Homework 6, Math 189, 2022

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I supply solutions to all problems, but I am only marking every other problem. I have to catch up with my backlog.

There is a bug in which problems I graded: I graded problem 4 instead of problem 7 in section 2.2; the grid on your paper may say 7, but its problem 4. Failed to alternate exactly, but met my goal of grading half the problems.

Levin 2.1:

2, a: $3 + 2^n$; b: $(n+1)^2 - 1 = n^2 + 2n$; c: $(n+2)(n+3)$; d: $n^2 + \frac{n^2+n}{2}$: square numbers plus triangular numbers equal house numbers.

4, a: $2n-1$: notice that the notation tells you that indexing starts with 1.

b: the sequence is 1,4,9,.... It is confusing that he suggests starting indexing of b_n with $n = 2$, then actually gives it with indexing starting at 1 (and the first partial sum just the first term).

5, a: 0,1,2,4,7,12,20, 33

b: compare with 1,2,3,5,8,13,21,34,..., that is F_{n+2}.

we get the sequence of partial sums by subtracting one from this, so $F_{n+2} - 1$. It is quite wrong to write something like $F_n = F_{n+2} + 1$: the sequence of partial sums, if we give it a name, will not have the same name F.

10,

$$7a_{n-1} - 10a_{n-2} = 7(2^{n-1} - 5^{n-1}) - 10(2^{n-2} - 5^{n-2})$$

$$= 14\cdot 2^{n-2} - 35\cdot 5^{n-2} - 10\cdot 2^{n-2} + 10\cdot 5^{n-2} = 4\cdot 2^{n-2} - 25\cdot 5^{n-2} = 2^n - 5^n = a_n$$

$a_0 = 0; a_1 = -3$

15, $R_0 = 1$; $R_1 = 2$. When you draw the $(n+1)$st line, it intersects each of the other n lines in a point: n points determine $n+1$ line segments. Each of these segments divide one of the R_n regions in two, so $R_{n+1} = R_n + (n+1)$. $R_2 = 4$. $R_3 = 7$, $R_4 = 11$ follow this pattern.

A closed formula is $\frac{n^2+n+2}{2}$: one greater than the triangular numbers.

19; $t_1 = 1; t_2 = 2; t_3 = 3$ in general, $t_n = t_{n-2} + t_{n-1}$: think about how the top right domino is placed. if it is vertical what remains is to tile a $2 \times (n-1)$; if it is horizontal, what remains is to place a domino under it then tile a $2 \times (n-2)$.

Levin 2.2:

2, 32, $8 + 6n$, 30,500 $= \frac{a_0 + a_99}{2} \cdot 100 = \frac{8+(8+(6)(99))}{2} \cdot 100$

4, $n+2$, since indexing has to start with -1. $6n+1$ is the second to last term. The sum of all the terms is $\frac{1+(6n+7)}{2} \cdot (n+2) = \frac{(6n+8)(n+2)}{2}$.

Note that the sum of an arithmetic sequence is the average of the first term and the last term multiplied by the number of terms. This is how I am generating my calculations here.

7, This is $a + ar + ar^2 + \ldots + ar^n = \frac{a - ar^{n+1}}{1-r}$ where $a = 1, r = -\frac{2}{3}, n = 30$.

This is $\frac{1+\frac{2}{3}^{31}}{1+\frac{2}{3}} = \frac{3}{5}(1 - (\frac{2}{3})^{31})$

14, 1,5,13,25 This is enough to see that it is neither arithmetic or geometric: 1+4 = 5, but 5+4 is not 13, and $1 \cdot 5 = 5; 5 \cdot 5 \neq 13$

it is 1, 1+4, 1+4+8, 1+4+8+12, 1+4+8+12+16, which is 1 + a sum of multiples of 4, which is not the sequence of partial sums of any arithmetic or geometric sequence; it differs by one from such a sum.

it is the partial sum of a sequence starting 1,4,8,12 which again can be seen to be neither arithmetic or geometric just by looking at these terms. However, the terms of this sequence turn out to go up by 4 at each subsequent step.

so it is $1 + 4\frac{n(n-1)}{2} = 1 + 2n(n-1) = 1 + 2n^2 - 2n$

This can be seen to work by realizing that each diagonal square can be reduced to the previous one by removing the middle row and the row below it; then notice that the sum of the middle row and the row below it goes up by 4 at each step.

It can also be seen, by thinking of the diagonal square as a sum of two triangles (vertically, one on top of the other)and using the result about the sum'of the first n odd numbers being n^2, that the nth diagonal square number is $n^2 + (n-1)^2$: I'll be happy to illustrate this in class if you dont see it. $n^2 + (n-1)^2 = n^2 + n^2 - 2n + 1 = 2n^2 - 2n + 1$.

15. a: 4^6 contain no numerals.

$3 \cdot 4^5$ contain one numeral.

$3^2 \cdot 4^4$ contain two numerals, etc.

This is a geometric sequence with $a = 4^6$ and $r = \frac{3}{4}$.

The answer is then $\sum_n = 0^6 4^6 \cdot (\frac{3}{4})^n$, and this, by results in this section, is $\frac{4^6 - 4^6 \cdot (\frac{3}{4})^7}{1 - \frac{3}{4}}$ and this simplifies straightforwardly to $4^7 - 3^7$.

If you did not use some recognizably "geometric series" technique, you got 3 points out of 5. The instructions tell you that it is a geometric series problem.