Testing

Proposition 1.6

For any $m, n, p \in \mathbb{Z}$

$$(m+n)\cdot p = m\cdot p + n\cdot p$$

$(m+n)\cdot p =$ by Ax 1.1 iv comm ×

$p\cdot(m+n) =$ by Ax 1.1 iii dist

$p\cdot m + p\cdot n =$ by Ax 1.1 iv dance

$m\cdot p + n\cdot p$ completing the proof

$$\boxed{(m+n)\cdot p \overset{\text{comm}\times}{=} p\cdot(m+n) \overset{\text{Ax 1.1 iii dist}}{=} p\cdot m + p\cdot n \overset{\text{Comm}\times\ \text{dance}}{=} m\cdot p + n\cdot p}$$

Proposition 1.9

Let m, n, p be integers.

If $m+n = m+p$ then $n = p$

Assume (1) $m+n = m+p$

Goal: $n = p$

(1) $m+n = m+p$ assumption

(2) $(-m)+(m+n) = (-m)+(m+p)$ property of equality

substitution using (1)

(3) $(-m+m)+n = (-m+m)+p$ assoc +

(4) $0+n = 0+p$ prop 1.8

(5) $n = p$ prop 1.7

which is the goal to be proved.

Type $\alpha \to \beta$

| Assume ① α
| ⋮
| ⓝ β

ⓝ⓫ $\alpha \to \beta$

$m \cdot 0 = 0$

1	$m \cdot n + 0 = m \cdot n$	ax 1.2
2	$m \cdot n = m \cdot (n+0)$	ax 1.2
3	$m \cdot (n+0) = m \cdot n + m \cdot 0$	dist
4	$m \cdot n = m \cdot n + m \cdot 0$	trans = 2, 3
5	$m \cdot n + 0 = m \cdot n + m \cdot 0$	chain of equations 1, 4 (trans =)
6.	$0 = m \cdot 0$	prop 1.9, line 5

Definition m goes into n

$n \mid m$ (m is divisible by n)

means there is an integer p such that

$$np = m.$$

$2 \mid 4$ because $2 \cdot \underline{2} = 4$

$4 \mid 2$ is false - there is no integer x such that $4x = 2$.

$0 \mid 5$ $0 \cdot \_\_ \neq 5$

$5 \mid 0$ true $5 \cdot 0 = 0$

$0 \mid 0$ true $0 \cdot 117 = 0$

$a|b$ is <u>not a fraction</u>.

$a|b$ is <u>almost</u> equivalent to "$\frac{b}{a}$ is an integer".

$a|b$ is exactly equivalent to "$\frac{b}{a}$ is an integer or $a=b=0$".

Theorem:

If $d|x$ and $d|y$ then $d|(x+y)$

Proof:

Let d, x, y be integers.

Assume (1) $d|x$ and (2) $d|y$

Goal: $d|(x+y)$

Because $d|x$, (3) there is $k \in \mathbb{Z}$ s.t. $dk = x$.

Because $d|y$ (4) there is $l \in \mathbb{Z}$ s.t. $dl = y$.

Our goal $d|(x+y)$ means "There is p such that $dp = x+y$" So we want to find such a p.

$x+y = dk + dl$ by (3)

$x+y = d(k+l)$ dist

Let $p := k+l$

$x+y = dp$ which what we want -

$d|(x+y)$

1.9. if $m+p = m+n$ then $p = n$

Corollary of 1.9

if $m+x_1 = 0$ and $m+x_2 = 0$

then $x_1 = x_2$

Proof: Assume $m+x_1 = 0$ and $m+x_2 = 0$.

then $m+x_1 = m+x_2$ trans =

so $x_1 = x_2$ prop 1.9.

Corollary: if $m+x_1 = 0$ then $x_1 = -m$

Proof: Assume $m+x_1 = 0$.

we also have $m + -m = 0$

so by the prev corollary, $x_1 = -m$.

Theorem: $-(-m) = m$

Proof:

① $m + -m = 0$ add inv

② $-m + -(-m) = 0$ add inv

③ $-(-m) + -m = -m + -(-m) = 0$ comm +

so $m + -m = -(-m) + -m$ 1, 2, 3 above =

so $m = -(-m)$ by prop 1.9.

Prop 1.20

for all $m, n \in \mathbb{Z}$, $(-m)(-n) = mn$

Claim 1:

$m(-n) = -(mn)$

Proof of Claim 1:

notice that

$m \cdot (-n) + m \cdot n = m(-n + n) = \vdots$

$m \cdot 0 \overset{1.14}{=} 0$

$-(m \cdot n) + m \cdot n = 0$ obv

so by Prop 1.9 (sort of)

$m \cdot (-n) = -(m \cdot n)$

Prop 1.20 $(-m) \cdot (-n) \overset{\text{claim 1}}{=} -(m \cdot (-n))$