Blackboard :/

$\{0, 1, 2\}$

$+$	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

$*$	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

mod 3 arithmetic

more generally, mod p arithmetic p a prime satisfies these axioms

Ex 1.4

n	$-n$
0	0
1	2
2	1

$(m+n)p \overset{?}{=} mp + np$ $\qquad$ $p(m+n) = pm + pn$

$\| \text{ Ax 1.1} \quad \overset{?}{=} \quad mp + np$

$p(m+n)$

$\| \text{ Ax 1.1} \quad \overset{?}{=} \quad mp + np$

$pm + pn$

$\| \text{ Ax 1.1. commutative}$

$mp + np$

Theorem For all $m, n, p \in \mathbb{Z}$
if $m+p = n+p$ then $m = n$

Assume (1) $m+p = n+p$

Goal: $m = n$

(2) $(m+p)+(-p) = (n+p)+(-p)$ property of equality

(3) $m+(p+-p) = n+(p+-p)$ assoc + (1.1 ii)

(4) $m+0 = n+0$ ~~...~~ 3 inv +

(5) $m = n$ identity property of addition

Prop 1.14: $m \cdot 0 = 0 \cdot m = 0$

If $m + x_1 = 0$ and $m + x_2 = 0$

then $x_1 = x_2$

$m + x_1 = 0 = m + x_2$

$x_1 = x_2$

This shows that $-m$ is unique

We know by axiom that for $m \in Z$ we have $-m$ such that $m + -m = 0$.

Prop 1.10 shows that $-m$ is the only number for which this is true.

Suppose that for all m, $m+x = m$.
Then $m+x = m = m+0$ 1.12
so $m+x = m+0$
so $x = 0$

Suppose that for some m, $m+x = m$ 1.13
$m+x = m+0$
$x = 0$
so the weaker hypothesis is enough

I do want you to attempt 1.14.

$n \mid m$ means

for some $j \in \mathbb{Z}$, $nj = m$

m is even is denoted as
$2 \mid m$

$n \mid m$ is almost the same

as $\frac{m}{n} \in \mathbb{Z}$.

$0 \mid 0$ is true because for
any $j \in \mathbb{Z}$, $0j = 0$

but $0 \mid 0$ is not equivalent to "$\frac{0}{0} \in \mathbb{Z}$"

Factor Theorem

If $m \cdot n = 0$ then $m = 0$ or $n = 0$ [or both]

or for us is always
inclusive

Either $m = 0$ or $m \neq 0$

Case 1 $m = 0$ - then we have $m = 0$ or $n = 0$

Case 2 $m \neq 0$ - then we have $m \cdot n = 0 = m \cdot 0$ and $m \neq 0$

By Ax 1.5 (cancellation property of $*$) ~~then~~ we have $n = 0$

so we have $m = 0$ or $n = 0$.