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Physics 3700

Lab 3 Report

This lab's purpose was to demonstrate the Central Limit Theorem through two means: through an experiment involving stainless steel balls and a Gaussian distribution apparatus, and through simulation using a C. L. T. definition.

In the experiment, steel balls were sent through a Gaussian distribution apparatus, which landed the balls in one of 34 slots. The overall shape after dropping all the balls appeared to be Gaussian-shaped. The number of balls in each slot is shown as a scatter plot in Fig. 1. The standard Poisson error was used to calculate the uncertainty of each data point, which is the square root of the number of entries in each bin of the scatter plot. Using the experimental data, we calculated an experimental mean of 17.3 and an experimental variance of 36.5. Superimposed on the theoretical data is the expected Gaussian distribution for the apparatus. The experimental data fits the Gaussian distribution very well, with many values being close to the theorized distribution, and roughly 82% of the experimental data's error bars touch the theorized data points. There is a range from slots 20 to 23 where none of the error bars touch the Gaussian distribution, but it is plausible that this is due to statistical fluctuation.

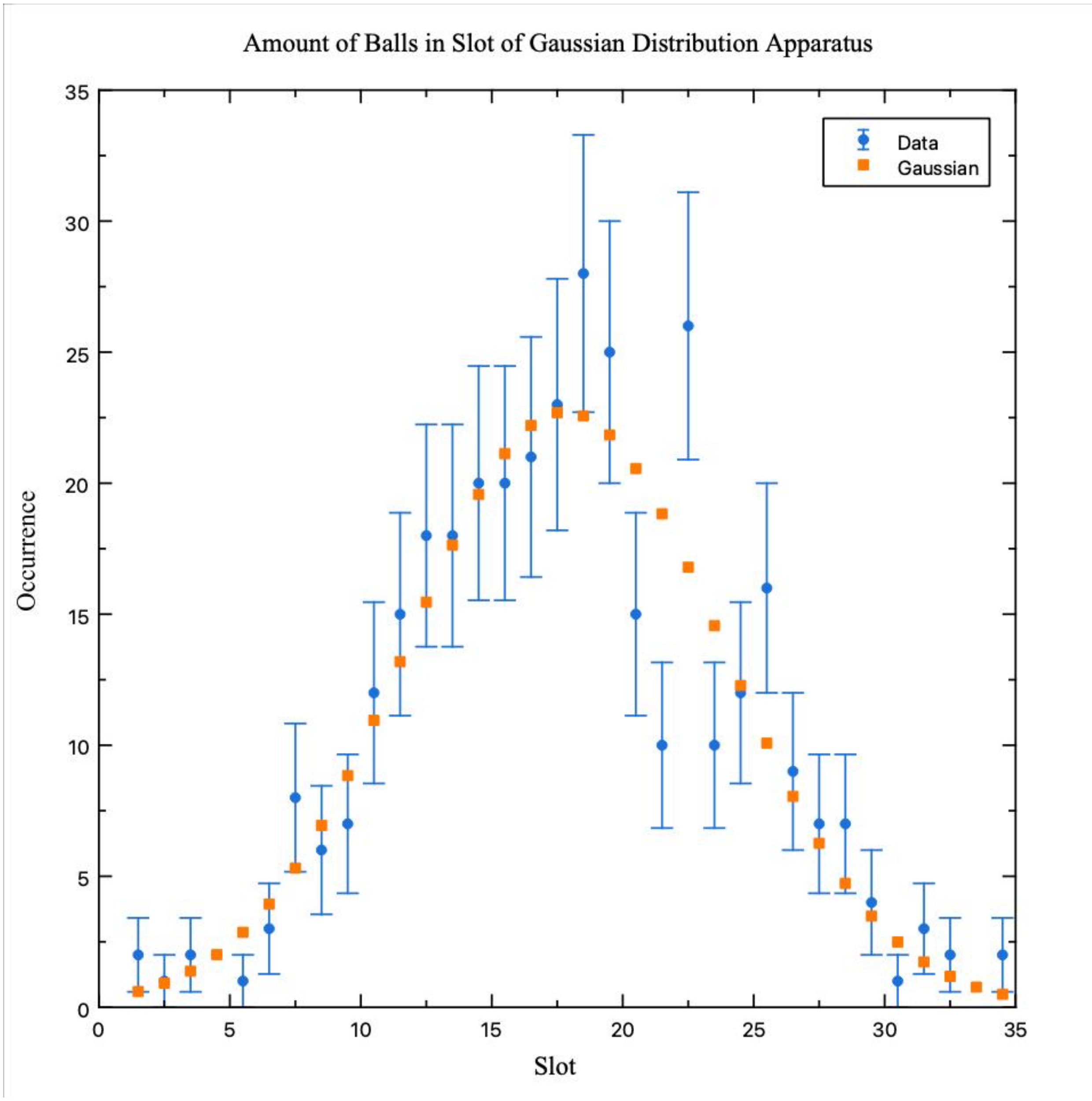

Figure 1: Experimental Data from Gaussian Distribution Apparatus

In the second part of the lab, we created a simulation that would demonstrate the C. L. T. First, we proved using the integral definitions of the mean and variance that the mean is 1/2 and the variance is 1/12 for a uniform distribution, as shown in Appendix A. Then, we wrote a program to test the following hypothesis: if adding twelve random variables between zero and one and then subtracting six approximates a Gaussian distribution with a mean of zero and a variance of one. We generated 10^6 values, with each value being the twelve random variables added together and then subtracted by six (we denote each generated value by Z). We plotted the simulated data as a histogram on Fig. 2. Superimposed on top of the histogram is the theoretical Gaussian distribution of the simulation, with a mean of zero and a variance of one. Our experimental data calculates a mean of zero and a variance of one, up to three significant figures. The histogram fits the theorized Gaussian distribution well, with all the theorized data points falling in or close to the counts displayed in the histogram. Additionally, the mean and variance of the experimental data and the theoretical distribution are effectively equal with values of zero and one respectively.

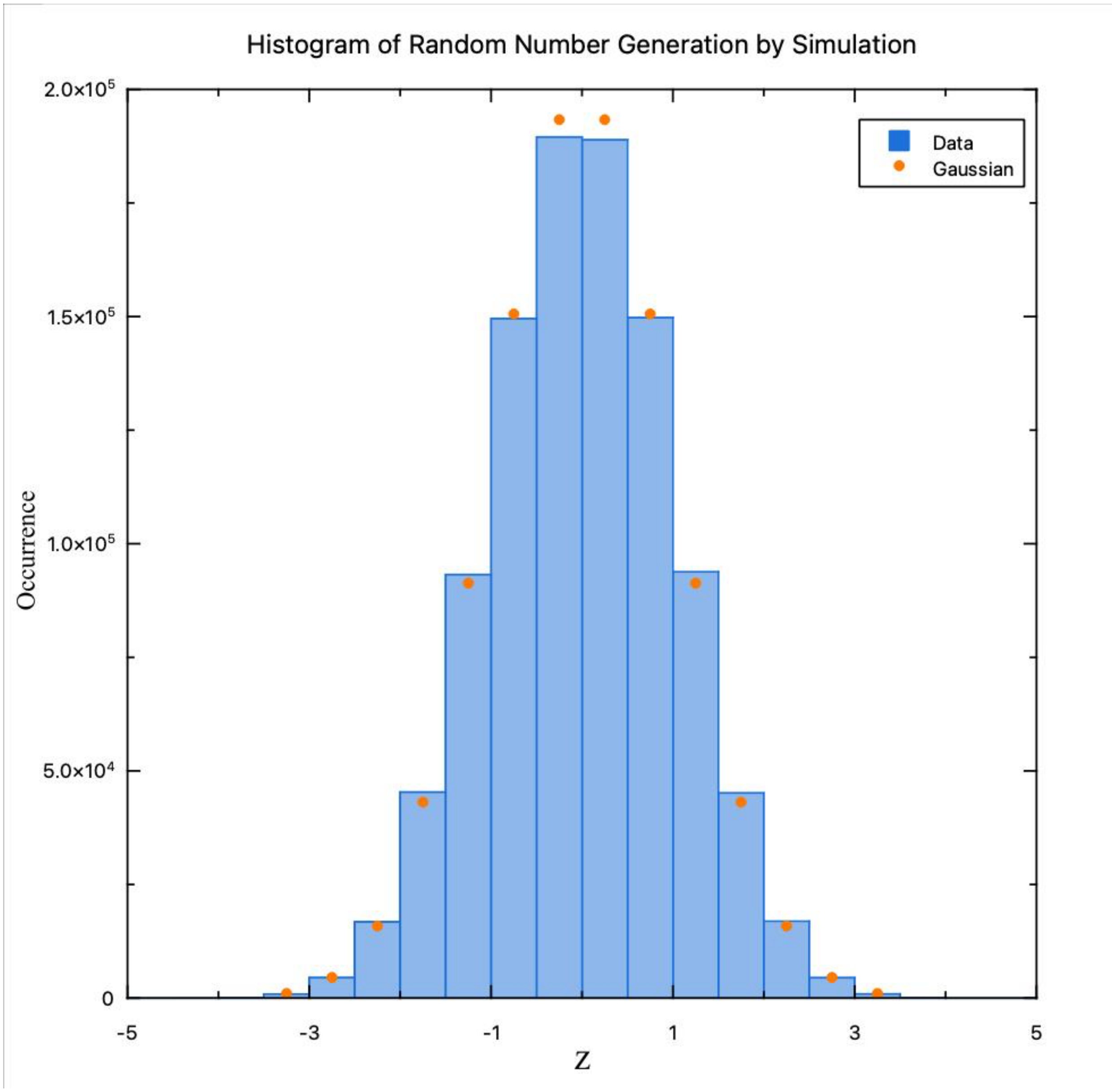

Figure 2: Gaussian Distribution of Random Number Generation by Simulation

Appendix A: Proofs for Uniform Distribution

Uniform Distribution:

$$p(x) = \begin{cases} 1 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

Mean of Uniform Distribution:

$$\begin{aligned} \mu &= \int_{-\infty}^{+\infty} xp(x)\ dx = \int_{-\infty}^{0} xp(x)\ dx + \int_{0}^{1} xp(x)\ dx + \int_{1}^{+\infty} xp(x)\ dx \\ &= \int_{-\infty}^{0} 0\ dx + \int_{0}^{1} x\ dx + \int_{1}^{+\infty} 0\ dx = \int_{0}^{1} x\ dx = \left[\frac{x^2}{2}\right]_{x=0}^{1} = \frac{1}{2} \end{aligned}$$

Variance of Uniform Distribution:

$$\begin{aligned} \sigma^2 &= \int_{-\infty}^{+\infty} p(x)(x-\mu)^2\ dx = \int_{-\infty}^{0} p(x)(x-\frac{1}{2})^2\ dx + \int_{0}^{1} p(x)(x-\frac{1}{2})^2\ dx \\ &+ \int_{1}^{+\infty} p(x)(x-\frac{1}{2})^2\ dx = \int_{-\infty}^{0} 0\ dx + \int_{0}^{1} (x-\frac{1}{2})^2\ dx + \int_{1}^{+\infty} 0\ dx \\ &= \int_{0}^{1} (x-\frac{1}{2})^2\ dx = \left[\frac{(x-\frac{1}{2})^3}{3}\right]_{x=0}^{1} = \frac{1}{24} + \frac{1}{24} = \frac{1}{12} \end{aligned}$$