Math 275 Test IV and Final, Summer 2013 (edited for Fall 2020 review)

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1 Cumulative Part

1. Compute the exact cosine of the angle between $\langle 1, 1, 1 \rangle$ and $\langle 2, 3, 4 \rangle$ (no calculator approximations) then compute the actual angle in degrees using your calculator (this answer will of course be approximate).

2. Find an equation for the plane parallel to $\langle 1, 1, 1\rangle$ and $\langle 2, 3, 4\rangle$ which passes through the point $(3, -1, 1)$.

3. For a particle whose position at time t is (t, t, t^2) determine the speed at time $t = 2$.

4. For the function $x^3y + 12x^2 - 8y$ find the critical point or points and classify them as local max, local min or saddle point using the second derivative test.

2 Test IV

1. 15.2 Evaluate
$$\int_0^2 \int_x^2 xy\,dy\,dx$$
   Sketch the region of integration.

2. 15.2 Evaluate the integral

$$\int_0^{\sqrt{\frac{\pi}{2}}} \int_x^{\sqrt{\frac{\pi}{2}}} \sin(x^2) dx\, dy$$

The region of integration is pictured.

First explain why you cannot evaluate it directly as written.

Then set up the integral with the order of the variables reversed, and evaluate it.

3. 15.4 polar coordinates Where the region D is the region $x^2 + y^2 \leq 1$, $x \geq 0$ (pictured) compute the integral of $(x^2 + y^2)^2$ over the region D by converting to polar coordinates and evaluating the resulting iterated integral.

4. 16.2 Calculate the work done by the field $\mathbf{F} = \langle y, -x \rangle$ in traversing the upper half of $x^2 + y^2 = 1$ from $(1, 0)$ to $(-1, 0)$. Why is this field not conservative?

   You may use the parameterization $\mathbf{c}(t) = \langle \cos(t), \sin(t) \rangle$, $0 \leq t \leq \pi$.

5. 16.3 One of the following vector fields is conservative and the other is not. Identify the one that is not, and explain why. For the one that is conservative, determine the potential function, and verify explictly that the gradient of the potential function is the vector field.

   (a)
   $$(1+y)\mathbf{i}+(x+2y)\mathbf{j}$$

   (b)
   $$(1-y)\mathbf{i}+(x-2y)\mathbf{j}$$