Prop 1.20

$$(-m)(-n) \overset{\text{claim 1}}{=} -((-m)\cdot n)$$

$$\overset{\text{comm} \times}{=} -(n\cdot(-m))$$

$$\overset{\text{claim 1}}{=} -(-(n\cdot m))$$

$$\overset{--x=x \text{ proved earlier}}{=} n\cdot m$$

$$\overset{\text{comm} \times}{=} m\cdot n$$

Prop. 1.23 [motivation of subtraction]

Given $m, n \in \mathbb{Z}$ there is exactly one x such that $m+x=n$.

Proof:

There is such an x: Let $x = n+(-m)$

then $m+x = m+(n+-m) = m+(-m+n)$

$= (m+-m)+n = 0+n = n$

There is only one such x:

if $m+x=n$ and $m+y=n$

then $m+x=m+y$ (draws =) and

$x=y$ (by prop 1.4)

So... if $m+x=n$

then $m+x=n$ and $m+(n+(-m))=n$

so $x=n+(-m)$.

$n+(-m)$

Definition:

$n-m$ is defined as $n+(-m)$.