Math 189 Fall 2023 Homework 4

Dr Holmes

September 14, 2023

These are modelled on exercises in Levin, 0.3, and you should read Levin section 0.3 and my set theory notes for background. One piece of notation which I did not use in lecture appears here: if A is a set, we use $|A|$ to represent the number of elements in A (which may also be called the size or cardinality of A), if A is a finite set. I may provide you with references to similar problems in Levin's 0.3 set; I also suggest doing the interactive examples in section 0.3, which are quite nice.

1. (modelled on problem 1 in the 0.3 exercises)

   Let $A = \{1, 2, 4, 7\}$ and let $B = \{1, 2, 4, 8, 16\}$. Find each of the following sets and present them in list notation.

   (a) $A \cup B$

   (b) $A \cap B$

   (c) $A \setminus B$ [Levin uses $A \setminus B$ instead of $A - B$, so I will as well. Expect the notes to be revised.]

   (d) $B \setminus A$

2. Find $|A|$ (the number of elements in A) for each of the following examples. If the set is infinite, say so. Modelled on problem 3.

   (a) $\{17, 18, 19, \ldots, 35\}$ [be careful that you don't make a fencepost error].

   (b) $\{x^3 : x \in \mathbb{Z}^+ \land x^3 < 200\}$ [read carefully, this is a set of perfect cubes and it is not very large]

(c) $\{1, 2, 3, 4, 5, 6, 7\} \times \{1, 3, 6, 9, 27, 81\}$ List a few elements of this set (they are ordered pairs) and tell me how many elements the set has. You don't want to list them all!

(d) $\{x \in \mathbb{Z}^+ : x|60\}$

(e) $\{x \in \mathbb{Z}^+ : 60|x\}$

3. This is problem 10 in Levin 0.3. Let $A = \{x \in \mathbb{N} : 3 \leq x \leq 13\}$, $B = \{x \in \mathbb{N} : x \text{ is even}\}$ and let $C = \{x \in \mathbb{N} : x \text{ is odd}\}$. Find the following sets. Write list or set builder notation for the sets which does not mention A, B, or C.

   (a) $A \cap B$

   (b) $A \cup B$

   (c) $B \cap C$

   (d) $B \cup C$

4. Let $A = \{1, 2, 3\}$. Write $\mathcal{P}(A)$ in list notation.

5. modelled on question 15. Draw Venn diagrams representing each of the following sets.

   (a) $A \setminus (B \cup C)$

   (b) $(A \setminus B) \cup (A \setminus C)$

   (c) $A \setminus (B \cap C)$

6. Present sets A, B of small positive integers in list notation such that $|A| = 3, |B| = 4$, and $|A \cup B| = 5$. (remember that $|X|$ means the number of elements in X when X is a set).

7. (extra credit puzzle question) Explain why no set A exists such that $A = \{2, |A|\}$. Give me two different sets B such that $B = \{1, 2, |B|\}$. The first part of this question is number 29 in 0.3.