# LAB 2

The goal of this lab is to do experiments in which the statistical fluctuation is governed by either binomial or Poisson statistics as discussed in Lecture 2. Your lab report should contain four plots. All data points should have error bars indicated, except the second plot in part 1. In the past, the average grade for this lab is 30% lower because this lab is much more complicated (reminder: show your plot to the instructor as it became available) and there are lot of questions and hence it is easier to miss some questions.

## 1. Measurement of $\pi$

In this exercise we will determine a value for $\pi$ by throwing darts:

a) Determine $\pi$ by throwing a dart 100 or more times. Use an 8x11 inch sheet of paper with a circle inscribed inside a square as the dart board target (a drawing is attached to this lab instruction sheet). How is the value of $\pi$ related to the ratio of the circle's area to the square's area? Unless you are really an expert dart thrower, your dart tosses should be uniformly distributed throughout the square. Thus $\pi$ can be determined from the *probability* of a dart falling within the circle. Calculate the value of your $\pi$ and its uncertainty (please show the formula used).

b) Determine $\pi$ by writing a computer program. One method of determining $\pi$ using the random number generator is as follows:

    i)   Generate 2 random numbers each between [0,1]. Call one random number x, the other y.
    ii)  Assume that (x,y) represents the coordinates of a point inside a square with sides = 1. Note: the units used to measure the size of the sides of the square are arbitrary.
    iii) Assume that there is a circle inscribed in this square with radius = 1/2 and calculate if (x,y) falls inside the circle.
    iv) Repeat the above procedure keeping track of the number of (x,y) points generated (= dart tosses) and the number of times the point lies within the circle.
    v)  Once all the points have been generated calculate $\pi$.

Determine $\pi$ using runs of $4\times10^2$, $10^3$, $4\times10^3$, $10^4$, $4\times10^4$, $10^5$, $4\times10^5$, and $10^6$ points. For each case, calculate the measured $\pi$ and its uncertainty. Plot the measured values and their uncertainties vs. the number of points (use semilog scales). Also display the result from part a) in the same plot. Draw a horizontal line indicating the true value of $\pi$. Comment on the precision of the measurements vs. sample size. How many measurements have their error bars touching the horizontal line? Is this consistent with the expectation that 68% of the measurements should be within one standard deviation of the expectation? How is your measurement of $\pi$ with $10^6$ points compare with the expectation? Explain the reasoning behind your answer.

Determine $\pi$ using 25 trials of 100 points. For each trial calculate $\pi$ and make a histogram of your 25 trials. Calculate the average value of $\pi$ and its standard deviation using your 25 trials. How is the uncertainty of $\pi$ estimated from part a) compare with the standard deviation? Indicate the result you obtained from tossing the darts on the histogram. Using the histogram as a probability distribution for obtaining $\pi$ from 100 dart tosses, comment on your expertise as a dart player.

## 2. Binomial Distribution

Take a dozen six-sided dice and toss them 50 times. For each toss record the number of times (e.g.) a two comes up.

a) Calculate the average and variance.

b) Make a histogram of your results (i.e. the number of times there were no two's in a toss, one two, etc.)

c) For each data point in the histogram, estimate its uncertainty (explain your calculation), e.g. what is the uncertainty on the number of times there were one two in a toss?

d) Compare your results with that expected from a binomial distribution. Superimpose the theoretical expectation at each data point (not as a curve). How are your measured average and variance compare with the theoretical expectations, i.e. what are the measured and expected values?

e) Compare your results from above with a Poisson distribution assuming the same average number of two's as above. Superimpose the Poisson expectation at each data point (not as a curve). How are your measured average and variance compare with the theoretical expectations, i.e. what are the measured and expected values? How would you change the dice and the experiment if you wanted to generate a Poisson distribution from the toss of dice?

## 3. Poisson Distribution

The rate of cosmic-ray particles passing through a Geiger counter is governed by Poisson statistics. The counter has a wire strung inside a chamber filled with gas. The wire (anode) is held at high voltage (~kV) with respect to the chamber (cathode). A traversing particle knocks electrons off a few gas molecules. The electrons accelerate toward the wire under the strong electric field, resulting in an avalanche of electrons. The electronic signal produces the well-known "click". The signal is also being sent to an electronic device that uses the LabView program, "Geiger with Timer", to count the number of particles traversing the chamber. Turn the switch on the Geiger counter to the "Audio" position. You should hear occasional "click", indicating that the counter is working.

a) Double click the "Geiger with Timer" icon in C:\LabView to start the LabVIEW program. You should see the following:
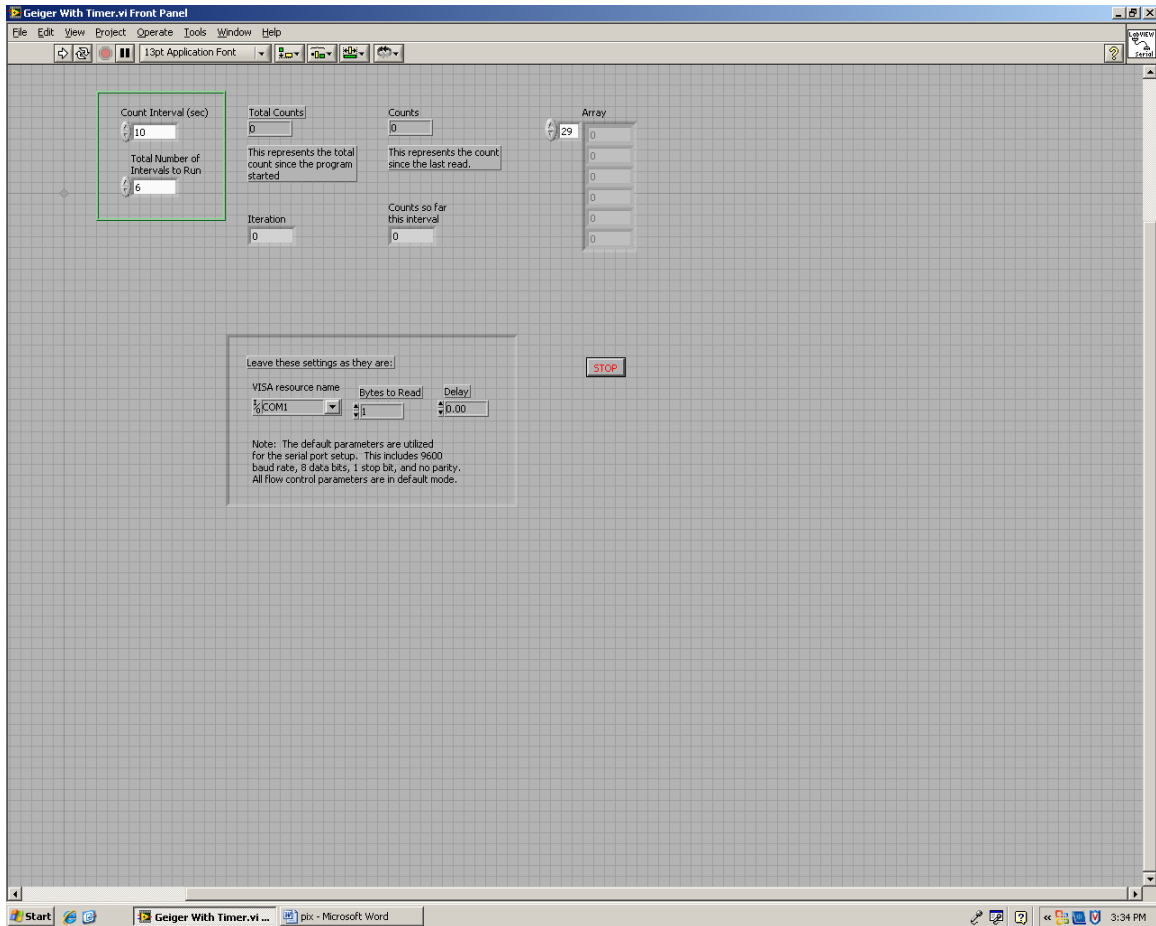
b) Set the "Total Number of Intervals to Run" to 60 and the "Count Interval (sec)" to 15. This instructs the program to record the number of counts in 15-second intervals for the duration of 15 minutes. Click the "OFF" button to turn on the program and then click the "→" button at the top of the window to start the counting. After 15 minutes, the program asks you for the name of the file to store the count information. If necessary, you can stop the counting by clicking on the icon that looks like a stop sign.

c) Use the data stored in the file to histogram the number of counts per 15 seconds interval. Calculate the uncertainty of each data point (explain your calculation) and display the uncertainty on the histogram. Calculate the mean of the histogram and use it to calculate the expectation at each data point based on Poisson statistics. Superimpose the results on the histogram (not as a curve). Is the histogram consistent with the Poisson expectations? How many 15-sec intervals have zero counts and how many have 10 or more counts? What are your Poisson predictions? Is your observation consistent with the predictions?