

## Math 189, Fall 2024, Homework 7

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This homework set is due on Wednesday October 16.

1. Consider the sequence 1,4,7,10,13....

Give a closed form definition for this sequence (assuming the indexing starts at 0).

Give a recursive definition of the sequence (assuming the indexing starts at 0).

Write these two definitions again assuming the indexing starts at 1.

2. Do Levin section 2.1 problem 9.

3. Do Levin 2.1 problem 10.  $\{n\}$

4. Write  $25+36+49+64+81+100$  in summation notation in the natural way.

Write out  $\sum_{i=3}^8 i^3 + i^2$  as a sum.

5. Do 2.1 problem 18 in Levin.

6. How many terms are there in the arithmetic sequence

2, 5, 8, ..., 152?

Compute

$$2 + 5 + 8 + \dots + 152$$

using our methods for summing arithmetic sequences.

7. Express the decimal number 3.333333 as the sum of a geometric sequence in the natural way. What is the first term of this sequence? What is the common ratio of this sequence? Write the sequence in summation notation.

Let den 3  
common ratio  $\frac{1}{10}$

$$3 + \frac{1}{3} + \frac{1}{30} + \frac{1}{300} + \frac{1}{3000} + \frac{1}{30000} + \frac{1}{300000} + \frac{1}{3000000} + \frac{1}{30000000} = \sum_{i=0}^{\infty} 3 \left(\frac{1}{10}\right)^i$$

8. Verify the calculus formula  $1 + x + x^2 + \dots = \frac{1}{1-x}$  ( $|x| < 1$ ) using the formula for the sum of a geometric sequence. Start by computing the sum  $1 + x + x^2 + \dots + x^n$  using the formula, then explain what the fact that  $|x| < 1$  does for you (what happens when  $n$  gets large?)

9. Do Levin, 2.2, problem 14

$$1 + x + x^2 + x^3 + \dots + x^n = \frac{1 - x^{n+1}}{1 - x} \text{ and as } n \rightarrow \infty, x^{n+1} \rightarrow 0$$

$$\frac{1 + 4(n-1)}{2} \cdot n = \frac{n(4n-3)}{2}$$

$$\frac{x^{n+1} - 1}{x - 1} \text{ and as } n \rightarrow \infty, \frac{1}{1-x}$$

1st term of the sequence is 1

$$1 + 4 + 8 + 12 + \dots + 4(n-1) = \frac{1 + \frac{4n(n-1)}{2}}{2}$$