MATH 101 Midterm Exam 2

Date: Monday, March 18, 2024 Duration: 45 minutes

- The test consists of 8 pages and 3 questions, worth a total of 18 marks.
- This is a closed-book examination. **None of the following are allowed**: documents, formula sheets, electronic devices of any kind (including calculators, cell phones, etc.).
- No work on this page will be marked.
- Fill in the information below before turning to the questions.

Student number								
Section								
Name								
Signature								

Rules governing UBC examinations:

1. Each candidate must be prepared to produce, upon request, a UBC card for identification.
2. No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
3. Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action:
   (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners;
   (b) Speaking or communicating with other candidates;
   (c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
4. Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
5. Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

Additional rules governing this examination:

1. This is a closed-book exam.
   (a) Calculators and other calculating devices may not be used.
   (b) Notes may not be used.
   (c) Watches must be removed and taken off the table.
   (d) Phones must be turned off and stored in an inaccessible location (like inside a backpack).
2. If an answer box is provided, you must write down your answer (but not its justification) in the box.
3. Simplification is an important skill to demonstrate. Answers must be simplified and calculator-ready. For example, write $\log\left(e^{\sqrt{2}}\right) = \sqrt{2}$, but do not write $\sqrt{2} \approx 1.414$. Any answer of the form "trig (arctrig)" (for example: $\sin(\arctan x)$) is incomplete and will not receive full marks.
4. You are expected to use proper notation throughout.
5. The evaluation of convergent improper integrals without proper limits will result in deduction of points. Correct use of limit notation in an improper integral demonstrates your understanding of the relevant definitions, and that you have recognized the integral as improper.
6. You must justify your answers unless an explicit exception is made.
7. You may use any result proven in class or on assignments.
8. You may not discuss this exam with anyone who has not yet taken their version of it.

1. (6 points) ★★☆☆ Evaluate $\int \frac{\log(25x^2 + 9)}{x^2}\,\mathrm{d}x$.

Use this page to continue your work on Question 1.

2. (6 points) ★★★☆ Let $f(x)$ be a differentiable function such that $f(x) > 0$ for all x, $f(1) = e^3$, and

$$\int_1^\infty \frac{f'(x)}{f(x)}\,dx = 6.$$

Compute the limit:

$$\lim_{x\to\infty} f(x).$$

Use this page to continue your work on Question 2.

3. (6 points) ★★★☆ An account has an initial balance of \$2,000. After t years of continuously compounded interest, with interest rate r, the account balance will be

$$2000 \cdot e^{rt}$$

dollars.

You want to know which value of r will result in a balance of \$3,000 after one year. You would like a **rational** number approximating r, with an error of no more than $\left(\frac{1}{12}\right)^3$.

(a) Find r exactly.

(b) Give a definite integral to use to approximate r with a rational number.

(c) How many intervals will you need if you use the trapezoid rule? (Remember to completely justify all your answers.)

For reference, the error bounds for numerical approximations of $\int_a^b f(x)\,\mathrm{d}x$ are given below:

Using the Trapezoidal method with n intervals,

$$\left|\text{actual} - \text{approximate}\right| \le \frac{M}{12}\frac{(b-a)^3}{n^2}$$

where $\left|f''(x)\right| \le M$ for all x in $[a,b]$.

Using Simpson's rule with n intervals,

$$\left|\text{actual} - \text{approximate}\right| \le \frac{L}{180}\frac{(b-a)^5}{n^4}$$

where $\left|f^{(4)}(x)\right| \le L$ for all x in $[a,b]$.