Animation of a Dot on a Wheel

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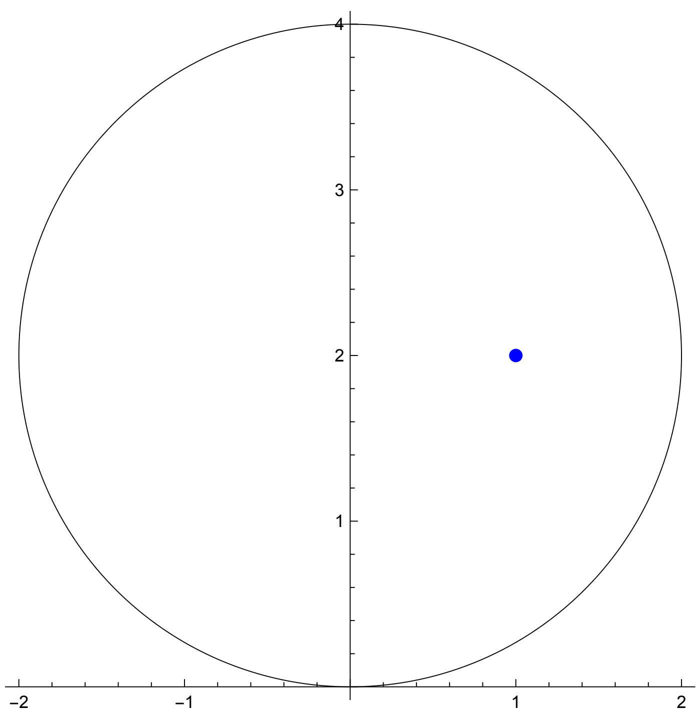
NOTE: IF YOU ARE READING THIS IN PDF FORMAT, PLEASE NOTE THAT THE ANIMATIONS MAY NOT SEEM TO HAVE STOPPED CORRECTLY. CHECK THE GIFS TO SEE THE ACTUAL ANIMATION

We are given a wheel with a constant angular velocity, $\dot{\theta}$, in the $-\hat{z}$ direction (that is, the wheel is moving in the $+\hat{x}$direction), with a radius R. There is a dot on the wheel at a radius ρ such that $0 \le \rho \le R$. Assume the wheel starts at (0, 0, 0). For the visualization, we will use $R = 2\,m$, $\rho = 1\,m$, and $\dot{\theta} = -2$ rad/s.

In[73]:=
```
R := 2;
p := 1;
thetaDot := -2;
```

In[76]:=
```
circle = Graphics[Circle[{0, R}, R], Axes → True];
dot = Graphics[{PointSize[Large], Blue, Point[{1, R}]}, Axes → True];
Show[circle, dot]
```

Out[78]=



So, as this thing rotates, the y position of the dot will oscillate up and down. Note that we are taking the ground to be y=0. The x position of the dot will vary as the wheel will move right while the dot oscillates left and right as well. We can derive the following equations for the dot:

$x_{\text{dot}}(t) = \rho \cos(\dot{\theta}\, t) - R\, \dot{\theta}\, t$

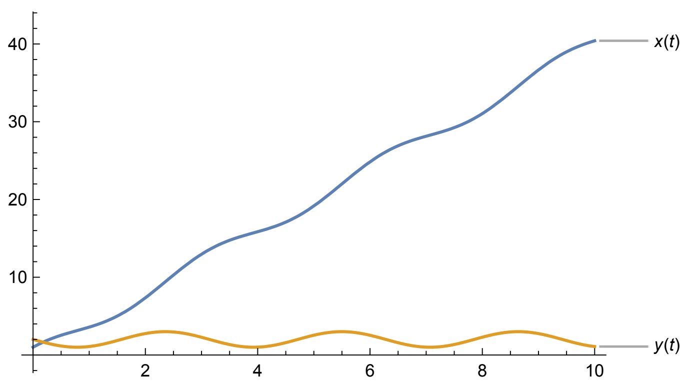
$y_{\text{dot}}(t) = R + \rho \sin(\dot{\theta}\, t)$

For x_{dot}, the first term takes into account if the dot is on the left or right side of the wheel, and the second part takes into account how far the dot has travelled in a $\hat{x}$ direction. The second term is negative since a negative rotation (rotation in $-\hat{z}$) means a movement in the $+\hat{x}$ direction. In the y_{dot} equation, the first term takes into account that y=0 is the ground, not the center of the wheel, and the second term takes into account where on the wheel the dot is.

We can graph $x(t)$ and $y(t)$ over time:

In[79]:=
```
x[t_] := p Cos[thetaDot t] - R thetaDot t
y[t_] := R + p Sin[thetaDot t]
Plot[{x[t], y[t]}, {t, 0, 10}, PlotLabels → "Expressions"]
```

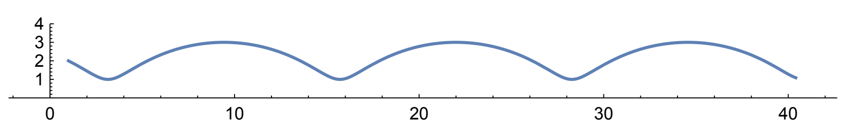
Out[81]=



That’s no fun, nor is it easy to understand, so it’s better to plot y vs. x, or a parametric plot:

In[82]:=
```
ParametricPlot[{x[t], y[t]}, {t, 0, 10}, PlotRange → {Automatic, {0, 4}}]
```

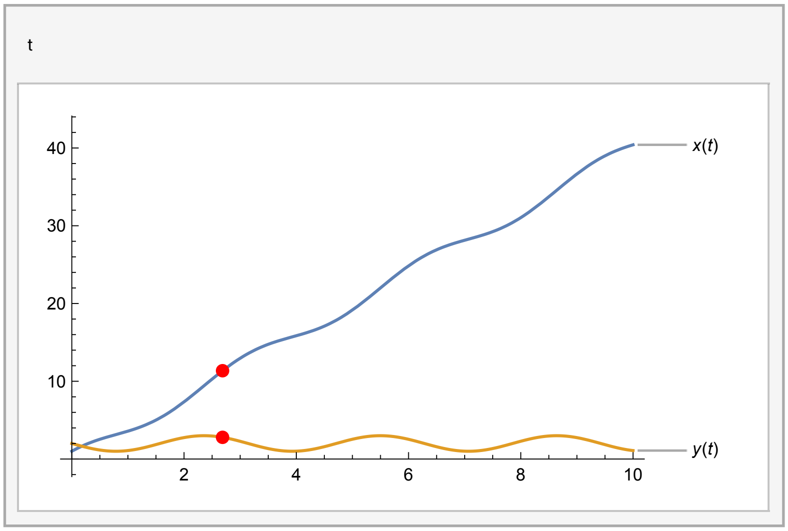
Out[82]=



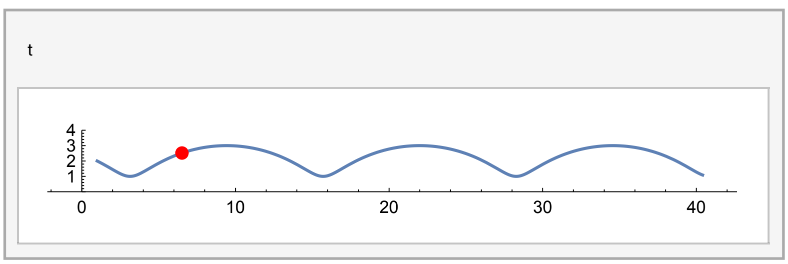
We can animate both of these together, so we can see how the dot moves over time (plot_xtyt.gif and paramplot_xtyt.gif):

In[83]:=
```
plot = Plot[{x[t], y[t]}, {t, 0, 10}, PlotLabels → "Expressions"];
paramPlot =
  ParametricPlot[{x[t], y[t]}, {t, 0, 10}, PlotRange → {Automatic, {0, 4}}];
Animate[
 Show[plot, Graphics[{PointSize[Large], Red, Point[Dynamic[{t, x[t]}]]}],
  Graphics[{PointSize[Large], Red, Point[Dynamic[{t, y[t]}]]}]],
 {t, 0, 10, AppearanceElements → None}
]
Animate[
 Show[paramPlot, Graphics[{PointSize[Large], Red, Point[Dynamic[{x[t], y[t]}]]}]],
 {t, 0, 10, AppearanceElements → None}
]
```

Out[85]=



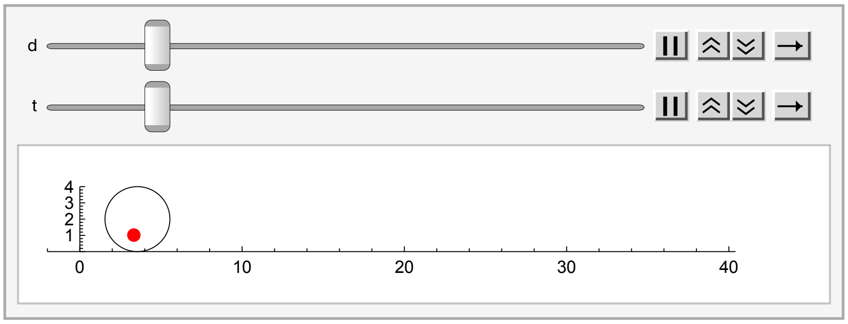
Out[86]=



This is cool, but it would be better to see the wheel rolling, so we will animate that directly (wheelroll1.gif).

In[87]:=
```
xend = x[10];
Animate[
 Show[Graphics[Circle[{d, R}, R], Axes → True, PlotRange → {{-R, xend}, {0, 4}}],
  Graphics[{PointSize[Large], Red, Point[Dynamic[{x[t], y[t]}]]}]],
 {d, 0, xend}, {t, 0, 10}
]
```

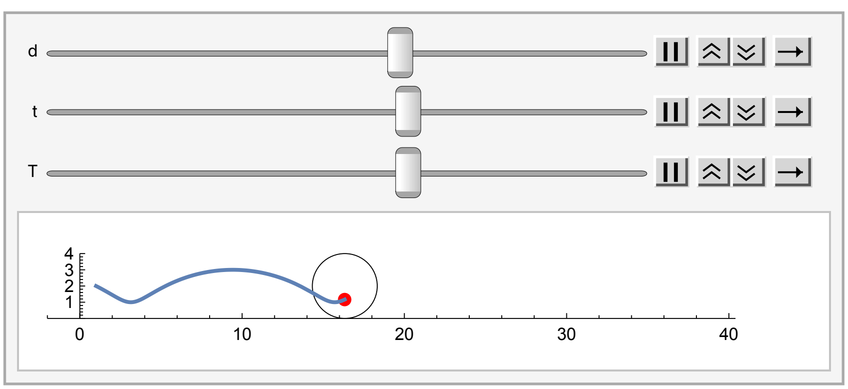
Out[88]=



We use two variables. The variable t is the standard time with $0 \leq t \leq 10$. The variable d, however, is just the x-distance the wheel is travelling. The distance the wheel is travelling has not been parameterized in terms of t, thus we use two variables. We abuse the fact that the circle moves from 0 to the x-position of the dot at 10 seconds, $x_{\text{dot}}(10)$. Now let's use this animation to trace the path of the dot (wheelroll2.gif).

In[89]:=
```
Animate[
 Show[Graphics[Circle[{d, R}, R], Axes → True, PlotRange → {{-R, xend}, {0, 4}}],
  Graphics[{PointSize[Large], Red, Point[Dynamic[{x[t], y[t]}]]}],
  ParametricPlot[{x[a], y[a]}, {a, 0, T}, PlotStyle → Thick]],
 {d, 0, xend}, {t, 0, 10}, {T, 0, 10}
]
```

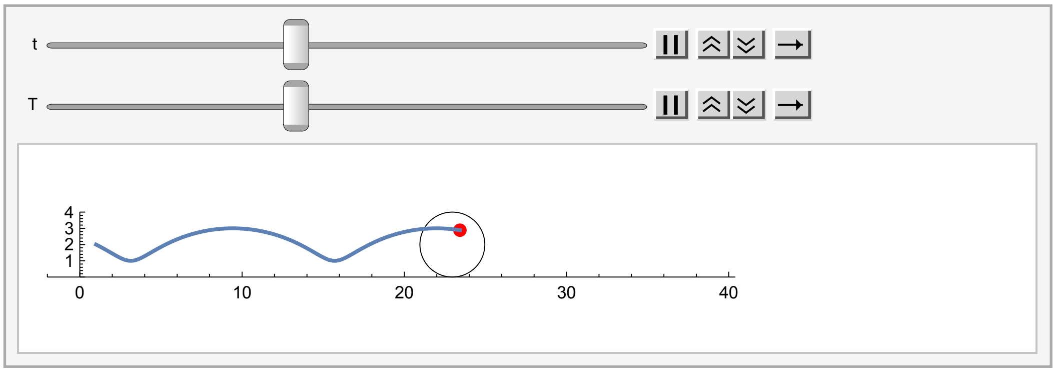
Out[89]=



For parametric purposes, we introduced a t and a T. It is important to note that both of these are the same t, and we just defined them differently to not confuse Mathematica (i.e. $0 \leq t = T \leq 10$). Just by the nature of Mathematica and our approximation of distance for the circle rolling, we do not have a perfect animation, and it does not appear that ρ stays at 1 (wheelroll3.gif).

In[90]:=
```
xcm[t_] := -R thetaDot t
Animate[
 Show[
  Graphics[Circle[{xcm[t], R}, R], Axes → True, PlotRange → {{-R, xend}, {0, 4}}],
  Graphics[{PointSize[Large], Red, Point[Dynamic[{x[t], y[t]}]]}],
  ParametricPlot[{x[a], y[a]}, {a, 0, T}, PlotStyle → Thick]],
 {t, 0, 10}, {T, 0, 10}
]
```

Out[91]=



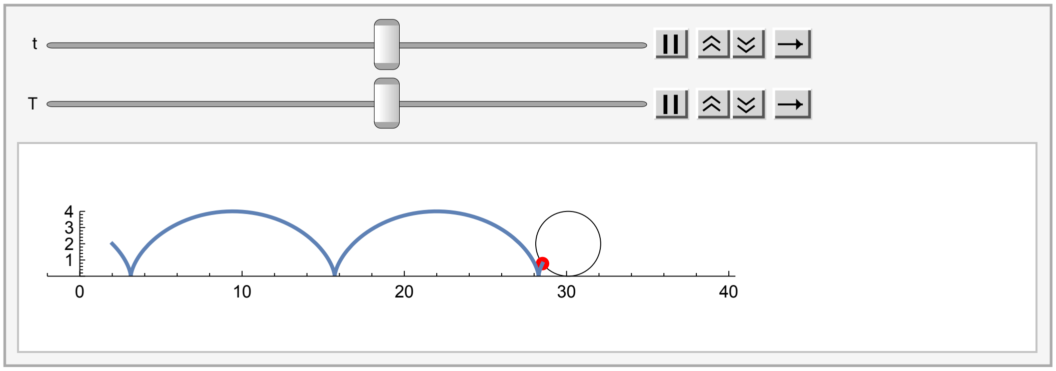
Now, instead of making an approximation of d, we can just define a new function that gets the exact position of the center of the circle:

$x_{\text{cm}}(t) = -R\,\dot{\theta}\,t$

Now we use the same t parameterization for the circle as well, and this animation looks much smoother. Let’s see what we get if $\rho = 2$ and if we place our dot “outside” the circle at $\rho = 4$. Since we manually define the y-axis, we will need to do some tweaking on the hard-coded values (wheelroll_rho2.gif).

In[92]:=
```
x2[t_] := 2 Cos[thetaDot t] - R thetaDot t
y2[t_] := R + 2 Sin[thetaDot t]
Animate[
 Show[
  Graphics[Circle[{xcm[t], R}, R], Axes → True, PlotRange → {{-R, xend}, {0, 4}}],
  Graphics[{PointSize[Large], Red, Point[Dynamic[{x2[t], y2[t]}]]}],
  ParametricPlot[{x2[a], y2[a]}, {a, 0, T}, PlotStyle → Thick]],
 {t, 0, 10}, {T, 0, 10}
]
```

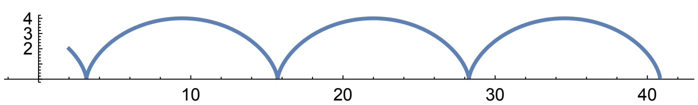
Out[94]=



You can see that when $\rho = 2$, we get the expected “semi-circle” (a better phrasing would be semi-circle-*esque*).

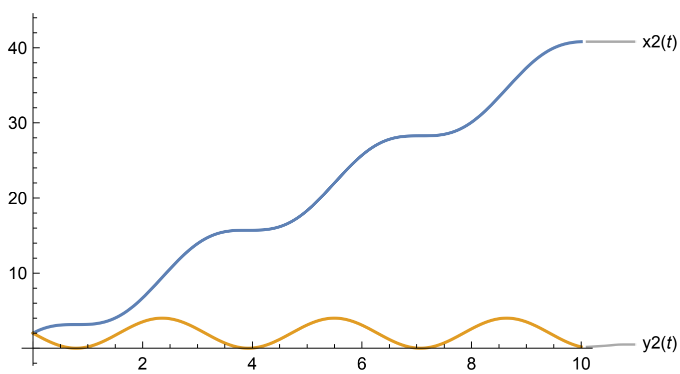
In[95]:=
```
ParametricPlot[{x2[t], y2[t]}, {t, 0, 10}, PlotStyle → Thick]
```

Out[95]=



In[96]:=
```
Plot[{x2[t], y2[t]}, {t, 0, 10}, PlotLabels → "Expressions"]
```

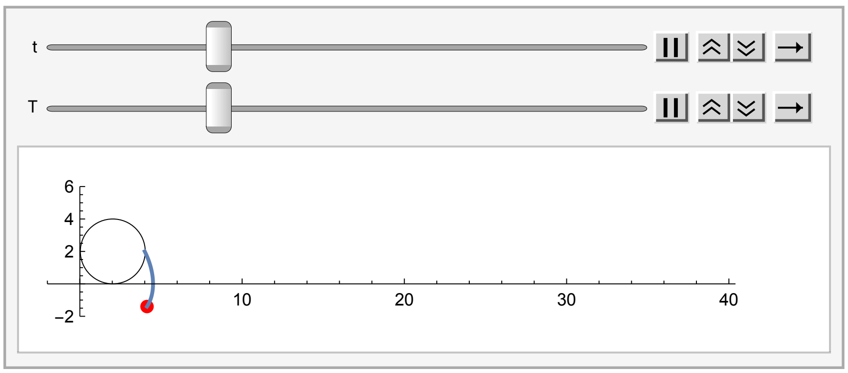
Out[96]=



Now let’s try $\rho = 4$ (wheelroll_rho4.gif):

In[97]:=
```
x3[t_] := 4 Cos[thetaDot t] - R thetaDot t
y3[t_] := R + 4 Sin[thetaDot t]
Animate[
 Show[
  Graphics[Circle[{xcm[t], R}, R], Axes → True, PlotRange → {{-R, xend}, {-2, 6}}],
  Graphics[{PointSize[Large], Red, Point[Dynamic[{x3[t], y3[t]}]]}],
  ParametricPlot[{x3[a], y3[a]}, {a, 0, T}, PlotStyle → Thick]],
 {t, 0, 10}, {T, 0, 10}
]
```

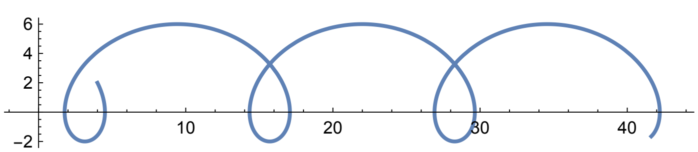
Out[99]=



Because the dot is now outside the wheel, we get this interesting non-trivial behavior where the dot is sometimes moving backwards instead of continuously forward.

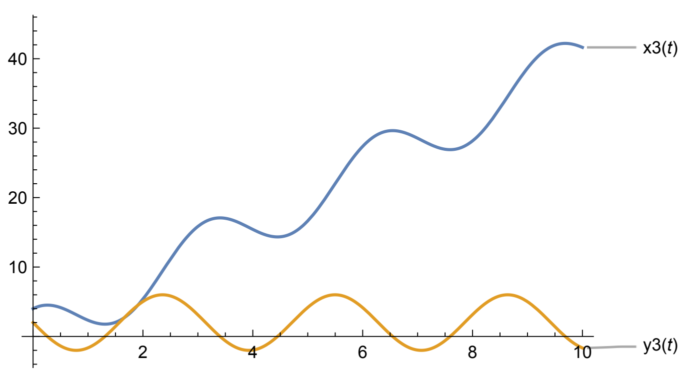
In[100]:=
```
ParametricPlot[{x3[t], y3[t]}, {t, 0, 10}, PlotStyle → Thick]
```

Out[100]=



In[101]:=
```
Plot[{x3[t], y3[t]}, {t, 0, 10}, PlotLabels → "Expressions"]
```

Out[101]=



We define these as new functions $x_2(t)$, $y_2(t)$, $x_3(t)$, and $y_3(t)$ so Mathematica doesn’t mess with our other animations.

Thank you for taking interest in this project!

In[102]:=