Circled problems relevant for Fall '24 review

Math 189 Fall 2023 Test III and Final

Randall Holmes

December 13, 2023

The exam lasts from 230 to 430 pm officially. Unless someone raises an objection, I will allow it to continue until 445.

The first part is Test 3; the second part is made up the questions for earlier parts of the course for the final.

You will receive a Test 3 grade based on the Test 3 questions, and a final exam grade in which the cumulative questions will count for three quarters of the value and your Test 3 performance for one quarter of the value.

You are allowed to use your test paper, your writing instrument, your scientific calculator without graphing or symbolic computation capabilities and two sheets of standard sized notebook paper with notes.

Happy Holidays!

1 Test 3

1. Chinese remainder theorem

   Solve the system of equations

$$x \equiv_{121} 24$$

$$x \equiv_{211} 179$$

   State the smallest positive solution and the general solution.

2. Do the complete setup for your RSA key with $p = 13, q = 19$ and encryption exponent $r = 7$.

   State what N is, and compute the decryption exponent s.

   Encrypt my favorite message 42.

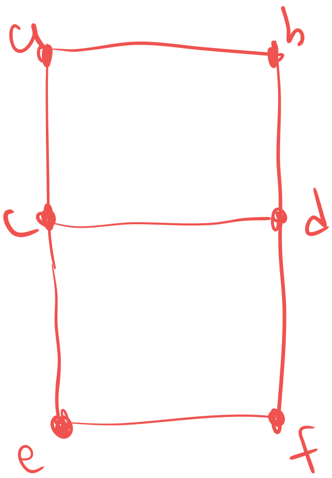
   I send you the message 77. Decrypt it. (Something funny happens...)

3. Eulerian walk (seven bridges problem) Two graphs are pictured. One has an Eulerian walk (a walk visiting each edge exactly once); one does not.

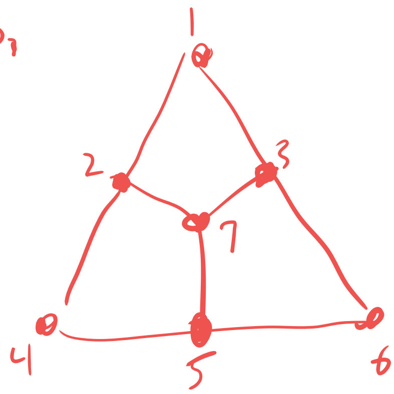
   For the one that does, describe an Eulerian walk for it as a list of vertices (in which vertices can be repeated, but a vertex will not appear more than once preceded or followed by the same vertex).

   For the one that doesn't, give a brief explanation of why such a walk is impossible.

a.



b.



4. Degree sequences – draw graphs with given degree sequences. If no such graph is possible, explain.

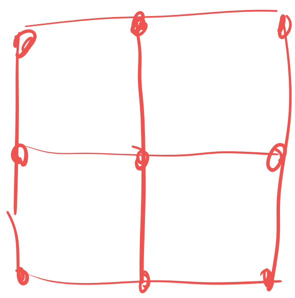
   (a) 1,2,3,3,3

   (b) 1,1,2,3,4

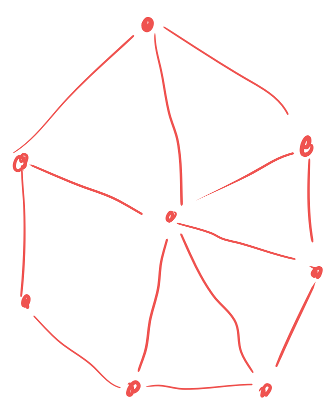
   (c) 2,2,2,3,3,3,3 (draw two non-isomorphic graphs with this degree sequence)

5. One of the graphs given can be colored with two colors (remember, we are coloring the vertices in such a way that the ends of any edge are of different colors), and one requires four colors. Give colorings of the two graphs with two colors for the two-colorable one and four colors for the four colorable one. Give an explanation of why the one that needs four colors cannot be colored with fewer colors.

a.



b.



label each vertex with a letter from R, B, G, Y to represent the coloring.

2 Cumulative Part

1. Euclidean algorithm, modular reciprocal, and solve an equation

   (a) Use the extended Euclidean algorithm to find $\texttt{gcd}(211, 121)$ and to find integers x and y such that $211x + 121y = \texttt{gcd}(211, 121)$.
   The way you write your answer should make it clear that you know what the gcd is, what x is and what y is.

   (b) State the multiplicative inverse of 121 in mod 211 arithmetic. Include a calculation checking that your answer is correct using the fact that the product of a number and its reciprocal is 1 (up to congruence mod 211 in this case).

   (c) Solve the equation $121x \equiv_{211} 5$. Your answer should be a remainder mod 211.

2. Four part counting problems. Four problems involving k choices from n alternatives are given. Solve the problems, and classify them by whether the order of choices matters and whether repeated choices are allowed. Show your calculations and give a final numerical answer.

   (a) How many four letter “words” (they do not need to be in the dictionary or pronounceable) can you make from the 26 letter alphabet?

   (b) 15 runners are in a race. First second and third place winners are recorded. There are no ties. How many possible outcomes are there?

(c) I order a dozen roses for my sweetie...the choices I have are pink, purple, red and white. How many possible bouquets do I have to consider?

(d) How many possible poker hands are there? (five cards are drawn from a 52 card deck)

3. Math induction proofs: do one of two. If you do both, you get credit for your best work.

   In both parts, clearly identify the basis step. induction hypothesis and induction goal, and be clear about where the induction hypothesis is used.

   (a) Prove that $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$. In words, the sum of the first n positive integers is $\frac{n(n+1)}{2}$

(b) Prove that for all nonnegative integers n, $n^3 + 5n$ is divisible by 3.