Homework 5, Math 189, Fall 2023

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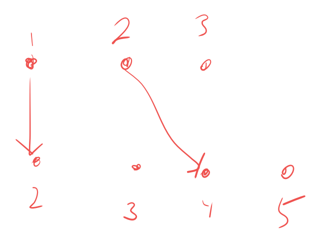
This assignment is based on Levin's 0.4 exercises, and I may refer to exercises in that set in connection with these, which you are welcome to look at (he has solutions to most of his). Interactive examples in the section also might be helpful.

1. based on example 0.4.3

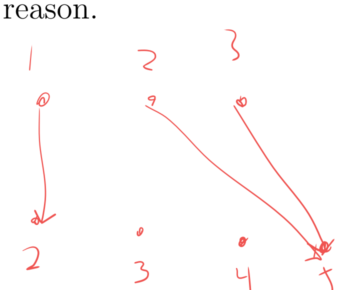
   Lwt $A = \{1, 2, 3\}$ and $B = \{2, 3, 4, 5\}$. Some arrow diagrams are given. Tell me which diagrams are diagrams of functions from A to B. Of the functions, tell me which ones are injective. Why don't I ask which ones are surjective? When you say that something is not a function, or that a function is not injective, give a brief reason.

a.



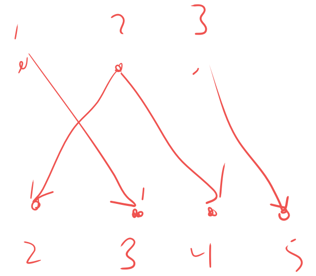
not a function, domain is not all of A

b

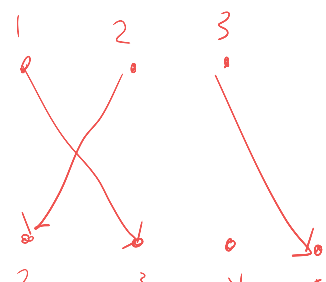


a function, not injective as both 2 and 3 map to 5

c.



not a function, 2 is related both to 2 and to 4



a function, and injective

I didn't ask about surjection because B is larger than A.

Its an accident that all inverses are actually the same function!

2. based on 0.4 problem 2

The following functions each have domain and codomain $\{1, 2, 3, 4\}$. For each one, say whether it is injective, surjective, neither, or both. If it is both, please write out its inverse function in the same format.

(a)

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}$$

both

inverse $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}$ (the same function)

(b)

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 2 & 1 & 1 \end{pmatrix}$$

neither 1, 2 map to 2 (for example)

nothing maps to 3

(c)

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix}$$

both $\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{pmatrix}$ (the same function!)

(d)

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 3 & 3 & 3 \end{pmatrix}$$

neither nothing maps to 1, 2, 4

1, 2, 3, 4 all map to 3

3. Do problem 5 and problem 6 in section 0.4 (I know problem 5 has a solution, so you can check your work on that part). In problem 5, where you have answers, also write three of the functions as triples in the style of the set theory notes.

$8 = 2^3$

$\{1,2,3\} \to \{a,1\}$

123	123	123	123	123	123	123	123
aaa	aab ✓	aba ✓	abb ✓	baa ✓	bab ✓	bba ✓	bbb

None are injective, 6 are surjective

$9 = 3^2$

$\{a,b\} \to \{1,2,3\}$

ab	ab	ab	ab	ab	ab	ab	ab	ab
11	12 ✓	13 ✓	21 ✓	22	23 ✓	31 ✓	32 ✓	33

None are surjective, 6 are injective

The second one out from problem 5 in triple notation:

$(\{1,2,3\},\{a,b\},\{(1,a),(2,a),(3,b)\})$

4. Based on example 0.4.8

~~5.~~

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ a & b & c & a & e \end{pmatrix}$$

is a function f with codomain $\{a, b, c, d, e\}$

Compute the following

(a) $f(4) = a$

(b) $f[\{1, 2, 4\}] = \{a, b\}$

(c) $f^{-1}[\{a, c\}] = \{1, 3, 4\}$

(d) $f^{-1}[\{d\}] = \emptyset$