime in the factorization of a is less than two thirds of the exponent of that prime in the factorization of b, so less than the exponent of that prime in the factorization of b, so $a|b$.

problem 13: $36 = 2^2 3^2; 756 = 2^2 3^3 7^1; 1001 = 7^1 \cdot 11^1 \cdot 13^1$ So the gcd of 36 and 756 is $2^2 3^2 7^0 = 36$, the gcd of 36 and 1001 is $2^0 3^0 7^0 11^0 13^0 = 1$ and the gcd of 756 and 1001 is $2^0 3^0 7^1 11^0 13^0 = 7$

problem 20: The number of zeroes will be determined by the number of factors of 5, because factors of 2 pile up faster. At 5 and thereafter you have at least one zero. At 10 and thereafter you have at least two zeroes. At 15 and thereafter you have at least three zeroes. At 20 and thereafter you have at least four zeroes. There are no more than four zeroes in any factorial up to 24! because there are only four factors of 5. 25! and every larger factorial has six zeroes because there are two factors of 5 in 25. So no factorial has exactly five zeroes.