Math 189 Spring 2022 Homework 3 Solutions

September 11, 2022

Levin 1: a: 64 subsets,

b: 4 of them have $\{2, 3, 5\}$ as a subset,

c: $64 - 8 = 56$ of them contain at least one odd number (there are 8 sets just of even numbers, take those out).

d: To count ones with exactly one even number, choose an even number (3 choices) and a set of odd numbers (8 choices): there are 24 of these.

2: a: There are $\begin{pmatrix} 6 \\ 4 \end{pmatrix}$ sets of size 4, that is, 15.

b: There are three subsets of size 4 containing $\{2, 3, 5\}$ as a subset.

c: All subsets of size 4 (all 15 of them) contain at least one odd number.

d: Three subsets of size 4 contain exactly one even number (choose one of the three evens to put in with the three odds).

3: (accidentally left this out at first)

a: $2^9 = 512$

b: 9 choose $5 = 126$

c: 16

d: 256

4: a: $2^{9-3} = 64$.

b:

b:after the 101 we will have a string of length 6, weight 3, and there are $\begin{pmatrix} 6 \\ 3 \end{pmatrix} = 20$ of these c: 2^6 start with 101, 2^7 end with 11, 2^4 have both features, so $64 + 128 - 16 = 176$

d: Weight 5 starting with 101, there are 20. Weight 5 ending with 11, there are $\left(\begin{array}{c} 7 \\ 3 \end{array} \right)$ of these, that is 35, and there are 4 strings with both properties (put a single 1 in one of the four remaining positions). so the answer is $20 + 35 - 4 = 51$,

6: 386. Add up $\left(\begin{array}{c} 10 \\ n \end{array} \right)$ where n ranges from six to ten.

8: $3640 = \left(\begin{array}{c} 15 \\ 12 \end{array} \right) \cdot 2^3$

12: there are 11 choose 3 = 165 possible pizzas.

Of these, 10 choose 3 = 120 have pineapple

and 10 choose 2 = 45 do have pineapple

and 120 + 45 = 165.

Lovasz 4.7: Prove $\left(\begin{array}{c} n \\ 2 \end{array} \right) + \left(\begin{array}{c} n+1 \\ 2 \end{array} \right) = n^2$ algebraically, then as a counting argument.

Algebraically:

$\left(\begin{array}{c} n \\ 2 \end{array} \right) + \left(\begin{array}{c} n+1 \\ 2 \end{array} \right) =$

$\frac{n!}{2!(n-2)!} + \frac{(n+1)!}{2!((n+1)-2)!} =$

$\frac{(n-1)(n)}{2} + \frac{(n)(n+1)}{2} =$

$\frac{n^2-n}{2} + \frac{n^2+n}{2} = \frac{2n^2}{2} = n^2$

Combinatorially, n^2 is the number of ordered pairs (a, b) where a, b belong to $\{1, \ldots, n\}$.

There are $\left(\begin{array}{c} n \\ 2 \end{array} \right)$ pairs (a, b) with $a > b$.

This leaves the pairs (a, b) with $a \leq b$. These correspond exactly to pairs $(a, b + 1)$ which are all the pairs of distinct increasing numbers from $\{1, \ldots, n+1\}$ and there are $\left(\begin{array}{c} n+1 \\ 2 \end{array} \right)$ of these. I thought that was a bit tricky!

There might be other solutions, and if students come up with others Ill add them here.