70092 ExerciseTypes.CW1

Logic Coursework

Submitters

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Logic coursework 1

elements: $\forall \exists \rightarrow \neg \wedge \vee \neq X_1 \equiv$

a.

i) $\exists X \forall E(\mathrm{comp}(X) \wedge \mathrm{event}(E) \wedge \mathrm{enter}(X, E) \rightarrow \mathrm{won}(X, E))$ ✓ 2/2

ii) $\forall E(\mathrm{event}(E) \rightarrow \exists X(\mathrm{comp}(X) \wedge \mathrm{enter}(X, E) \wedge \mathrm{won}(X, E))) \wedge \exists E, X_1, X_2(\mathrm{event}(E) \wedge \mathrm{comp}(X_1) \wedge \mathrm{comp}(X_2) \wedge \mathrm{enter}(X_1, E) \wedge \mathrm{enter}(X_2, E) \wedge \mathrm{won}(X_1, E) \wedge \mathrm{won}(X_2, E) \wedge X_1 \neq X_2)$ ✓ 3/3

You want e.g. $\forall x(\mathrm{won}(X,E_1) \Leftrightarrow \mathrm{won}(X,E_2))$, as there can be more winners

iii) $\exists X, E_1, E_2(\mathrm{comp}(X) \wedge \mathrm{event}(E_1) \wedge \mathrm{event}(E_2) \wedge \mathrm{enter}(X, E_1) \wedge \mathrm{enter}(X, E_2) \wedge \mathrm{won}(X, E_1) \wedge \mathrm{won}(X, E_2) \wedge E_1 \neq E_2)$ ... This says there exists a competitor who won at least two different events 1.5/3

iv) $\forall X(\exists E_1, E_2(\mathrm{comp}(X) \wedge \mathrm{event}(E_1) \wedge \mathrm{event}(E_2) \wedge \mathrm{enter}(X, E_1) \wedge \mathrm{enter}(X, E_2) \wedge \mathrm{won}(X, E_1) \wedge \mathrm{won}(X, E_2) \wedge E_1 \neq E_2) \rightarrow \forall P(\mathrm{press}(P) \rightarrow \mathrm{invited}(X, P))$ ✓ 3/3

v) $\forall \cancel{\neg}\exists X(\neg \exists E(\mathrm{comp}(X) \wedge \mathrm{event}(E) \wedge \mathrm{enter}(X, E) \wedge \mathrm{won}(X, E)) \vee \cancel{\forall X}(\exists E_1, E_2(\mathrm{event}(E_1) \wedge \mathrm{event}(E_2) \wedge \mathrm{comp}(X) \wedge \mathrm{enter}(X, E_1) \wedge \mathrm{enter}(X, E_2) \wedge \mathrm{won}(X, E_1) \wedge \mathrm{won}(X, E_2) \rightarrow E_1 = E_2)) \rightarrow (\neg \exists P, \cancel{X}(\mathrm{press}(P) \wedge \mathrm{invited}(X, P)) \,)\,)$ 1/4

You can't have multiple quantifiers for the same variable

missing 2 closing brackets, I'm guessing they go here

b.

Conclusion: (i), (iii), (iv), (v) are correct. ✓ 2.5/2.5

To show that (i) and (iv) are equivalent, we only need to demonstrate that $\neg \forall X \forall E(\neg \mathrm{inv}(X, P) \vee \neg \mathrm{ev}(E) \vee \neg \mathrm{won}(X, E))$ *is the same as* $\exists X \exists E(\mathrm{inv}(X, E) \wedge \mathrm{ev}(E) \wedge \mathrm{won}(X, E))$. According to Equivalences in predicate logic, $\neg \forall X p(X) \equiv \exists X \neg p(X)$, then $\neg \forall X \forall E(\neg \mathrm{inv}(X, P) \vee \neg \mathrm{ev}(E) \vee \neg \mathrm{won}(X, E)) \equiv \exists X \exists E(\neg(\neg \mathrm{inv}(X, P) \vee \neg \mathrm{ev}(E) \vee \neg \mathrm{won}(X, E)))$. According to De Morgan Rule and Double Negation Rule, $\exists X \exists E(\neg(\neg \mathrm{inv}(X, P) \vee \neg \mathrm{ev}(E) \vee \neg \mathrm{won}(X, E))) \equiv \exists X \exists E(\mathrm{inv}(X, P) \wedge \mathrm{ev}(E) \wedge \mathrm{won}(X, E))$. So, (i) and (iv) are equivalent. ✓ 2.5/2.5

15.5/20