Math 275, Test IV (selected problems for Fall 2020 review)

Dr. Holmes

December 5, 2020

The test will begin at 8:40 am and end at 9:35 am. A plain scientific calculator with no graphing or symbolic capability may be used. You may have one standard notebook sheet of notes. Youy do not need to give me your note sheet unless you are taking the exam late, in which case I do want it. Cell phones and PDAs must be turned off and out of sight.

Evaluation of any integral you are asked to set up but not asked to evaluate is worth a little extra credit.

1. Set up but do not evaluate an integral representing the surface area of the solid bounded below by the xy plane and above by the paraboloid $x^2 + y^2 = 9$. You may use polar coordinates if you wish (but carefully!)

2. Determine the volume of the tetrahedron bounded by the coordinate planes and the plane $x + y + z = 1$ by setting up **and evaluating** a triple integral.

3. Convert the triple integral of the function x over the upper half (a solid hemisphere) of the sphere $x^2 + y^2 + z^2 = 4$ of radius 2 centered at the origin to an integral in spherical coordinates. You do not need to evaluate it. Remember that the integrated function also needs to be converted.

4. Sketch part of the vector field $y\mathbf{i} - x\mathbf{j}$ by drawing the actual vectors at the nine points with each coordinate equal to 0, 1 or -1. You may draw the field vectors in a smaller scale.

5. Compute

$$\int_C ydx - xdy$$

where C is the unit circle traversed once counterclockwise, starting and ending at (1,0).

6. One of the functions

$$2xy\mathbf{i} + (x^2 + 3y^2)\mathbf{j}$$

and

$$2xy\mathbf{i} - (x^2 + 3y^2)\mathbf{j}$$

is a gradient and one is not. Explain how you can tell which one is a gradient,and find a function of which it is the gradient by a process of partial integration as in the book. Show all work.