Math 275 1,5,6 Fall 2020 Week 1 and 2 Quiz

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This is due on Sunday 9/6 at 11:55 pm, the same due time as your Webassign. There are four problems, one on each page. One way to complete the quiz is to do it by hand on a printed copy of the quiz, which is why I give a full page for each problem, then scan the results and return them to me. But you can do it in other ways too. However you do it, please show all work.

1. Find real numbers s, t such that $s\langle 1,2\rangle + t\langle 3,-1\rangle = \langle 11,2\rangle$. Hint: you will be solving two equations in two unknowns.

   From the first coordinates

   $$(1) : s + 3t = 11$$

   From the second coordinates

   $$(2) : 2s - t = 2$$

   So (multiply both sides of (1) by two)

   $$(3) : 2s + 6t = 22$$

   So (subtract equation (2) from equation (3))

   $7t = 20$ so $t = \frac{20}{7}$

   Now by (2) $s = \frac{t}{2} + 1$ so $s = \frac{1}{2}(\frac{20}{7}) + 1 = \frac{17}{7}$.

   It's a good idea to check that the values you get actually work in the original equations; I did this in my head. The solution is $s = \frac{17}{7}; t = \frac{20}{7}$.

2. Determine the angle between the vectors $\langle 1,2,1\rangle$ and $\langle 1,-1,3\rangle$.

$$\cos(\theta) = \frac{\langle 1,2,1\rangle \cdot \langle 1,-1,3\rangle}{(||\langle 1,2,1\rangle||)(||\langle 1,-1,3\rangle||)}$$

$$= \frac{1-2+3}{\sqrt{6}\sqrt{11}}$$

so $\theta = \arccos(\frac{2}{\sqrt{66}}) \approx 75.75° \approx 1.32$ radians.

3. Find the equation of the plane through the points (1,2,3), (3,0,-1), (2,2,1). [It really should be “find *an* equation” since variations are possible].

   Let $P = (1, 2, 3), Q = (3, 0, -1), R = (2, 2, 1)$.

   The vector $\mathbf{u}$ from P to Q is $\langle 2, -2, -4 \rangle$.

   The vector $\mathbf{v}$ from P to R is $\langle 1, 0, -2 \rangle$

   The cross product $\mathbf{u} \times \mathbf{v}$ will be orthogonal to both these vectors and so suitable to be a normal vector.

   $$\begin{array}{ccccc} \mathbf{i} & \mathbf{j} & \mathbf{k} & \mathbf{i} & \mathbf{j} \\ 2 & -2 & -4 & 2 & -2 \\ 1 & 0 & -2 & 1 & 0 \end{array}$$

   The cross product is then $\langle 4, 0, 2 \rangle$ and an equation is

   $$4(x - 1) + 0(y - 2) + 2(z - 3)$$

   or $4x+2z = 10$. Oddly, this plane is parallel to the y-axis (the geometric meaning of the fact that the coefficient of y in the equation is 0)!

   I could equally well have used Q or R as the selected point.

   You can check that

   $$4(x - 3) + 0(y - 0) + 2(z + 1),$$

   which uses Q, is the same.

4. Find symmetric equations for the line of intersection of the planes

$$2x - y + z = 10$$

and

$$x + 2y - 3z = 1.$$

Normal vectors of the two planes are $\langle 2, -1, 1 \rangle$ and $\langle 1, 2, -3 \rangle$. Any vector perpendicular to both of these will be parallel to the line we are looking for. The cross product will give us such a vector.

$$\begin{array}{ccccc} \mathbf{i} & \mathbf{j} & \mathbf{k} & \mathbf{i} & \mathbf{j} \\ 2 & -1 & 1 & 2 & -1 \\ 1 & 2 & -3 & 1 & 2 \end{array}$$

The cross product is $\langle 1, 7, 5 \rangle$ (I checked using the dot product that it is actually orthogonal to the two original vectors, and you should, too).

So all we need now is a point which is on both planes.

Set $x = 0$. So we need $-y + z = 10$ and $2y - 3z = 1$. From the first equation, $-2y + 2z = 20$. Add the second and third equations to get $-z = 21$, or $z = -21$. $-y - 21 = 10$ gives $y = -31$. So $(0, -31, -21)$ is a point on both planes. I checked that it is, and also that $(0 + 1, -31 + 7, -21 + 5)$ is on both planes, which reassures me that my calculations are correct.

So a set of symmetric equations is

$$\frac{x}{1} = \frac{y + 31}{7} = \frac{y + 21}{5}$$