Math 189, Fall 2024, Homework 6

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Because I was slow posting this, I am making it due next Friday.

1. You want to buy a dozen roses for your significant other. There are pink white, yellow and purple roses, and the very rare genetically engineered blue rose.

   How many bouquets are possible?

   How many are possible if you insist on including one rose of each color?

   How many are there which contain no more than three of any color (hint: this is an inclusion-exclusion question).

   How many are there which contain no more than two blue roses? (genetic engineering is expensive)?

2. Consider functions from $\{1,2,3,4,5,6\}$ to $\{1,2,3\}$?

   How many of these functions are strictly increasing?

   How many of them are nondecreasing (this is a stars and bars question)?

   How many of them are surjections (this is an inclusion-exclusion question)?

3. Baskin Robbins has 31 flavors of ice cream and you are going to buy a triple scoop ice cream cone.

   How many cones are possible if...

   (a) you won't use the same flavor more than once.

   (b) you won't use the same flavor more than once, and in addition you care what order the scoops are stacked in.

(c) you will go for repeated flavors. 5456 $\binom{33}{3}$

(d) you will go for repeated flavors but in addition you care what order the scoops are stacked in. 29791 31^3

1000
−500
−333
−200
−90
+166
+100
+45
+66
+30
+18

−6
−9
−15
−33
+3
=212

4. How many of the numbers from 1 to 1000 are divisible by at least one of the numbers 2, 3, 5, and 11? This is an inclusion-exclusion question.

5. I amend the combination lock question I did in class so that there are six buttons on the lock: a combination is made up of three actions, each action being pressing a single button or pressing two buttons at the same time. A combination may not use a number more than once. How many possible combinations are there (this is an elaborate problem combining combinations and the multiplicative principle: no stars and bars or inclusion-exclusion involved).

cashews

111 $6 \cdot 5 \cdot 4$ +

112 ×3 $6 \cdot 5 \binom{4}{2} \cdot 3$ +

122 ×3 $6 \binom{5}{2} \binom{1}{1} 3$ +

222 $\binom{6}{2}\binom{4}{2}\binom{2}{2}$

$= 1290$