### MELVIN RANDALL HOLMES

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Citizenship: U.S.A.

#### Education

University of Mons-Hainaut, Mons, Belgium Postdoctoral Research with Maurice Boffa, 1990-91.

Cornell University, Ithaca, N.Y. Postdoctoral Research with Anil Nerode, 1989-90

State University of New York at Binghamton Ph.D. in Mathematics (1990)

**Advisor:** Louis F. McAuley

**Dissertation:** Systems of Combinatory Logic Related to Quine's "New Foundations"

State University of New York at Binghamton M. A. in Mathematics (1985)

**Advisor:** Prabir Roy

**Thesis:** The Universal Separable Metric Space of Urysohn

## **Employment History**

Boise State University, Boise, Idaho Professor of Mathematics, 2014-2025 (retired in the latter year)

Boise State University, Boise, Idaho Associate Professor of Mathematics, 1997-2014

Hughes Hall, Cambridge, UK Visiting Scholar, fall 1998 (sabbatical appointment).

Boise State University, Boise, Idaho Assistant Professor of Mathematics, 1991-7

Cornell University, Ithaca, N.Y. Teaching Associate, Department of Mathematics 1989-90

State University of New York at Binghamton Adjunct Lecturer, Teaching Assistant, Department of Mathematics 1983-89

# Masters Students Supervised:

- advisor to Morgan Sinclaire: who defended a thesis titled "Formally Verifying Peano Arithmetic" successfully on March 15, 2019.
- advisor to Joanna Guild: who defended a thesis titled "Theorem Proving in Elementary Analysis" successfully on June 7, 2007.
- advisor to Cap Petschulat: who defended a thesis titled "Transparency in Formal Proof" successfully on May 7, 2009.
- on the committee for another Masters student: I was a member of the committee that examined Trevor Jack May 16, 2008.

# **Research Grants**

- "EFTTP: an interactive equational theorem prover and programming language", Army Research Office proposal no. P-33580-MA-DPS (funded by grant no. DAAH04-93-G-0247, starting date July 1, 1994)
- 2. An REU (Research Experience for Undergraduates) award through NSF EPSCoR during summer 1995, supporting a BSU computer science undergraduate who implemented an important subset of the prover in C++.
- 3. (with Jim Alves-Foss of the University of Idaho): "Automated Reasoning using the Mark2 Theorem Prover", Army Research Office proposal no. P-36291-MA-DPS (funded by grant no. DAAH04-96-1-0397, starting date August 1, 1996)
- **4.** "The Mark2 Theorem Prover", Army Research Office proposal number P-37735-MA-DPS (funded by grant no. DAAG55-98-1-0263, starting date May 15, 1998).

- **Invited Research Presentations** I have confined myself to presentations of papers given by invitation. I have been invited this year to speak at a conference on topics related to the Alternative Set Theory of Vopenka in Prague in October.
  - Summer 2022: I was present in Cambridge for a month to assist students working with Dr Thomas Forster on the formal verification of my proof of the consistency of New Foundations, which has since been completed. This isn't precisely an invited address, but this seems as good a category as any.
  - NF76 workshop, Cambridge University, 27 Mar-2 April 2013: This workshop at Cambridge University was organized by Thomas Forster; I was the principal speaker, and spoke for several days, as the primary purpose was for me to outline my claimed solution of the consistency problem for New Foundations.
  - NF 75th Anniversary Workshop, 26-31 March 2012, University of Cambridge: I gave an invited talk titled "On the Canonical Model of a Symmetric Fragment of New Foundations".
  - University of Alberta, Sept 11 2012: I was invited to give a talk on a paper in preparation to the Department of Philosophy at the University of Alberta. I presented "A treatment of (some) functions in three types".
  - University of Cambridge, June 26-27 2010 I was invited to serve as external examiner at the PhD defense of Dang Vu and give a talk to a seminar on New Foundations, titled "Symmetry Motivates a New Consistent Fragment of NF and an Extension of NF with Semantic Motivation".
  - PM 100th Anniversary Workshop, Cambridge, November 27th-28th 2010:

This was held at Trinity College, Cambridge, in honor of the 100th anniversary of the publication of the *Principia Mathematica* of Russell and Whitehead, a historical milestone for mathematical logic. I was surprised and very honored to be invited to this occasion, and gave a presentation titled "Automating the ramified theory of types of Principia", reporting on information found in my publication 20 below, which I believe earned me the invitation.

- NF in the Bay Area, Stanford, June 25-27 2008: Invited to participate, and gave talks on "Symmetry as a criterion for comprehension motivating NF" (item 24 in my pubs list) and on the paper now titled "The usual model construction for NFU preserves information" (item 29).
- NF 70th Anniversary Meeting, 25-27 May 2007, University of Cambridge: I presented the paper "A Forster Term Model of TST" (item 27).
- Workshop on the Urysohn Space, 21-24 May 2006 The invitation to this workshop came as a bolt from the blue. I had not thought much about this topic since submitting the research paper summarizing the results of my Masters thesis in 1992, but apparently it has attracted a fair amount of interest. My presentation titled "The Urysohn Space embeds in Banach Spaces in Just One Way" (item 25) was an exposition based on my research paper from 1992 on this topic (item 2).
- presentation on New Foundations at Idaho State, 2005: Given by invitation from the COLD project there.
- AMS SE section meeting, Atlanta, March 8-10 2002: "Mechanization of the System of the Principia Mathematica of Russell and Whitehead (preliminary report)", invited talk given to a special session on Automated Reasoning.
- Istanbul Bilgi University, Istanbul, May 2002: invited to give talks on NF, "Two talks on an alternative set theory" ("Set Theory with a Universal Set The Quine-Jensen Theory NFU" and "Foundations of Mathematics in Polymorphic Type Theory" (given May 31; one of these was pitched toward students and one toward faculty) and a talk titled "The Watson Theorem Prover" for computer scientists. I was there for a week.
- Free University of Brussels, Belgium Sept. 18-22, 2002: I was invited by the university to serve as an examiner at the Ph.D. defense of Armin Rigo and gave two talks "The double extension set theory of Kiseielewicz is inconsistent" to the Logic Contact Group of the FNRS (Belgian national research organization) and "Foundations of mathematics in polymorphic type theory" to the Belgian Society for Logic and Philosophy.

### List of Publications

- 1. "There is a Continuum which is Connected by Uniformly Short Paths but not Uniformly Path Connected". Topology and Its Applications, 42 (1991) pp. 17-23.
- 2. "The Universal Separable Metric Space of Urysohn and Isometric Embeddings Thereof in Banach Spaces", Fundamenta Mathematicae 140 (1992), pp. 199-223.
- **3.** "Systems of Combinatory Logic Related to Quine's 'New Foundations'", Annals of Pure and Applied Logic, 53 (1991), pp. 103-33.
- 4. "The Axiom of Anti-Foundation in Jensen's 'New Foundations with Ur-Elements'", Bulletin de la Societe Mathematique de la Belgique, series B, 43 (1991), no. 2, pp. 167-79.
- 5. "Modelling Fragments of Quine's 'New Foundations'", Cahiers du Centre de Logique, no. 7, Institut Supérieur de Philosophie, Université Catholique de Louvain, Louvain-la-Neuve, 1992, pp. 97-112.
- 6. "Systems of combinatory logic related to predicative and 'mildly impredicative' fragments of Quine's 'New Foundations'". Annals of Pure and Applied Logic 59 (1993), 45-53.
- 7. The set theoretical program of Quine succeeded (but nobody noticed), *Modern Logic*, vol. 4 (1994), pp. 1-47.
- 8. The equivalence of NF-style set theories with "tangled" type theories; the construction of  $\omega$ -models of predicative NF (and more), Journal of Symbolic Logic, vol. 60, no. 1 (1995), pp. 178-190.
- 9. "Untyped  $\lambda$ -calculus with relative typing", in Dezani and Plotkin, eds., Typed Lambda-Calculi and Applications, the proceedings of TLCA '95, Springer, 1995.
- 10. "Disguising recursively chained rewrite rules as equational theorems", in Hsiang, ed., *Rewriting Techniques and Applications*, the proceedings of RTA '95, Springer, 1995.
- 11. "Brief observations on software architecture and an examination of the type system of Spec", in the proceedings of the Monterey Workshop on software architecture at the Naval Postgraduate School, 1995.

- 12. Elementary Set Theory with a Universal Set, volume 10 of the Cahiers du Centre de logique, Academia, Louvain-la-Neuve (Belgium), 1998. 241 pages. ISBN 2-87209-488-1. This book is an elementary set theory text at the advanced undergraduate or graduate level using NFU.
- **13.** "Subsystems of Quine's "New Foundations" with Predicativity Restrictions", *Notre Dame Journal of Formal Logic*, vol. 40, no. 2 (spring 1999), pp. 183-196.
- 14. "A strong and mechanizable grand logic", in *Theorem Proving* in Higher Order Logics: 13th International Conference, TPHOLs 2000", Lecture Notes in Computer Science, vol. 1869, Springer-Verlag, 2000, pp. 283-300. (refereed conference proceedings).
- **15.** "Strong axioms of infinity in *NFU*", *Journal of Symbolic Logic*, vol. 66, no. 1 (March 2001), pp. 87-116. ("Errata in 'Strong Axioms of Infinity in *NFU*", *JSL*, vol. 66, no. 4 (December 2001), p. 1974, reports some errata and provides corrections).
- **16.** (with Jim Alves-Foss) "The Watson theorem prover", *Journal of Automated Reasoning*, vol. 26 (2001), no. 4, pp. 357-408.
- 17. "Foundations of mathematics in polymorphic type theory", *Topoi*, vol. 20 (2001), pp. 29-52.
- **18.** "Tarski's Theorem and *NFU*", in C. Anthony Anderson and M Zeleny (eds.), *Logic, Meaning and Computation*, Kluwer, 2001, pp. 469–478.
- **19.** "Forcing in *NFU* and *NF*", in M. Crabbé, C. Michaux and F. Point, eds., *A tribute to Maurice Boffa*, Belgian Mathematical Society, 2002. (refereed proceedings of a conference in honor of the 60th birthday of Maurice Boffa).
- **20.** "Polymorphic type-checking for the ramified theory of types of *Principia Mathematica*", in Fairouz Kamareddine, ed., *Thirty-five Years of Automating Mathematics*, Kluwer, 2003, pp. 173-215.
- 21. "Paradoxes in double extension set theories", *Studia Logica*, vol. 77 (2004), pp. 41-57.
- 22. "The structure of the ordinals and the interpretation of ZF in double extension set theory",  $Studia\ Logica$ , vol. 79 (2005), pp.

357-372.

- 23. "Alternative Axiomatic Set Theories", article in the Stanford Encyclopedia of Philosophy (online) at the URL http://plato.stanford.edu. The exact URL for the article is http://plato.stanford.edu/entries/settheory-alternative/. This article was refereed and accepted by the editors for inclusion in 2006. It was reviewed and slightly revised in 2010.
- **24.** "Symmetry as a criterion for comprehension motivating Quine's "New Foundations", *Studia Logica*, vol. 88, no. 2 (March 2008).
- 25. "The Urysohn Space Embeds in Banach Spaces in Just One Way", Topology and its Applications Volume 155, Issue 14, 15 August 2008, Pages 1479-1482 Special Issue: Workshop on the Urysohn space. I was invited to give a talk on my work on the Urysohn space at this workshop, which was held in May 2006 at the University of the Negev in Beersheba. This is not a new research paper, but an exposition of my older results in this area.
- **26.** "Permutation methods in *NF* and *NFU*", with Thomas Forster, in *Proceedings of the 70th anniversary NF meeting in Cambridge*, Cahiers du Centre de Logique no. 16, Academia-Bruylant, Louvain-la-Neuve, 2009.
- 27. "There is a Forster model of simple type theory" in *Proceedings* of the 70th anniversary NF meeting in Cambridge, Cahiers du Centre de Logique no. 16, Academia-Bruylant, Louvain-la-Neuve, 2009.
- 28. M. Randall Holmes, Thomas Forster and Thierry Libert. "Alternative Set Theories", in *Handbook of the History of Logic*, vol. 6, "Sets and Extensions in the Twentieth Century", 2012, Elsevier/North-Holland, Dov Gabbay, Akihiro Kanamori, and John Woods, eds., pp. 559-632.
- **29.** "The usual model construction for *NFU* preserves information". *Notre Dame Journal of Formal Logic*, vol 53, no 4 (2012), 571-580.
- **30.** "The Axiom Scheme of Acyclic Comprehension", with Zuhair al-Johar and Nathan Bowler, *Notre Dame J. Formal Logic* Volume 55, Number 1 (2014), 11-24.

- **31.** M. Randall Holmes, "On hereditarily small sets in ZF", *Mathematical Logic Quarterly* vol 60 issue 3 pp. 228-229.
- **32.** M. Randall Holmes, "Representation of functions and total antisymmetric relations in third-order logic", *Journal of Philosophic Logic*, vol. 48 (2019), pp.263-278.
- **33.** M. Randall Holmes, "Symmetric comprehension revisited", accepted for inclusion in (refereed) College Publications multi-volume collection titled *Research Trends in Contemporary Logic* (edited by Melvin Fitting, Dov Gabbay, Massoud Pourmahdian, Adrian Rezus, and Ali Sadegh Daghighi), to appear.
- **34.** Thomas Forster and M. Randall Holmes, "Synonymy questions concerning the Quine systems", to appear in the *Journal of Symbolic Logic*.

Work in Progress These notes are miscellaneous and of varying levels of importance in my mind.

1. "New Foundations is consistent", paper in preparation, with Sky Wilshaw.

My proof of the consistency of New Foundations has been formally verified under Lean (and so is generally viewed as correct). The verifier, Sky Wilshaw, was a Part III student in mathematics at Cambridge at the time that the formal verification was completed, and is now a PhD student at the University of Nottingham. She was one of the workers in the Summer 2022 effort along these lines mentioned above.

Solving this problem is the capstone of my career: it is what I have been thinking about the entire time (with other parallel trains of thought running on nearby tracks!) I am hoping to submit the paper to the Journal of Symbolic Logic this spring.

- 2. I have produced a theorem prover based on dependent types, similar to Automath, called Lestrade, for which I have papers and extensive sample proofs in preparation. This includes a formalization of Zermelo's proof of the well-ordering theorem, 1908. In connection with this project, I also have an implementation of the old Automath system itself.
- 3. I am planning to write a paper and have been writing testing software on an efficient algorithm for bracket abstraction (implementation of lambda calculus using combinators) which I developed in 2005. I have already communicated with Henk Barendregt, an expert in this field, who seems to think that my approach has some interest.
- 4. I have developed a sequent calculus prover implementing NFU (following a formal presentation by Marcel Crabbé). I used this prover extensively (the logical rather than the set theoretical aspects) in teaching Math 502, the graduate Logic and Set Theory course, and in our second logic and discrete math course. I used it to a lesser extent to reinforce the initial review of formal proof in our introductory real analysis course, and introduced it briefly in our first logic and discrete math course. I directed a graduate

student who wrote a Master's thesis about proof development in elementary real analysis using the prover. My experience suggests that the prover does help at least some students to understand formal proof, both in propositional logic and in the logic of quantifiers. The built in set theory has had some applications for some students, but the fact that it is NFU has never actually been important to a student user. I have a project in mind to develop a formal proof of Specker's theorem that Choice is false in NF under the prover.

- 5. I have a project of implementation of the logic of Frege in a stratified version which avoids paradox, with an associated theorem prover. It is interesting to observe that the underlying type system used in this prover is essentially the same as that of Marcel though the provers appear quite different. These two systems might be covered in a single paper.
- 6. An expanded version of "Forcing in NFU and NF", listed above as appearing in a Festschrift in honor of Maurice Boffa, should at some point be submitted to a refereed journal.
- 7. I project a revision of my book *Elementary set theory with a universal set*: currently it is out of print but I have permission from the publishers to post it on the web. The web version contains various revisions and I am soliciting advice from others about needed corrections. I do not intend to extend this book, but write a second monograph on a higher level; see the next item. I did substantial revisions and corrections to the book in fall 2012, and will continue working on this.
- 8. A book on the consistent subsystems of NF: my current plan is to write this as an extension of the existing notes I used to teach foundations of mathematics to Boise State graduate students starting with type theory rather than set theory. I made some additions to this text during my sabbatical in Fall 2012.
- 9. The paper on the ramified theory of types of *Principia* is accompanied by software implementing a complete polymorphic type checker for the ramified theory of types. I have considered writing a proposal to implement some significant part of *Principia* using

- this system. I was invited in November 2010 to speak at a work-shop commemorating the 100th anniversary of the publication of *Principia Mathematica*, due to my work in this area, and gave a presentation whose slides should soon appear on my web page.
- 10. I have a new paper in preparation (as of 2022, I ought to revisit this) presenting models of NFU related to the very strong rank-into-rank large cardinal axioms.