

Uniform Distribution:

$$p(x) = \begin{cases} 1 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

Mean of Uniform Distribution:

$$\begin{aligned} \mu &= \int_{-\infty}^{+\infty} xp(x) \, dx = \int_{-\infty}^0 xp(x) \, dx + \int_0^1 xp(x) \, dx + \int_1^{+\infty} xp(x) \, dx \\ &= \int_{-\infty}^0 0 \, dx + \int_0^1 x \, dx + \int_1^{+\infty} 0 \, dx = \int_0^1 x \, dx = \left[\frac{x^2}{2} \right]_{x=0}^1 = \frac{1}{2} \end{aligned}$$

Variance of Uniform Distribution:

$$\begin{aligned} \sigma^2 &= \int_{-\infty}^{+\infty} p(x)(x - \mu)^2 \, dx = \int_{-\infty}^0 p(x)(x - \frac{1}{2})^2 \, dx + \int_0^1 p(x)(x - \frac{1}{2})^2 \, dx \\ &\quad + \int_1^{+\infty} p(x)(x - \frac{1}{2})^2 \, dx = \int_{-\infty}^0 0 \, dx + \int_0^1 (x - \frac{1}{2})^2 \, dx + \int_1^{+\infty} 0 \, dx \\ &= \int_0^1 (x - \frac{1}{2})^2 \, dx = \left[\frac{(x - \frac{1}{2})^3}{3} \right]_{x=0}^1 = \frac{1}{24} + \frac{1}{24} = \frac{1}{12} \end{aligned}$$