Math 189 Fall 2023 Homework 4

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These are modelled on exercises in Levin, 0.3, and you should read Levin section 0.3 and my set theory notes for background. One piece of notation which I did not use in lecture appears here: if A is a set, we use $|A|$ to represent the number of elements in A (which may also be called the size or cardinality of A), if A is a finite set. I may provide you with references to similar problems in Levin's 0.3 set; I also suggest doing the interactive examples in section 0.3, which are quite nice.

1. (modelled on problem 1 in the 0.3 exercises)

   Let $A = \{1, 2, 4, 7\}$ and let $B = \{1, 2, 4, 8, 16\}$. Find each of the following sets and present them in list notation.

   (a) $A \cup B$ $\{1, 2, 4, 7, 8\}$

   (b) $A \cap B$ $\{1, 2, 4\}$

   (c) $A \setminus B$ [Levin uses $A \setminus B$ instead of $A - B$, so I will as well. Expect the notes to be revised.] $\{7\}$

   (d) $B \setminus A$ $\{8, 16\}$

2. Find $|A|$ (the number of elements in A) for each of the following examples. If the set is infinite, say so. Modelled on problem 3.

   (a) $\{17, 18, 19, \ldots, 35\}$ [be careful that you don't make a fencepost error]. $19 = 35 - 17 + 1$

   (b) $\{x^3 : x \in \mathbb{Z}^+ \wedge x^3 < 200\}$ [read carefully, this is a set of perfect cubes and it is not very large] $|\{1, 8, 27, 64, 125\}| = 5$

(c) $\{1,2,3,4,5,6,7\} \times \{1,3,6,9,27,81\}$ List a few elements of this set (they are ordered pairs) and tell me how many elements the set has. You don't want to list them all! *7·6 = 42* *(1,1),(2,1),(1,3)...*

$2^2 \cdot 3 \cdot 5$ 12

(d) $\{x \in \mathbb{Z}^+ : x|60\}$ *$\{1,2,3,4,5,6,10,15,20,30,60\} = 12$*

(e) $\{x \in \mathbb{Z}^+ : 60|x\}$ *∞*

3. This is problem 10 in Levin 0.3. Let $A = \{x \in \mathbb{N} : 3 \leq x \leq 13\}$, $B = \{x \in \mathbb{N} : x \text{ is even}\}$ and let $C = \{x \in \mathbb{N} : x \text{ is odd}\}$. Find the following sets. Write list or set builder notation for the sets which does not mention A, B, or C.

(a) $A \cap B$ *$\{4,6,8,10,12\}$*

(b) $A \cup B$ *$\{2,4,5,6,7,8,9,10,11,12,13,14,16,18,20...\}$*

(c) $B \cap C$ *∅*

(d) $B \cup C$ *ℕ* *$\{11,12,13,15,17,19,21...\}$*

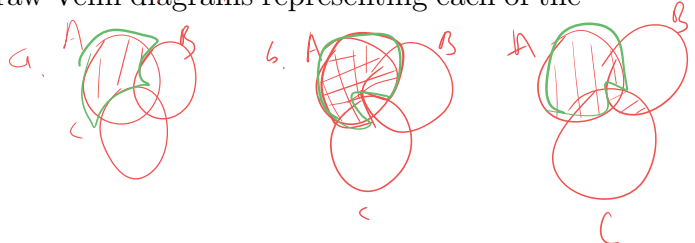4. Let $A = \{1,2,3\}$. Write $\mathcal{P}(A)$ in list notation.
$\{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}, \{1,2,3\}\} = 8$

5. modelled on question 15. Draw Venn diagrams representing each of the following sets.

(a) $A \setminus (B \cup C)$

(b) $(A \setminus B) \cup (A \setminus C)$

(c) $A \setminus (B \cap C)$

6. Present sets A, B of small positive integers in list notation such that $|A| = 3, |B| = 4$, and $|A \cup B| = 5$. (remember that $|X|$ means the number of elements in X when X is a set).
$A = \{1,2,3\}$ $B = \{2,3,4,1\}$

7. (extra credit puzzle question) Explain why no set A exists such that $A = \{2, |A|\}$. Give me two different sets B such that $B = \{1,2,|B|\}$. The first part of this question is number 29 in 0.3.

Second part

$|\{1,2,|B|\}|$ is

2 in either case

$\{1,2,2\} = \{1,2\} = B$

3 in either case

$\{1,2,3\} = B$

$A = \{2,|A|\}$

$\{2,|A|\}$ is of size either 1, if $2 = |A|$

or 2 if $|A| \neq 2$

In neither case can $|A| = \{2,|A|\}$, so the two cases show directly, so A cannot be $\{2,|A|\}$