Math 275 Test III, Summer 2013

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1. Determine the linearization $L(x, y)$ of $f(x, y) = \frac{x}{y}$ for x near 2 and y near 1 (a function).

   Write an equation of the tangent plane to $z = \frac{x}{y}$ at $(2, 1, 2)$.

   Estimate $\frac{1.98}{1.03}$ using the linearization.

2. 14.5 Compute the directional derivative of $f(x, y) = x^2y$ in the direction of $\langle 1, 1\rangle$ at the point $(2, 3)$

Give a vector pointing in the direction in which this function is increasing fastest at $(2, 3)$. What is this fastest rate of increase?

3. 14.6 Determine the partial derivatives of $f(x, y) = 2x - y$ with respect to the polar coordinates r and θ, using the usual relations $x = r\cos(\theta)$, $y = r\sin(\theta)$ and the Chain Rule.

4. 14.7 Find all critical points of

$$f(x, y) = 4x^3 + y^3 - 12x - 3y.$$

There are four of them. Classify each of the critical points as a local maximum, local minimum, or saddle point, using the second derivate test.

5. 14.8 Use the method of Lagrange multipliers to find the values of the coordinates x and y on the circle $x^2 + y^2 = 1$ which will maximize the area of the pictured rectangle. You have the advantage that you ought to know something basic about the answer intuitively! You do need to use the method of Lagrange multipliers (section 14.8) to get credit.

6. 15.1 Compute the integral of $f(x, y) = x + 2y$ over the pictured region by setting up and evaluating an iterated integral. Set it up and evaluate it in both possible ways and verify that you get the same answer in both cases.