> restart; libname;

"D:\Program Files\lib" **(1)**

In order for Maple to load the GRTensorIII library, first add the directory it was installed in to the Maple libpath variable.

> libname := "D:/grtensor/released/git/grtensor/lib", libname;

libname := "D:/grtensor/released/git/grtensor/lib", "D:\Program Files\lib" **(2)**

Now the GRTensorIII module can be loaded using the with() command.

> with(grtensor);

```
s=0000021231F1BA48, LENGTH=7, invalid FOR length
```

"GRTensor has detected correct length for inert FOR. Disregard the above error"
"

GRTensor III v2.2 Oct 1, 2018"

"Copyright 2018, Peter Musgrave, Denis Pollney, Kayll Lake"

"Latest version is at http://github.com/grtensor/grtensor"

"For help ?grtensor"

"Support/contact grtensor3@gmail.com"

[*Asym*, *KillingCoords*, *PetrovReport*, *Sym*, *autoAlias*, *cmcompare*, *difftool*, *grDalias*, *grF_strToDef*, *gralter*, *gralterd*, *grapply*, *grarray*, *grassign*, *grcalc*, *grcalc1*, *grcalcalter*, *grcalcd*, *grclear*, *grcomponent*, *grconstraint*, *grdata*, *grdebug*, *grdef*, *grdisplay*, *grdump*, *greqn2set*, *grinit*, *grload*, *grload_maplet*, *grmap*, *grmetric*, *grnewmetric*, *grnormalize*, *groptions*, *grsaveg*, *grt2DG*, *grtestinput*, *grtransform*, *grundef*, *hypersurf*, *join*, *kdelta*, *makeg*, *nprotate*, *nptetrad*, *qload*, *spacetime*] **(3)**

Help for the GRTensorIII commands is now available by entering `?<command name>`. For a list of commands use `?grt_commands`.

The `spacetime()` command is used to specify the input for GRTensor. The most common use is to enter the line element of the spacetime you want to calulate objects in.

In this example we use a general metric and define f(r) such that the spacetime metric becomes the charged spherical vacuum; the Reisner-Nordstrom metric. This spacetime has a non-vanishing Einstein tensor since it is not pure vacuum and has some mass-energy due to the charge.

> $f(r) := 1 - \frac{2 \cdot m}{r} + \frac{q^2}{r^2};$

$$f := r \mapsto 1 - \frac{2\, m}{r} + \frac{q^2}{r^2} \quad \textbf{(4)}$$

> $spacetime\left(spherical, coord = [r, \mathrm{theta}, \mathrm{phi}, t], ds = -f(r) \cdot d[t]^2 + + \frac{d[r]^2}{f(r)} + r^2 \cdot \left(d\left[\mathrm{theta}\right]^2 + \sin(\mathrm{theta})^2 \cdot d[\mathrm{phi}]^2 \right) \right);$

```
Calculated ds for spherical (0.000000 sec.)
```

CPU Time = 0.

For the spherical spacetime:

Line element

$$ds^2 = \frac{d\,r^2}{1 - \frac{2\,m}{r} + \frac{q^2}{r^2}} + r^2\,d\,\theta^2 + r^2 \sin(\theta)^2\,d\,\phi^2 + \left(-1 + \frac{2\,m}{r} - \frac{q^2}{r^2}\right) d\,t^2 \tag{5}$$

> *grcalcd*(*G*(*dn*, *up*));

```
Created definition for G(dn,up)
Calculated g(dn,dn,pdn) for spherical (0.000000 sec.)
Calculated Chr(dn,dn,dn) for spherical (0.000000 sec.)
Calculated detg for spherical (0.000000 sec.)
Calculated g(up,up) for spherical (0.000000 sec.)
Calculated Chr(dn,dn,up) for spherical (0.000000 sec.)
Calculated R(dn,dn) for spherical (0.000000 sec.)
Calculated Ricciscalar for spherical (0.000000 sec.)
Calculated G(dn,dn) for spherical (0.000000 sec.)
Calculated G(dn,up) for spherical (0.000000 sec.)
```

CPU Time $= 0.$

For the spherical spacetime:

G(dn,up)

G(*dn*, *up*)

$$G_a^{\ b} = \begin{bmatrix} -\frac{q^2}{r^4} & 0 & 0 & 0 \\ 0 & \frac{q^2}{r^4} & 0 & 0 \\ 0 & 0 & \frac{q^2}{r^4} & 0 \\ 0 & 0 & 0 & -\frac{q^2}{r^4} \end{bmatrix} \tag{6}$$

We now introduce the useful GRTensorIII command `grdef()`. grdef allows new tensor objects to be defined. It makes use of a string expression in which tensor indices are places inside curly-braces {} and "upstairs" indices are prefixed with a ^ (caret) symbol. Symbols after a semi-colon are taken to be indices of covariant derivitive in alignment with the usual mathematical notation. A new object definition consists of a left-hand side (LHS) which defines the tensor name and indices (e.g. "Z{a ^b}") . The right-hand side (RHS) must refer to known tensor objects and must have the same tensor indices as the LHS. The RHS may also have dummy indices to perform summations. For example to define the contracted divergenece of the Einstein tensor "G{a ^b; b}".

As an example, we define the contracted divergence of the Einstein tensor.

> *grdef*("divG{a} := G{a ^b; b}");

```
Created a definition for G(dn,up,cdn)
Created definition for divG(dn)
```

Notice that GRTensorIII determined what is needed to define a new object G(dn, up, cdn). This object definition can be created algorithmically, GRTensorIII can create covariant derivitives of known objects as needed.

Note also that indices indicating covariant derivitives are designated using cdn and cup. Indices for partial derivitives are designated pdn. pup.

We now calculate the divergence (even though we know this must be zero, since it is a mathematical identity).

> grcalcd(divG(dn));

```
Calculated G(dn,up,cdn) for spherical (0.000000 sec.)
Calculated divG(dn) for spherical (0.000000 sec.)
```

$$CPU\ Time\ = 0.$$

For the spherical spacetime:

divG(dn)

$$divG(dn)$$

$$divG_a = All\ components\ are\ zero \tag{7}$$

>

As must be true, all components of this new entity are zero. To confirm the intermediate values were computed we can examine the intermediate values:

> grdisplay(G(dn, up, cdn));

For the spherical spacetime:

G(dn,up,cdn)

$$G_r{}^{r}{}_{;r} = \frac{4\,q^2}{r^5}$$

$$G_\theta{}^{\theta}{}_{;r} = -\frac{4\,q^2}{r^5}$$

$$G_\phi{}^{\phi}{}_{;r} = -\frac{4\,q^2}{r^5}$$

$$G_t{}^{t}{}_{;r} = \frac{4\,q^2}{r^5}$$

$$G_\theta{}^{r}{}_{;\theta} = \frac{2\,q^2\,(2\,m\,r - q^2 - r^2)}{r^5}$$

$$G_r{}^{\theta}{}_{;\theta} = -\frac{2\,q^2}{r^5}$$

$$G_\phi{}^{r}{}_{;\phi} = \frac{2\,q^2\,(2\,m\,r - q^2 - r^2)\,\sin(\theta)^2}{r^5}$$

$$G_r{}^{\phi}{}_{;\phi} = -\frac{2\,q^2}{r^5} \tag{8}$$

>