X

Troll 1 and Troll 2 disagree, so one of them is a knight and one of them is a knave.

Troll 3 is a knight - since either Troll 1 or Troll 2 is a knave.

Troll 1 is a knight
Troll 2 is a knave
Troll 3 is a knight

We are working in a context where what matters about a statement is just whether it is true or false. (is "truth value")

$P \wedge Q$

P	Q	P and Q
T	T	T
T	F	F
F	T	F
F	F	F

$\wedge$ is a truth function

and	$P \wedge Q$	P and Q	binary
or	$P \vee Q$	P or Q	binary
if...then...	$P \to Q$	if P then Q	binary
iff	if and only if	$P \leftrightarrow Q$	P if and only if Q
not	$\neg P$		unary

and

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P or Q

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

$P \oplus Q$
F
T
T
F

inclusive or

This is the official meaning of $\vee$

exclusive or

XOR

and $\wedge$

or $\vee$

John (and) Mary love ice cream

John loves ice cream $\wedge$ Mary loves ice cream

John reads and writes.

~~P~~ John reads and John writes

2 and 3 are prime

John and Mary carried the ~~two~~ half ton safe.

15 and 29 are relatively prime

If ... then ...

P	Q	P → Q
T	T	T
T	F	F
F	T	T
F	F	T

Anything implies a true statement

A false statement implies anything.

F

If Napoleon conquered China, then French is widely spoken in Beijing. T

F

If Napoleon conquered China, then $2+2=5$. T

F

Implication and Pizza

mom says to son, If you clean your room (Q) we will have pizza (P)

Q	P	Q → P
T	T	T
T	F	F
F	T	T
F	F	T

fink

chump

Our official definition of if P, then Q follows this table.

$P \leftrightarrow Q$ P if and only if Q

$P \to Q \wedge Q \to P$

P	Q	$P \to Q$	$\wedge$	$Q \to P$	$P \leftrightarrow Q$
T	T	T	T	T	T
T	F	F	F	T	F
F	T	T	F	F	F
F	F	T	T	T	T

P	$\neg P$
T	F
F	T

not P

It is not the case that P...

The anatomy of implication

$P \to Q$

P is the hypothesis antecedent

Q is the conclusion consequent

---

if $P \to Q$ is an implication

we refer to $Q \to P$ as its converse

$\neg P \to \neg Q$ as its inverse

$\neg Q \to \neg P$ as its contrapositive

P	Q	$P \to Q$	$Q \to P$	$\neg P \to \neg Q$	$\neg Q \to \neg P$
T	T	T	T	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	T	T

logically equivalent

~~logically equivalent~~

P is necessary for Q means $Q \to P$

P is sufficient for Q means $P \to Q$

P is necessary and sufficient for Q $P \leftrightarrow Q$

An argument is a list of statements in the format

$$\left.\begin{array}{l} P_1 \\ \vdots \\ P_n \end{array}\right\} \text{premises}$$

$$\overline{\quad C \quad} \Big\} \text{conclusion}$$

$$\begin{array}{l} A \to B \\ B \to C \\ \hline A \to C \end{array}$$

"correct"

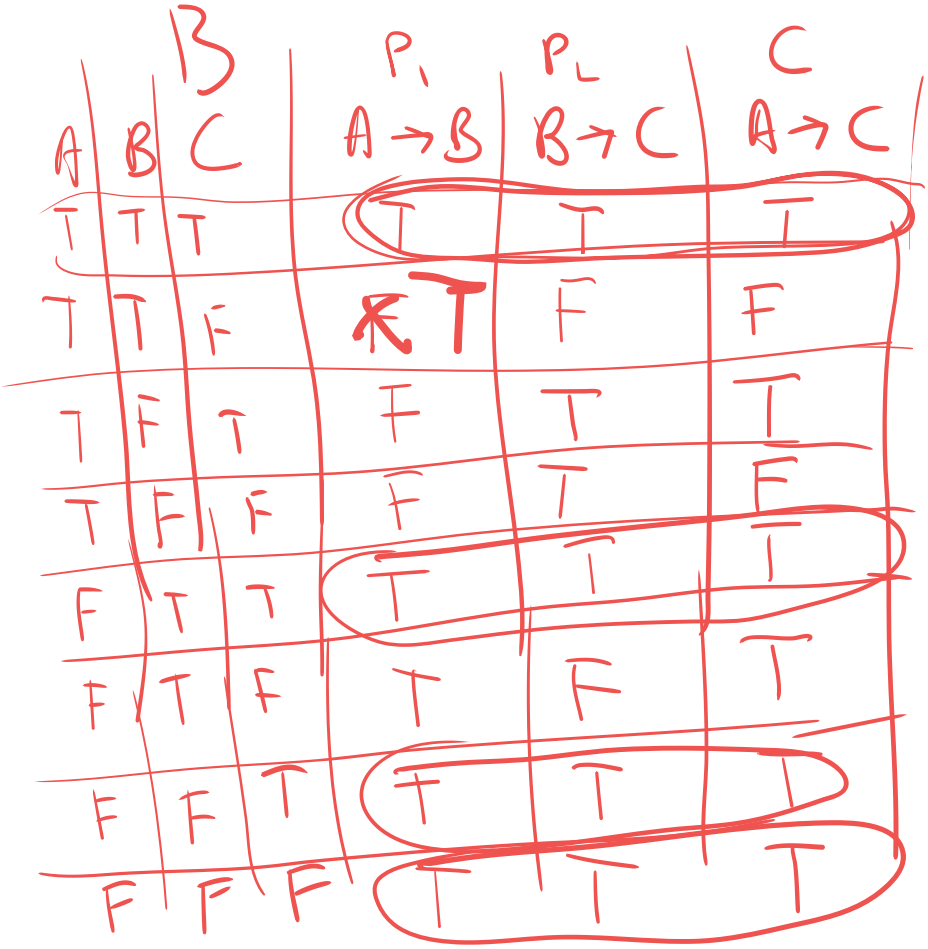
valid

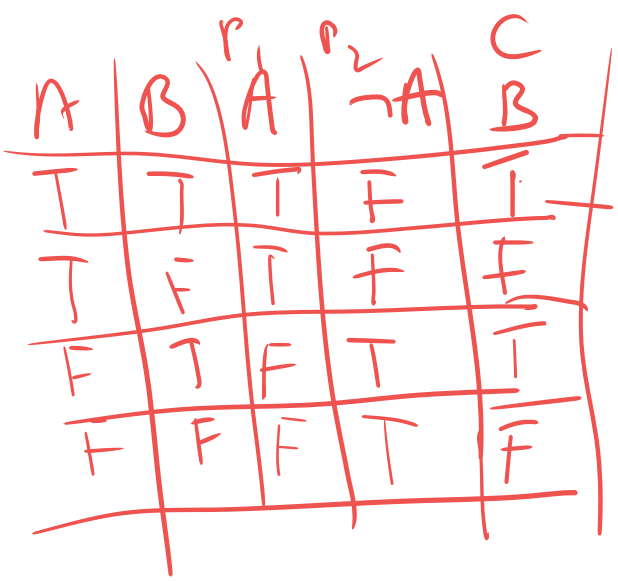
A valid argument is one in which there is no assignment of values to variables appearing in the argument which makes the premises all true and the conclusion false.

A

¬A

B





Valid ?!?

every assignment of values making all premises true makes the conclusion true - because there are no such assignments.

All men are mortal

(For any x) x is a man → x is mortal

All wombats in this room are named Alice

F

For any x if x is a wombat in this room

then x is named Alice

?

True no matter what x is


¬
¬P
¬P
~P
~P