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Physics 3700

Lab 2 Report

This lab's purpose was to experiment with statistical fluctuations to implement Binomial and Poisson statistics. We approximated the value of pi through throwing a dart 100 times at a circle inscribed within a square, and we created a simulation to produce an approximation for larger number of throws. We investigated the binomial distribution by tossing twelve six-sided dice 50 times and recorded the number of times a two was face-up. Finally, we used Poisson statistics when measuring the rate of cosmic-ray particles passing through a Geiger counter.

The first experiment involved throwing darts at a circle of radius R inscribed within a square of side-length 2R. The ratio of the circle's area to the square's area is exactly $\frac{\pi}{4}$. Since the ratio of the circle's area to the square's area is a constant ratio, we can use the following formula to gain an approximation for pi: $\pi \approx 4\frac{n_{in}}{n}$, where n_{in} is the number of darts landing inside the circle and n being the total number of throws. Using this formula, throwing 100 darts at the shape described yielded an approximation for pi as 3.36 ± 0.15. We calculated the uncertainty using the following formula: $4(\frac{\sqrt{Npq}}{N})$, where p is the probability the dart landed in the circle, q is the probability the dart didn't land in the circle, and N is the total number of throws. We created a simulation to produce approximations for the following total throws: 4×10^2,10^3,4×10^3,10^4,4×10^4,10^5,4×10^5, and 10^6. As seen in Fig. 1, as the sample size increased, the fluctuation between the expected value for pi and the experimental value decreased. Out of the nine measurements taken, six have their error bar touching the horizontal line representing the true value of pi, thus the data is consistent with the expectation that 68% of the measurements are within one standard deviation of the expectation, since six divided by nine is roughly 67%. The recorded measurement of pi with 10^6 points was 3.14, which is comparable to the true value of pi (3.141), . The second part of this experiment used 25 trials of 100 points and creating a histogram of the approximations of pi, as seen in Fig. 2. Using the data from the figure, the average value of pi was 3.13 with an n-1 standard deviation of 0.13. This is comparable to the uncertainty of pi obtained from the first set of 100 throws, with a calculated uncertainty of 0.15. Since my 100 throws resulted in an approximation of pi of 3.36, I was able to get a sufficient spread of darts landing inside and outside the circle, but I am not a good enough darts thrower to get a better approximation as I subconsciously threw more darts inside the circle than outside.

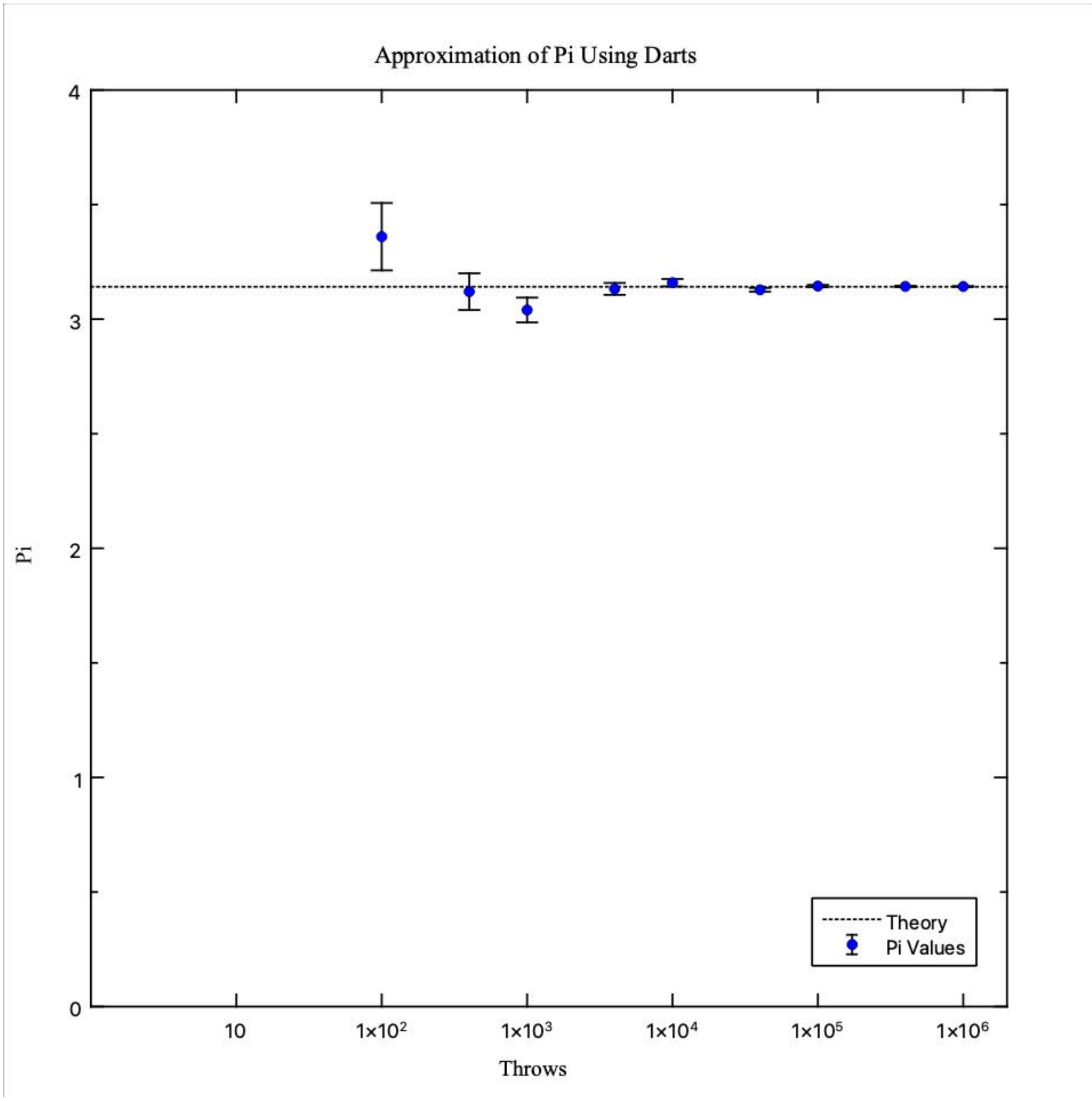

Figure 1: Approximating Pi Using Darts, From 100 to 1,000,000 Times

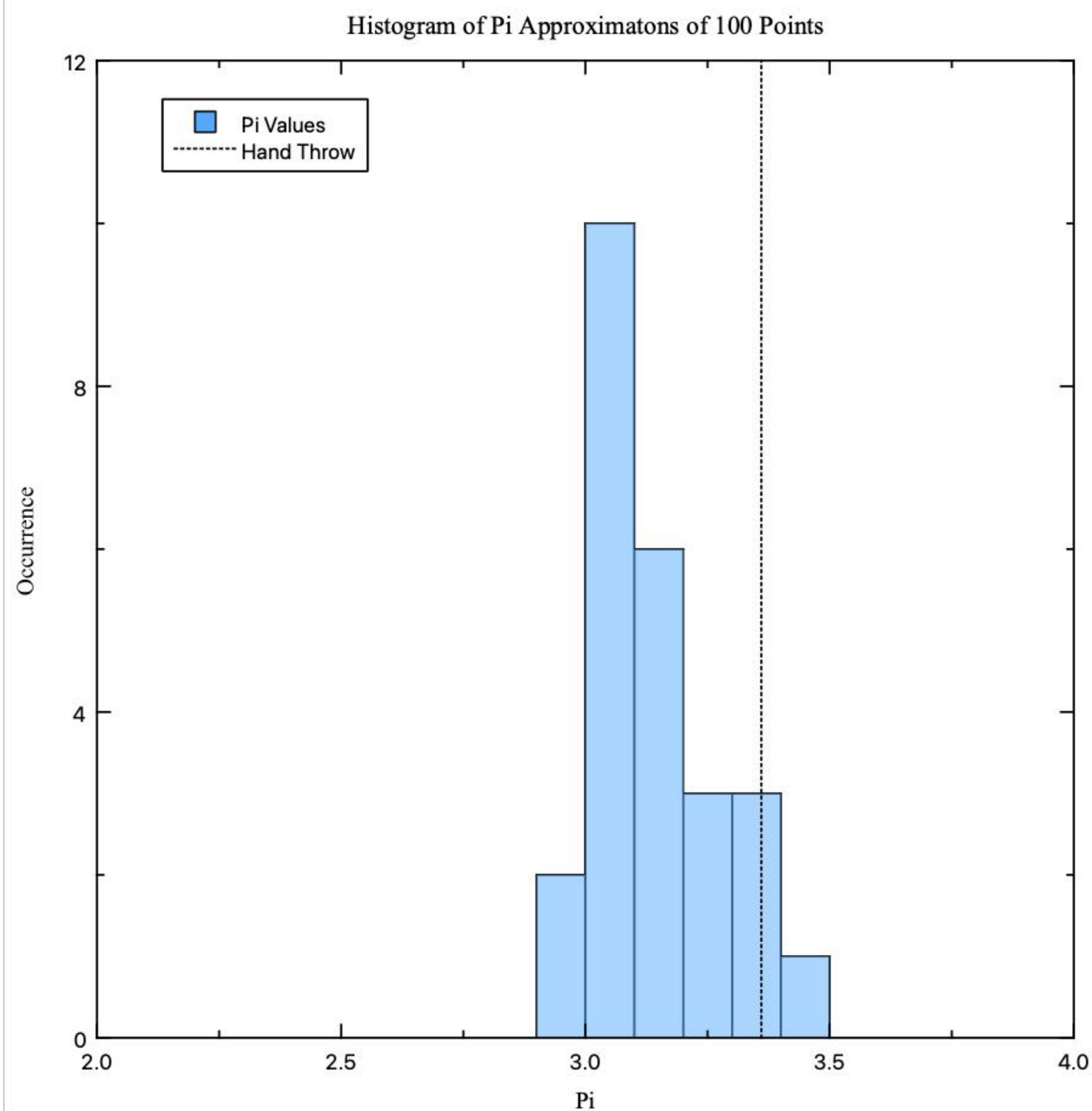

Figure 2: Histogram of Pi Approximations for 25 Trials of 100 Darts

The second experiment involved tossing twelve six-sided dice for 50 trials and recording the number of times a two was face-up. The mean for our distribution was 1.76 with a variance of 1.74. The uncertainty calculated for the number of times a single two was face-up was ±3.1. The equation used to calculate uncertainties was the equation $\sqrt{m}$, where m represents the number of entries for each bin. The experimental data as a histogram and the theoretical values using a Binomial and Poisson distribution are displayed on Fig. 3. Using a Binomial distribution, we calculate a mean of 2 and a variance of 1.66. We used our experimental mean to estimate a Poisson distribution. Therefore, it is not surprising that the mean and variance for the Poisson expectation are both 1.76, as the mean was used to create the expectation, and the variance equals the mean for a Poisson distribution. The Poisson distribution yields a mean and variance very similar to our experimental data, while the Binomial distribution has a mild discrepancy for the mean specifically. In general, the Poisson distribution better fits our experimental data. To generate a better Poisson distribution for this experiment we could dramatically increase our sample size. Another way would be to reduce the probability, p, of our distribution. We could do this by tossing more dice than the twelve we used.

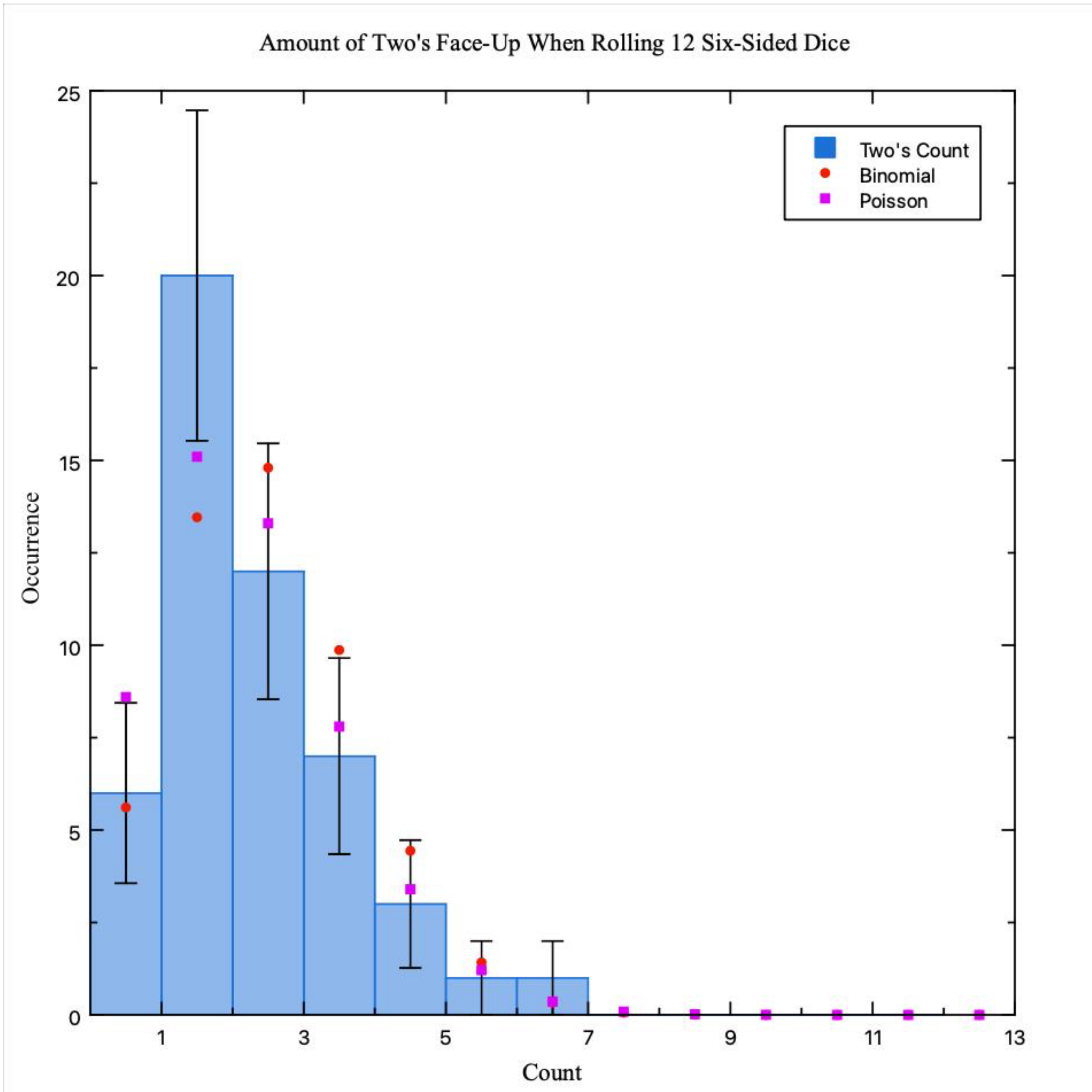

Figure 3: Histogram of Two's Face-Up When Rolling 12 Six-Sided Dice 50 Times

The third experiment observed the rate of cosmic-ray particles passing through a Geiger counter. After collecting the data, we calculated an experimental mean value of 4.23. We used this value to estimate the Poisson distribution for the data using the formula $N\frac{e^{-\mu}\mu^{m}}{m!}$, where μ is the mean, N is the sample size, and m is the number of counts per fifteen seconds. The histogram of the experimental data along with the Poisson distribution is displayed on Fig. 4. Overall, the histogram is consistent with the predicted Poisson distribution, as most of the theorized Poisson values fall within the error bars of the experimental data, with the error bars being calculated using the typical Poisson error using the square root of the number of entries per bin. In our experimental data, no entries are recorded having higher than ten counts per fifteen seconds, which follows the Poisson distribution, as it only predicts 0.6 counts per fifteen seconds. In general, the shape of the experimental data follows the theorized Poisson distribution, with the only notable exception being the bin with four counts per fifteen seconds having seven experimental occurrences, while the Poisson distribution predicts 11.6.

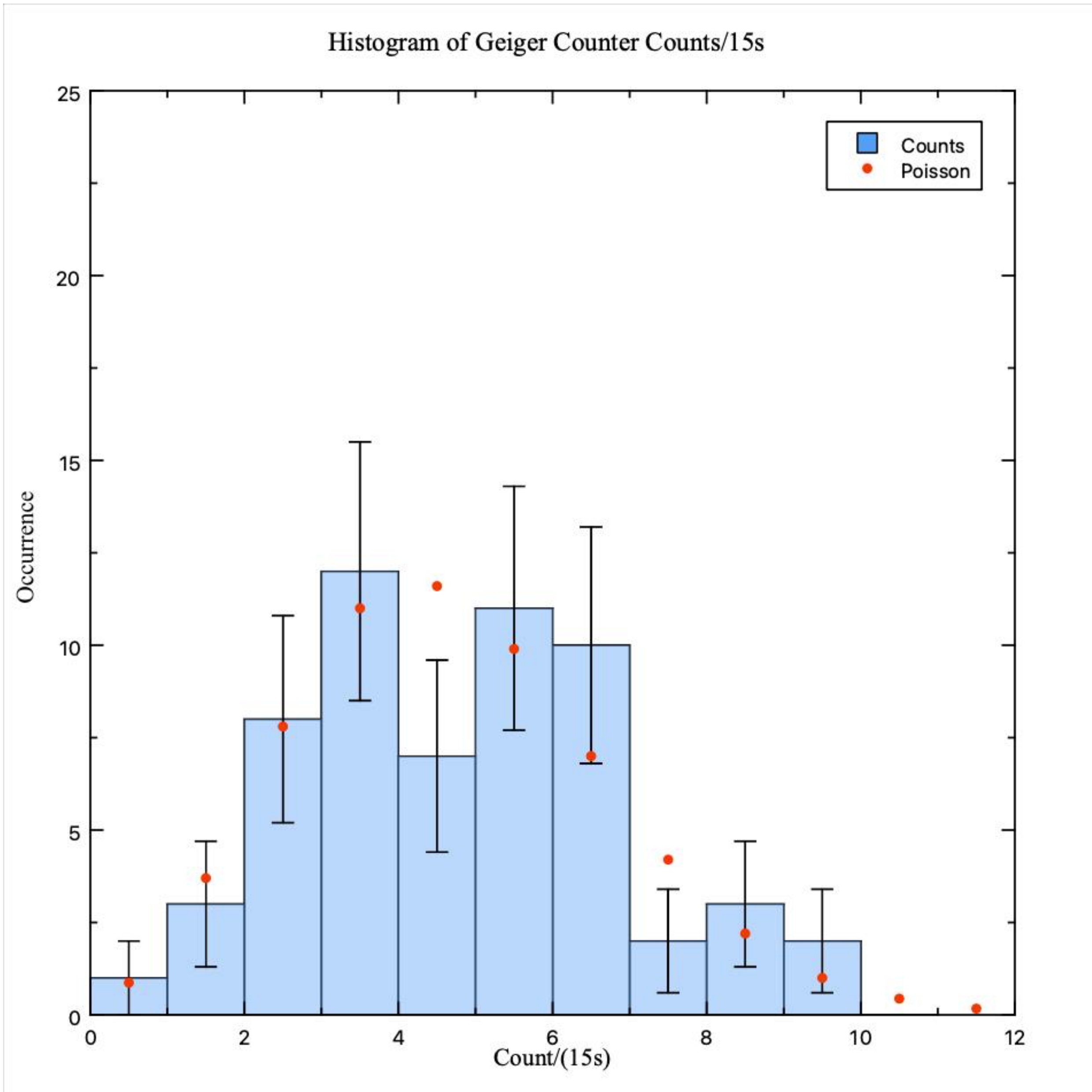

Figure 4: Histogram of 60 Geiger Counter Counts per 15s