Q1)

a)

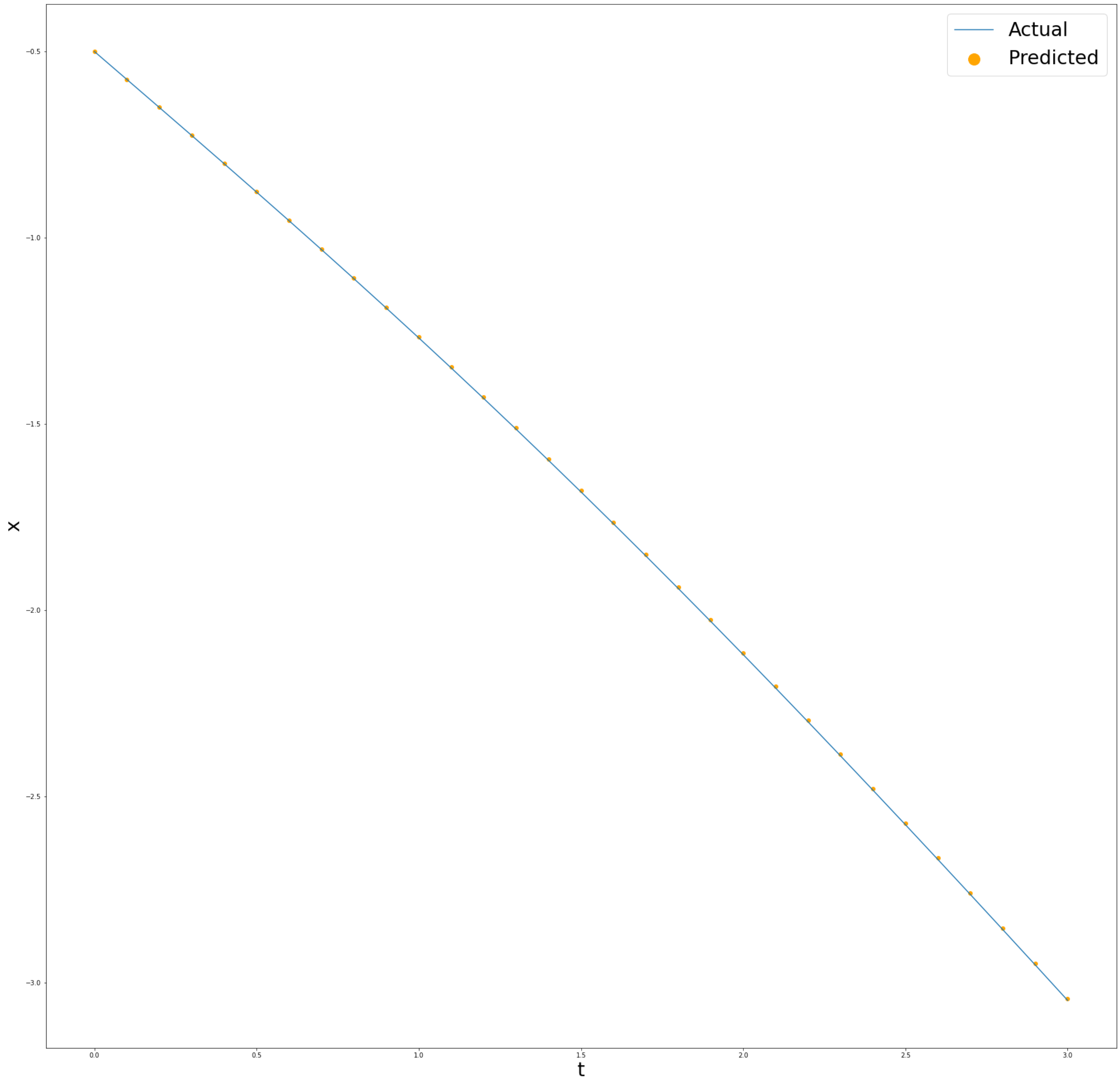
Graph obtained by plotting the actual curve vs the points predicted by Euler's first order scheme.

Time, $0 \leq t \leq 3$

$x(0) = -1/2$

$$x(t) = -t - \frac{1}{e^t + 1}$$

taking time-steps, h = 0.01



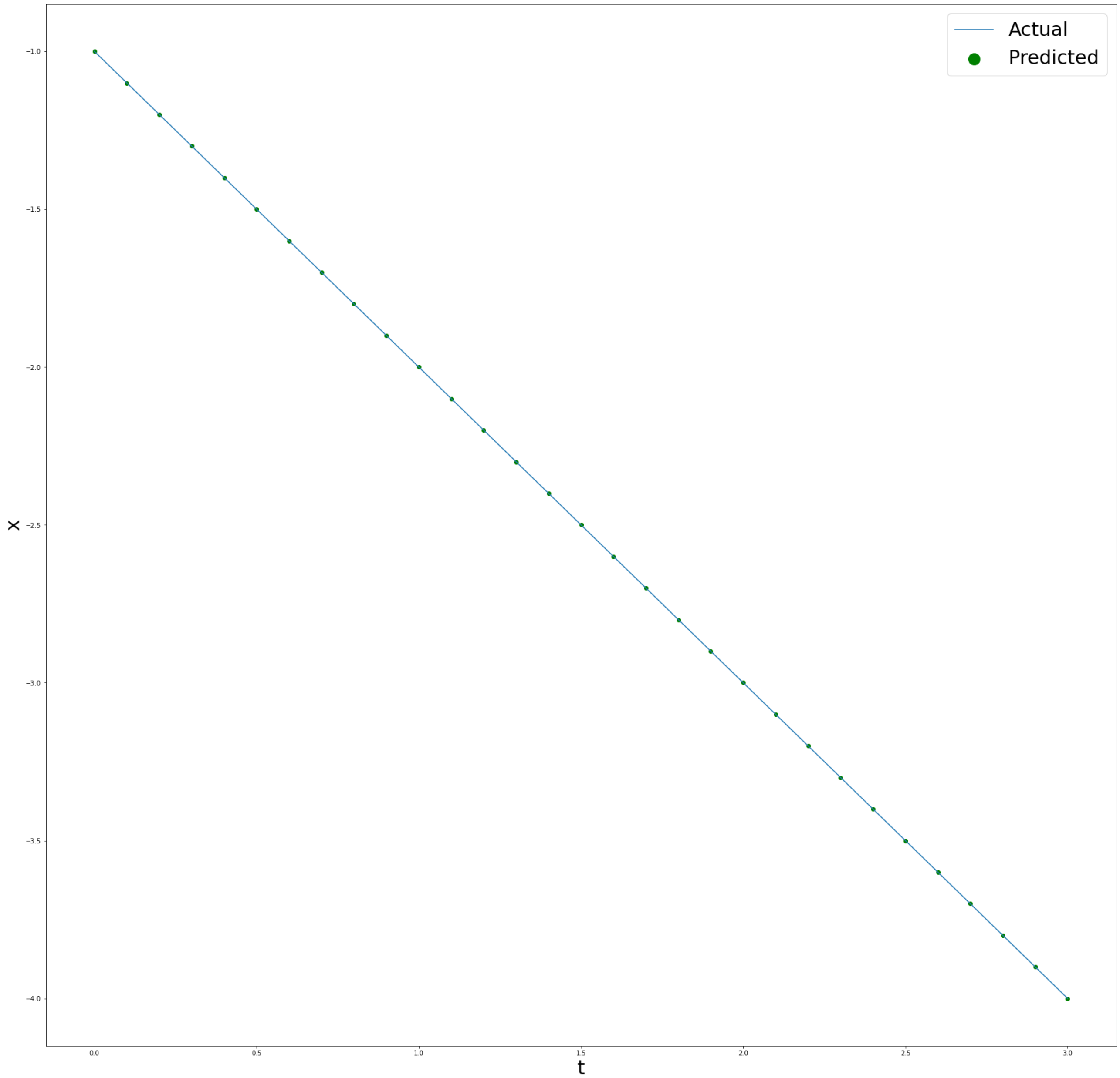
b)

Graph obtained by plotting the actual curve vs the points predicted by Euler's first order scheme.

Time, $0 \leq t \leq 3$

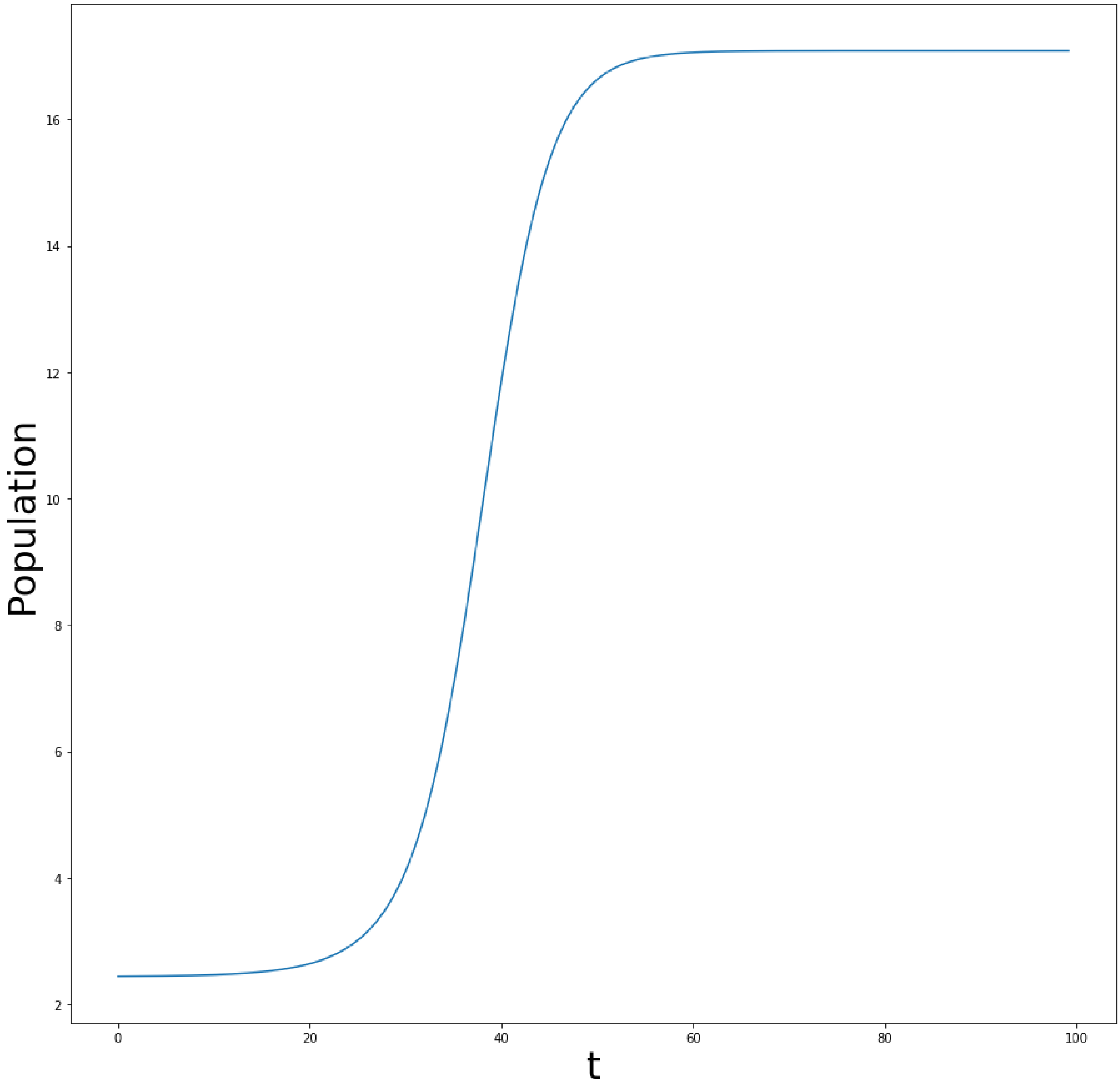
$x(0) = -1$

$x(t) = -t - 1$
taking time-steps, h = 0.01



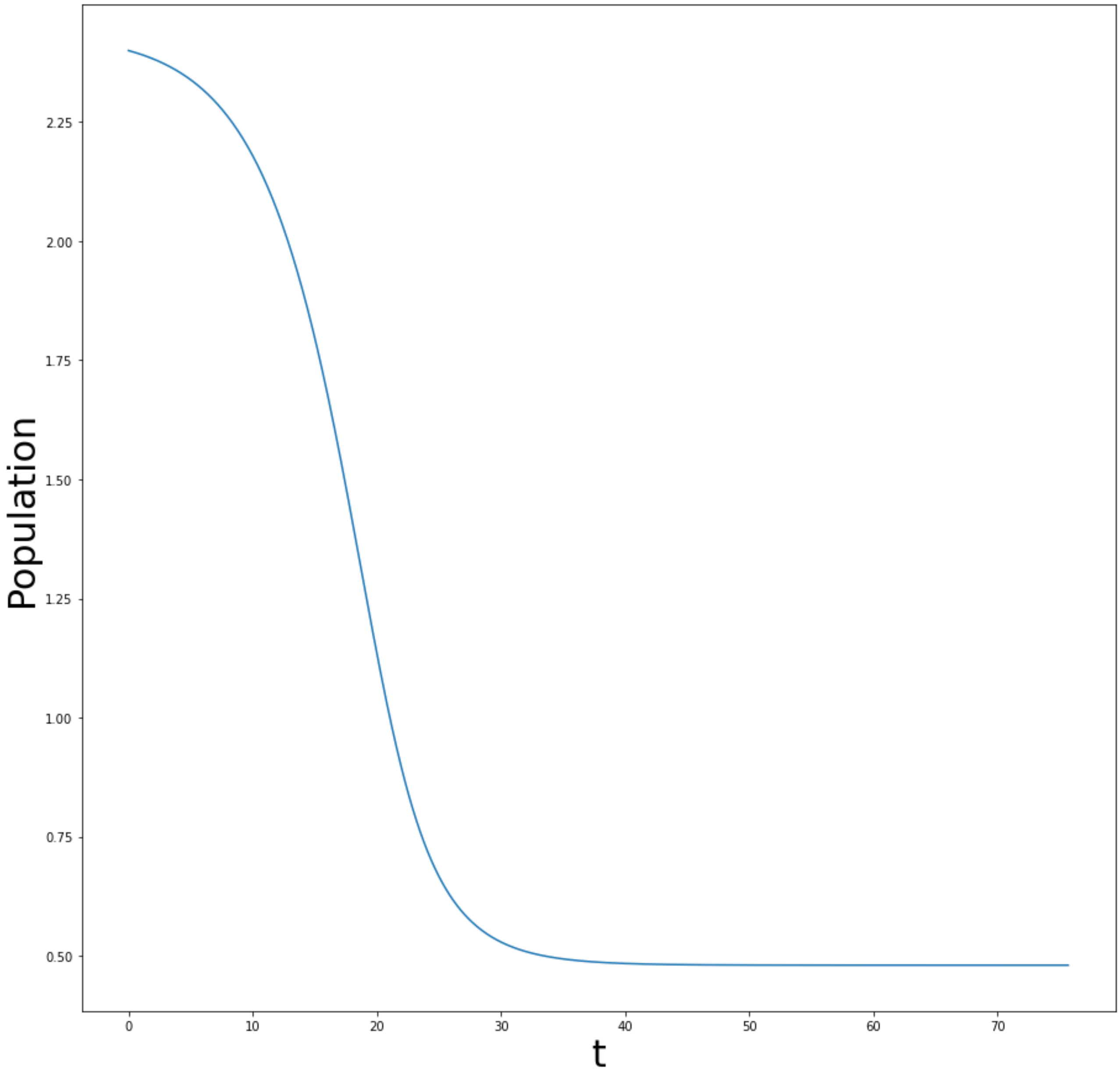
2)

For initial population = 2.44
Graph obtained by plotting the points predicted by Euler's first order scheme.



The population increases and eventually converges to 17.0831307

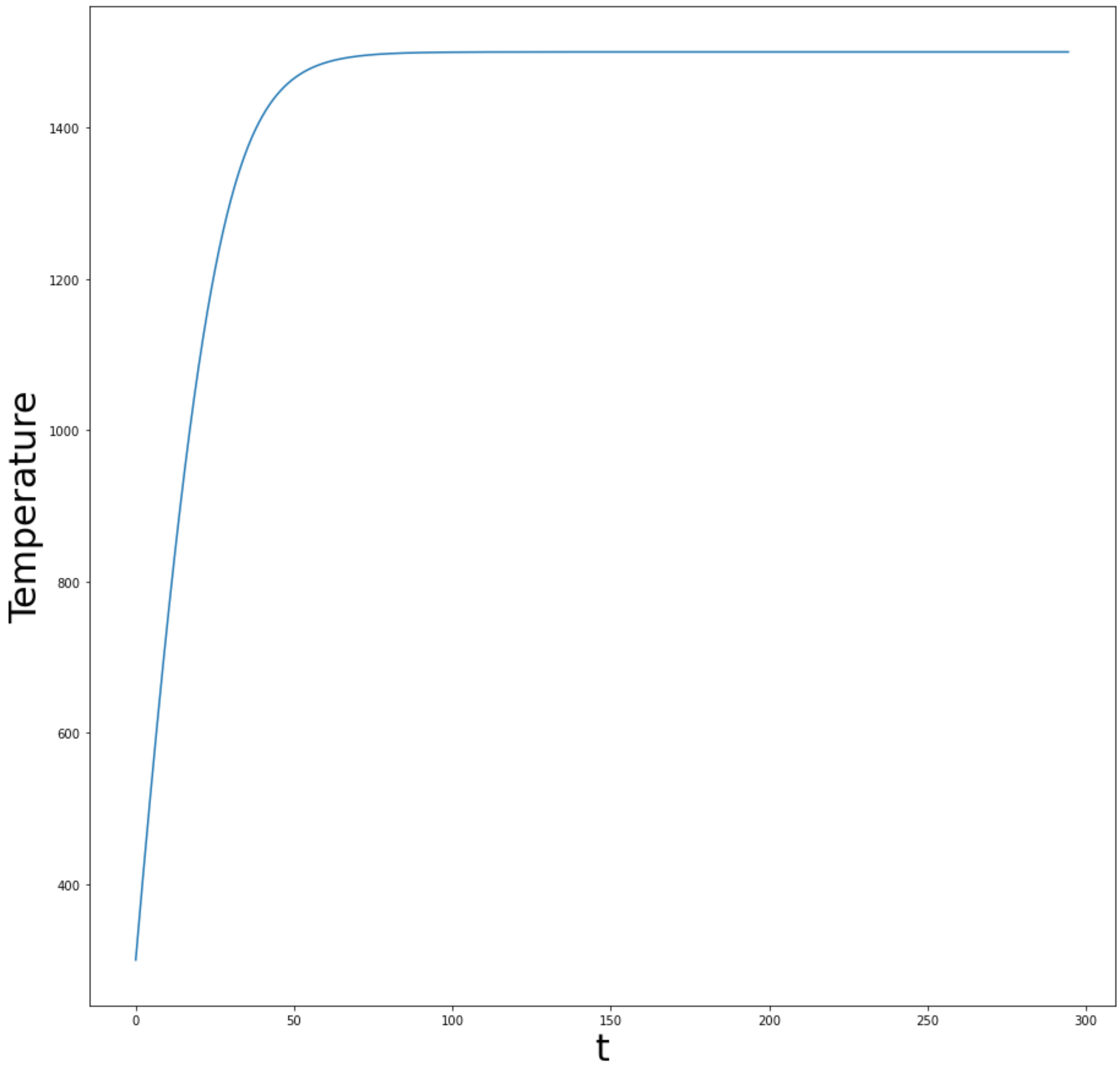
For initial population = 2.40
Graph obtained by plotting the points predicted by Euler's first order scheme.



The population decreases and eventually converges to 0.480536317.

3)

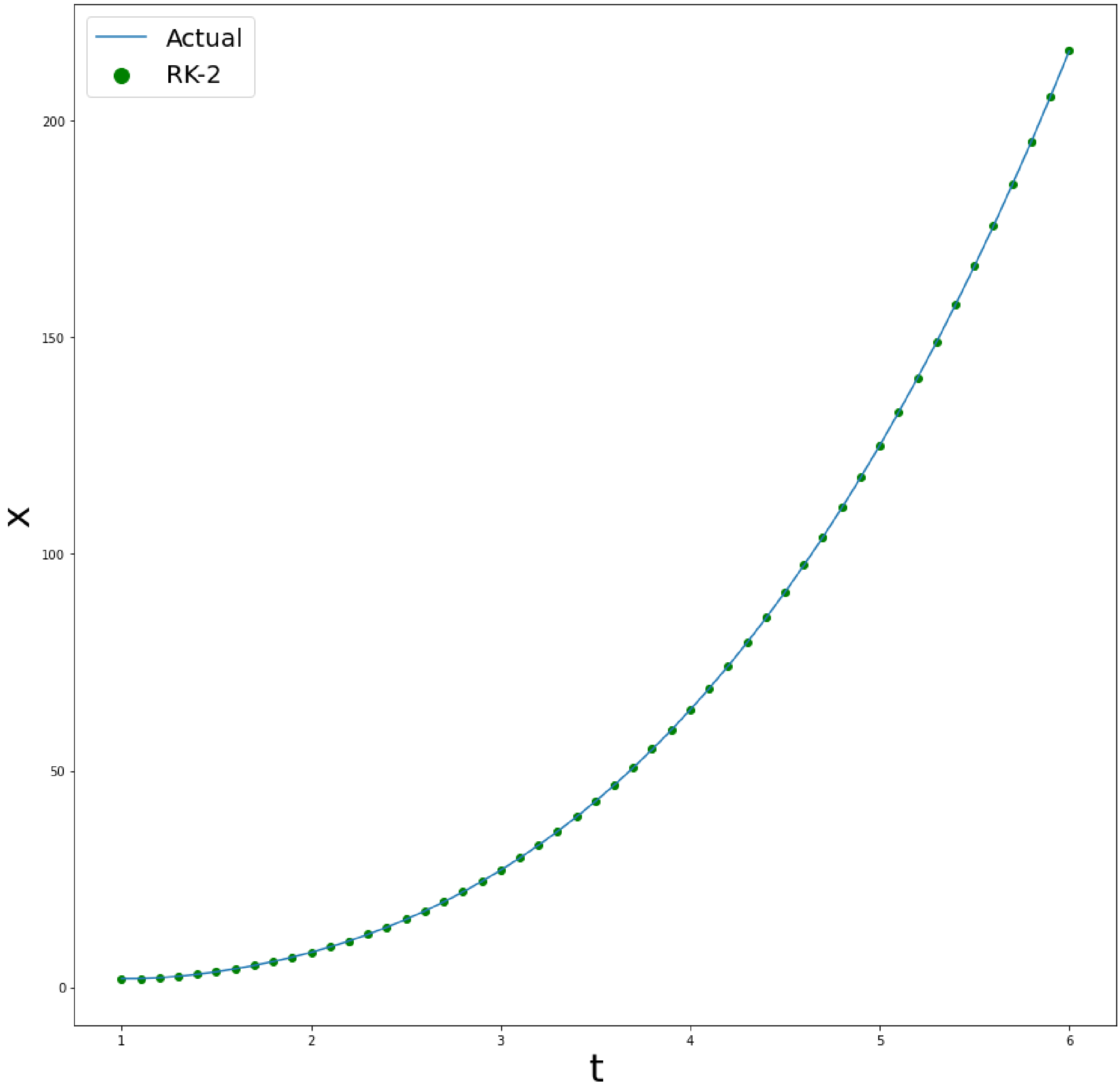
For initial temperature = 0 K
Graph obtained by plotting the points predicted by RK - 4 scheme.



Time taken, t = 294.4 s

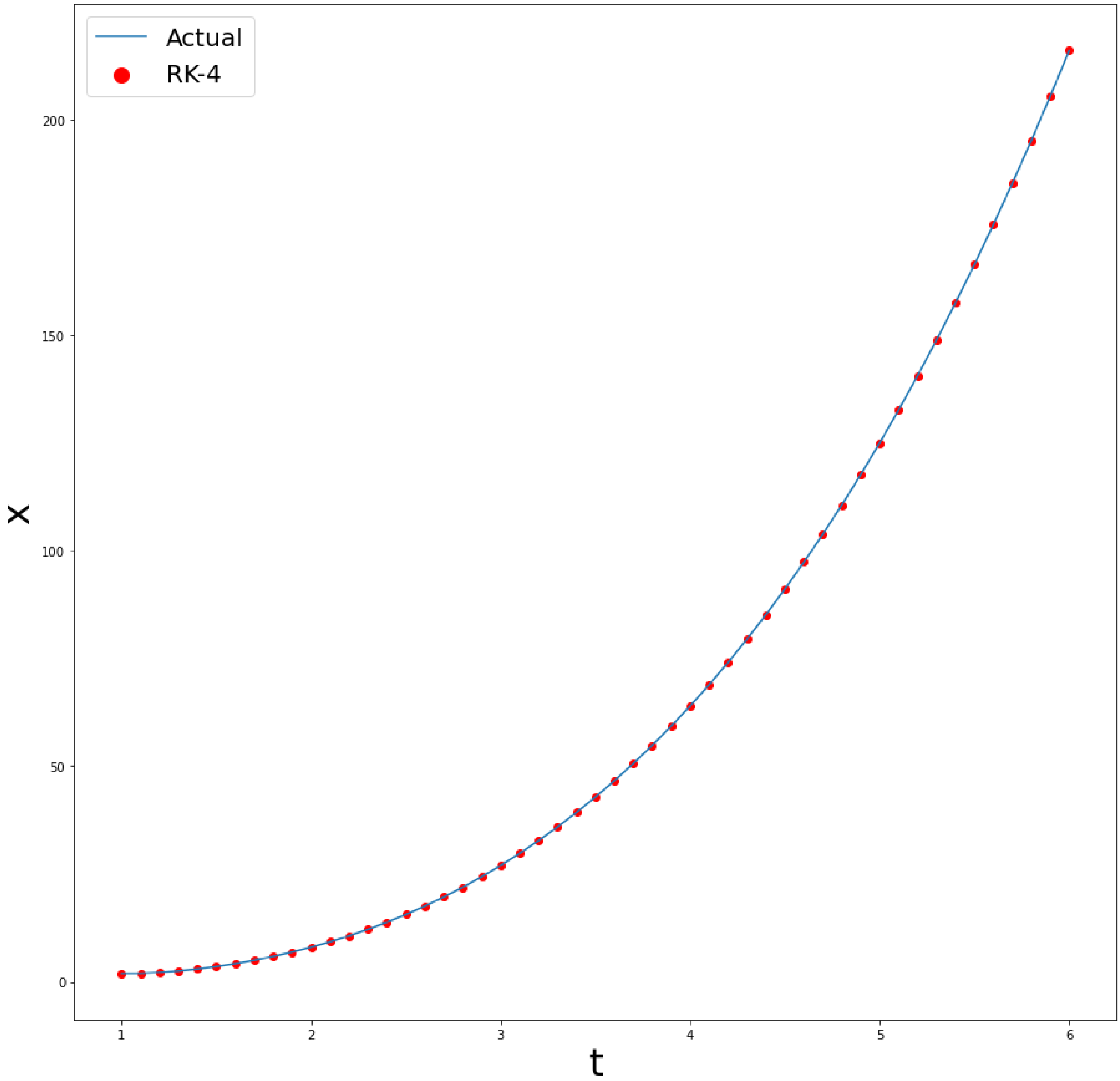
4)

Using RK-2:

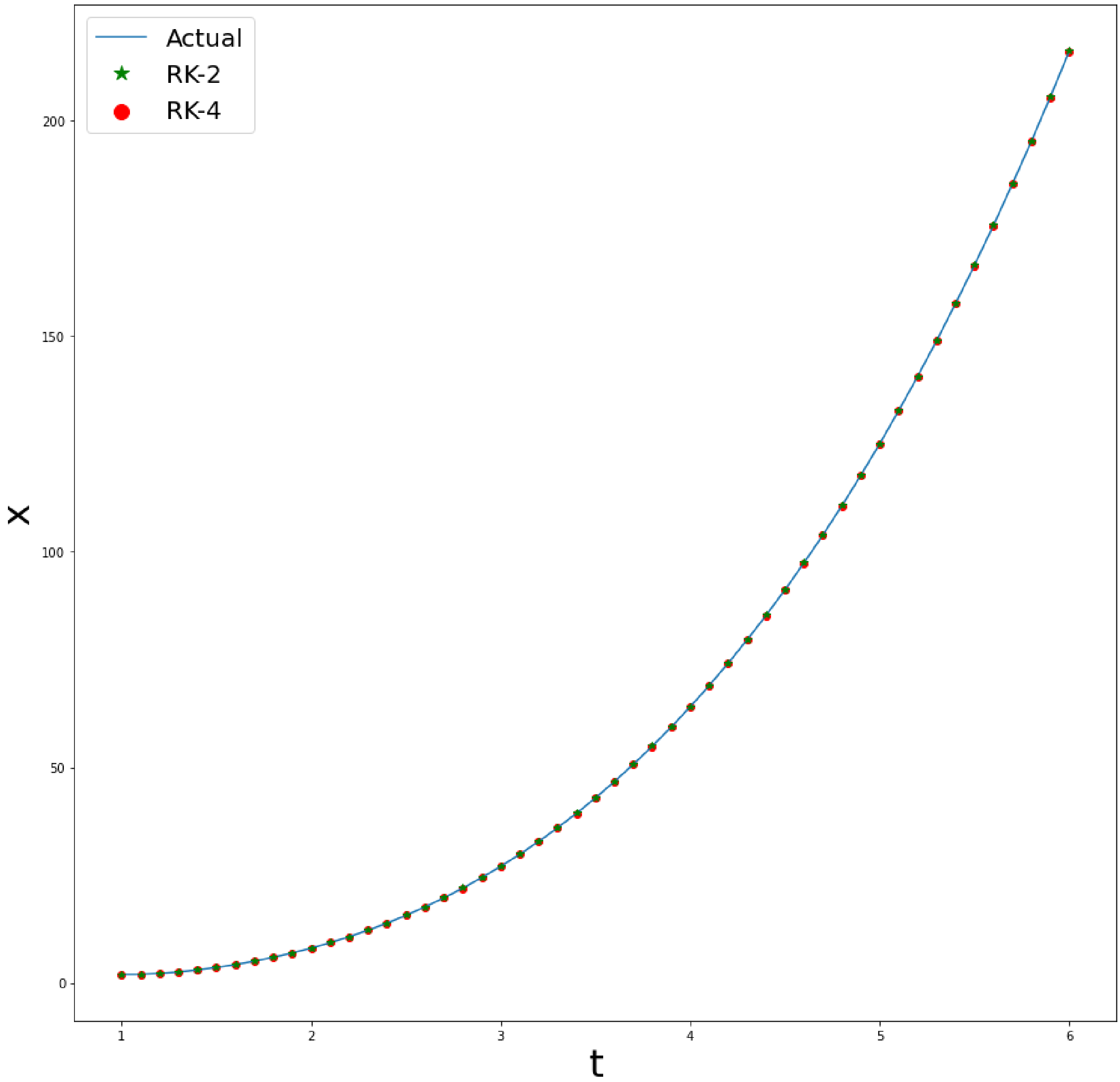


Taking time-steps of size 0.1, we get a percentage error of -0.03720518543% with RK-2.

Using RK-4:



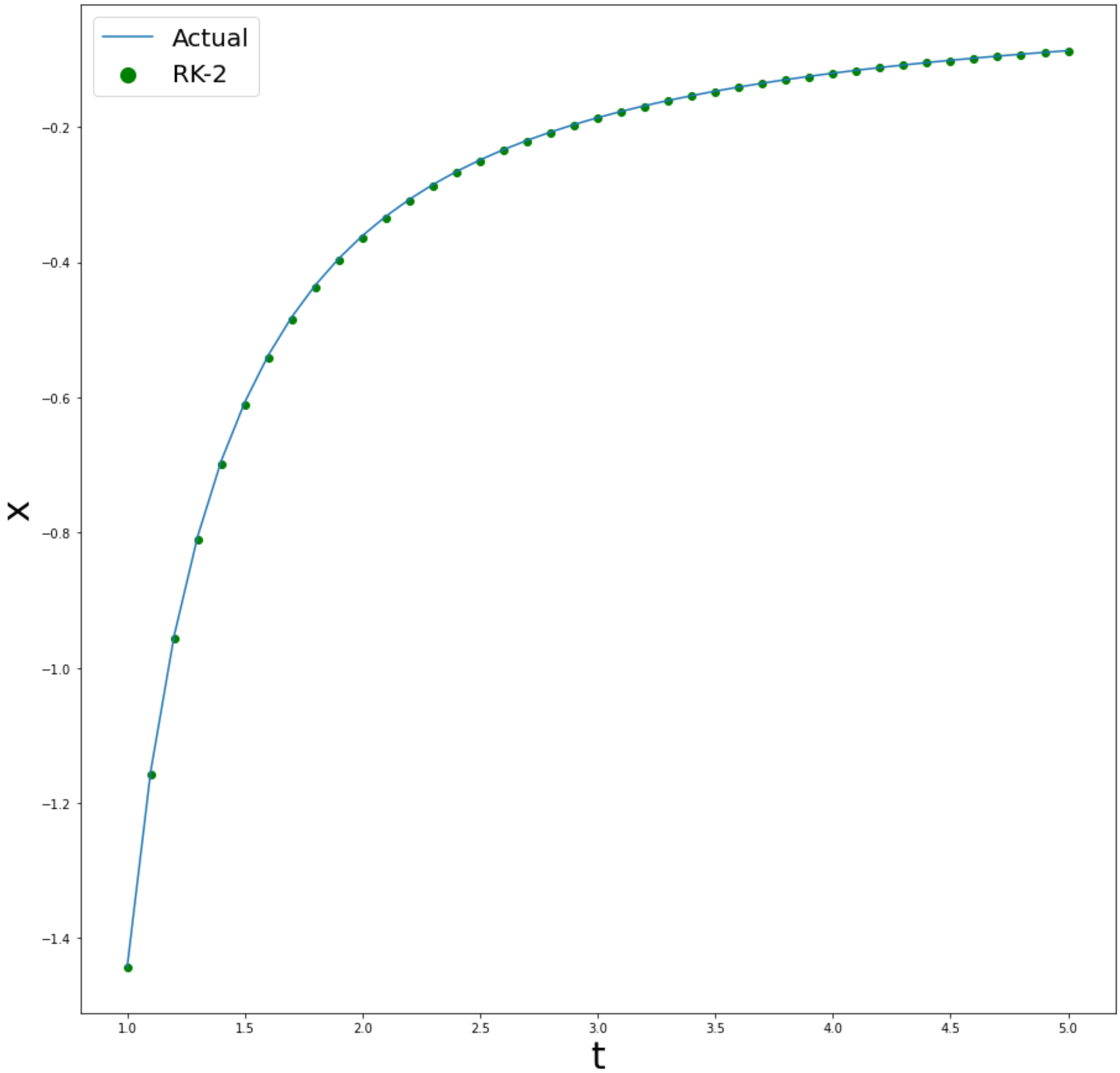
Taking the same time-steps of size 0.1, we get a percentage error of -8.66159974e-06% with RK-4.



Hence, we can see that RK-4 converges much faster in the same number of iterations.

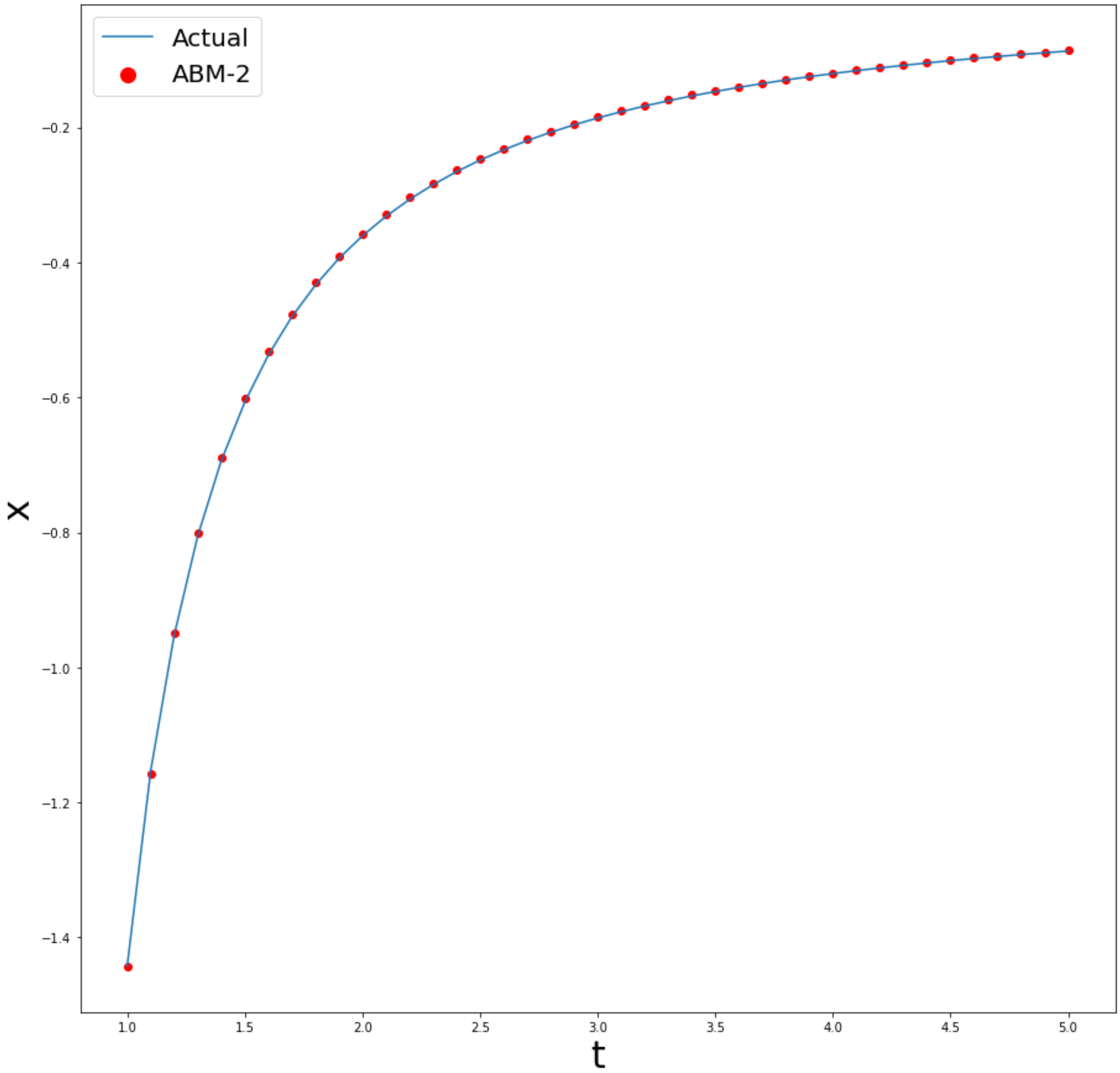
5)

Using RK-2:

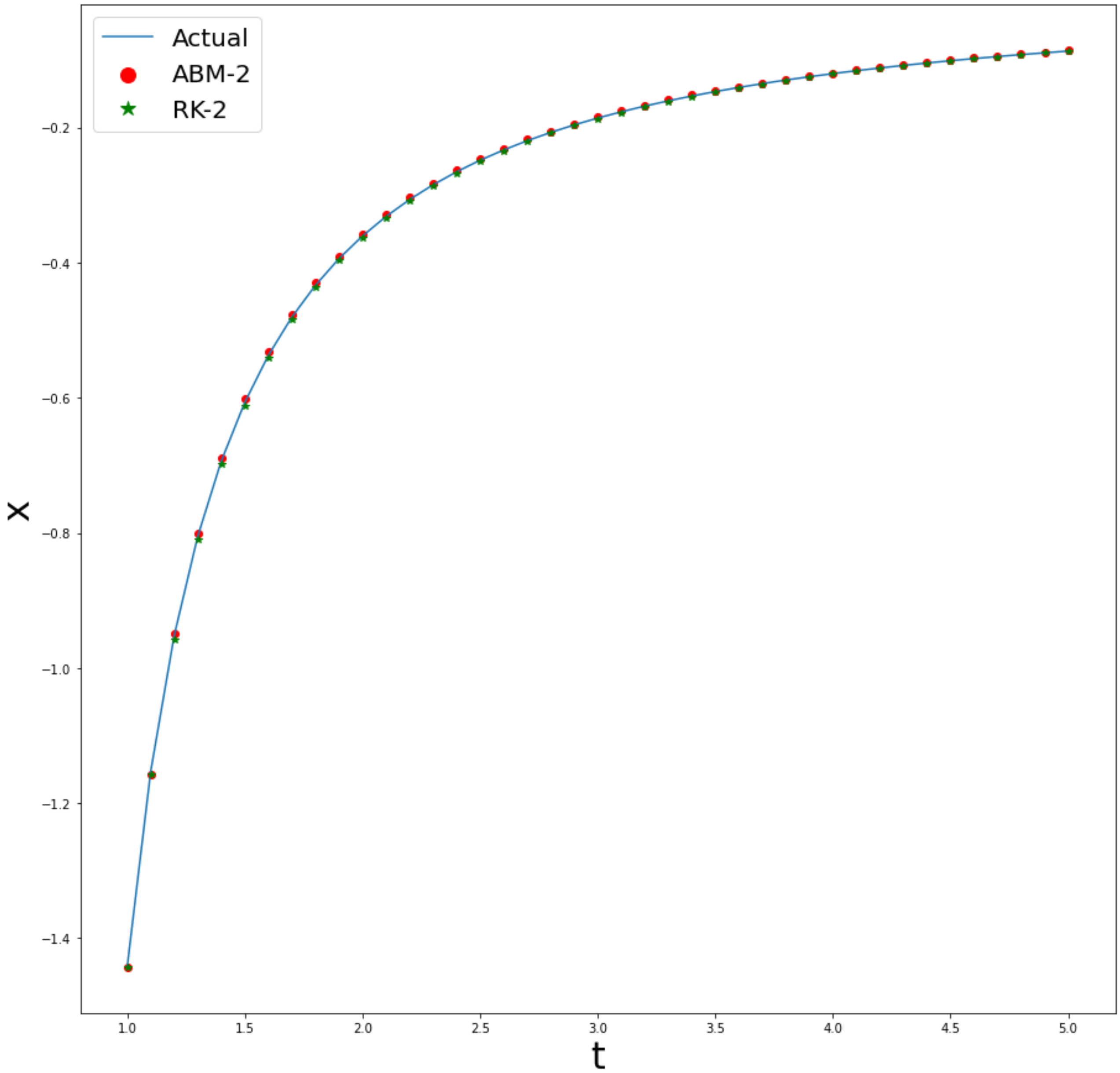


Taking time-steps of size 0.1, we get a percentage error of -0.5357938563% with RK-2.

Using ABM-2:



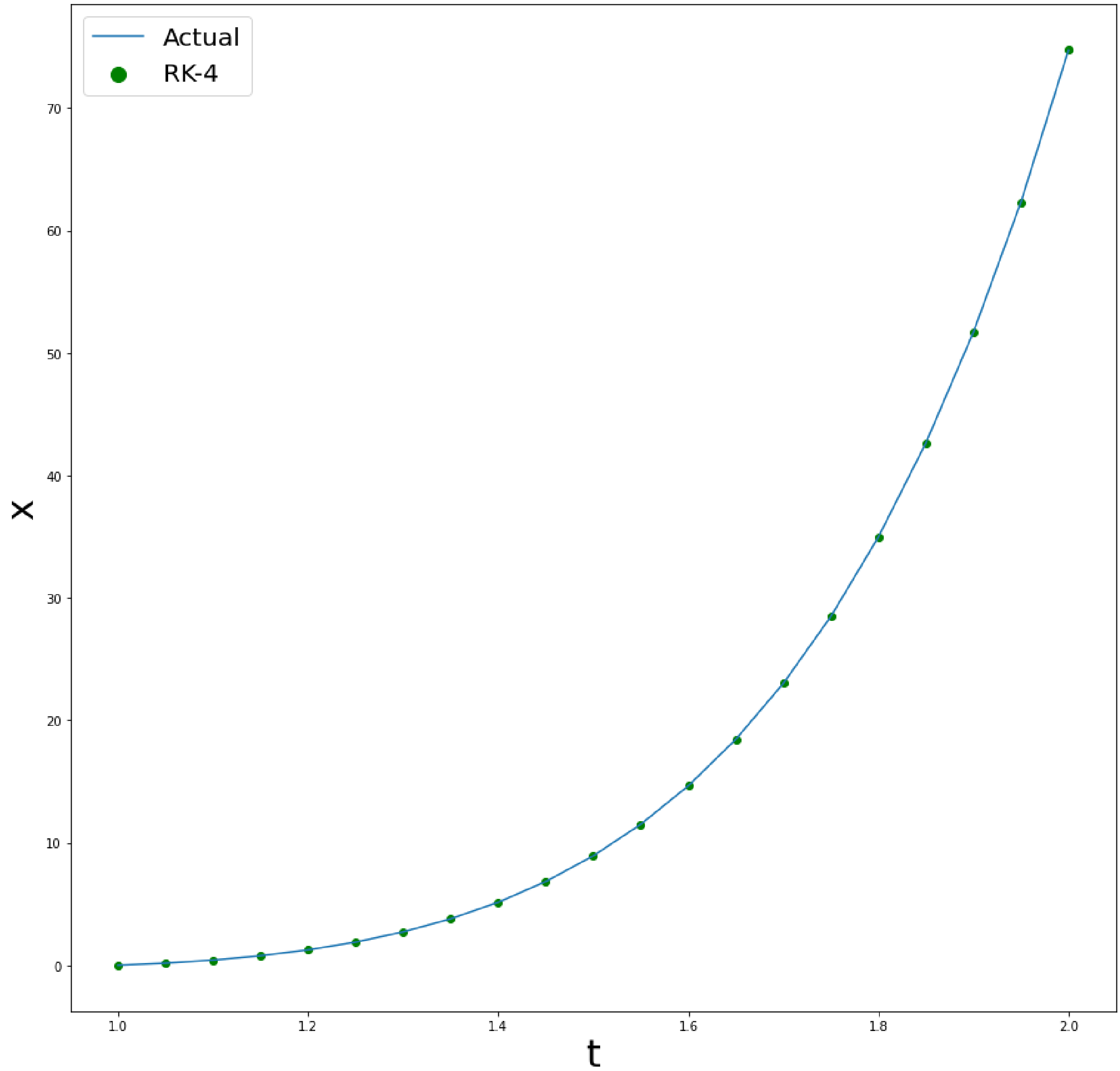
Taking the same time-steps of size 0.1, we get a percentage error of 0.7384963899% with ABM-2.



Hence, we can conclude that RK-2 has better convergence than ABM-2 for same time-steps.

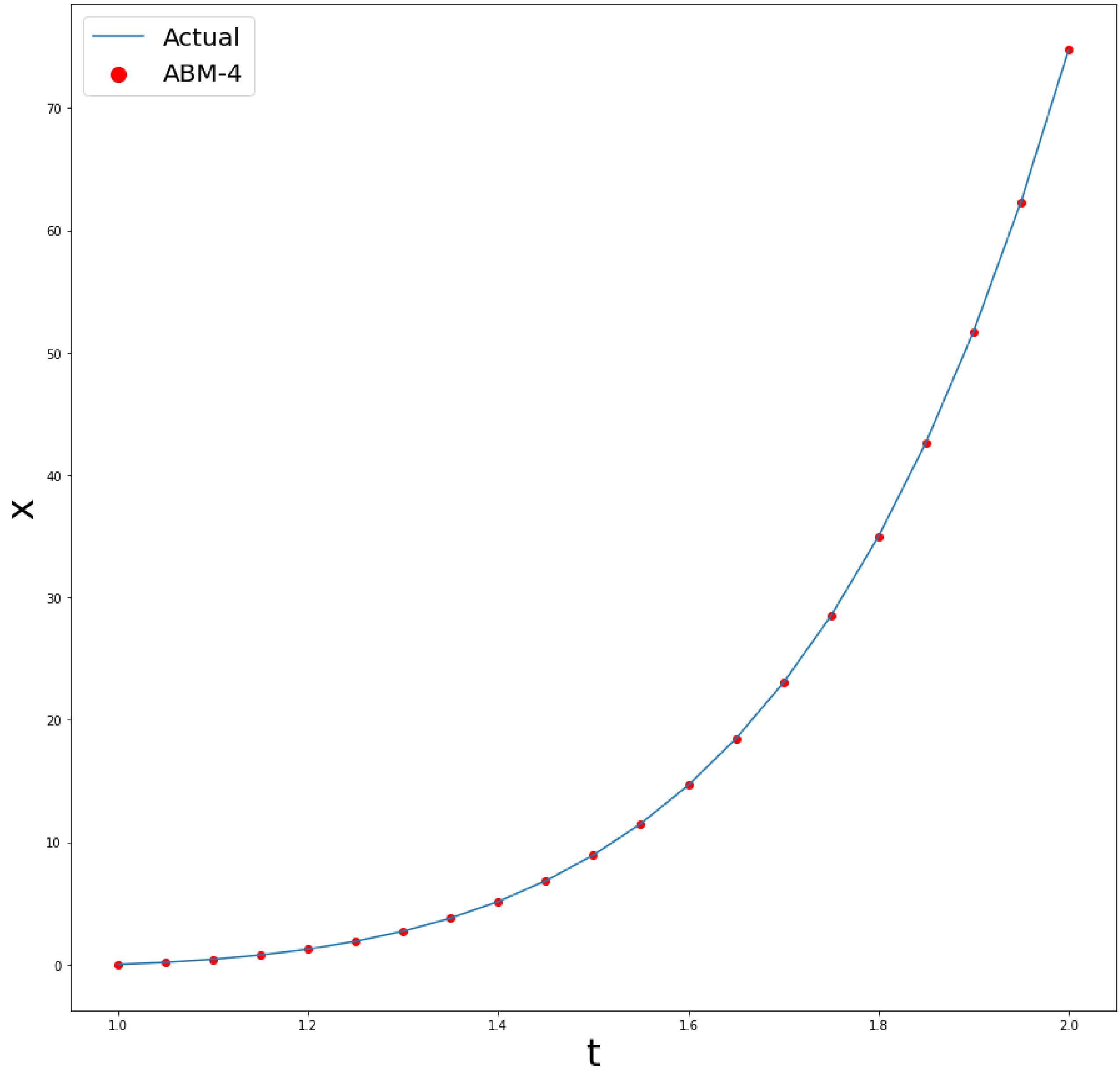
6)

Using RK-4:

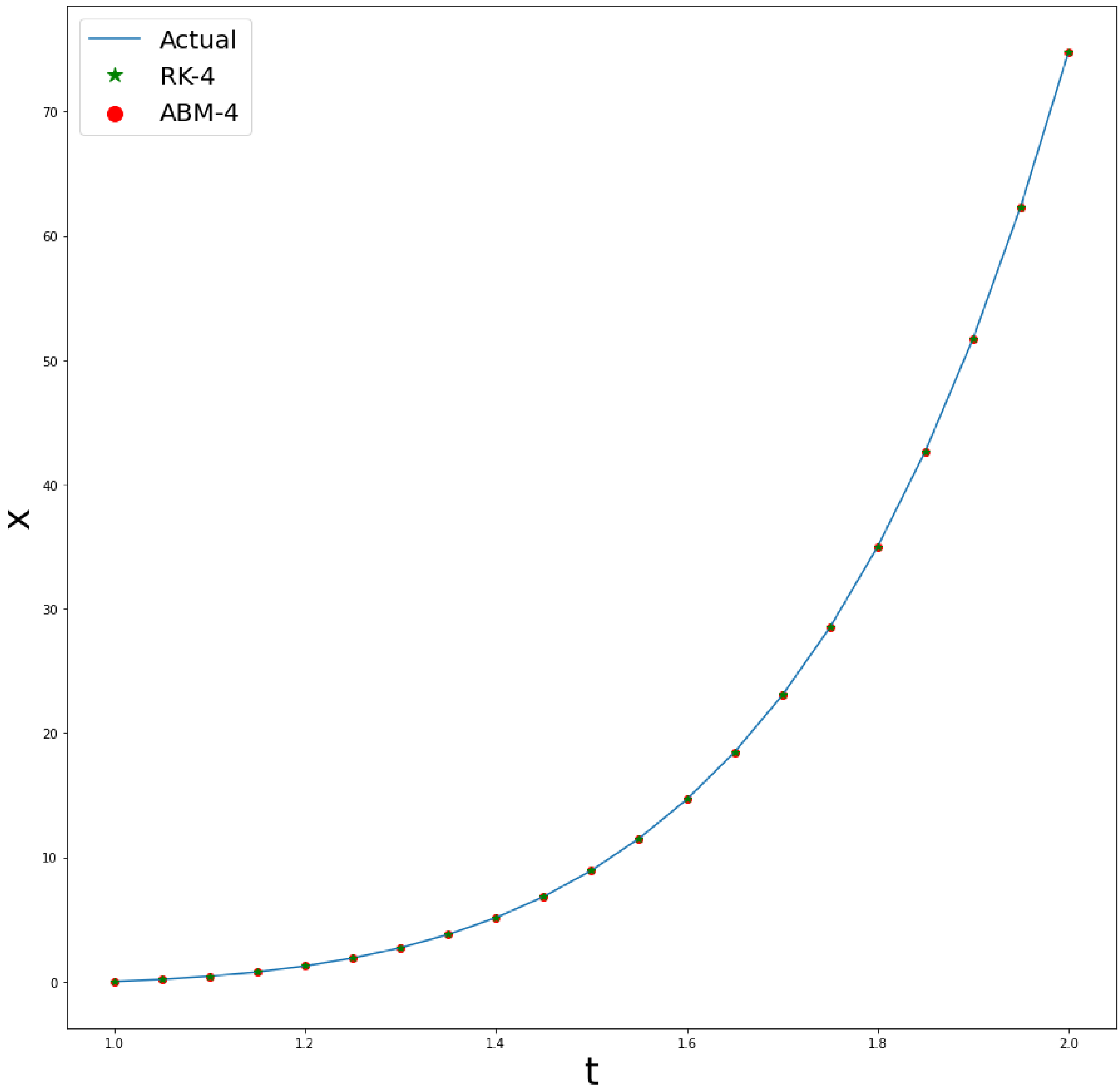


Taking time-steps of size 0.05, we get a percentage error of 0.0008569493606% with RK-4.

Using ABM-4:



Taking the same time-steps of size 0.05, we get a percentage error of -0.0005567053846% with ABM-4.



Hence, we can conclude that ABM-4 has better convergence than RK-4 for same time-steps.

7)

$X'' + (X^2 - 1) + X = 0$

$putting U_1 = X, U_2 = X', we get:$

$U_1{}' = U_2$ --(1)

$U_2{}' + (U_1^2 - 1) * U_2 + U_1 = 0$ --(2)

Taking time-steps, h = 0.1

Solving (1) and (2) simultaneously using RK-4, we get the following curve: