Math 311 Test I, Spring 2013

Dr. Holmes

February 20, 2013

The test begins at 10:30 and ends at 11:45. You are allowed your test paper, your writing instrument and drawing tools if you wish. You may ask the instructor for scratch paper if you feel the need of it.

Your grade will be posted on the web (on the class announcements page) using a number written on the first inside page of your paper. Make a note of it.

1. Logic

   (a) Write down the converse and contrapositive of the assertion "If the Euclidean Parallel Postulate holds, then there are at least four points". Label them as converse and contrapositive.

   (b) Write the negation of each of these sentences in a natural form, making use of the deMorgan laws and rules of negation of quantifiers where appropriate.

      i. All triangles are isosceles.

      ii. Some polygon has angle sum greater than 180 degrees.

      iii. Angle α is acute or obtuse.

2. Four interpretations of incidence geometry are presented. One of them is a model. Identify the one which is a model, and for each of the others explain which axioms fail and explain how they fail. Your explanation should mention specific points and lines where appropriate.

   (a) A social bridge night with two tables. The points are players; the lines are pairs of partners. A, B, C, D sit at table 1, and A, C are partners and B, D are partners. E, F, G, H sit at table 2, and E, G are partners and F, H are partners.

   (b) Three dice are sitting on a table. A point is a die. A line is a pair of dice.

(c) Points are the letters A B C D E. A line is any set of four of these points: $\{A, B, C, D\}, \{A, C, D, E\}, \{A, B, D, E\}, \{A, B, C, E\}, \{B, C, D, E\}$ are the lines.

(d) The model consists of one point, and a single line on which just that point lies.

3. Present three interpretations of incidence geometry which are not models. In the first, IA1 should fail and the other two axioms should hold; in the second, IA2 should fail and the other two axioms should hold. In the third, IA3 should fail and the other two axioms should hold.

   In each part you should describe your model in words (a picture by itself doesn't work) and you should explain concretely why the axiom that fails, fails, mentioning specific points and lines where appropriate. You may use parts of the previous problem, at your own risk (what you say about them needs to be correct).

   (a) model where IA1 fails:

   (b) model where IA2 fails:

   (c) model where IA3 fails:

4. A model is pictured in which none of the Parallel Postulates that we have considered holds. Explain why each of the parallel postulates fails (give a specific line and point not on it which is a counterexample and say why it is a counterexample).

5. Two models of incidence geometry are presented which are isomorphic. Present a correspondence between points and lines of the two models (listing points AND listing lines) which witnesses the isomorphism.

6. Prove using the axioms of incidence geometry that there are at least three distinct lines.

7. Present two models of incidence geometry, each with exactly four points, which are not isomorphic. State a reason why they cannot be isomorphic.

8. Prove using the axioms of incidence geometry and assuming the Euclidean Parallel Postulate that there must be at least four points. Hint: you will use IA2, something we have seldom done in our proofs.

1 Axioms and Postulates

Axioms of Incidence Geometry

IA1: For each pair of distinct points P, Q, there is exactly one line L such that P lies on L and Q lies on L.

IA2: For each line L, there are at least two distinct points P, Q which lie on L (there may be more).

IA3: There are three distinct points P, Q, R such that there is no line L such that P, Q, R all lie on L.

Parallel Postulates

Euclidean Parallel Postulate: For any line L and point P not on L, there is exactly one line M such that P lies on M and M is parallel to L.

Hyperbolic Parallel Postulate: For any line L and point P not on L, there are two distinct lines M, N such that P lies on M and M is parallel to L and P lies on N and N is parallel to L (there may be more such lines).

Elliptic Parallel Postulate: For any line L and point P not on L, there is no line M such that P lies on M and M is parallel to L (i.e., there are no parallel lines).

Useful definitions

parallel: Lines L and M are parallel iff there is no point P such that P lies on L and P lies on M.

collinear: Points A, B, C are collinear iff there is a line L such that A, B, C all lie on L.