Math 275 Test IV and Final Exam, Fall 2018

Dr Holmes

December 5, 2020

The exam will begin at 9:30 am and ends officially at 11:30 am. I will allow it to continue to 11:45 am unless someone raises an objection (no one ever has, but I must say this).

You are allowed your book, one standard sized sheet of notebook paper for notes, your non-graphing calculator and a writing instrument.

1 Cumulative Part

1. Find an equation of the plane passing through the points (1,2,3), (2,3,4), and (3,5,7), in the form $ax + by + cz = d$.

2. Find an equation for the line which is the intersection of the planes $2x + 3y + 4z = 9$ and $x + y + z = 3$. Hint: take a cross product to find a vector parallel to this line. If you are alert you may notice a point which is on both planes without computation (certainly you can find such a point by computation).

What is the angle between these two planes?

3. Find the critical points of the function

$$f(x, y) = 2x^2 + y^4 + 4xy.$$

Use the second derivative test to classify them as local maxima, local minima and saddle points.

Hint: two of the three critical points are (0,0) and (1,−1). You do need to show work deriving all three.

4. Set up and evaluate $\int\int_D xy\,dA$ as an iterated integral, where D is the triangle bounded by $x = 0$, $y = 1$, and $y = x$ (picture provided). Set up as an iterated integral using both orders of integration ($dxdy$ and $dydx$) Evaluate one of these iterated integrals.

5. Determine the volume of the cone whose lower boundary is $z = \sqrt{x^2 + y^2}$ and whose upper boundary is $z = 3$, using cylindrical coordinates. (the region of integration is of course $x^2 + y^2 \leq 9$). The setup of a triple integral using cylindrical coordinates is worth a lot of the credit, but I do want you to evaluate it.

6. Find an equation for the tangent plane to the paraboloid $x^3+y^3+z^3 = 36$ at (1,2,3). Hint: this can be thought of as a level surface of the function $f(x,y) = x^3 + y^3 + z^3$: find the normal vector to the plane using a gradient calculation.

2 Test IV

1. Compute $\int_C \frac{x}{y}\,ds$ (a scalar line integral) where C is parameterized by $t^3\mathbf{i} + t^4\mathbf{j}$. I guarantee, the integral comes out easy if you set it up correctly. In any event, you will get most of the credit if you set it up correctly.

2. Compute the vector line integral $\int_C y dx - x dy$, where C is the circle $x^2 + y^2 = 1$ traversed counterclockwise from (1,0) back to (1,0), in two different ways:

   (a) compute it directly,

   (b) and set up an equivalent double integral using Green's Theorem and evaluate it (using high school geometry).

3. The field $\mathbf{g}(x, y) = (3x^2y + 1)\mathbf{i} + (x^3 - 2y)\mathbf{j}$ is conservative. Verify this using a suitable test with partial derivatives.

Find a function $f(x, y)$ such that $\nabla f = \mathbf{g}$.

Then compute $\int_C \mathbf{g} \cdot \mathbf{dr}$ (a vector line integral) where C is parameterized by $\langle t, \ln(t) \rangle$ where $0 \leq t \leq 1$. Hint: you don't have to do any integrals involving `ln` to do this. In fact, you don't have to do any integration at all.

4. Compute the surface area of the cone $z = \sqrt{x^2 + y^2}, 0 \leq z \leq 3$, by setting up and evaluating a scalar surface integral. A parameterization of this surface is given by $\langle v\cos(u), v\sin(u), v\rangle\,; 0 \leq u \leq 2\pi; 0 \leq v \leq 3$.

5. Compute the vector surface integral of the field $\langle x + 2y, y + 2z, z + 2x\rangle$ over the closed surface $x^2 + y^2 + z^2 = 4$ with the standard outward orientation, using the Divergence Theorem. By the theorem, the surface integral is equal to a certain triple integral. Describe it (being sure to say what the region of integration is). Then evaluate it using high school geometry.