

Induction Homework (Homework 11) Math 189 Fall 2023

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Because I vagued out about producing this, the due date is one week from today, Wednesday November 1. The extended time is provided not so that you can put it off but so that you have plenty of time to ask questions. Please use lots of paper: these are proofs in English with supporting calculations, they take up space.

1. Prove by mathematical induction that for every $n \geq 0$, $n^3 + 2n$ is divisible by 3.
2. Prove by mathematical induction that the sum of the first n cubes is $\left(\frac{n(n+1)}{2}\right)^2$. Use properties of summation notation that I have talked about.
3. For all large enough numbers, $n^5 < 2^n$. Find the largest n for which this is not true, and then prove by induction that this inequality holds for all larger numbers.
4. Prove by strong induction that every positive integer can be expressed as a sum of distinct Fibonacci numbers. The proof is quite similar to the similar result for powers of 2 which is an exercise in Levin with solution.
5. Do problem 23, section 2.5 in Levin.
6. Do problem 29, section 2.5 in Levin.