Math 275 Final Exam, Fall 2013, revised for Fall 2020 review

Dr. Holmes

December 5, 2020

The exam will start at 2:30 pm and officially end at 4:30 pm. If no one objects, the exam will continue until 4:45 pm. You are allowed to use your book, one standard sized sheet of paper with whatever notes you like written or printed on it, and your non-graphing scientific calculator.

1. Compute the angle between $\langle 1, 1, 1 \rangle$ and $\langle 1, 3, 5 \rangle$ to two decimal places (in degrees or radians).

2. Determine an equation for the plane passing through the points $(1, 1, 1)$, $(-1, 1, 2)$, and $(1, 3, 5)$.

3. Find a parametric equation for the tangent line to the twisted cubic $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$ at the point $(3, 9, 27)$.

4. Determine the linearization of the function $f(x, y) = \sqrt{x^2 + y}$ near the values $x = 4, y = 9$. Use the linearization to estimate $\sqrt{(3.8)^2 + 9.1}$.

5. Find the critical points of $f(x, y) = x^2 + 2y^2 - x^2y$ and classify each of them as local maximum, local minimum, or saddle point.

6. Set up the integral of $f(x, y) = xy$ over the region pictured using both orders of integration, and evaluate one of the integrals. (region bounded by $y = 2x$, $y = 2$, $x = 0$).

7. An object is bounded above by $z = 1 - x^2 - y^2$, below by the xy-plane and its density is given by $\rho(x, y, z) = z$. Set up the integral which evaluates to the mass of this object using cylindrical coordinates. You may for a little additional credit evaluate the integral (don't do additional credit until finished with the rest of the exam!)

8. State and verify the three mixed partial conditions which tell us that $\langle yz^2 + z, xz^2, 2xyz + x \rangle$ is a conservative vector field. You can also state this in terms of the curl (but thin that case compute the curl and verify the needed fact about it).

Determine its potential function by a suitable process of integration.

9. Show calculations verifying that $\langle y + 1, x \rangle$ has potential function $xy + x$. Compute the line integral $\int_C (y + 1)dx + xdy$ where C is the path from $(-2, -2)$ to $(2, 4)$ consisting of the straight line segment from $(-2, -2)$ to the origin, followed by the portion of the graph of $y = e^x - 1$ going from the origin to $(1, e - 1)$, followed by the line segment from $(1, e - 1)$ to $(2, 4)$.

10. Compute the line integral of the vector field $y\mathbf{i} - x\mathbf{j}$ along the circle $x^2 + y^2 = 1$ counterclockwise from $(1, 0)$ back to $(1, 0)$. You may if you choose compute it directly (using the parameterization $\langle \cos(t), \sin(t) \rangle$ of this circle), but it is also straightforward to compute it using Green's Theorem: it is equal to a double integral (describe it) which you can compute using high school geometry. Credit for this depends on explaining your reasoning clearly.