Testing

We are proving

$$(A \to (B \to C)) \leftrightarrow ((A \land B) \to C)$$

Part I

Assume ① $A \to (B \to C)$ ✓

Goal: $(A \land B) \to C$ ✓

Assume ② $A \land B$

Goal: C

③ A simp 2

④ B simp 2

⑤ $B \to C$ mp 1, 3

⑥ C mp 5, 4

⑦ $(A \land B) \to C$ ✓✓ deduction 2-6

finish part 1

Part II:

Assume (8) $(A \wedge B) \to C$

Goal: $A \to (B \to C)$

Assume (9) A

Goal: $B \to C$

Assume (10) B

Goal C

(11) $A \wedge B$ conjunction 9, 10

(12) C mp. 8, 11

(13) $B \to C$ ded 10-12

(14) $A \to (B \to C)$ ded 9-13

finished part 2

(15) the theorem biconditional introduction 1-7, 8-14

bico

biconditional introduction

Or

$P \vee Q$ for P or Q

is always inclusive P or Q or both

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

$$\frac{A}{A \vee B}$$

$$\frac{B}{A \vee B}$$

addition

To prove $A \vee B$ prove A

To prove $A \vee B$ prove B

$A \vee \neg A$ is a theorem

how are we going to prove it?

To use a disjunction

Suppose we have assumed or proved $A \vee B$.

We want to prove a conclusion C

The strategy is, show that

C follows from A [case 1]

then show that C follows from B

$$\frac{P \vee Q \quad P \to C \quad Q \to C}{C}$$

proof by cases

1 $P \vee Q$

Goal: C

Case 1   1a Assume P

Goal C

⋮

m C

Case 2   Assume 1b Q

Goal C

⋮

n C

n+1 C proof by cases 1, 1a-m, 1b-m

$((A \to C) \land (B \to C)) \to ((A \lor B) \to C)$

Assume ① $(A \to C) \land (B \to C)$

Goal: $(A \lor B) \to C$

Assume ② $A \lor B$

Goal C

③ $A \to C$ simp 1

④ $B \to C$ simp 1

start proof by case using 1

Case 1 Assume 2a A

Goal C

3a C mp. 3, 2a

Case Assume 2b B

Goal C

3b C mp 4, 2b

⑤ C proof by cases 2, 2a-3a, 2b-3b