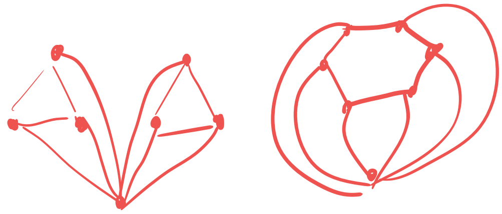
SOLUTIONS FOR GRAPH THEORY PROBLEMS

H11

Levin 4.1.1 sols in the book

4.1.2 sols in book

4.1.3

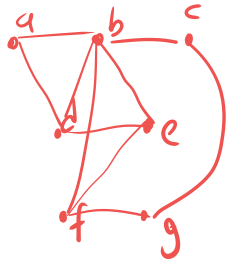


here I added one vertex and six edges to the class example to "connect" both.
They are not isomorphic - the one on the left does not contain a 6-cycle, for example.
They both have degree sequence 3, 3, 3, 3, 3, 3

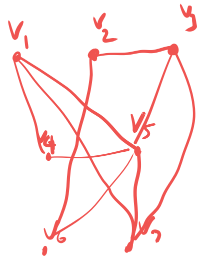
4.1.4 solution in book

4.1.5 a. It is not an isomorphism.
$\{a,d\}$ is an edge but $\{v_4, v_6\}$ is not.

b.
G_1



G_2



I'm solving this as I write, I'll tell you my thinking.

b must map to v_5, they are the only degree 5 vertices in each graph.

b has two degree two neighbors, a and c.
c has another degree 2 neighbor, a does not

c must map to v_6, which has degree 2 neighbor v_2

a must map to v_4, the other degree 2 neighbor of v_5

g (the degree 2 neighbor of c) must map to v_2, degree 2 neighbor of v_6

d, the other neighbor of a, must map to v_1, the other neighbor of v_4

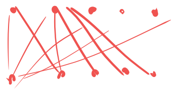
f (the other neighbor of g) must map to v_3, the other neighbor of v_2

by elimination, e maps to v_7

c. It can't be, no vertex of degree 5.

4.1.6 a. $\binom{10}{2}$

b. 25

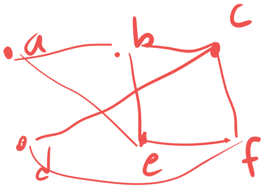


$K_{5,5}$ has $25 = 5 \cdot 5$ edges

c. 9

4.1.9 sols in book

4.1.12



$N(a) = \{b, e\}$

$N[a] = \{a, b, e\}$

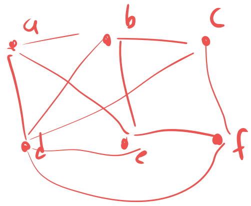
$N(c) = \{b, f, d\}$

$N[c] = \{b, f, d, c\}$

$|N(v)|$ is degree of v, largest value is 3

$|N[v]|$ is $d(v)+1$, largest value is 4

c.



$N[d] = V$

K_6 would have all vertices connected and $N[v] = V$ for all v

d. $N(v) = \emptyset$ says v we are not connected to any other vertex

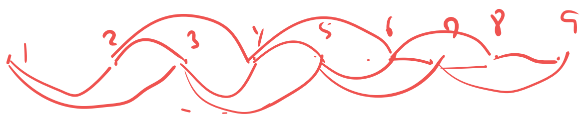
$N[v] = V$ says we are connected to all other vertices

v can't have two different vertices one with each of these properties - they would have to be adjacent AND not adjacent

4.1.13. a is a graph.

b. is not a graph - 4 is a multiple of 2 but 2 is not a multiple of 4

c is a graph



4.1.14 a. To not have a vertex with degree 2 there have to be $\frac{n}{2}$ or less edges. $\lfloor \frac{n}{2}+1 \rfloor$ is the exact expression,

$\boxed{\frac{n}{2}+1 \text{ if } n \text{ is even, } \frac{n}{2}+\frac{1}{2} \text{ if } n \text{ is odd.}}$

b. If a graph does not have a vertex of degree >2 it must be a union of paths or cycles. One can keep adding edges, preserving this condition, until there are n edges, then the next one must create a vertex of degree 3 $\boxed{n+1}$

4.1.15 no graph proofs on exam.

4.1.16 If G has $n-1$ edges and $n>1$ then its total degree is $2n-2$ (sum of all degrees). If it has no vertex of degree 1, it also has no degree 0 vertex because it is connected, so total degree of all vertices must be $\geq 2n > 2n-2$, a contradiction.