计算机辅助手术讲座（16）
Image Guided Surgery (16)

Level Set Methods

水平集算法

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WHAT IS LEVEL SET?

- Level set is a set of points with same height such as water level or geodesic line
- Level set method as a front propagation theory was first proposed by Sethian in 1982
- In 1995, Malladi introduced it to image analysis domain, to find image boundary

WHAT IS SEGMENTATION?

- Separate object from background
- Broadly speaking, it is to use a model whose boundary representation is matched to the image to recover the object of interest.
- Or simply, it is object recover from raw data

HOW SNAKE WORKS?

- Initialize a guess contour clicking points in image
- Digitize the contour
- Move the contour under the internal and external forces
- Problems in snake:
  - ➢ Sensitive to initial guess of shape
  - ➢ Difficult to recover complex structure
  - ➢ Difficult to track multi-object automatically

FRONT PROPAGATION: ANOTHER UNDERSTANDING OF SNAKE

- A closed interface moving in a plane
- Or more broadly, a front moves from initial contour to image boundary along its normal vector with a speed of **F**
- Two different representations in front:
  - Parametric representation
  - Level set (or geodesic) representation

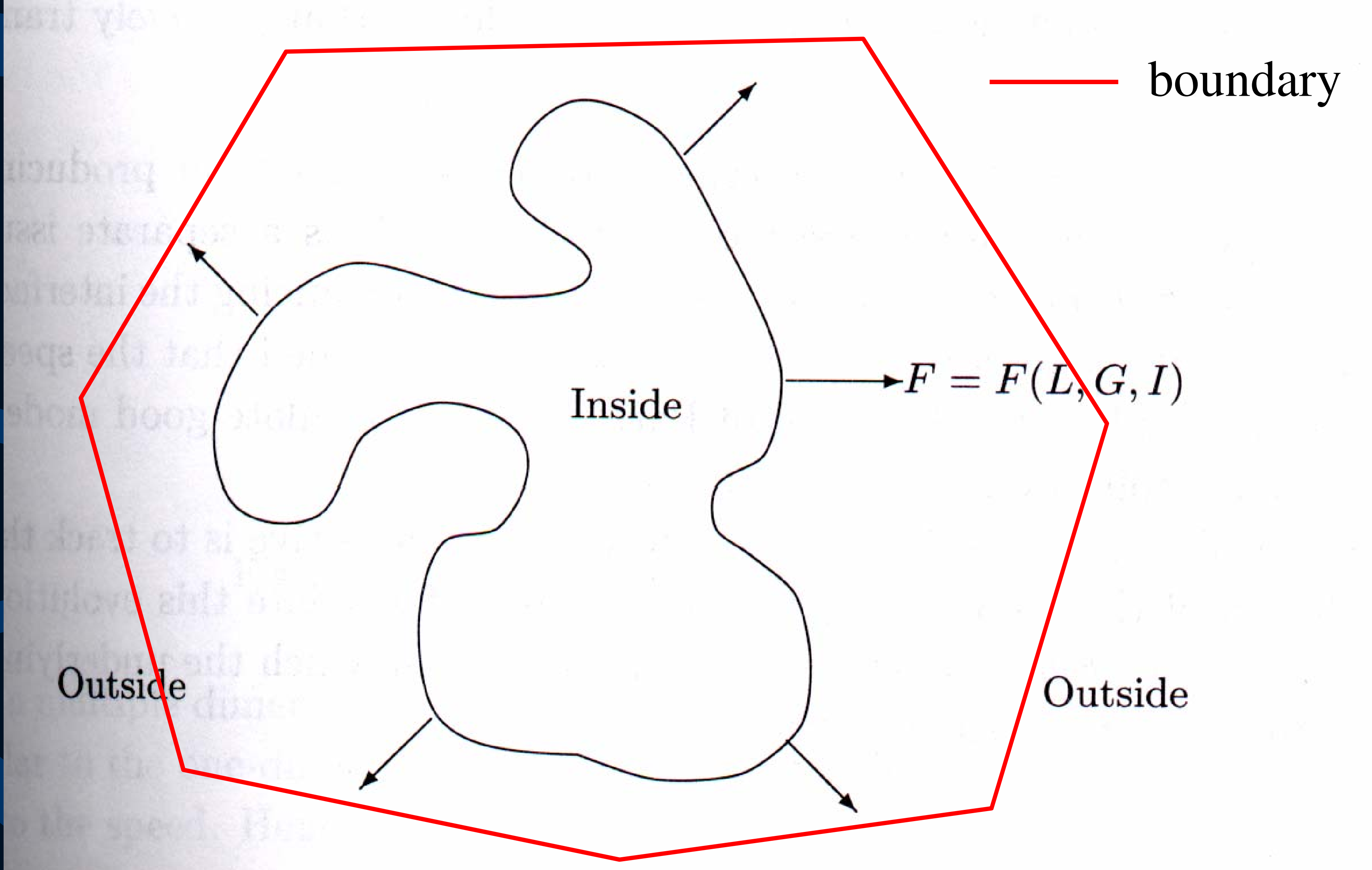


Fig. 1.1. Curve propagating with speed F in normal direction.

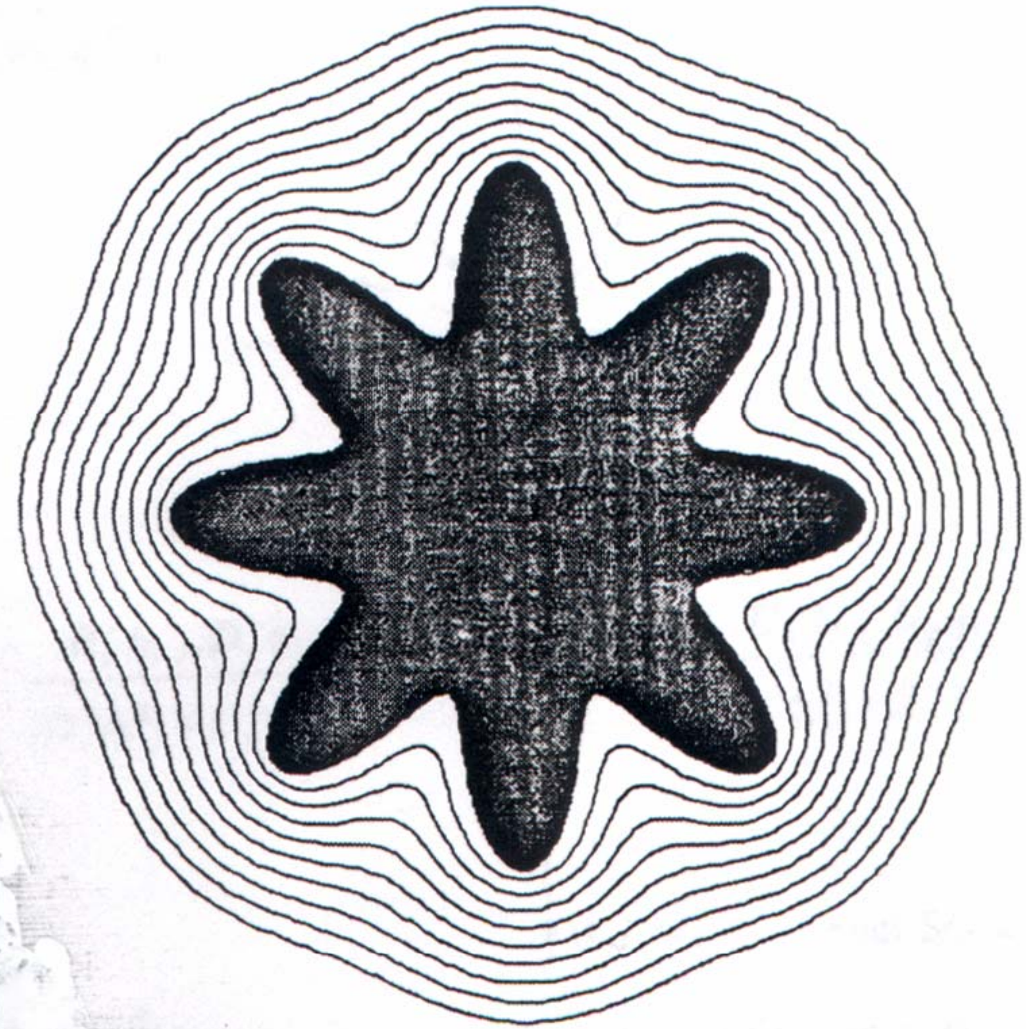
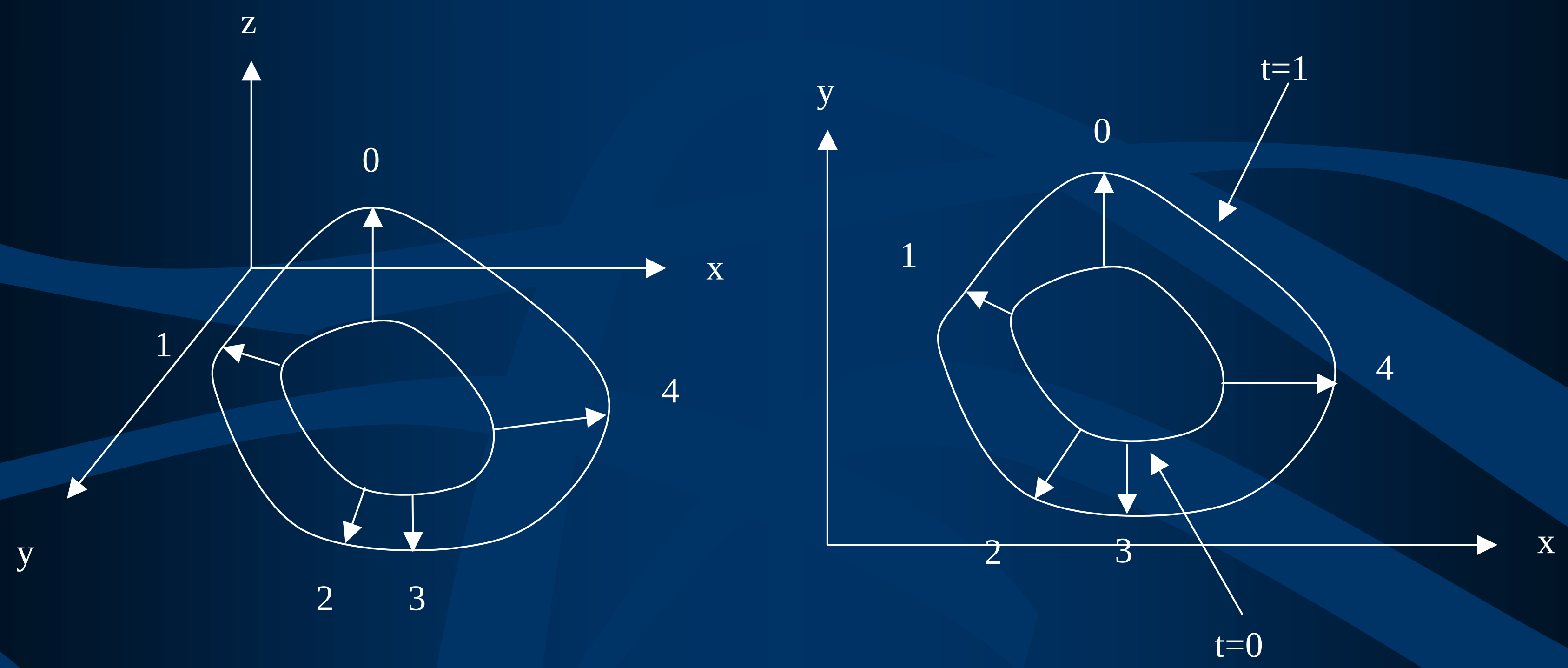


FIG. 5. Expanding star, $F(K) = 1 - 0.01\ K$.

FRONT REPRESENTATION

- Drawbacks in 2D parametric function:
  - The function definition dependent on the different objects
  - t is not a single value function when the front moved back and forth
  - Difficult to express the complex curve
- Level set: use one dimension higher function to represent the curve.

LEVEL-SET SURFACE, f(x,y,z)=0



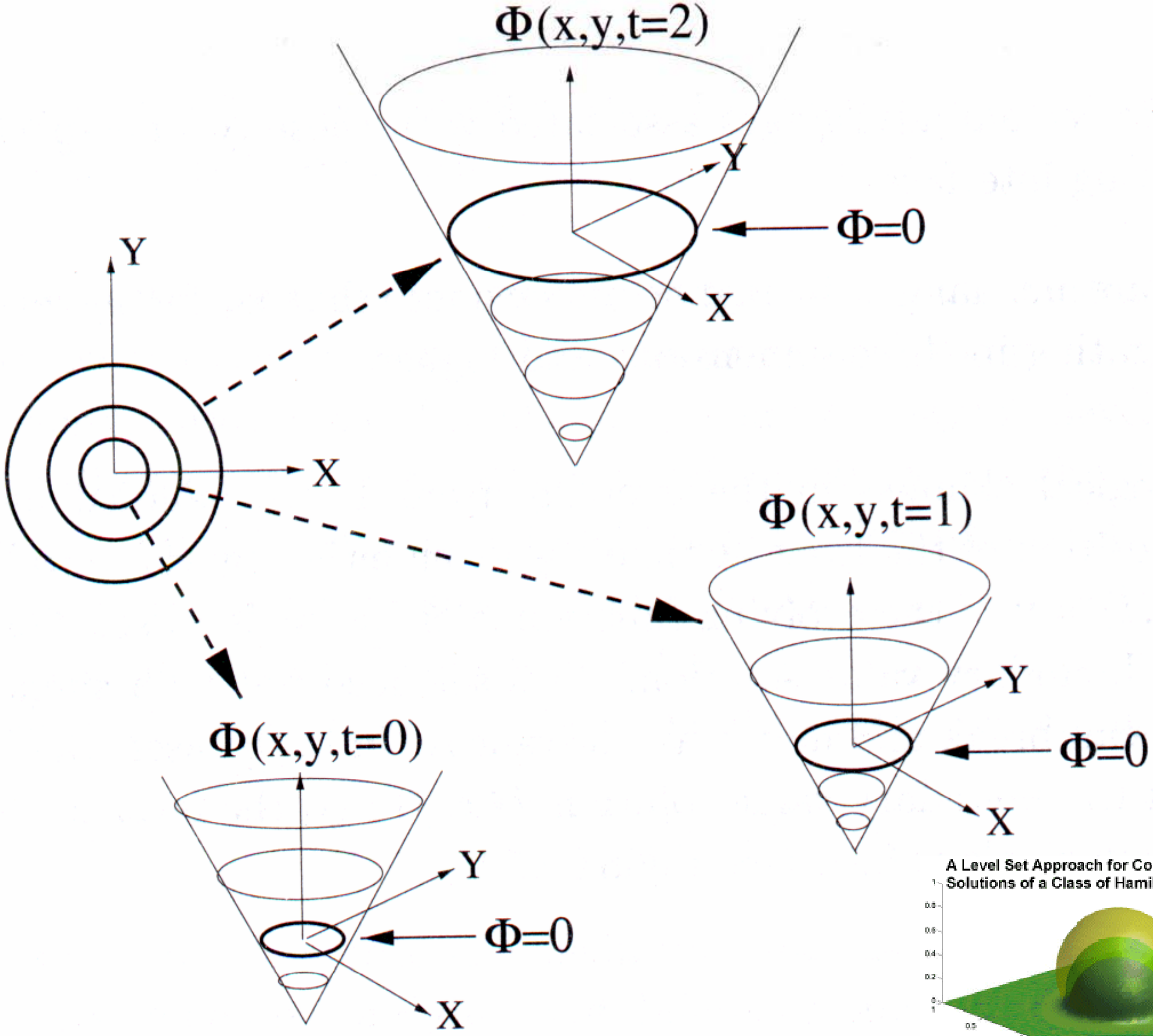
$$z_i' = \pm \left| (x_i, y_i, 0), (x_i', y_i', 1) \right|, i = 0,1,2,3,4$$

$(x_i', y_i', z_i'), i = 0,1,2,3,4$ are the points on level-set surface

FRONT: ZERO LEVEL SET

- To avoid complex 3D contour, we always suppose current contour has zero height. This is called *zero level set*.
- Dynamic coordinate system:

  The plane of *Oxy* is defined dynamically overlapped with the evolving front.



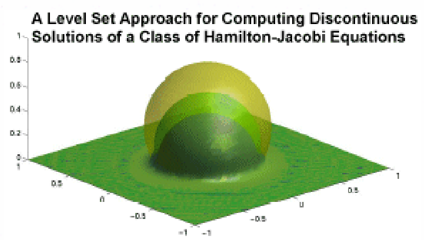
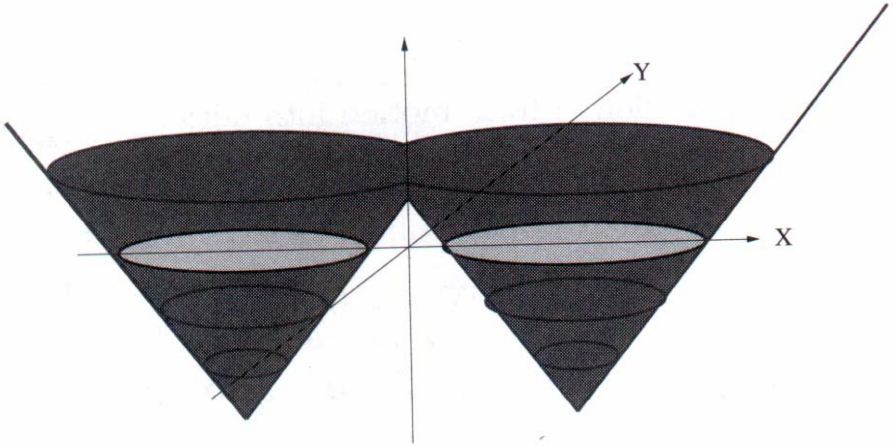
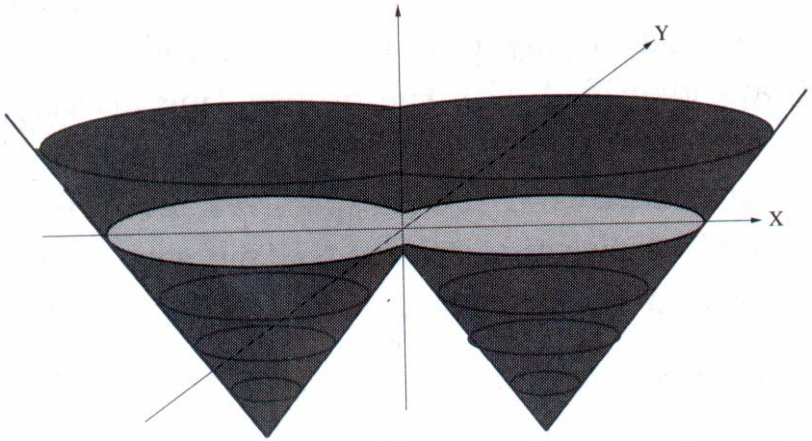


Fig. 1.5. Transformation of front motion into initial value problem.



The level set surface ϕ (dark gray):
Two separate initial fronts (in light gray).



Later in time: the interface topology has changed, yielding a single curve as the zero level set.

Fig. 1.6. Topological change.

DETERMINATION OF IMAGE BOUNDARY

- Snake:
  - Determine a energy function *C* so that the initial contour can fit to boundary when the C is minimized.
- Level set method:
  - Solve a Partial Differential Equation (PDE) , in which the interface is a zero level set and constrained by the initial contour.

HAMILTONG-JACOBI EQUATION

- Propagating hyper-surface: $\phi(\mathbf{X}(t),t)=0$
- By using the chain rule, we have

$$\phi_t + \sum_{i=1}^{N} \phi_{x_i} x_{i_t} = 0 \qquad (1)$$

- Because

$$\sum_{i=1}^{N} \phi_i x_{i_t} = (\phi_{x_1}, \phi_{x_2}, \cdots, \phi_{x_N}) \cdot (x_{1_t}, x_{2_t}, \cdots, x_{N_t}) \qquad (2)$$

Hamilton-Jacobi equation

$$\phi_t + F(\mathbf{X}(t)) \cdot \left| \nabla \phi \right| = 0$$

SWALLOWTAIL REMOVAL

- In front propagation, a swallowtail problem in corner may appear when we let the boundary pass itself
- Huygens' principle construction or a entropy satisfying solution, *i.e.*, we only expand the boundary which consists of the points located a distance, *t* ,from the initial curve

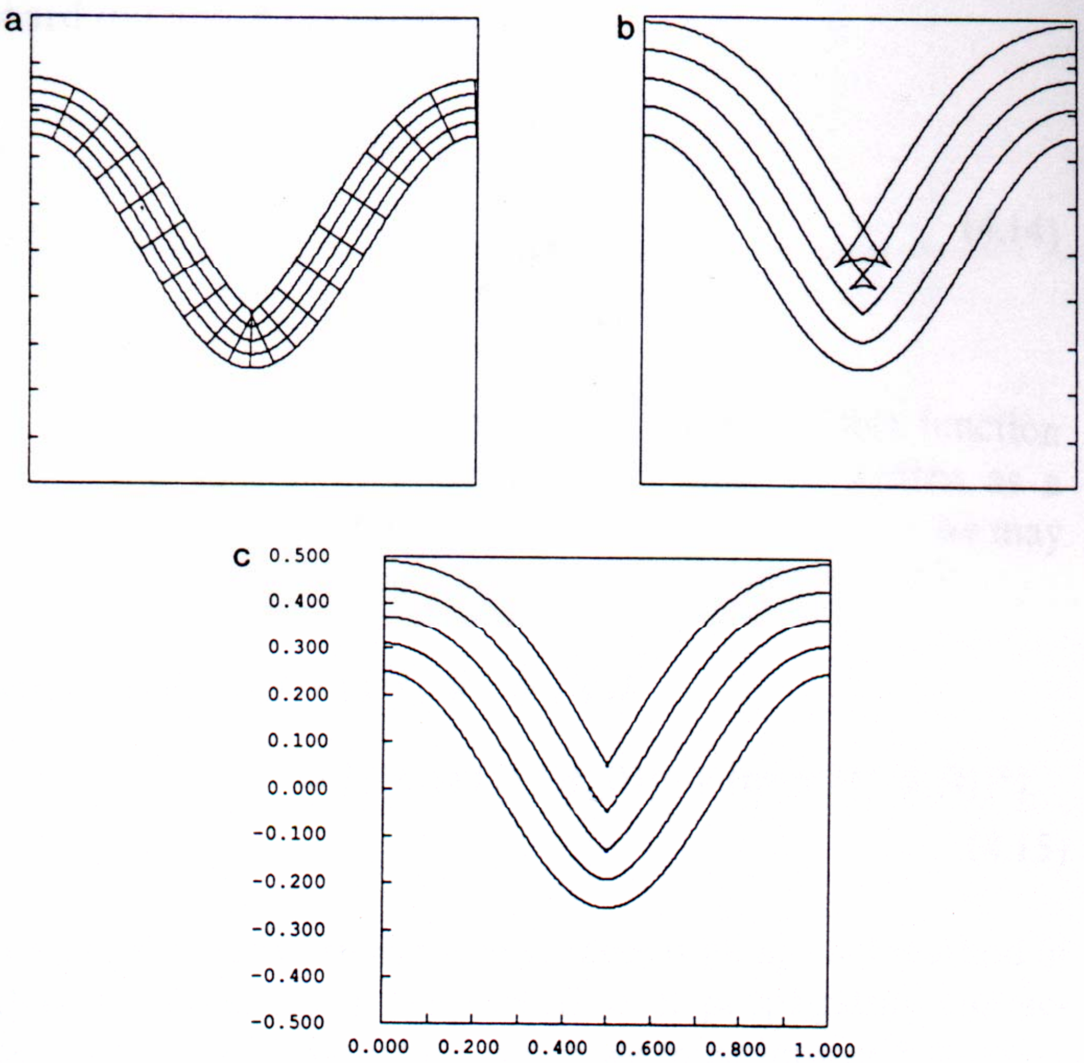


FIG. 4. Corner formation and the entropy condition: (a) propagating curve until singularity forms; (b) entropy-violating swallowtail solution; (c) entropy-satisfying solution from Huyghen's construction.

NUMERICAL APPROXIMATION

- Suppose we use a uniform mesh of spacing h and a time step of Δt, the Hamilton-Jacobi equation will be

$$\frac{\phi_i^{n+1} - \phi_i^n}{\Delta t} + F \cdot \nabla_i \phi_i^n = 0$$

where $\nabla_i \phi_i^n$ is the appropriate finite difference operator for the spatial derivative

HOW TO DETERMINE THE SPEED?

- The normal vector speed $F=F(L,G,I)$ is determined by
  - Local properties such as curvature and normal direction
  - Global properties of the front like PDE
  - Independent properties. For instance an underlying fluid velocity. $R(x,y)= 2\ I(x,y)-1$.

DETERMINATION OF SPEED

- We invoke the ENTROPY CONDITION and HYPERBOLIC CONSERVATION LAWS:

$$F(\mathbf{X}(t),t) = F_0 + F_1(k)$$

where K is the curvature of hyper-surface,

F_0 is a constant inflation term and $F_1(K)$ is a term depending on the geometry of front

DEFINITION OF SPEED TERMS

- For example, we can choose

  $$F(\mathbf{X}(t),t) = F_0 + F_1(k)$$
  $$= \pm 1 - \varepsilon k$$

  where ε is a constant acted as an advection term while the uniform expansion speed, 1 (or -1), corresponds to the inflation (or shrink) force.

DEMO OF LEVEL SET MOVING

FRONT STOPPING CRITERION

- In order to let the front halting on the boundary, we must define such a speed that acts as a stopping criterion for this speed function by multiplying the term:

$g_I(x_{i,j})$, where $x_{i,j}$ is the gradient at (i, j)

DEFINITION OF STOPPING CRITERIONS

- Different definitions of stopping term:

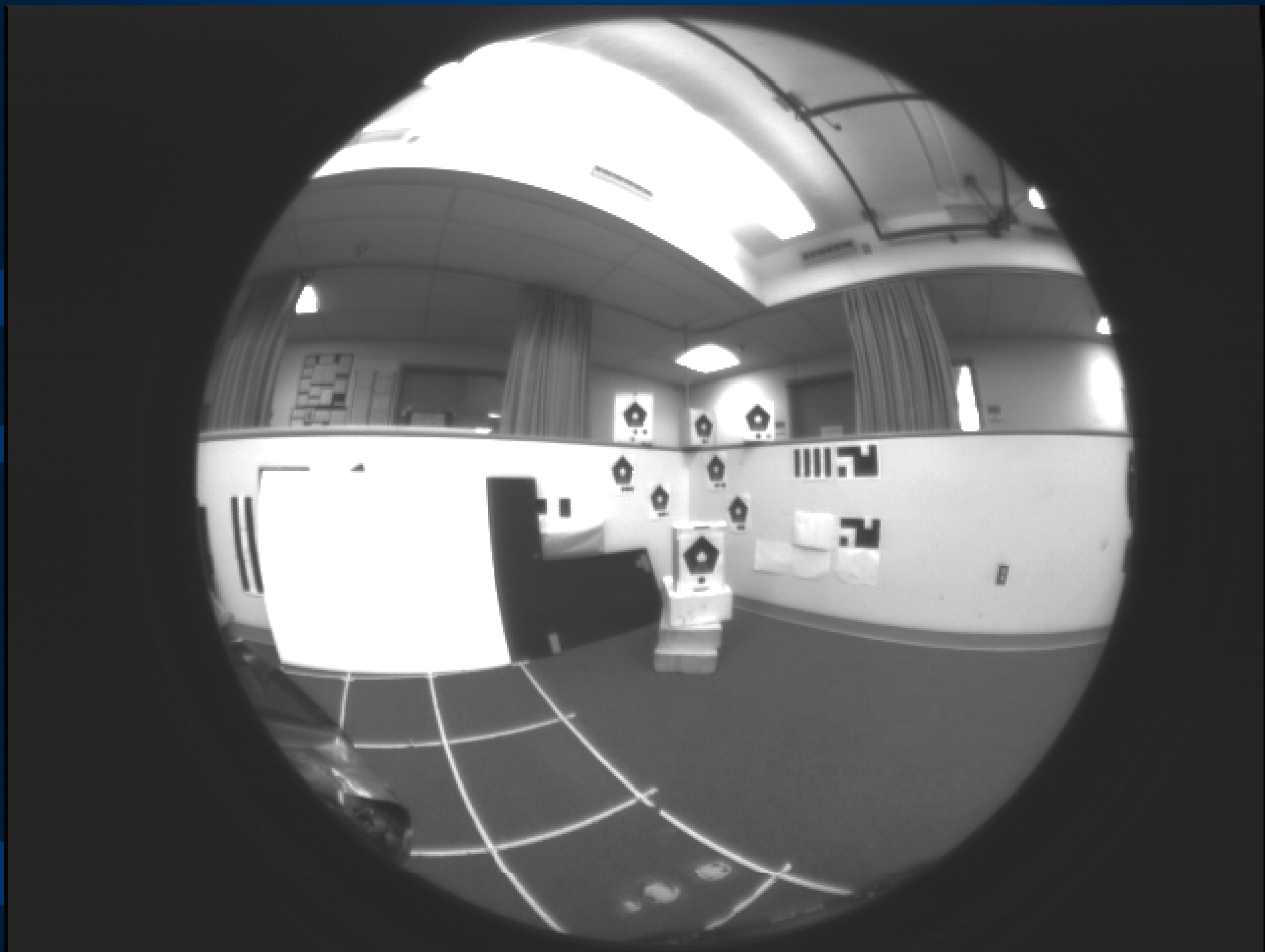
$$1)\ g_I(x_{i,j}) = \frac{1}{1 + x_{i,j}}$$

$$2)\ g_I(x_{i,j}) = \frac{1}{1 + x_{i,j}^2}$$

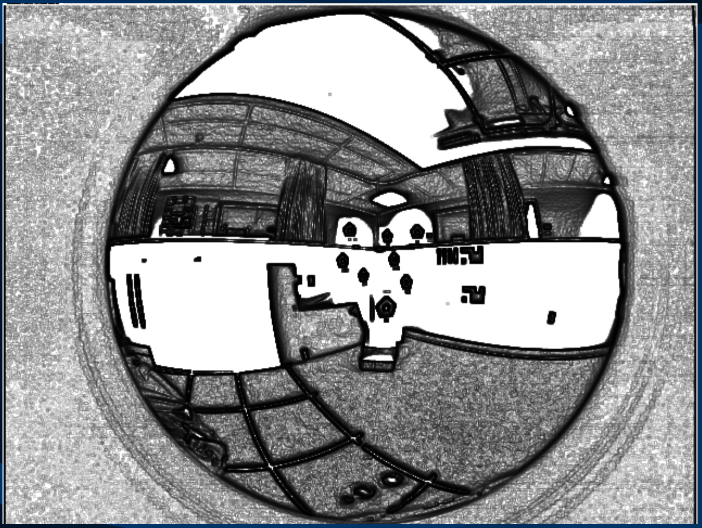
$$3)\ g_I(x_{i,j}) = e^{-x_{i,j}}$$

where $x_{i.j} = \left|\nabla G_\sigma * I(i,j)\right|$

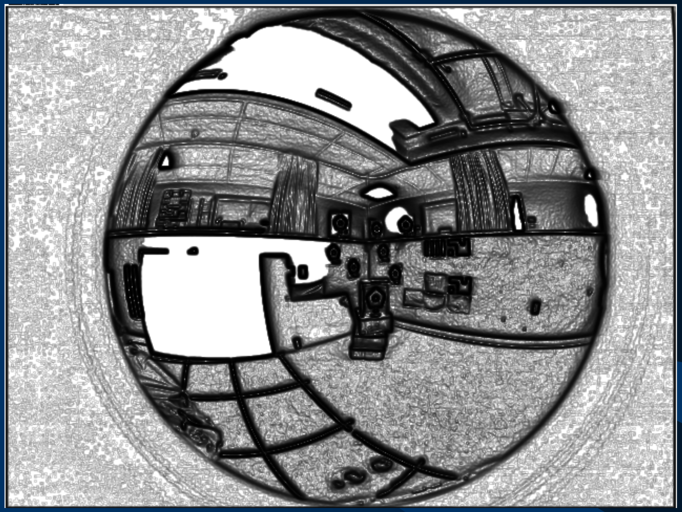
ORIGINAL IMAGE



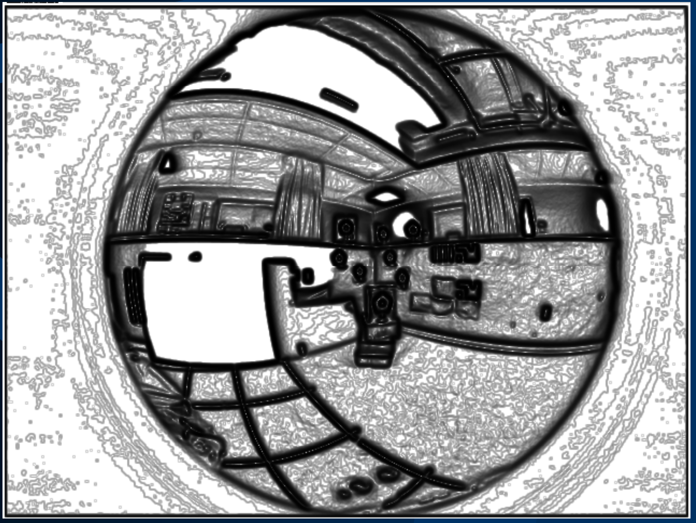
EFFECT OF THE VALUES OF SIGMA



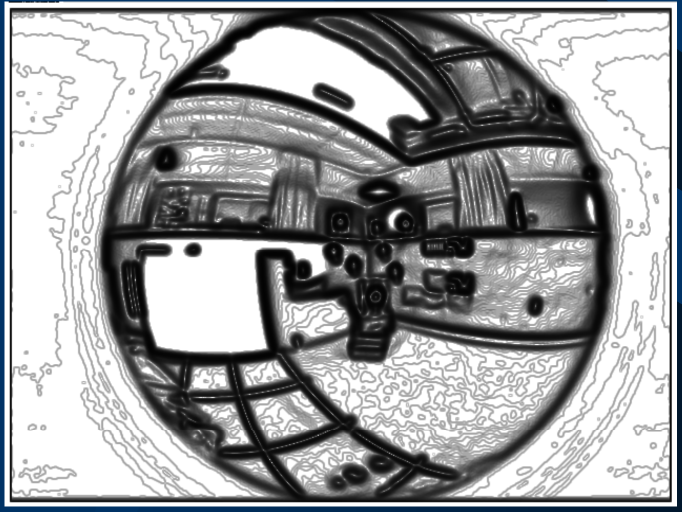
(a) sigma=0.3



(b) sigma=0.5

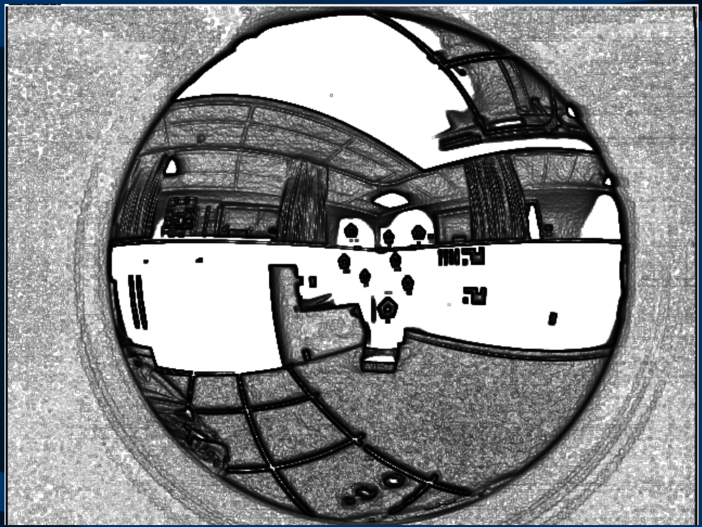


(c) sigma=1.0

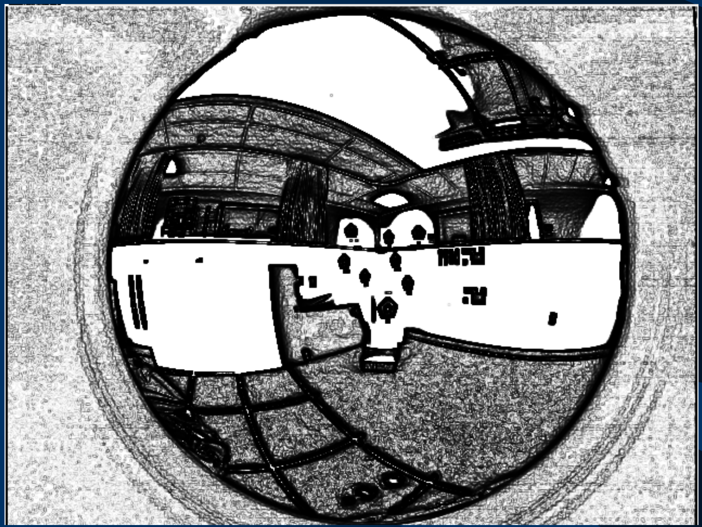


(d) sigma=2.0

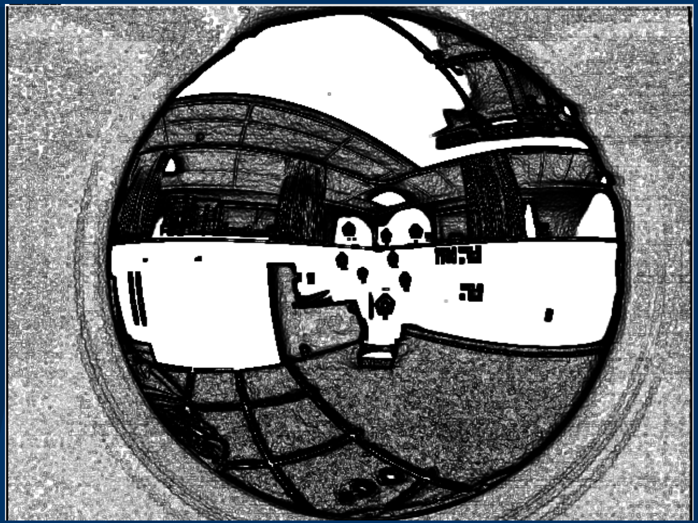
IMAGE-BASED SPEED COMPARISON



(a) Reciprocal function(N=1)



(b) Reciprocal function(N=2)

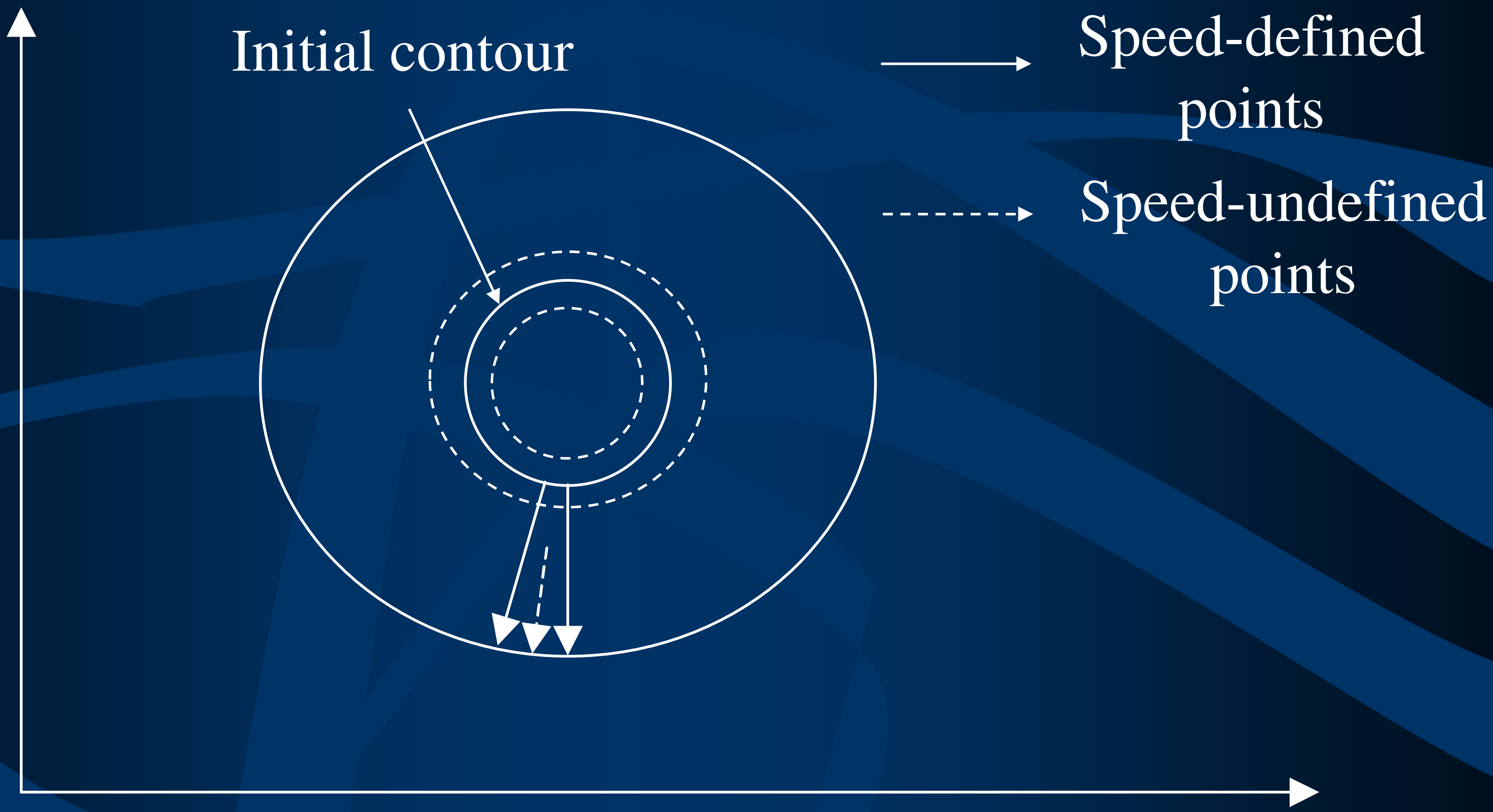


(c) Exponential function

EXTENDING SPEED FUNCTION

- The speed is locally defined along the boundary but not globally defined
- Requirements for extension:
  - Level set moving under this speed function cannot collide
  - Computation efficient

SPEED EXTENSION PROBLEM



EXTENDING SPEED FUNCTION

- There are different ways to extend the speed function to the neighboring level sets:
  - Global extension: nearest speed point
  - Global extension with re-initialization
  - Narrow-band extension
  - Narrow-band extension with re-initialization

NUMERICAL SOLUTION OF HAMILTON-JACOBI EQUATION

- We can get the entropy-satisfying weak solution of Hamilton-Jacobi equation by the following iteration:

$$\phi_i^{n+1} = \phi_i^n - \Delta t \left[\left\{ \left(\max(D_x^- \phi_i, 0) \right)^2 + \left(\min(D_x^+ \phi_i, 0) \right)^2 \right\}^{1/2} - F \, \nabla \phi_i^n \right]$$

$$where \qquad D_x^- \phi_i = \frac{\phi_i^n - \phi_{i-1}^n}{\Delta x}, \quad D_x^+ \phi_i = \frac{\phi_{i+1}^n - \phi_i^n}{\Delta x}$$

NUMERICAL SOLUTION OF HAMILTON-JACOBI EQUATION

- Similarly, in 2-D case, the solution is

$$\phi_{ij}^{n+1} = \phi_{ij}^{n} - F_A \Delta t \{ \left(\max(D_x^- \phi_{ij}, 0) \right)^2$$

$$+ \left(\min(D_x^+ \phi_{ij}, 0) \right)^2 + \left(\max(D_y^- \phi_{ij}, 0) \right)^2$$

$$+ \left(\min(D_y^+ \phi_{ij}, 0) \right)^2 \}^{1/2} - \Delta t F_G \left| \nabla \phi_{ij} \right|$$

where $$D_x^- \phi_{ij} = \frac{\phi_{ij}^n - \phi_{i-1j}^n}{\Delta x}, \quad D_x^+ \phi_{ij} = \frac{\phi_{i+1j}^n - \phi_{ij}^n}{\Delta x}$$

$$D_y^- \phi_{ij} = \frac{\phi_{ij}^n - \phi_{ij-1}^n}{\Delta y}, \quad D_y^+ \phi_{ij} = \frac{\phi_{ij+1}^n - \phi_{ij}^n}{\Delta y}$$

FINDING THE FRONT, *X(t)*

- Given a cell of (i,j), if

$$\max(\phi_{i,j}, \phi_{i+1,j}, \phi_{i,j+1}, \phi_{i+1,j+1}) < 0 \quad \text{or}$$

$$\min(\phi_{i,j}, \phi_{i+1,j}, \phi_{i,j+1}, \phi_{i+1,j+1}) > 0$$

the cell cannot contain the front ***X****(t)*

- Otherwise, find the entrance and exit points by linear interpolation which is one of our approximation to ***X****(t)*
- Collection of all such line segments consists of our approximation to ***X****(t)*

INNER (HOLE) BOUNDARY SEGMENTATION

- Temporarily relax the stop criterion and allow the front to move past the outer boundary
- Once it occurs, the stopping criterion is turned back on.
- Resume the level set front evolving

FAST MARCHING METHODS

- In level set methods, in order to avoid the missing of boundary, a very small time step should be adopted, leading a large number of iterations.
- Fast marching methods can be used to greatly accelerate the initial propagation from the seed structure to the near boundary

LEVEL SET SEGMENTATION ALGORITHM

1: Initialize a contour $\boldsymbol{X_0}$

2: Calculate the speed along $\boldsymbol{X_0}$

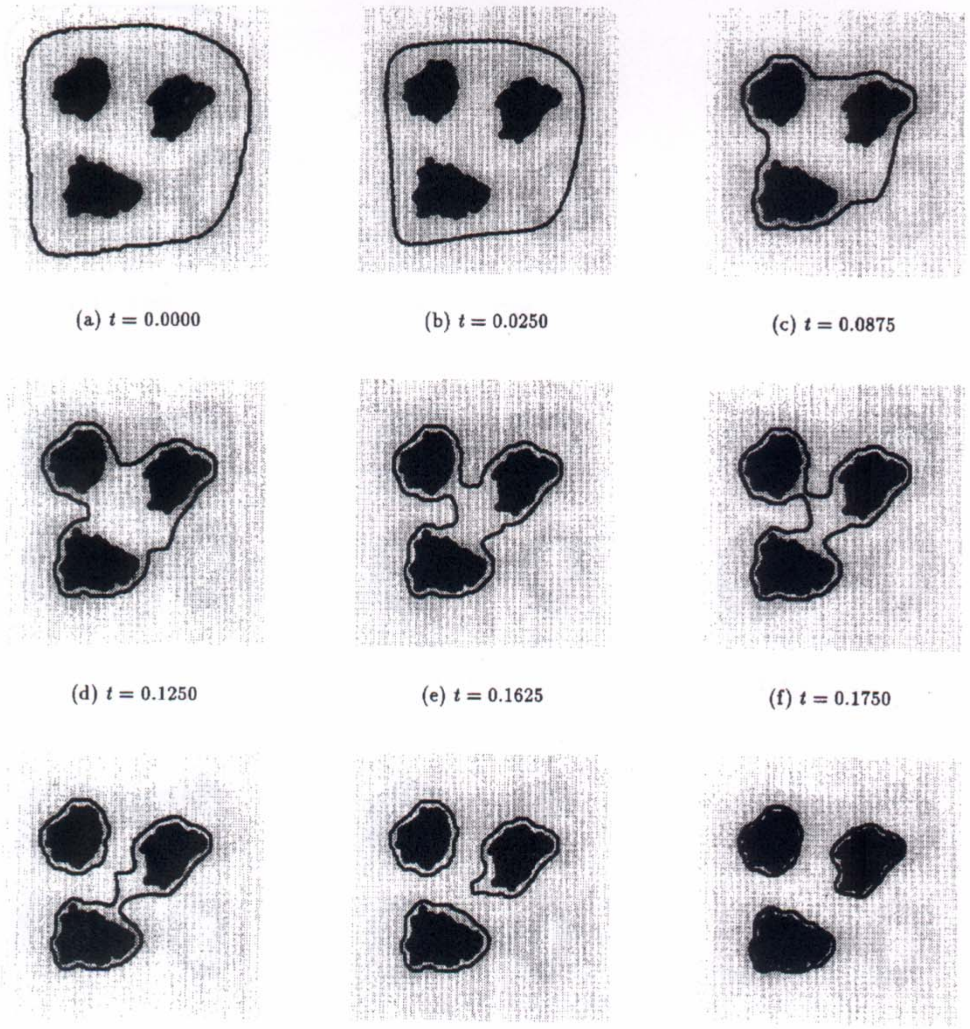
3: Extend the speed calculation

4: Level set function calculation

5: Find the evolving front

6: If speed is near 0, stop. Otherwise go to Step 2

7: If no front point moved, end the segmentation



(a) $t = 0.0000$ (b) $t = 0.0250$ (c) $t = 0.0875$

(d) $t = 0.1250$ (e) $t = 0.1625$ (f) $t = 0.1750$

(g) $t = 0.1875$ (h) $t = 0.2000$ (i) $t = 0.2500$

Fig. 12. Topological split: A single instance of the shape model splits into three instances to reconstruct the individual shapes. Computation was done on a 64×64 mesh with a time step $\Delta t = 0.00025$.

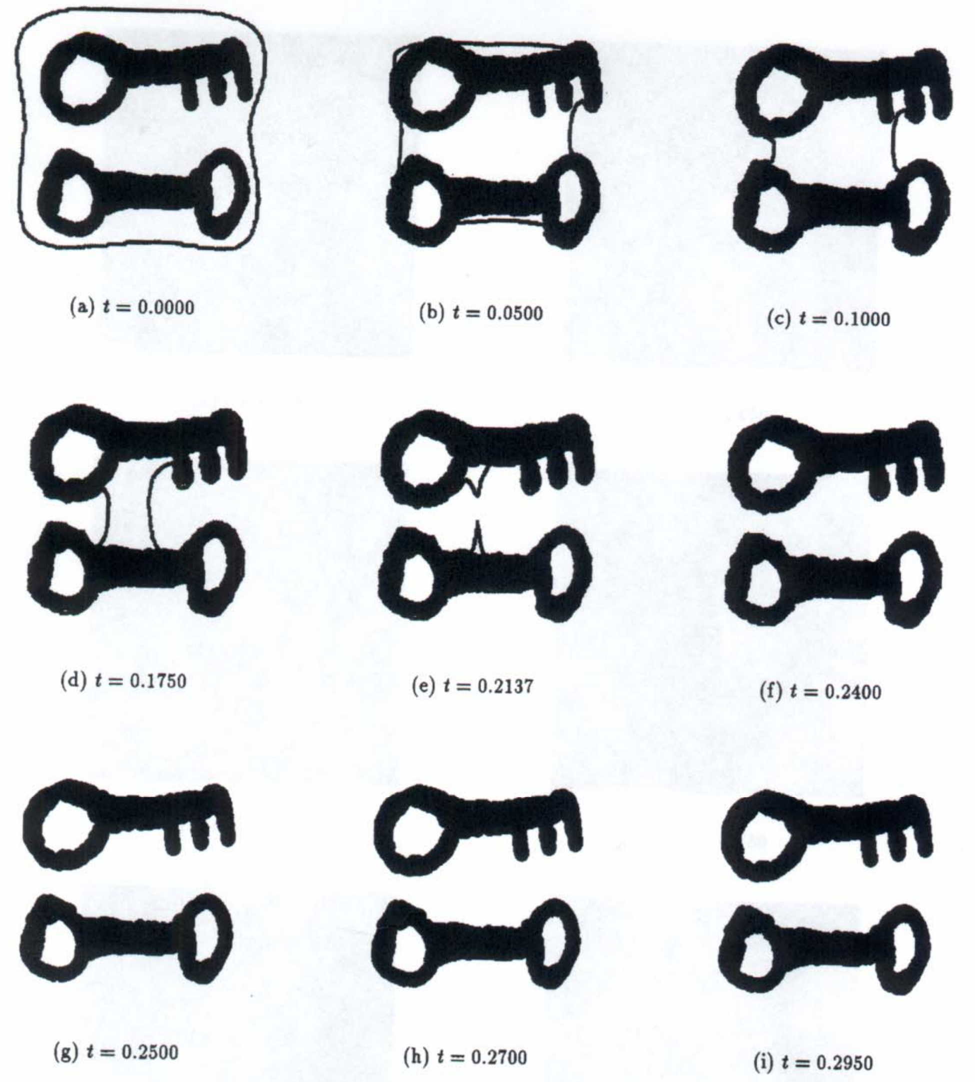
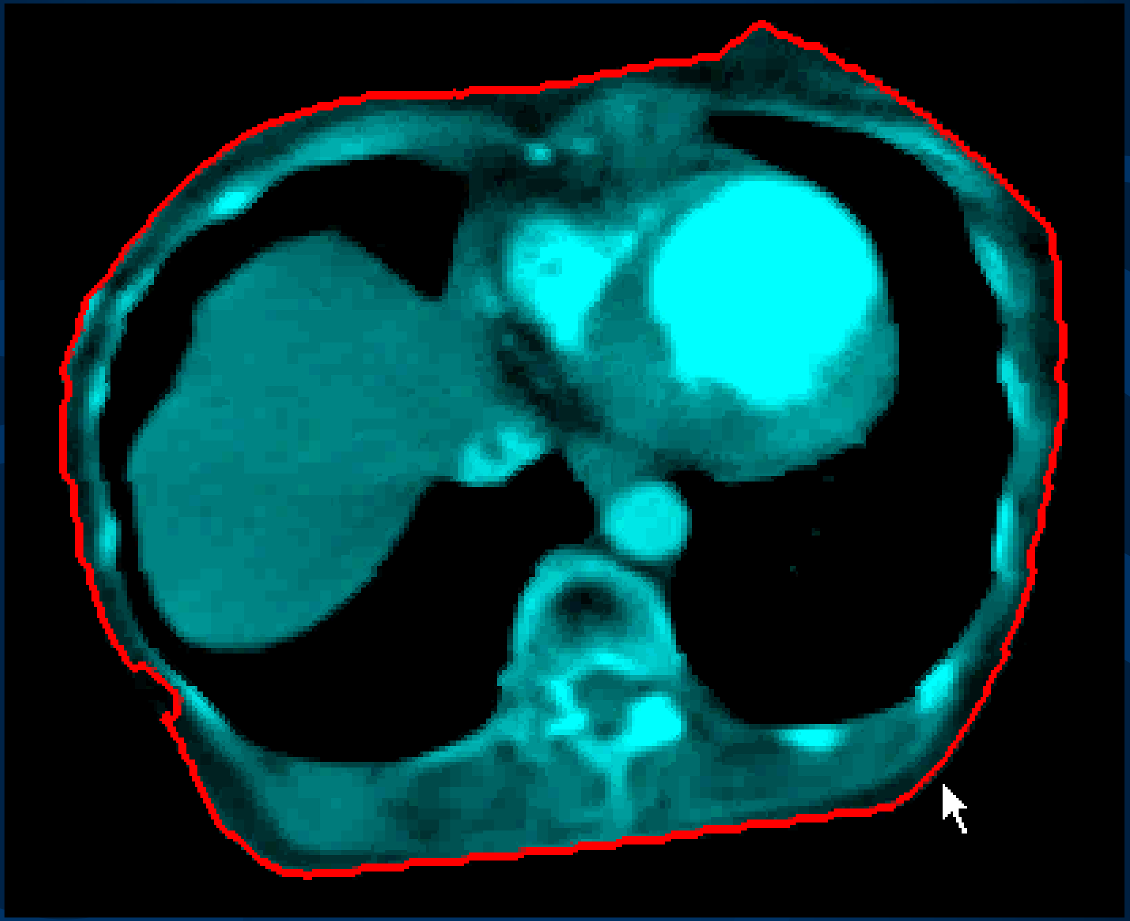


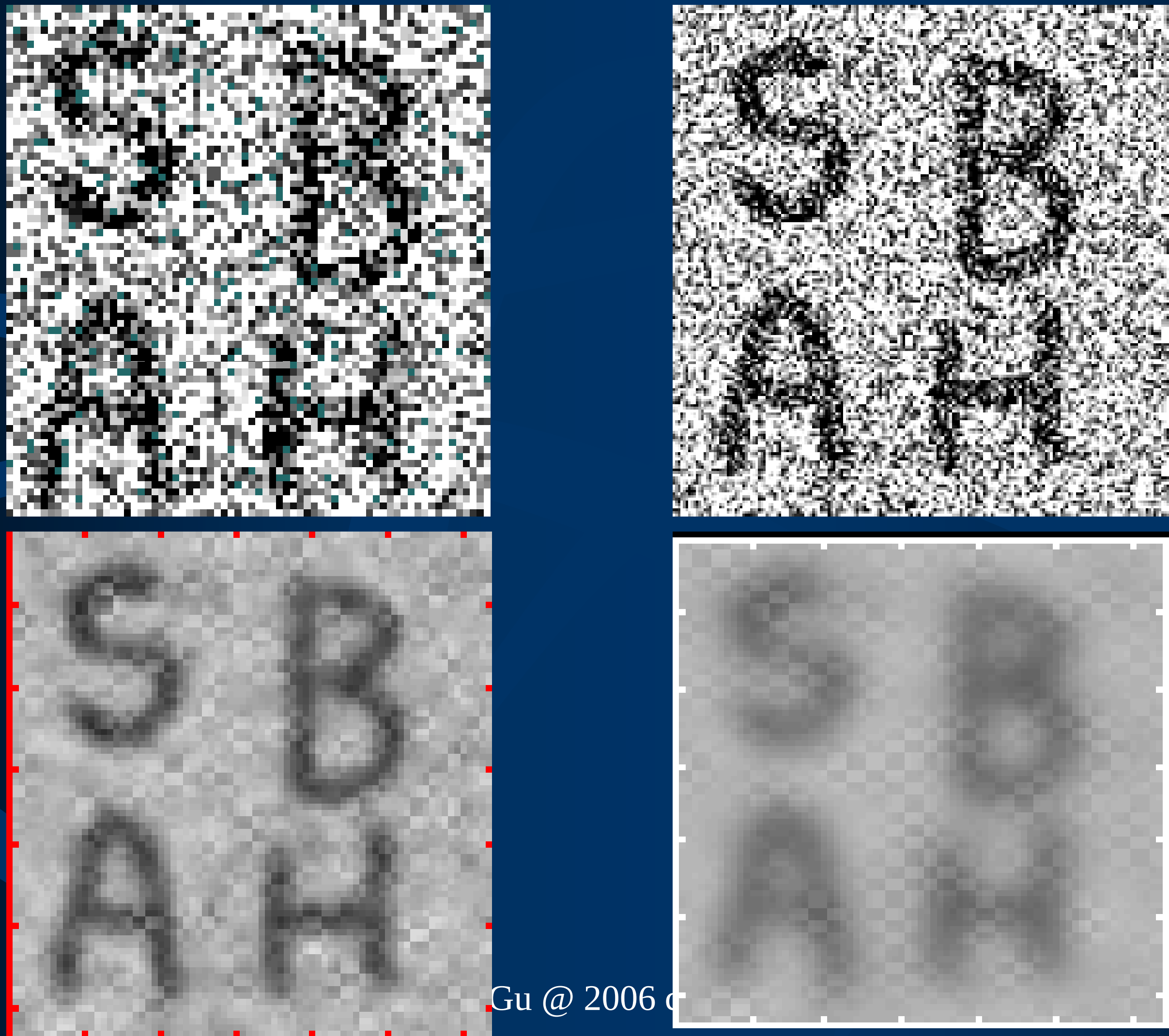
Fig. 13. Shapes with holes: A two-stage scheme is used to arrive at a complete shape description of both simple shapes and shapes with holes. Computation was performed on 128×128 grid and the time step Δt was set to 0.00025.

ARTERY BOUNDARY TRACKING

CONTOUR DETECTION BY CLICKING



NOISE REMOVAL WITH EDGE PRESEVING



ITK

- **LevelSetCurvatureFunction (itk)**
- **LevelSetFunction (itk)**
- **LevelSetFunction::GlobalDataStruct (itk)**
- **LevelSetFunctionBase (itk)**
- **LevelSetImageFilter (itk)**
- **LevelSetNeighborhoodExtractor (itk)**
- **LevelSetNode (itk)**
- **LevelSetTypeDefault (itk)**
- **LevelSetVelocityNeighborhoodExtractor (itk)**

CONCLUSION

- Level set is a new methodology for segmentation and different application. It has the following features:
  - Insensitive to the initial contour guess
  - Fast and easy to be extended to high dimension
  - Complex topological structure
  - Can be processed in parallel

CONCLUSION

- Open problems
  - Sensitive to sharp corners, cusps and topological changes
  - Segmentation result greatly depending on the design of stopping criteria
  - Complexity in speed extension
  - … …

BOOKS

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WEBSITES

- //math.berkeley.edu/~sethian/level_set.html
- http://www.math.ucla.edu/~sjo/
- http://www.levelset.com/lss.html

PAPERS

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Discussion