Math 387 Homework 1

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1. Seven teams send their baseball teams to a tournament in which each time must play each other exactly once. Find the answer in two different ways (same thing I did in problem 1).

$6+5+4+3+2+1 = \frac{7 \cdot 6}{2} = 21$

2. You can have a hamburger, a fishburger or a turkey burger. You can have a plain bun or a sesame seed bun. There are four toppings: bacon bits, onions, ketchup, and mustard. You may have two toppings, which must be different.

   How many kinds of burger are possible?

$3 \cdot 2 \cdot \frac{4 \cdot 3}{2} = 72$, 36

there is an argument for $3 \cdot 2 \cdot 4 \cdot 4$

if we count just one topping (or no toppings)

3. Explain why the ordered pair (a, b) cannot simply be defined as the two element set $\{a, b\}$.

$(1,2) \neq (2,1)$, but $\{1,2\} = \{2,1\}$

4. $A = \{1, 2, 3, 4\}$ and $B = \{5, 6, 7\}$.

(a) How many functions from A to B are there?

(b) How many functions from B to A are there?

(c) How many functions from A to B are injections (one-to-one)?

(d) How many functions from B to A are injections (one-to-one)?

(e) How many functions from A to B are surjective (onto)? This may be quite hard, depending on how you approach it.

(f) How many functions from B to A are surjective (onto)?

a. 3^4

b. 4^3

c. none

d. $4 \cdot 3 \cdot 2 = 24$

e. Have one of 5, 6, 7 to be mapped to two vals: $3 \cdot \binom{4}{2} \cdot 2 = 36$ pick two elts of A that are mapped to where the other two elts go

f. none

5. If six flavors of ice cream are available, and you are making a triple cone,

   (a) and the order in which the scoops are placed makes a difference, how many cones are possible?

   (b) If order matters and the flavors all have to be different, how many cones are possible?

   (c) If order does not matter and the flavors all have to be different, how many cones are possible?

   (d) If order doesnt matter and flavors can be repeated, how many cones are possible?

a. 6^3

b. $6 \cdot 5 \cdot 4 = 120$

c. $\frac{6 \cdot 5 \cdot 4}{3!} = 20$

d. $\binom{8}{3} = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} = 56$

6. How many partitions are there of the set $\{1, 2, 3, 4, 5, \}$? We worked this out for a four element set in class.

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