Probability Density Functions

Generic PDF Function

The generic formula for a PDF is as follows: $P(a \le x \le b) = \int_a^b f(x)\,\mathrm{dx}$, where $f(x)$is the probability function describing the distribution.

In[∘]:=
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P[f_, x_, a_, b_] := NIntegrate[f, {x, a, b}];
Default[P, 3] = -∞;
Default[P, 4] = ∞;
```

Gaussian Distribution

Gaussian Distribution: $G(y, \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{(y-\mu)^2}{2\sigma^2}}$

A standard Gaussian distribution has a mean of 0 and a standard deviation of 1.

In[∘]:=
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Gaussian[y_, μ_, σ_] := 1/(σ Sqrt[2 π]) E^(-(y-μ)^2/(2 σ^2));
Default[Gaussian, 2] = 0;
Default[Gaussian, 3] = 1;
```

Chi-Square Distribution

Chi-Square Distribution: $P(\chi^2) = \frac{1}{2^{n/2}\,\Gamma(n/2)}\,(\chi^2)^{n/2-1}\, e^{-\chi^2/2}$

Where n represents the number of degrees of freedom for the distribution.

This distribution includes the gamma function: $\Gamma(x) = \int_0^\infty e^{-t}\, t^{x-1}\,\mathrm{dt}$

In[∘]:=
```
ChiSquare[chi2_, n_] := 1/(2^(n/2) Gamma[n / 2]) (chi2)^((n/2)-1) E^(-chi2/2);
```

Binomial Distribution

Binomial Distribution: $P(m, N, p) = \binom{N}{m} p^m (1-p)^{N-m}$

This distribution represents the probability of getting m successes out of N trials, where the probability of success is p.

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In[]:= Bin[m_, N_, p_] := Factorial[N] / (Factorial[m] Factorial[N - m]) p^m (1 - p)^(N-m);
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$$\texttt{Bin[m\_, N\_, p\_]} := \frac{\texttt{Factorial[N]}}{\texttt{Factorial[m] Factorial[N - m]}} \texttt{p}^{\texttt{m}} \texttt{(1 - p)}^{\texttt{N-m}};$$

Poisson Distribution

Poisson Distribution: $P(m, N, p) = \frac{N^m}{m!} p^m e^{-p N}$ or $P(m, \mu) = \frac{e^{-\mu} \mu^m}{m!}$

In this distribution, m represents the number of successes out of N trials, and p the probability of success. For a Poisson distribution, $\mu = N p$.

$$\texttt{Poisson[m\_, } \mu\texttt{\_] :=} \frac{\texttt{E}^{-\mu} \mu^{\texttt{m}}}{\texttt{Factorial[m]}};$$