Math 189 section 3 Fall 2021: Test I

Randall Holmes

October 4, 2021

You will have the period (12 pm -1:15 pm) as the official time to take the test. At 1:15 pm I will actually give a five minute warning to get your paper to me.

Laws of propositional logic from the zybook section on calculation with logical equivalences are attached, as is a set of rules from the manual of logical style. You are welcome to detach these sections from the test to use them more easily.

You are allowed to bring one standard sized sheet of notebook paper to the test with whatever you like written or printed on it. There is no use for a calculator on this test (calculators will be allowed on future exams, with restrictions).

Each of the eight numbered problems on the test has the same weight as any other.

Lettered or numbered parts of problems have equal weight except that in problem 4 (manual of logical style proofs) and problem 8 (even/odd and divisibility proofs), each of which is a problem with two parts, the weight of the part you do best in will be much higher (70 percent) then the weight of the part you do worse on (30 percent).

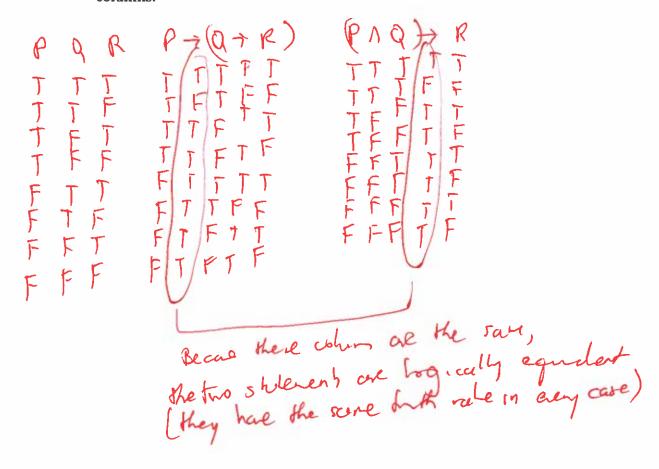
I assert that if Poltroon wins the third race, I will eat my hat.	
(a) State in English the inverse of this conditional statement. (b) State in English the converse of this conditional statement. If I cat what, Poldon all we the the me (c) State in English the contrapositive of this	had
(d) We have four statements now (including the original one): indicate which ones are equivalent to each other.	~
(e) State the exact conditions under which the contrapositive of this statement will be true and its converse will be false, if this is possible.	
If Polders does not un the third race, and	
I do cat my hat.	

1. Inverse converse and contrapositive

2. Use the method of truth tables to show that $P \to (Q \to R)$ is logically equivalent to $(P \land Q) \to R$.

Make sure that you put the table in the standard format with letters in alphabetical order, for ease of grading.

Be sure to state in English the fact about rows or columns of the table which verifies the logical equivalence, highlighting all relevant rows or columns.



3. One of the proposed logical rules

$$\frac{P}{Q \to P}$$

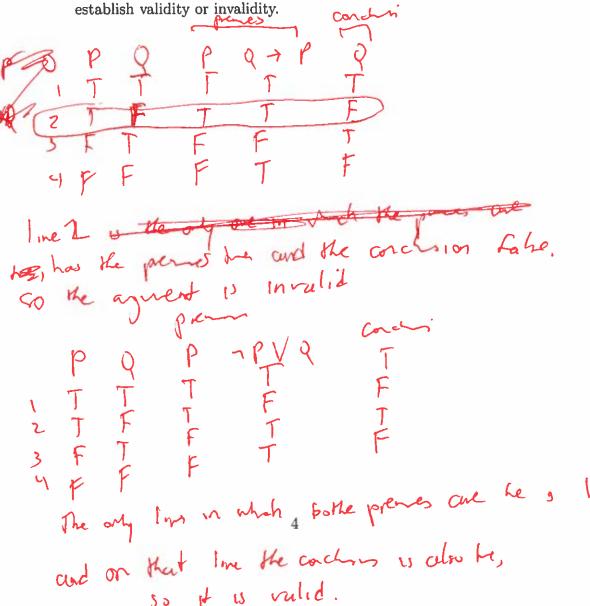
and

P

 $\frac{\neg P \lor Q}{Q}$ is valid and one is invalid.

Use truth table methods to show that the one is valid and the other is invalid.

Highlight relevant rows and/or columns and say in English why they



(extra page if wanted for problem 3)

nether to Hower

4. Prove two theorems using the mathod of the manual of logical style
Using the rules in the manual of logical style, prove the two theorems.

(a)
$$((P \to Q) \land (Q \to R)) \to (P \to R)$$

Assue, O(P+Q) 1 (Q+R)

Col: P+R

Msee O P

God R

Op+Q Simp

Op+Q

(b)
$$(P \vee \neg Q) \wedge (Q \vee R) \to (P \vee R)$$

Hint: use deduction, then alternative elimination, then applications of disjunctive syllogism.

Assoc O(PV-1Q)A(QVR)

God: PVR

God: R

OpV-1Q spp1

OQVR simp!

OQVR simp!

R

PVR

Ass, 4,5

PVR

PVR

Alkenhe elimahn 2-6

8. the these deductor 1-7

5. Prove something using the laws of propositional logic

Use the laws of propositional logic from section 1.5 (in the second appendix to the test) to show that $P \to (Q \to R)$ is logically equivalent to $(P \land Q) \to R$. You are welcome to use Boolean algebra notation in working this problem if you prefer it. Show all steps (make sure for example that you write all parentheses so that you are not skipping applications of the associative law). Name the rule you use at each step of the calculation.

 $P \rightarrow (Q \rightarrow R) \equiv \text{cond. identry}$ $\neg P \lor (Q \rightarrow R) \equiv \text{cond. identry}$ $\neg P \lor (\neg Q \lor R) \equiv \text{cassare} \lor$ $\neg P \lor (\neg Q \lor R) \equiv \text{de Mogan}$ $\neg (P \land Q) \lor R \equiv \text{de Mogan}$ $\neg (P \land Q) \lor R \equiv \text{cond. identry}$ $P \land Q \rightarrow R$ 6. Read some statements about numbers in quantifier language and class as true or false; write some statements about numbers in quantifier language

Follow the instructions in each part.

(a) The statement $(\forall xy : (\exists z : x+z=y))$ is true if the domain of the quantifiers is the set of real numbers. How would you describe z in terms of x and y?

Z= Y-X

Does it remain true if the domain is the set of integers? If it is false, give a counterexample.

Does it remain true if the domain is the set of positive real numbers? If it is false, give a counterexample.

18 counterexample (1-2 is anywhe)

(b) One of the statements $(\forall x : (\exists y : y = x^2) \text{ and } (\forall x : (\exists y : y^2 = x))$ is true if the quantifiers range over all real numbers, and one is false.

Translate each sentence into a natural statement in mathematical English (no variables!). Say which one is true, which one is false, and give a counterexample to indicate why the false one is false. How could the domain of the variables be restricted to make both statements true?

Every real wher has a sque part the Every real wher has a sque part false: if x<0 ker is to real y s.t. y2 = x. So x = -1 is a contactagle.

The down and be rested to porte reals,

(c) Write the statement which we express in English as "There is no largest integer" using nothing but <, letters, and logical symbols. Hint: think of it as saying, for every integer, there is a larger one.

Exer: Byer:x<y)

- 7. de Morgan's law for quantifiers
 - (a) How would you express the negation of the sentence

$$(\forall x: \exists y: P(x,y) \to P(y,x))$$

in a form in which all negations are directly in front of P(x, y) or P(y, x)? (you need to remember how to negate an implication).

(3x: Yy: P(x,y) An P(y,x))

Extra credit: without knowing anything about P, this sentence actually must be true and its negation must be false. If you can briefly state why, extra points might be awarded.

For each x, tale y = x and ve here P(x,y) + P(y,x) actually heary P(x,x) + P(x,x), which

x=y & grs contateuxagles to the negation.

(b) Suppose we are in a context in which people who don't love each other, necessarily hate each other. We use x L y for "x loves y".
Write the following symbolic expressions and their negations as natural English sentences about love or hate (with no variables).

i. $(\forall x : (\exists y : x L y))$

Every body love) somehody The negation v Someone hastes every are

ii. $(\exists y : (\forall x : x L y))$

Sovere is loved by every one
Everyone is hasted by someone
one and do synthe calculation
be chech she would for the negations.

- 8. Write a proof about even and odd numbers or divisibility

 Prove the two theorems in the style presented in the zybook.
 - (a) The square of any odd integer is odd. The definition of "x is odd" is "There is an integer k such that x = 2k + 1".

lot x he un arbitany chen intege. Assue that x is odd, an integer
By cosempta, we can almore to s.t. x = 2ht1. Our god u to And an integer n such that Plugging 12, we get 12 = (2ht1) = 4h2+ 4h+1 = 2(7h2+2h)+1. 2nH = x2. Now by the $2h^2 + 2h$ is an inlight, so

Now by the $2h^2 + 2h$ is an inlight, so $x^2 = 2nH$, or $n: = 2h^2 + 2h$ gus we $x^2 = 2nH$, or

an inlight, so x^2 is add. and we have shows this for an only it (Yx; x a dd > x2 a odd).

(b) For any integers x, y, z, if x|y and x|z then x|(y+z). The definition of x|y ("x goes into y") is "There is an integer k such that y = kx".

lat x, y, z be abiday ratiges. Suppre 0 x /y and 0 x /=. Then 3 ve can an or onlyer h such had y = hx and @ re can down on integer I such that == lx So (plugging in from 3, 4) æytz= hx+lx= (h+l)x. hthis an integer by due purples so if we duke now kether here # Z = nx, and X/Z follow by defents. We have shown that for any x, y, to if x/y and y x/z ne hae x/(y+E).