

Math 189, fall 2024, Homework 5

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This is due Wednesday, Oct 2, after the exam, but the material is on Test I, so be sure you attack it promptly.

Give both exact answers computed with your calculator and a description of how they are computed.

I would in addition work problems from sections 1.1, 1.2 and 1.3, completing each problem before looking at the solution.

1. A certain state has license plates consisting of three letters followed by five digits.

(a) How many possible plates are there if there are no additional restrictions? $26^3 \cdot 10^5 = 1757600000$

(b) How many possible plates are there if no letter or digit is repeated? $26 \cdot 25 \cdot 24 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 = 235872000$

(c) How many plates have exactly one 8 on them? $26^3 \cdot 5 \cdot 9^4 = 576580680$

(d) How many plates have at least one 8 on them (one or more 8's?) $26^3 \cdot 10^5 - 26^3 \cdot 9^5 = 719754776$

2. How many divisors does 1800 have?

$1800 = 2^3 \cdot 3^2 \cdot 5^2 = 2^3 \cdot 3^2 \cdot 5^2$ has $(3+1)(2+1)(2+1) = 36$ divisors

3. Of 25 children, 11 like chocolate ice cream, 11 like vanilla ice cream, and 11 like strawberry ice cream. 4 like chocolate and vanilla. 6 like chocolate and strawberry. 2 like vanilla and strawberry. 1 likes all three. How many of the children do not like any of the flavors? How many of the children will only eat strawberry ice cream?

$25 - 11 - 11 - 11 + 4 + 6 + 2 - 1 = 3$ do not like any. $11 - 6 - 2 + 1 = 4$ only like strawberry.

4. Expand $(x + y)^8$.

$x^8 + 8x^7y + 28x^6y^2 + 56x^5y^3 + 70x^4y^4 + 56x^3y^5 + 28x^2y^6 + 8xy^7 + y^8$

5. Using the digits 3 through 9, find the number of 4 digit numbers such that

- (a) no additional condition are imposed (digits may be repeated freely and may appear in any order) $7^4 = 2401$
- (b) No digit can appear more than once but they may appear in any order $7 \cdot 6 \cdot 5 \cdot 4 = 840$
- (c) The digits must be distinct and must appear in increasing order $\binom{7}{4} = 35$
6. A committee of 20 members wants to choose a subcommittee with six members. How many choices can be made if
- (a) There are no additional instructions (just choose six of the twenty people) $\binom{20}{6} = 38760$
- (b) How many ways are there to choose the subcommittee of six members and in addition choose a chair and secretary from the subcommittee? $38760 \cdot 6 \cdot 5 = 1162800$ or $20 \cdot 19 \cdot \binom{18}{4} =$ same value
- (c) There are ten men and ten women in the group of 20, and you are required to choose three men and three women to be on the subcommittee. $\binom{10}{3} \cdot \binom{10}{3} = 14400$
7. How many different anagrams of PARALLEL are there? (A good math word)
- $$\frac{8!}{2! 3!} = 3360$$
- 2 n's 3 L's