I can write.

Theorem

$((A \wedge B) \vee (B \wedge C)) \rightarrow B$

Assume ① $(A \wedge B) \vee (B \wedge C)$

Goal: B

Case 1 Assume (1a) $A \wedge B$
Goal: B
(2a) B simp 1a

Case 2 Assume (1b) $B \wedge C$
Goal B
(2b) B simp 1b

③ B proof by cases 1, 1a-2a, 1b-2b

Theorem $(A \rightarrow B) \rightarrow A \rightarrow (B \vee C)$

Assume ① $A \rightarrow B$

Goal: $A \rightarrow (B \vee C)$

Assume ② A

Goal: $B \vee C$

③ B mp 1,2

④ $B \vee C$ addition 3

⑤ $A \rightarrow (B \vee C)$ deduction 2-4

$(A \to B) \to (A \to (B \to C))$ deduction 1-5

Negation

$\neg P$ not P   P is false

it is not the case that P

P	$\neg P$
T	F
F	T

I add in $\bot$

this stands for a false statement

and it is read "the absurd".

using negate statements

$$\frac{\begin{matrix} P \\ \neg P \end{matrix}}{\bot}$$

contradiction

$$\frac{\neg\neg P}{P}$$

double negation elimination

dne

proving negative statements

To prove $\neg P$

$$
\begin{array}{|l}
\text{Assume } \textcircled{n}\ P \\
\text{Goal } \bot \\
\vdots \\
\textcircled{m}\ \bot
\end{array}
$$

$\textcircled{m+1}$ $\neg P$ negation introduction n-m

introduction

To prove P (anything at all!)

$$
\begin{array}{|l}
\text{Assume } \textcircled{n}\ \neg P \\
\text{Goal: } \bot \\
\vdots \\
\textcircled{m}\ \bot
\end{array}
$$

$\textcircled{m+1}$ $\neg\neg P$ neg intro n-m

$\textcircled{m+2}$ P dne m+1

$\textcircled{m+1}$ P reducto n-m

reductio ad absurdum

"proof by contradiction"

$$\frac{P}{\neg\neg P}$$

double negation introduction

To prove $\neg Q$

Assume Q

Goal $\perp$

① P premise

Goal: $\neg\neg P$

| ② Assume $\neg P$
| Goal: $\perp$
| ③ $\perp$ contradiction 1, 2

④ $\neg\neg P$ negation introduction 2-3

Ex falso

$$\frac{\bot}{P}$$

(1) $\bot$ premise

Goal: P Try to prove this by reductio

Assume (2) $\neg P$

Goal: $\bot$

(3) $\bot$ copy (1)

(4) P reductio ad absurdum

---

(1) $\bot$ premise

Goal: P

Assume (1) $\neg P$

Goal: $\bot$

Goal: $\neg\neg P$

Assume (?) $\neg P$

Goal: $\bot$

(4) $\bot$ copied from (1)

(5) $\neg\neg P$ 1-4 neg intro

(6) P dne 5

Theorem: $(P \to Q) \Leftrightarrow (\neg Q \to \neg P)$

Part I Assume (1) $P \to Q$

Goal: $\neg Q \to \neg P$

Assume (2) $\neg Q$

Goal: $\neg P$

Assume (3) P

Goal: $\bot$

(4) Q mp 1,3

(5) $\bot$ contra 2,4

(6) $\neg P$ neg intro 3-5

(7) $\neg Q \to \neg P$ deduction 2-6

finishes part I

Part II: Assume (8) $\neg Q \to \neg P$

Goal: $P \to Q$

(9) Assume P

Goal: Q

(10) Assume $\neg Q$

Goal: $\bot$

(11) $\neg P$ mp 8 10

(12) $\bot$ 9, 11 contra

(13) Q reductio 10-12

(14) $P \to Q$ deduction 9-13

(15) $(P \to Q) \leftrightarrow (\neg Q \to \neg P)$ bii, 1-8, 9-13

I NEED TO FIX LINE 11 JUSTIFICATION IN MANUAL

To prove $P \to Q$

(m)	Assume $\neg Q$
	Goal: $\neg P$
	⋮
(n)	$\neg P$

(n+1) $\neg Q \to \neg P$ deduction m–n

(n+2) $(P \to Q) \leftrightarrow (\neg Q \to \neg P)$ contrapositive theorem

(n+3) $P \to Q$ b.mp. n+1, n+2

(n+2) $P \to Q$ indirect proof m–n

$$\frac{X \leftrightarrow Y \qquad X}{Y}$$

$$\frac{X \leftrightarrow Y \qquad Y}{X}$$

$A \leftrightarrow B$

$A \leftrightarrow B$

$A \to B$ $\qquad$ $A \to B$

$\neg A \to \neg B$ $\qquad$ $B \to A$

Modus tollens

$$\frac{P \to Q \qquad \neg Q}{\neg P}$$

modus tollens

① $P \to Q$ premise

② $\neg Q$ premise

Goal: $\neg P$

| Assume ③ P
| Goal: $\bot$
| ④ Q impl. 1, 3
| ⑤ $\bot$ 2, 5 contradiction

⑥ $\neg P$ neg intro 3-5