The perceptron

Types of learning

1. Learning with a teacher: supervised learning.
2. Learning with a critic: reinforcement learning.
3. Learning on your own: unsupervised learning.

- How we study learning in neural networks:

The perceptron (Rosenblatt 1957)

Notation:

- x_i: activity of input unit i (binary or [0,1]).
- w_i synaptic weight from unit i.
- $z = \sum_i w_i x_i$ total input to the output unit.
- y: activity of output unit.
- t: desired output (of use later). (μ superscript denotes training sample.)
- $f(\cdot)$: transfer function; $y = f(z)$.

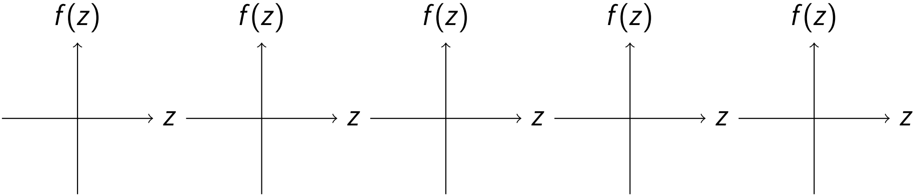
Training set

Sample	x_1	x_2	t
$\mu = 1$	0	0	0
$\mu = 2$	0	1	0
$\mu = 3$	1	0	0
$\mu = 4$	1	1	1

Transfer function

Given total weighted input to neuron, what is its output?

1. identity: $f(z) = z$.
2. threshold: $f(z, \theta) = \begin{cases} 1 & \text{if } z \geq \theta \\ 0 & \text{otherwise} \end{cases}$
3. sigmoidal: $f(z) = \frac{1}{1+\exp(-kz)}$
4. tanh: $f(z) = tanh(z)$
5. rectified linear unit (ReLU): $f(z) = max(0, z)$



How to choose? One key property: differentiable.

Perceptron decisions

Whether a perceptrons output is 0 or 1 depends on whether $\sum_{i=1}^{N} w_i x_i$ is less or greater than θ.

The equation

$$\sum_{i=1}^{N} w_i x_i = \theta$$

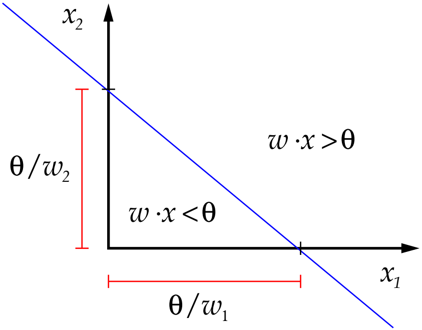
defines a hyperplane in N-dimensional space.

This hyperplane cuts the space in two.

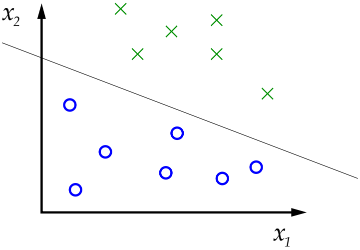
$$w_1 x_1 + w_2 x_2 = \theta$$

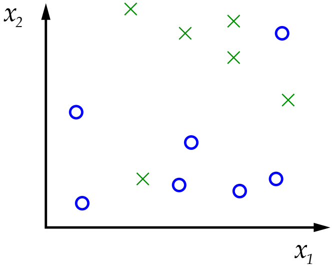
$$x_2 = \left(\frac{-w_1}{w_2}\right) x_1 + \frac{\theta}{w_2}$$

This is an equation for a straight line.



Linearly separable problems and the perceptron

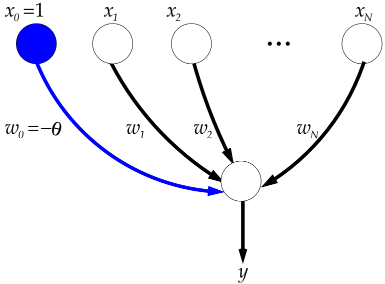




Learning involves adjusting the values of w and θ so that the decision plane can correctly divide the two classes.

Threshold & Bias

The threshold, θ, can be treated as just another weight from a new input unit which always has a value of +1 (or -1). This new input unit is called the **bias** unit.

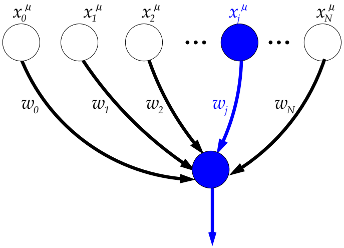


Instead of comparing $\sum_{i=1}^{N} w_i x_i$ with θ, we compare $\sum_{i=0}^{N} w_i x_i$ with 0, or

$$y = \begin{cases} 1 & \text{if } \sum_{i=0}^{N} w_i x_i > 0, \\ 0 & \text{if } \sum_{i=0}^{N} w_i x_i \leqslant 0. \end{cases}$$

Now, learning is about jiggling **weights** only.

Intuitively... (positive valued inputs)



$$y^\mu = \text{step}\left(\sum_{i=0}^{N} w_i x_i^\mu\right)$$

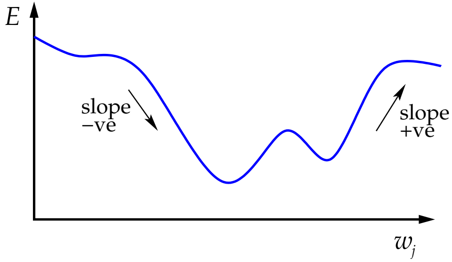
Consider w_j's contribution to $\sum_i w_i x_i^\mu$ in different cases:

1. $y^\mu = t^\mu$ Perceptron has classified input μ correctly – change nothing
2. $x_j^\mu = 0$ Changing w_j will not affect the $\sum_i w_i x_i^\mu$ – change nothing
3. $x_j^\mu \neq 0, y^\mu < t^\mu$ The sum $\sum_i w_i x_i^\mu$ is too low – so increase it
4. $x_j^\mu \neq 0, y^\mu > t^\mu$ The sum $\sum_i w_i x_i^\mu$ is too high – so decrease it

The local rule:

$$\boxed{\Delta w_j \propto (t^\mu - y^\mu)\, x_j^\mu}$$

Perceptron learning rule



$$y = f(\mathbf{w} \cdot \mathbf{x}) \qquad \text{e.g.} \quad f(z) = 1/(1+\exp(-z)), \quad f(z) = z$$

$$E = \frac{1}{2}(t-y)^2 \qquad t \text{ is target output}$$

$$\Delta w_j = -\epsilon \frac{\partial E}{\partial w_j} = -\epsilon \frac{\partial E}{\partial y} \frac{\partial y}{\partial w_j} =$$

This is the method of gradient descent with learning-rate parameter ϵ.

Example of a perceptron learning

Perceptron pros and cons

- The *perceptron convergence theorem* (Dayan and Abbott, page 327) guarantees that solution will be found iff there is a linearly separable solution.
- Many complex problems are not linearly separable, e.g. XOR problem.

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	0