Four Vectors

For physics purposes, a four vector is displayed using the following notation: $A^\mu = (A_0, A_1, A_2, A_3)$. If we consider that the first component is the “timelike” component, and that the other three components following are “spacelike”, then we can rewrite this expression as $A^\mu = \left(A_0, \vec{A}\right)$.

Defining a vector is pretty easy in Mathematica, as we just have to use an array. The main functionality of this notebook is the provided physics equations using four vectors in special relativity.

Lorentz Transformation

In[1]:=

```
γ[β_] := 1/Sqrt[1 - β^2];
Λ[β_] := {{γ[β], -γ[β] β, 0, 0}, {-γ[β] β, γ[β], 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}};
Λ[β] // MatrixForm
```

Out[3]//MatrixForm=

$$\begin{pmatrix} \frac{1}{\sqrt{1-\beta^2}} & -\frac{\beta}{\sqrt{1-\beta^2}} & 0 & 0 \\ -\frac{\beta}{\sqrt{1-\beta^2}} & \frac{1}{\sqrt{1-\beta^2}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Example

In[16]:=

```
A = {0, 1, 2, 3};
Λ[3 / 5].A // MatrixForm
```

Out[17]//MatrixForm=

$$\begin{pmatrix} -\frac{3}{4} \\ \frac{5}{4} \\ 2 \\ 3 \end{pmatrix}$$

In Lorentz transformation terms, this would be the value of the above four vector with respect to a frame going three-fifths the speed of light in relation to the original four vector’s rest frame.

Four Vector Length Squared (In Special Relativity)

In[18]:=
```
η = {{1, 0, 0, 0}, {0, -1, 0, 0}, {0, 0, -1, 0}, {0, 0, 0, -1}};
η // MatrixForm
```

Out[19]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

In[20]:=
```
FVLength[A_] := (A.η.A)
```

Example

In[21]:=
```
FVLength[A]
```

Out[21]=

-14

In Einstein’s summation notation, this length squared can be written as follows: $A^2 = A \cdot A = A^\mu \eta_{\mu\nu} A^\nu$. Further mathematical simplifications can be made to this, depending on the scenario.