Math 275 Test III (all of this is relevant to the Fall 2020 final of course: some of it is relevant to Test III

Dr. Holmes

You will have from 11:40 to 12:35 to do this exam. As announced in class on Wednesday, you are allowed a single sheet of notebook paper with whatever notes you like, as well as your plain scientific calculator without graphing or symbolic computation capability. Cell phones must be turned off and out of sight.

1. Chain Rule

   Let $y = u^2 + sin(uv)$, where $u = st + e^t$ and $v = \sqrt{1 + s + t^2}$.

   Write out the form of the Chain Rule needed to compute the partial derivative of y with respect to s.

   Compute the partial derivative of y with respect to s. You may leave u's and v's in your answer.

2. Compute the gradient of the function $f(x, y, z) = xy^2z^3$.

   Compute the directional derivative of the function $f(x, y, z) = xy^2z^3$ at (1,1,1) in the direction of the vector $\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$. You should set up your calculation as the dot product of a value of the gradient of f with an appropriate unit vector.

   Determine the equation of the tangent plane to the graph of $xy^2z^3 = 1$ at (1,1,1).

3. Find all critical points of $f(x, y) = -1 + 2xy - 2y - 2x^2 + 4x - y^2$ and classify them as local maxima, local minima, or saddle points.

4. Use the method of Lagrange multipliers to find the closest point to the origin on the plane $x + 2y + 3z = 2$. Hint: find the critical point for $x^2 + y^2 + z^2$ (the square of the distance from the origin) subject to the constraint given by the equation of the plane. There is only one critical point, and you do not have to verify that it is actually a minimum (this is obvious from geometry).

5. Find the volume of the solid bounded below by the triangle in the xy plane with corners (0,0,0), (0,1,0), and (1,1,1) (and on the sides by the vertical planes through the sides of this triangle), and above by $z = xy^2$. Set up the double integral for this problem in both orders of integration; you only need to evaluate one of the two forms, though.

6. Convert the integral

$$\int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} x^2\, dy\, dx$$

to polar coordinates. You do not need to evaluate it. It is essential to draw the picture of the region of integration as part of the process!

7. Find the area of the vertical surface bounded below by the part of the unit circle in the first quadrant and above by $z = xy^2$. (this is a line integral problem leading to an easy trigonometric integral).