Test I, Math 502, Spring 2017

March 7, 2021

The exam will begin at 12 and end at 1:15 officially: I will actually give a five minute warning at that point.

You should not need anything but your writing instrument and your test paper. A section of logical rules is appended to the test, which you are free to tear off.

1. Write a proof of

$$((P \to Q) \wedge (Q \to R)) \to (P \to R),$$

complete with line numbers, stated goals, and documentation of what rules are applied to get each line.

2. Write a proof of

$$\neg(P \vee Q) \to (\neg P \wedge \neg Q).$$

You may not use the other de Morgan law (or any other theorem) to prove this: just the rules (notice that this is just one direction of one of the de Morgan laws).

3. Verify the rule of destructive dilemma, with the same level of formality (line numbers, stated goals, state rules used to get each line).

$$\frac{\begin{array}{c} P \to Q \\ R \to S \\ \neg Q \vee \neg S \end{array}}{\neg P \vee \neg R}$$

Again, you may use rules but not theorems.

4. Prove

$$(\forall x : P[x]) \wedge (\forall x : P[x] \rightarrow Q[x]) \rightarrow (\forall x : Q[x])$$

(same level of formality: be sure to be especially careful to indicate all uses of quantifier rules).

5. Prove

$$(\forall x : P[x]) \wedge (\exists x : P[x] \rightarrow Q[x]) \rightarrow (\exists x : Q[x])$$

(same instructions)

6. Prove

$$(\forall x : \neg P[x]) \leftrightarrow \neg(\exists x : \neg P[x])$$

Same instructions.

7. Prove $(A \cap B)^c = A^c \cup B^c$. You may be somewhat informal about propositional logic steps but you should be explicit about uses of definitions of set operations and the strategy for proving that two sets are equal.

8. Prove the theorem

$$A \subseteq B \to \mathcal{P}(A) \subseteq \mathcal{P}(B)$$

(same instructions as previous question)

9. Give an assignment of types to the constants, variables, and sets in the equation

$$1 = \{x \mid (\exists y : (\forall u : u \in x \leftrightarrow u = y))\}$$

and in the equation

$$(A + 1) = \{(p \cup \{q\}) \mid p \in A \wedge q \notin p\}$$

(in this one, be sure to assign a type to each variable, to each set expression in braces, and also to the expressions I have put in parentheses).

You can do this by making tables or just by writing numbers (neatly) in the appropriate places.

10. List the elements of the set

$$\{\{p, q\} : p \in A \wedge q \in B\}$$

where $A = \{a, b, c\}$ and $B = \{c, d, e\}$. a, b, c, d, e are five distinct individuals. Remember that $\{x, y\} = \{y, x\}$ and $\{x, x\} = \{x\}$. How many elements does this set have?