MATH 101 Midterm Exam 1

Date: 07 February, 2024 Duration: 45 minutes

- The test consists of 8 pages and 3 questions, worth a total of 18 marks.
- This is a closed-book examination. **None of the following are allowed**: documents, formula sheets, electronic devices of any kind (including calculators, cell phones, etc.).
- No work on this page will be marked.
- Fill in the information below before turning to the questions.

Student number								
Section								
Name								
Signature								

Rules governing UBC examinations:

1. Each candidate must be prepared to produce, upon request, a UBC card for identification.
2. No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
3. Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action:
   (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners;
   (b) Speaking or communicating with other candidates;
   (c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
4. Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
5. Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

Additional rules governing this examination:

1. This is a closed-book exam.
   (a) Calculators and other calculating devices may not be used.
   (b) Notes may not be used.
   (c) Watches must be removed and taken off the table.
   (d) Phones must be turned off and stored in an inaccessible location (like inside a backpack).
2. If an answer box is provided, you must write down your answer (but not its justification) in the box.
3. Simplification is an important skill to demonstrate. Answers must be simplified and calculator-ready. For example, write $\log\left(e^{\sqrt{2}}\right) = \sqrt{2}$, but do not write $\sqrt{2} \approx 1.414$. Any answer of the form "trig (arctrig)" (for example: $\sin(\arctan x)$) is incomplete and will not receive full marks.
4. You must justify your answers unless an explicit exception is made.
5. You may use any result proven in class or on assignments.
6. You may not discuss this exam with anyone who has not yet taken their version of it.

1. (6 points) ★★☆☆ Find

$$\int \sin^2(7x)\cos^3(7x)\,\mathrm{d}x$$

Use this page to continue your work on Question 1.

2. (6 points) ★★★☆ Evaluate the following integral:

$$\int e^{2x}\sqrt{1-e^{4x}}\,\mathrm{d}x.$$

Use this page to continue your work on Question 2.

3. (6 points) ★★★☆ We are shopping for cylindrical columns of length 25 cm to use in a building project. One supplier offers columns whose linear density at distance x from one end is given by

$$\rho(x) = 2\log(35 - x) \text{ kg/cm} \quad \text{for } 0 \le x \le 25.$$

We know that the total mass of such a column is given by

$$M = \int_0^{25} \rho(x)\, dx.$$

Our project has an upper limit on the total weight, so we need to make sure that the exact value of M is not too big. Let's calculate an approximate total mass M_{app} such that $M \le M_{\text{app}}$: if M_{app} is low enough, we can buy from this supplier. For reasonable accuracy with reasonable effort, our approximation can use the numbers $\rho(0), \rho(5), \rho(10), \ldots, \rho(25)$.

(a) Which of the approximation methods below will make $M \le M_{\text{app}}$? Circle one. Explain your answer.

(i) a left-endpoint Riemann sum, (ii) a right-endpoint Riemann sum, (iii) the Trapezoidal Rule.

(b) Write out a calculator-ready form of the expression you would evaluate to find M_{app}. (An explicit sum is acceptable; simplification is not expected; your answer must not include ρ.)

Use this page to continue your work on Question 3.