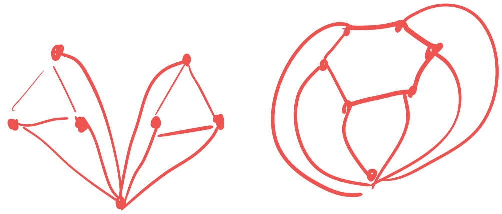
SOLUTIONS FOR GRAPH THEORY PROBLEMS

H11

Levin 4.1.1 sols in the book

4.1.2 sols in book

4.1.3



here I added one vertex and six edges to the class example to "connect" both.

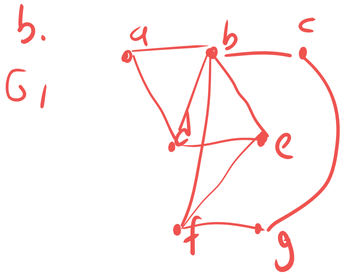
They are not isomorphic - the one on the left does not contain a 6-cycle, for example.

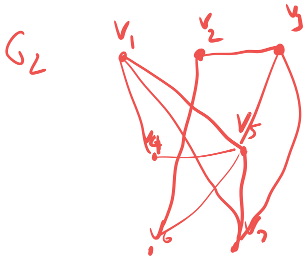
They both have degree sequence 3, 3, 3, 3, 3, 3

4.1.4 solution in book

4.1.5 a. It is not an isomorphism.
$\{a,d\}$ is an edge but $\{v_4, v_6\}$ is not.

b.





I'm solving this as I write, I'll tell you my thinking.

b must map to v_5, they are the only degree 5 vertices in each graph.

b has two degree two neighbors, a and c.
c has another degree 2 neighbor a does not

c must map to v_6, which has degree 2 neighbor v_2

a must map to v_4, the other degree 2 neighbor of v_5

g (the degree 2 neighbor of c) must map to v_2 degree 2 neighbor of v_6

d, the other neighbor of a, must map to v_1, the other neighbor of v_4

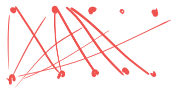
f (the other neighbor of g) must map to v_3 the other neighbor of v_2

by elimination, e maps to v_7

c. It can't be, no vertex of degree 5.

4.1.6 a. $\binom{10}{2}$

b. 25

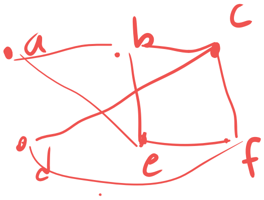


$K_{5,5}$ has $25 = 5 \cdot 5$ edges

c. 9

4.1.9 sols in book

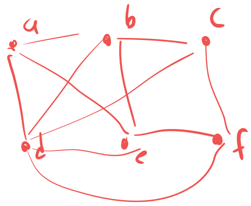
4.1.12



$N(a) = \{b, e\}$
$N[a] = \{a, b, e\}$
$N(c) = \{b, f, d\}$
$N[c] = \{b, f, d, c\}$

$|N(v)|$ is degree of v, largest value is 3
$|N[v]|$ is $d(v)+1$, largest value is 4

c.

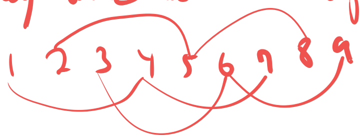


$N[d] = V$

K_6 would have all vertices connected and $N[v] = V$ for all v

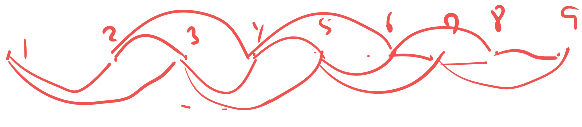
d. $N(v) = \emptyset$ says v is not connected to any other vertex
$N[v] = V$ says v is connected to all other vertices
v can't have two different values one with each of these properties - they would have to be adjacent AND not adjacent

4.1.13. a is a graph.



b. is not a graph - 4 is a multiple of 2 but 2 is not a multiple of 4

c is a graph



4.1.14 a. To not have a vertex with degree 2
there has to be $\frac{n}{2}$ or less edges. $\lfloor \frac{n}{2}+1 \rfloor$ is the exact expression,

$\boxed{\frac{n}{2}+1 \text{ if } n \text{ is even}, \ \frac{n}{2}+\frac{1}{2} \text{ if } n \text{ is odd.}}$

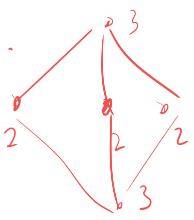
b. If a graph does not have a vertex of degree >2
it must be a union of paths or cycles. One
can keep adding edges, preserving this condition,
until there are n edges, then the next one must create
a vertex of degree 3 $\boxed{n+1}$

4.1.15 no graph proofs on exam.

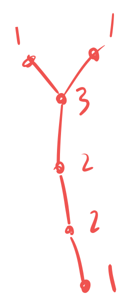
4.1.16 If G has $n-1$ edges and $n>1$ vertices
its total degree is $2n-2$ (sum of all degrees)
if it has no vertex of degree 1, it also has no
degree 0 vertex because it is connected, so total degree
of all vertices must be $\geq 2n > 2n-2$, a contradiction.

LAST HOMEWORK 4.2.3 4.2.13 4.2.15 4.3.1,2,4,5 4.4 3,6,10

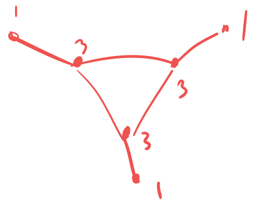
3. 33222



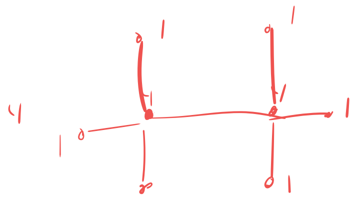
322111



3 3 3 1 1 1



d.



#13 did this in last lecture.

There are 24 spanning trees

You have to cut one of ab, ac, bc

<u>then</u> independent either cut ec then one of the others (4)

or don't cut ec and cut one other one below $(2 \cdot 2 = 4)$

$3 \cdot (4+4) = 24$ possible spanning trees

#15 consider any cycle in the tree

it will have at least 3 edges

cut one of these edges, giving at least 3 different graphs

Now, every cycle in each of these graphs has an edge <u>not</u> in the original cycle. Build spanning trees for each of them, always choosing edges not in the original cycle to cut.

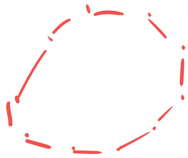
So we get distinct spanning trees from each possible cut of the original cycle.

No graph proofs on this test.

4.3 1, 2, 4, 5

1. solution in book.

2. Solution in book.

4. Yes. a cycle. 

5. If every vertex has degree 6, then ~~the~~ we have $6V = 2E$

so $3V = E$

$V - 3V + F = 2$

$F = 2 + 3V - V = 2 + 2V$ must be even

so No.

4.4 3, 6, 10

3 solution in book

6. Suppose G is a cycle with n vertices. Show by induction on n that G is 2-colorable.

$n=1$ • R color the one vertex red

$n=k+1$ 

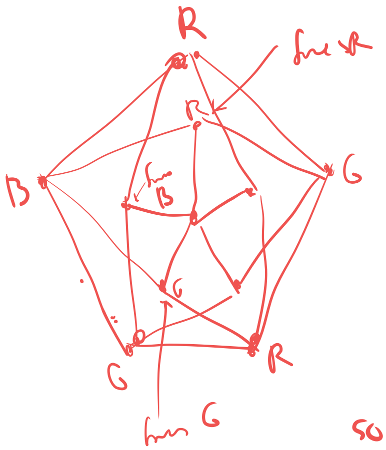
G has a vertex x of degree 1. Let G' be obtained by dropping x and its edge. Color G' with 2 colors.

If xy is the edge with x in it, color vertices in G' with same colors in G, then color x differently from y.

10. When C^n is odd if we try to color with two colors, we assign a color (R or G to one vertex), then the color of the next vertex is always forced, and when the cycle closes up, the forced colors disagree: if you

x is assigned color y and a path from x to y is of even length x, y have same color, if of odd length different and in an odd cycle there are paths of odd and even length from any x to any y.

#10



outer 5-cycle needs at least 3 colors

To color with 3 colors we must force a color in the middle pentagon when two colors appear, non-adjacent the third color is forced in the middle and every pair of the 3 colors occur nonadjacent so the middle vertex is forced to a fourth color.