# Math 275 Test IV, Fall 2013, revised for Fall 2020 review

Dr. Holmes

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1. Evaluate the double integral of the function $x^2 + y^2$ over the pictured region (the area in the first quadrant inside the circle $x^2 + y^2 = 4$ by converting the integral to polar coordinates (show the setup in polar coordinates) then evaluating the integral.

2. Compute the total mass of a thin plate occupying the pictured region (the upper half of the interior of the circle $x^2 + y^2 = 1$) with density $\sigma(x, y) = y$. Hint: you might want to convert to polar coordinates.

3. A vector field is pictured, and two points $P$ and $Q$ are shown. Two different paths from $P$ to $Q$ are shown. Which of them (upper or lower) has you doing more work against the field?

   Explain using the information just discussed why this field is not conservative.

4. Compute the mass of a wire following the helix $\langle \cos(t), \sin(t), t \rangle$ from $t = 0$ to $t = \pi$ with density function $\rho(x, y, z) = x + y + z$. [This is a scalar line integral question].

5. Compute the vector line integral $\int_C y dx - x dy$, where $C$ is the portion of the graph of $y = x^2$ extending from $(0, 0)$ to $(2, 4)$ (which we can parameterize by $\mathbf{c}(t) = \langle t, t^2 \rangle$). (Notice that this vector field is not conservative).

6. Two vector fields are listed. One of them is conservative and one of them isn't. Show calculations which verify these facts. Then compute the potential function for the one which is conservative.

   (a) $(1 + 3x^2y)\mathbf{i} + (2y - x^3)\mathbf{j}$

   (b) $(1 + 3x^2y)\mathbf{i} + (2y + x^3)\mathbf{j}$

7. Verify that the function $x^2y$ is a potential function for the vector field $2xy\mathbf{i} + x^2\mathbf{j}$.

Determine the vector line integral $\int_C 2xydx + x^2dy$ where $C$ is the path from $(-1, 1)$ to $(2, 4)$ consisting of the line segment from $(-1, 1)$ to the origin, the line segment from the origin to $(1, 1)$ and the part of the graph of $y - x^2$ between $(1, 1)$ and $(2, 4)$ (pictured).

8. Compute the line integral of $y^2\mathbf{i}+xy\mathbf{j}$ around the perimeter of the triangle with corners $(0,0)$, $(1,0)$ and $(1,1)$ (pictured) proceeding from the origin back to the origin counterclockwise. Hint: use Green's Theorem.