Last homework assignment

Dr. Holmes

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This is due at the time the final exam starts.

1. Read example 1 on page 75, then prove $\lim_{x\to 3} \frac{1}{x} = \frac{1}{3}$

2. Prove the subtraction rule for limits from the definition directly (not from the addition rule and the constant multiple rule): if $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} g(x) = M$ then $\lim_{x\to a}(f(x)-g(x)) = L-M$. This should look very much like the proof of the addition rule with slightly different manipulations of absolute values.

3. Prove that $|x||y| = |xy|$. $|x|$ is defined as x if $x \geq 0$ and $-x$ otherwise. This is a straightforward argument by cases: make sure you write out everything you need to say.

4. (depends on Thursday's lecture) Write out the proof that any nonempty set of real numbers which is bounded below has a greatest lower bound, using the Completeness Axiom, which asserts that each nonempty set of real numbers which is bounded above has a least upper bound.