Math 275 Test II, Fall 2020

Dr. Holmes

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Please do this exam without consulting anyone else but the instructor. It is due electronically at 11:55 pm on Sunday, 18 Oct, 2020: email it to me. You may work on a printed copy of the test paper or on your own paper.

1. 14.3 partial derivatives

   Let $f(x, y) = xy^4 + x^3y^2$. Compute the first partials of this function (two functions) and the second partials of this function (four functions). Clearly label each first and second partial so that I know that you know which one is which.

   What fact about your second partials is an example of Clairaut's thoerem?

2. 14.4 tangent planes and linearization

   Give the equation for the tangent plane to $z = xy^3$ at $(1, 2, 8)$.

   Give the linear function $L(x, y)$ which best approximates xy^3 when x is close to 1 and y is close to 2. You might notice that this is almost the same question.

   Give the linear approximation to $(1.01)(1.99)^3$ using this linear function.

3. Chain Rule 14.5

   Let $y = u^2 + 2uv + vw^3$ where $u = s + t$, $v = st^2$, $w = s^2 - t^2$

   Write out the form of the Chain Rule needed to compute the partial derivative of y with respect to t.

   Compute the partial derivative of y with respect to t. You may leave u's, v's, and w's in your answer.

4. 14.6 gradient and directional derivative

   Compute the gradient of the function $g(x, y) = x^3y - xy^3$ at the point $(1, 2)$. Compute the directional derivative of this function in the direction of the vector $\langle 5, 12\rangle$.

   In what direction does this function increase most rapidly at $(1, 2)$? What is the rate of change?

5. 14.7 critical points and local extrema

   Find all critical points of the function $f(x,y) = x^3 - y^3 - 12x + 3y$ and classify them as local maxima, local minima, or saddle points using the Second Derivative Test.

6. 14.8

   Use the method of Lagrange multipliers to find the closest point to the origin on the plane $x + 4y + 9z = 13$. Hint: find the critical point for $x^2 + y^2 + z^2$ (the square of the distance from the origin) subject to the constraint given by the equation of the plane. There is only one critical point, and you do not have to verify that it is actually a minimum (this is obvious from geometry).

7. 15.1

   Evaluate

$$\int_0^1 \int_1^2 x + y \, dy \, dx.$$

   Reverse the order of integration and evaluate it again.

8. 15.2

Sketch the region of integration of the iterated integral $\int_0^2 \int_{x^3}^8 x \, dy \, dx$.

Change the order of integration and set up the integral as $\int_{??}^{??} \int_{??}^{??} x \, dx \, dy$.

Evaluate both integrals and check that the same value is obtained