Math 275 Test III, Fall 2018 (edited for Fall 2020 review)

Dr Holmes

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You will be provided with two copies of this exam. You may take the second one home. Work done on the take-home paper which was not completed satisfactorily on the in-class paper will be worth half its value in-class (this fifty percent value might be adjusted upward based on class performance).

The take-home paper must be returned on Monday.

Both of the center of mass problems designated as take-home are optional, but both are good for substantial credit if completed successfully.

1. Evaluate the integral
$$\int_0^1 \int_1^3 x^2 y \, dy dx$$
. There is a trick which can be applied to evaluate this particular integral quickly, if you remember it, but direct evaluation should not be hard either.

2. Sketch the region of integration of the iterated integral $\int_0^2 \int_{x^2}^4 y\,dy\,dx$.

Change the order of integration and set up the integral as $\int_{??}^{??} \int_{??}^{??} y\,dx\,dy$.

Evaluate both integrals and check that the same value is obtained.

3. A thin plate occupying the triangular region with corners at (-1,0), (1,0), and (0,2) has density y^2 at each point (x, y) in this region.

   Determine the mass of this object. This will require you to set up and compute a suitable integral.

4. Set up the integral of $f(x, y) = x^2 + y^2$ over the region bounded by $x = \sqrt{1 - y^2}$ and the y-axis in Cartesian coordinates. I do not suggest trying to evaluate it.

   Set up the same integral in polar coordinates and evaluate it.

5. (a) Evaluate the triple iterated integral $\int_0^2 \int_x^2 \int_0^{x+y} x \, dz \, dy \, dx$

6. I provide for you the setup for an integral representing the volume of a sphere of radius a in cartesian coordinates:

$$\int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}}\int_{-\sqrt{a^2-x^2-y^2}}^{\sqrt{a^2-x^2-y^2}} 1dzdydx$$

Convert this to cylindrical coordinates and evaluate it. Hint: the region of integration in the plane is just the disk bounded by the circle of radus a with center at the origin, the world's simplest polar rectangle...

7. An object occupies the region in the first octant bounded by the coordinate planes and the sphere $x^2 + y^2 + z^2 = 4$. Set up but do not evaluate an integral which would evaluate to the mass of the object if the density function for the object at a point is proportional to the distance of the point from the origin, using spherical coordinates.

Formulas for spherical coordinates:

$x = \rho \sin(\phi) \cos(\theta)$

$y = \rho \sin(\phi) \sin(\theta)$

$z = \rho \cos(\theta)$

$\rho^2 = x^2 + y^2 + z^2$

$$\int \int \in_E f(x, y, z) dV$$

$$= \int_a^b \int_{f(\phi)}^{g(\phi)} \int_{h(\phi,\theta)}^{k(\phi,\theta)} f(\rho \sin(\phi) \cos(\theta), \rho \sin(\phi) \sin(\theta), \rho \cos(\phi))\, \rho^2 \sin(\phi)\, d\rho\, d\theta\, d\phi$$