计算机辅助手术讲座（6）
Image Guided Surgery (6)

数学形态学及其二值运算

Mathematical morphology and it′s binary operations

顾 力栩 (*Lixu Gu*)

上海交通大学 Med-X研究院

2009.12

Mathematical Morphology

- A methodology for the quantitative analysis of spatial structures which was initiated by G.Matheron and J.Serra at Paris School of Mines.
- It aims at the analyzing the shape and the forms of the objects.
- The initial theoretical work was done by Hadwiger [1957], and Serra[1982] produced the first systematic theoretical treatment of the subject.

Mathematical Morphology

- Originally it was applied to analyze images from geological or biological specimens. But its powerful function have propelled its widespread diffusion and adoption by many academic and industry groups as one of the dominant image analysis methodologies.
- It's mathematical origins stem from set theory, topology, lattice algebra, random functions, stochastic geometry, etc.
- Extremely useful, not yet often used

Application Areas

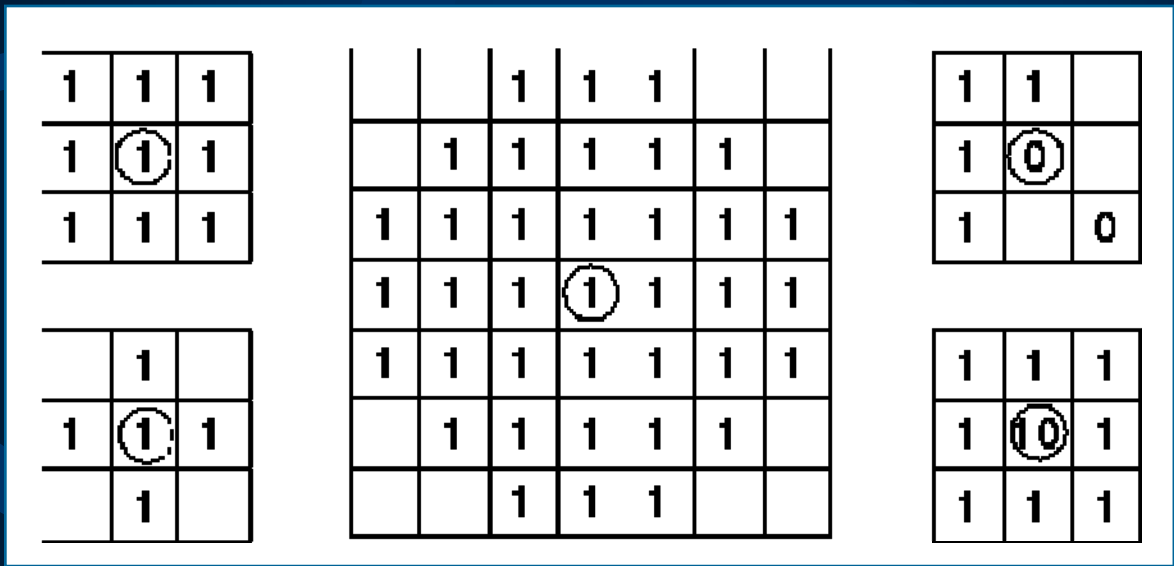
- image enhancement
- image segmentation
- image restoration
- edge detection
- texture analysis
- feature generation
- skeletonization
- shape analysis
- image compression
- component analysis
- curve filling
- general thinning
- feature detection
- noise reduction

References

- Homepage:
  - Center of Mathematical Morphology (http://cmm.ensmp.fr/index_eng.html) at Ecole des Mines de Paris.
- Book:
  - "Image analysis and mathematical morphology" by J. Serra (v.2 1988)
  - "Morphological Image Analysis" by Pierre Soille, 1999, Springer

Structure Element

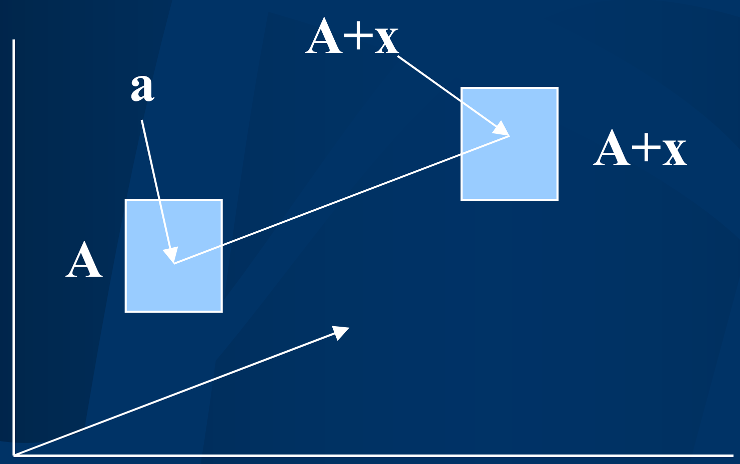
- Structuring element (SE): is also called the *kernel*, but I reserve this term for the similar objects used in convolutions
- Origin: the SE is typically translated to each pixel position in the image based on the origin.



Geometric Shift

- To shift a set A by the distance x can be described as A+x, and it's defined as:

$$A+x = \{a+x : a \in A\}$$

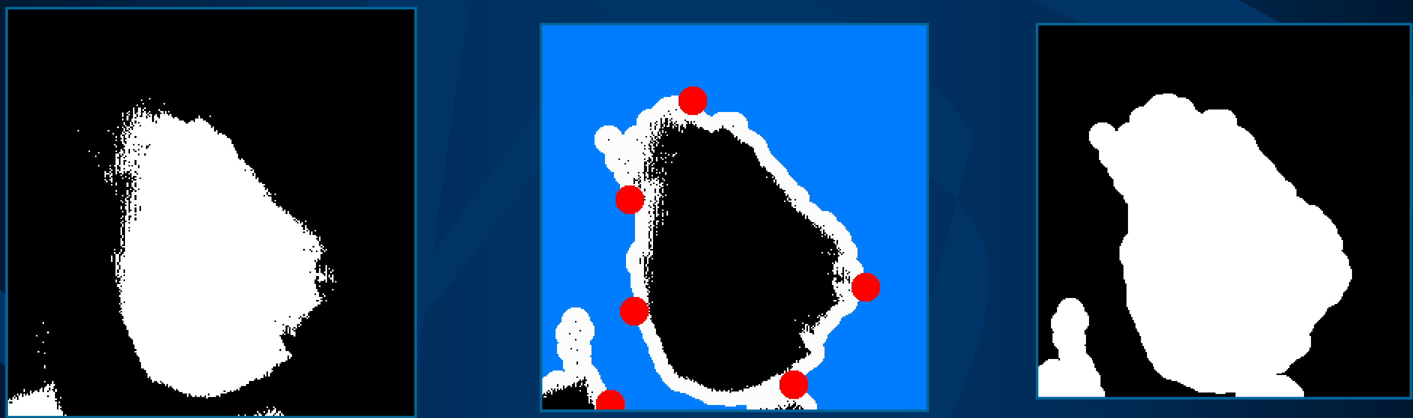


Binary Dilation

- Binary Dilation: also called Minkowski addition. An image *F* dilated by a SE *K* is defined as:

$$D(F, K) = F \oplus K = \bigcup_{b \in K} (\{a + b \mid a \in F\})$$

- It can be regarded as an expansion operation.

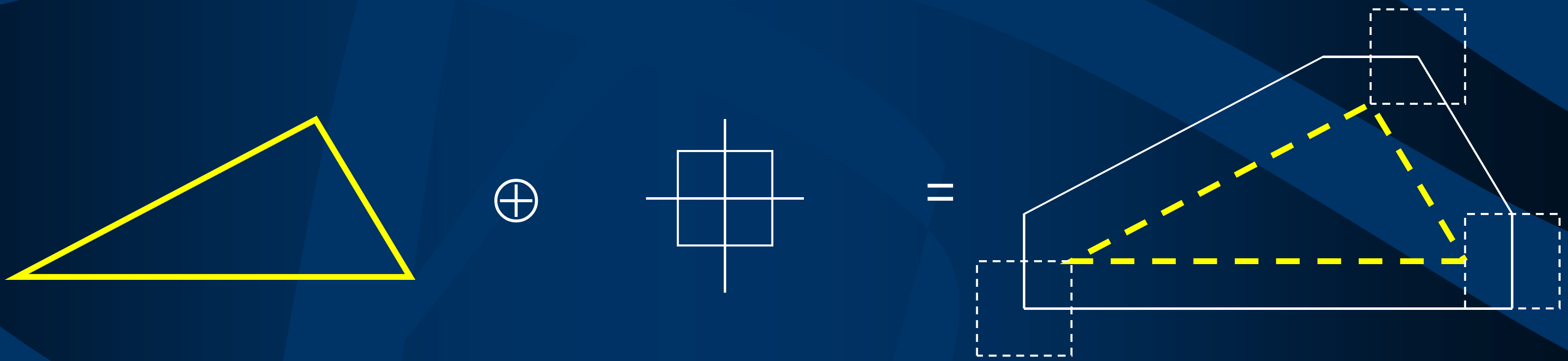


Binary Dilation

- Example:

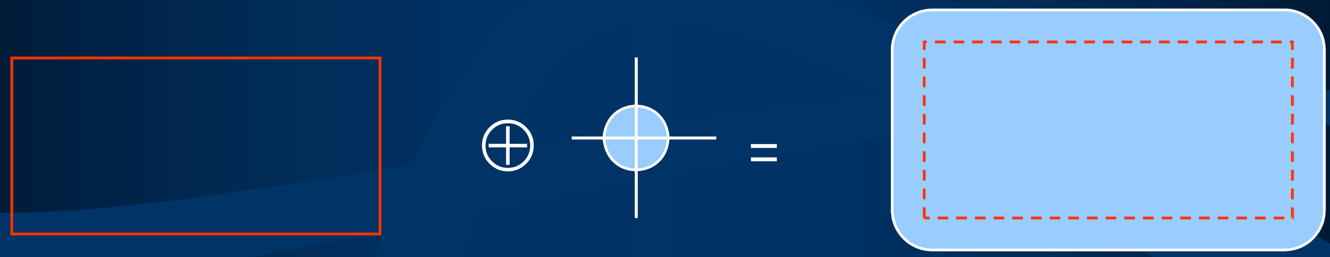
$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix} \oplus \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

$F \qquad K \qquad F \oplus K$

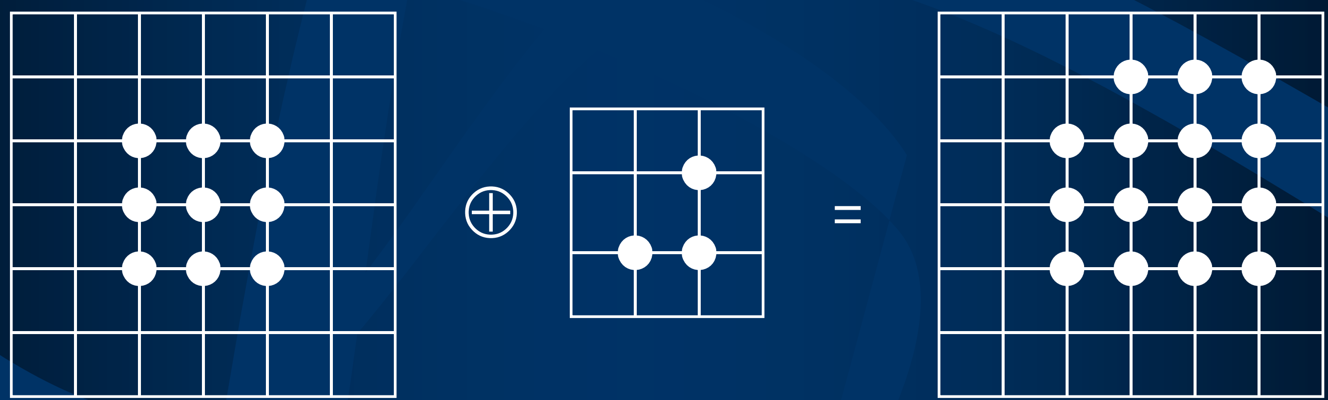


Binary Dilation

- Example:

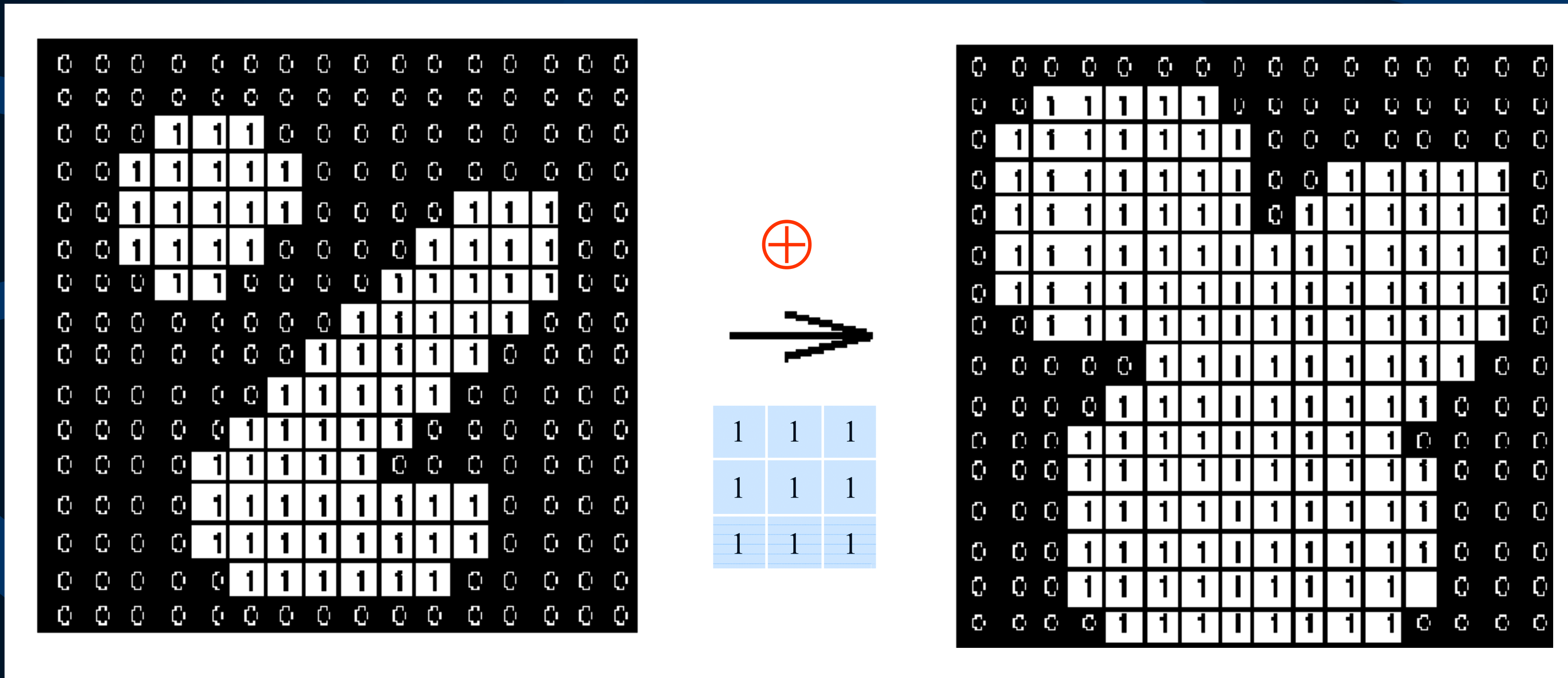


F $\qquad$ K $\qquad$ $F \oplus K$



Binary Dilation

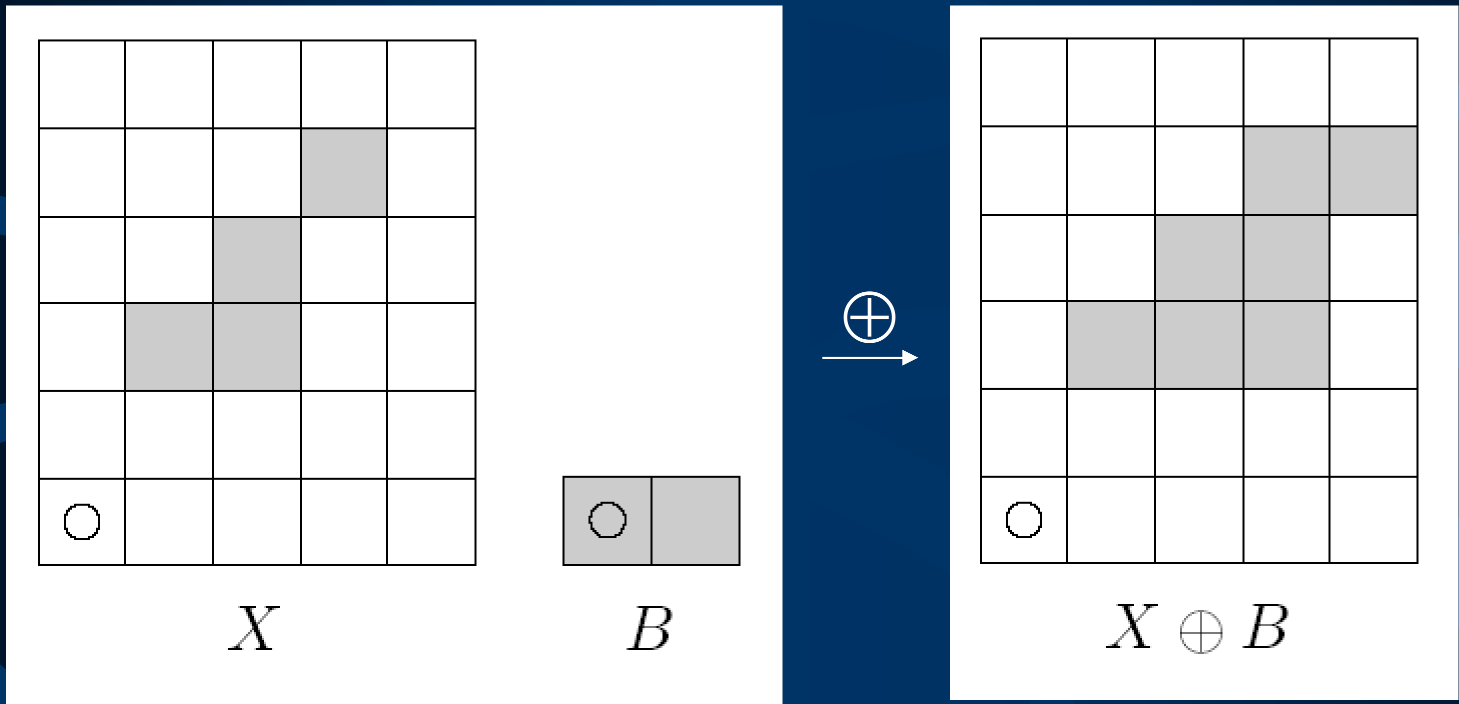
- Dilated set is the locus of points where the structuring element hit the points in the set



Binary Dilation

- Commutative: $D(A,B) = A \oplus B = B \oplus A = D(B,A)$
- Associative: $A \oplus (B \oplus C) = (A \oplus B) \oplus C$
- Translation Invariance: $A \oplus (B + x) = (A \oplus B) + x$
- Increasing: $A_1 \subseteq A_2 \Rightarrow (A_1 \oplus B) \subseteq (A_2 \oplus B)$
- Decomposition: $A \oplus (B \cup C) = (A \oplus B) \cup (A \oplus C)$
- Multi-Dilations: $nB = \underbrace{(B \oplus B \oplus B \oplus \ldots \oplus B)}_{n}$

Exercise

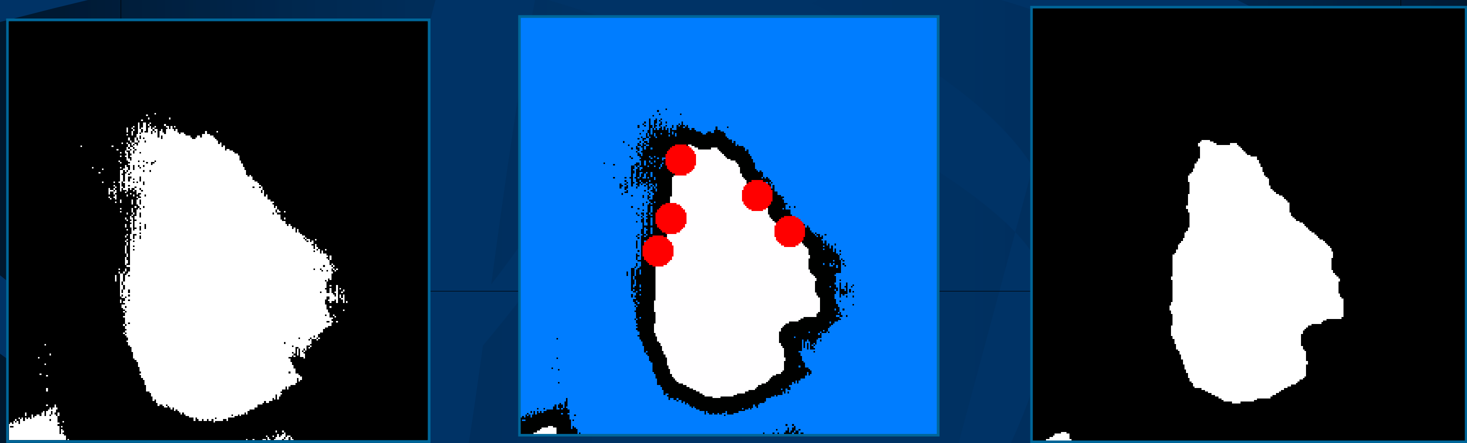


Binary Erosion

- Binary Erosion: also called Minkowski subtraction. An image F eroded by a SE K is defined as:

$$E(F, K) = F \$ K = \bigcap_{b \in K} (\{a - b \mid a \in F\})$$

- It can be regarded as an shrinking operation

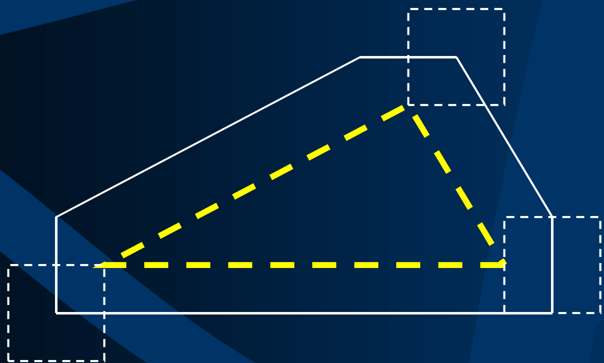


Binary Erosion

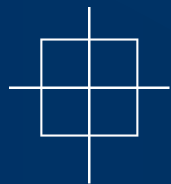
- Example:

$$\begin{bmatrix}0&1&0&1&0\\0&1&1&0&1\\0&1&1&1&0\end{bmatrix} \$ \begin{bmatrix}1&0\\1&1\end{bmatrix} = \begin{bmatrix}0&0&0&0&0\\0&0&1&0&0\\0&0&1&1&0\end{bmatrix}$$

F K F $ K



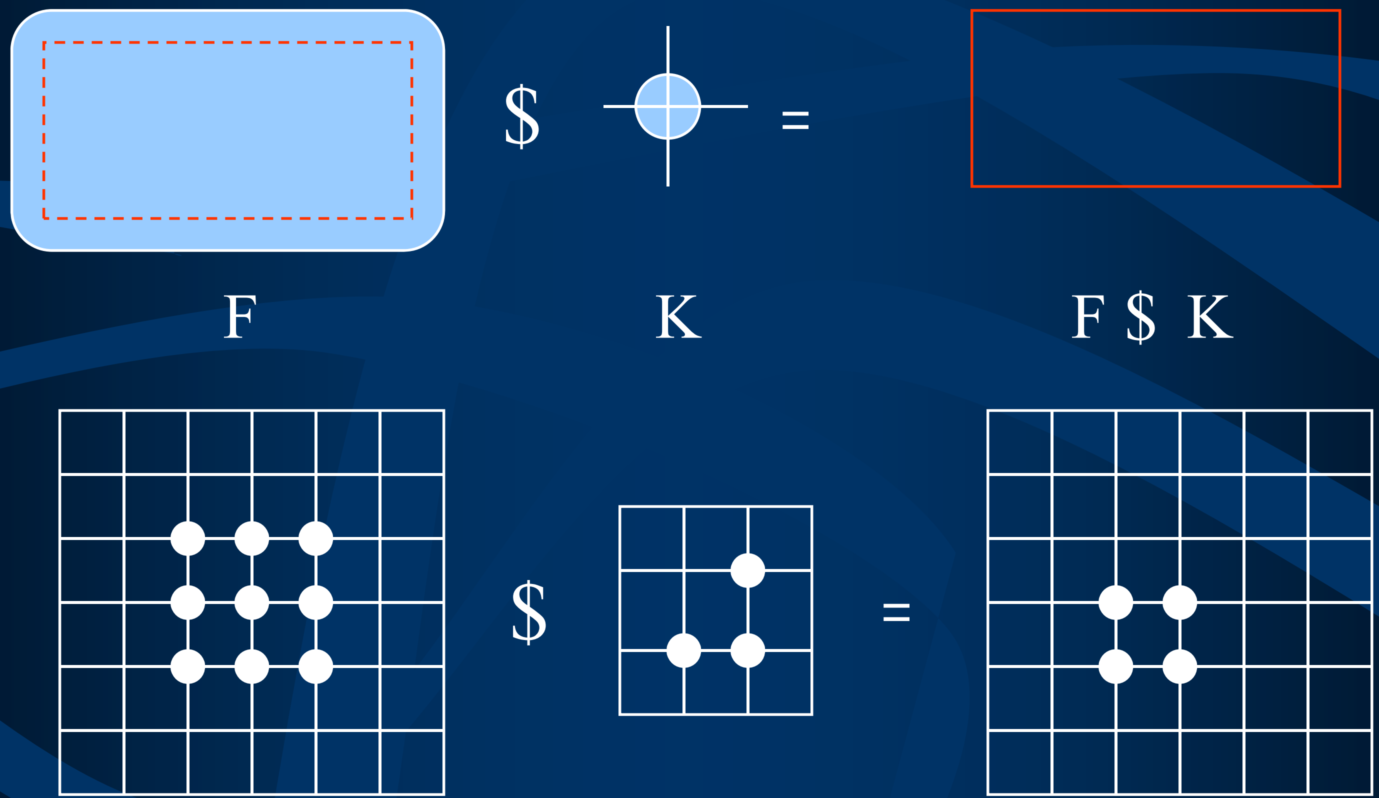
$



=

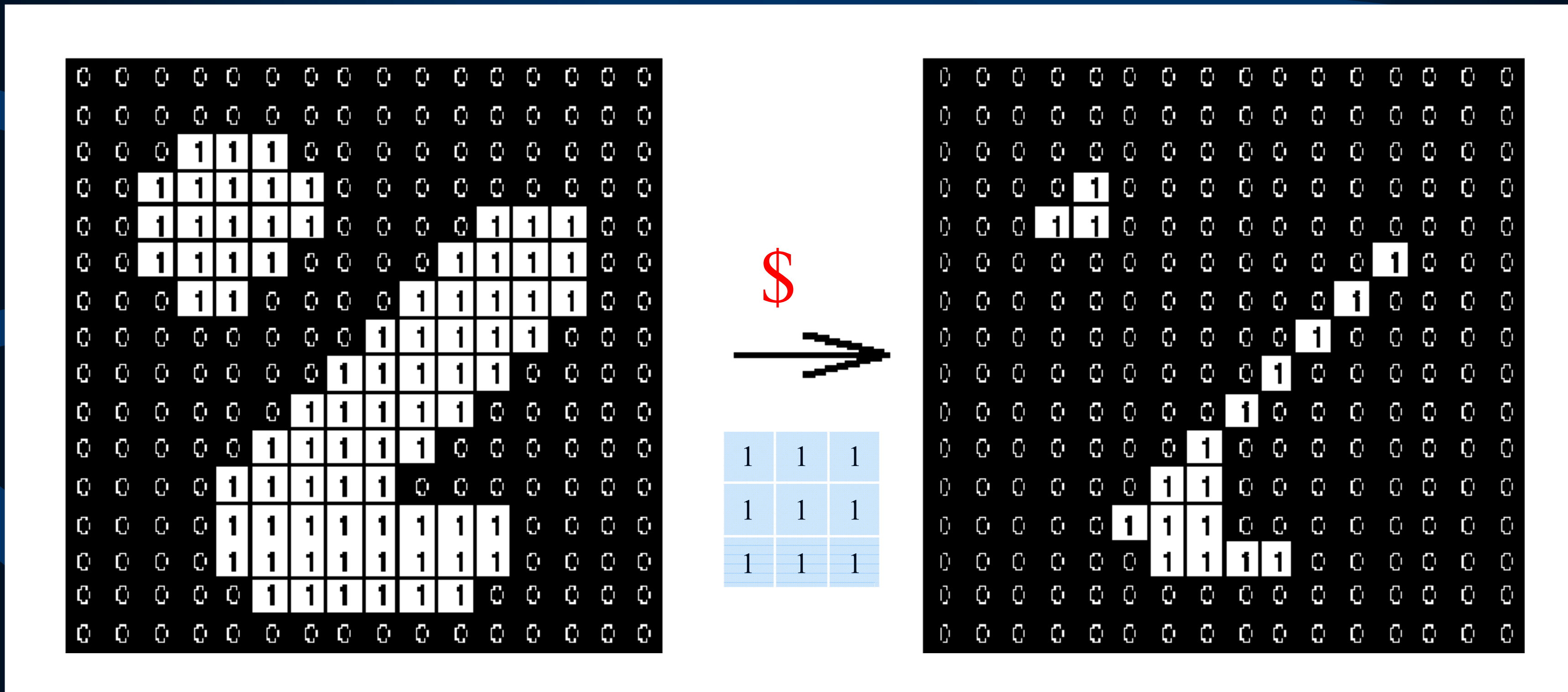
Binary Erosion

- Example:



Binary Erosion

- Eroded set is the locus of points where the structuring element fit the points in the set



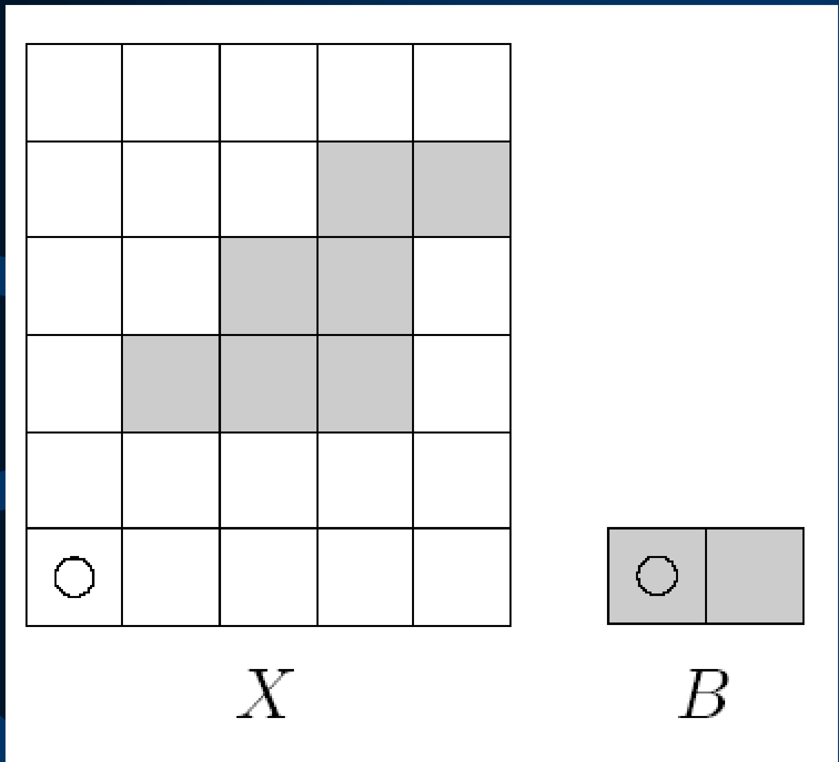
Binary Erosion

- Non-Commutative: $E(A,B) \neq E(B,A)$
- Non-Inverses: $D(E(A,B),B) \neq A \neq E(D(A,B),B)$
- Translation Invariance: $A\$\ (B+x) = (A\$\ B)+x$
- Increasing in A: $A_1 \subseteq A_2 \Rightarrow (A_1\$\ B) \subseteq (A_2\$\ B)$
- Decreasing in B: $B_1 \subseteq B_2 \Rightarrow (A\$\ B_1) \supseteq (A\$\ B_2)$
- Decomposition:

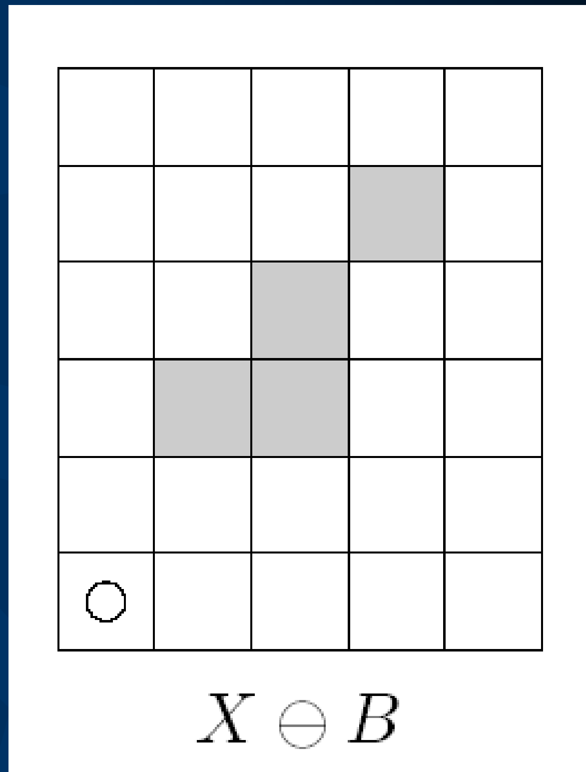
$$A\$\ (B \cup C) = (A\$\ B) \cap (A\$\ C)$$

$$(A\$\ B)\$\ C = A\$\ (B \oplus C)$$

Exercise



$\xrightarrow{\$}$



Fast Operations

- Based on the feature of decomposition :

$$A\$ \ (B \oplus C) = (A\$ \ B)\$ \ C$$

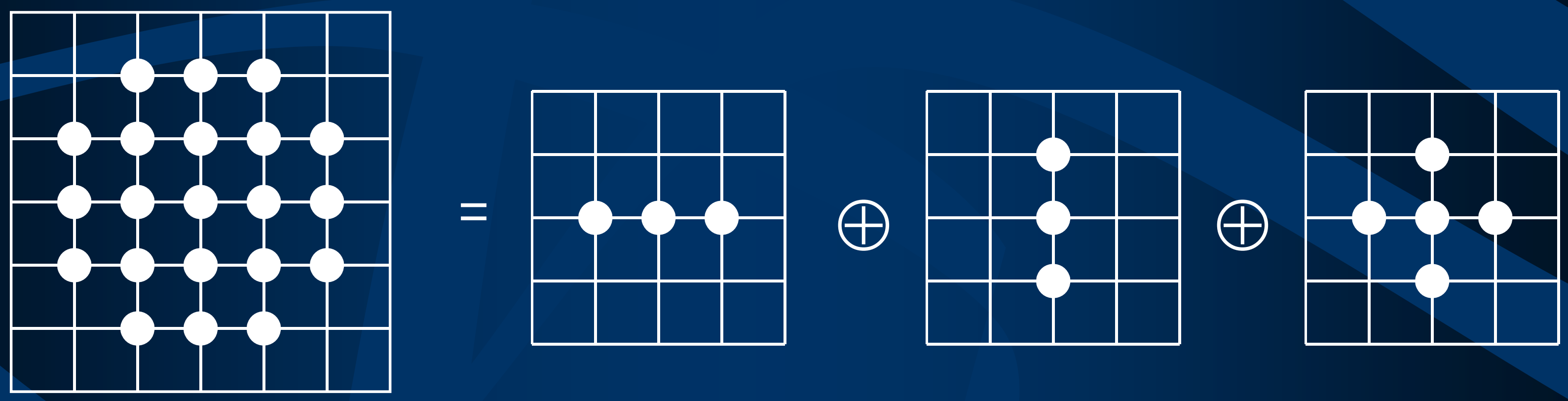
- Decomposition of the SE increase the efficiency of the computing (more than 50%)

$$A\$ \ 2B = A\$ \ (B \oplus B) = A\$ \ B\$ \ B$$

where, B is a disk with radius of 1

Fast Operations

- Many complicated structures can be decomposed to a set of simple elements.



Dilation and Erosion

- Dilation and erosion are not reverse operations but two dual operations.
- If A^C and –B stands for the complement of A and flip (turn 180 degree) of B, respectively,

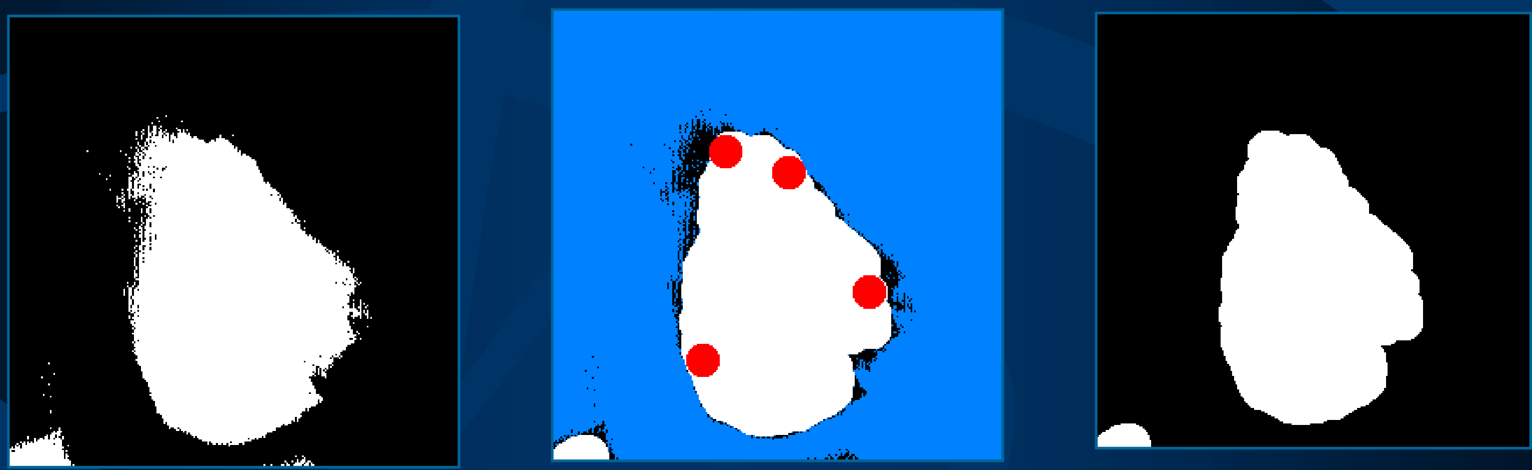
$$A \oplus B = [A^C \ \$ \ (-B)]^C$$

$$A \ \$ \ B = [A^C \oplus (-B)]^C$$

Binary Opening

- Binary Opening: An image F opened by a SE K is defined as:

$$O(F,K) = F \circ K = (F \ominus K) \oplus K$$



Binary Opening

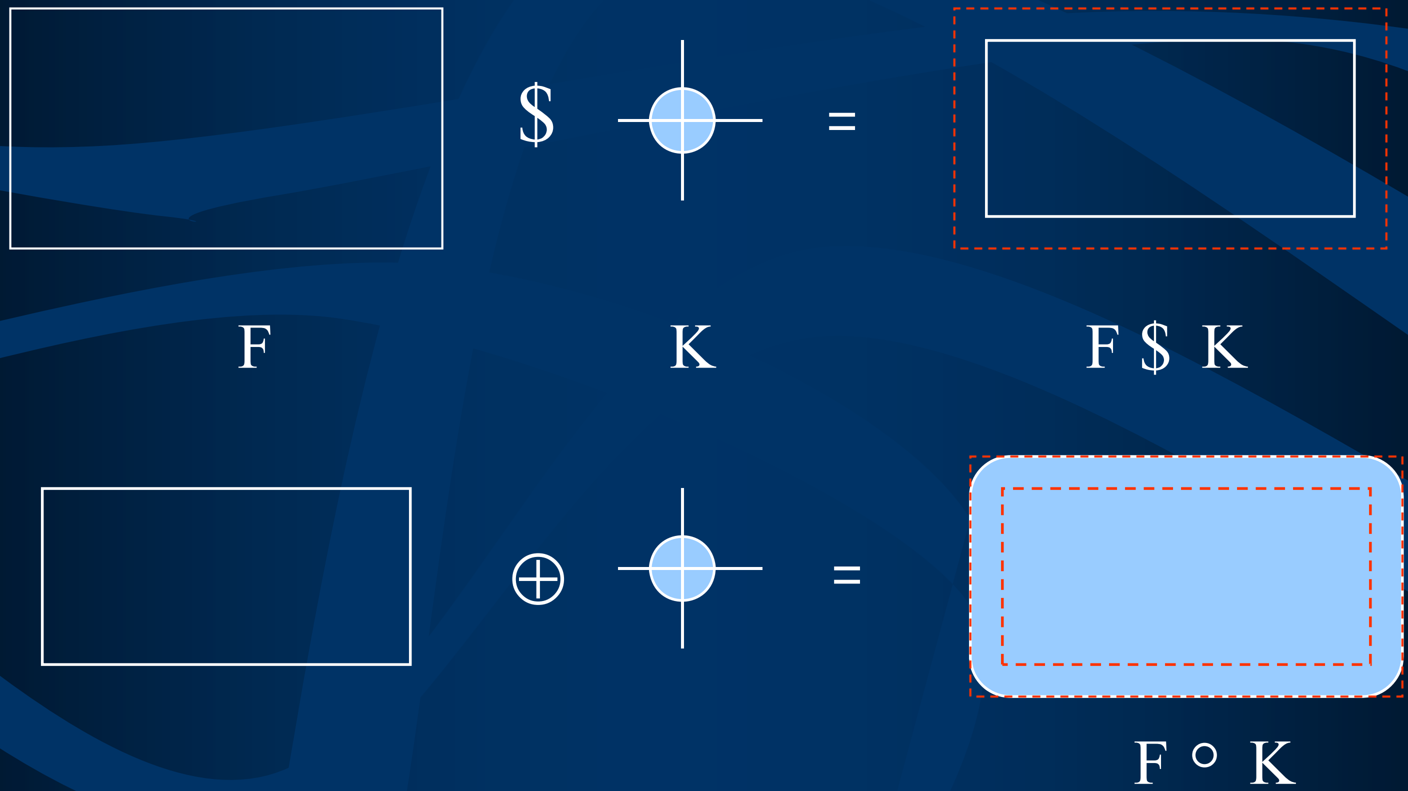
- Example:

$$\begin{bmatrix}0&1&0&1&0\\0&1&1&0&1\\0&1&1&1&0\end{bmatrix} \circ \begin{bmatrix}1&0\\1&1\end{bmatrix} = \begin{bmatrix}0&1&0&0&0\\0&1&1&0&0\\0&1&1&1&0\end{bmatrix}$$

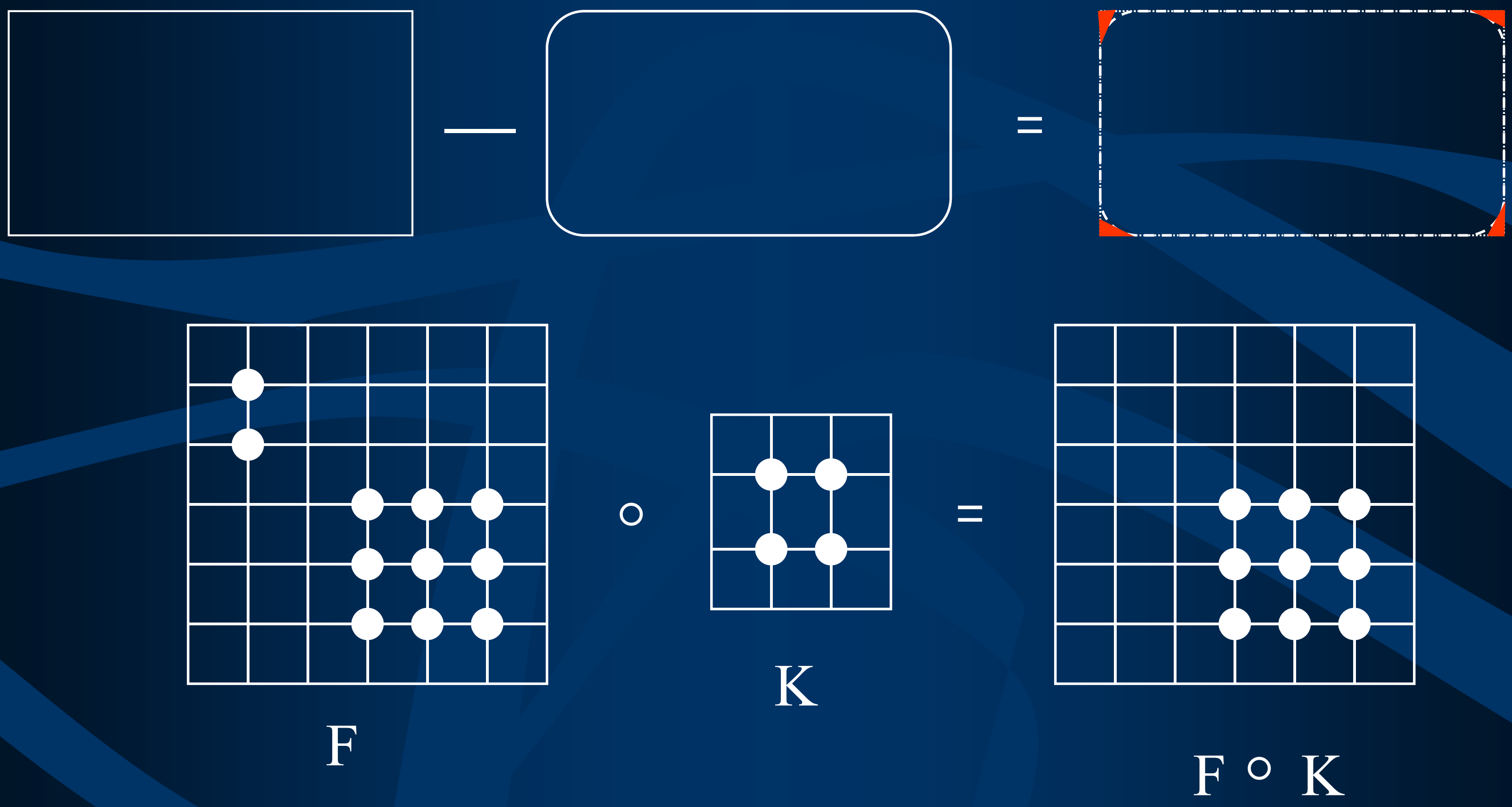
F K $F \circ K$

Binary Opening

- Example:

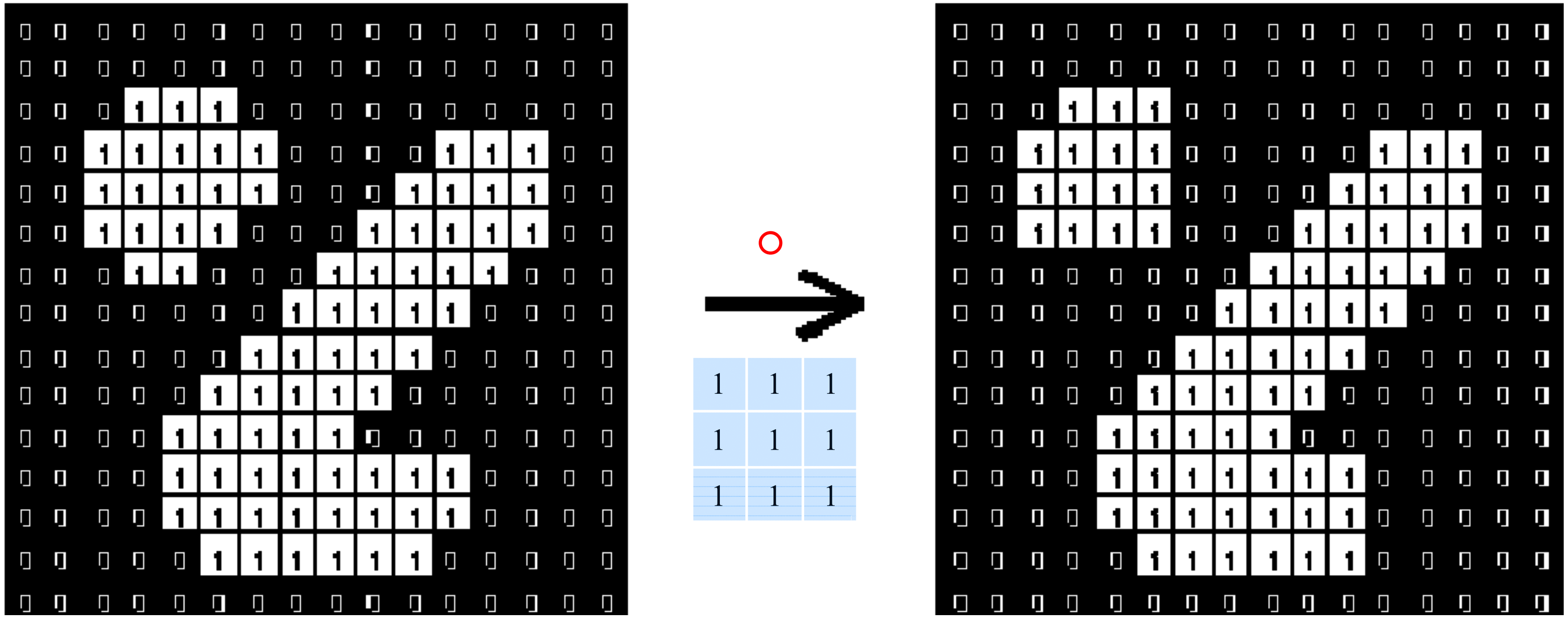


Binary Opening



Binary Opening

- Opening operation can remove the small regions which are smaller than the structuring element



Binary Opening

- Translation: $O(A+x, B) = O(A, B) + x$
- Antiextensivity: $O(A, B) \subseteq A$
- Increasing monotonicity:

$$A_1 \subseteq A_2 \Rightarrow (A_1 \circ B) \subseteq (A_2 \circ B)$$

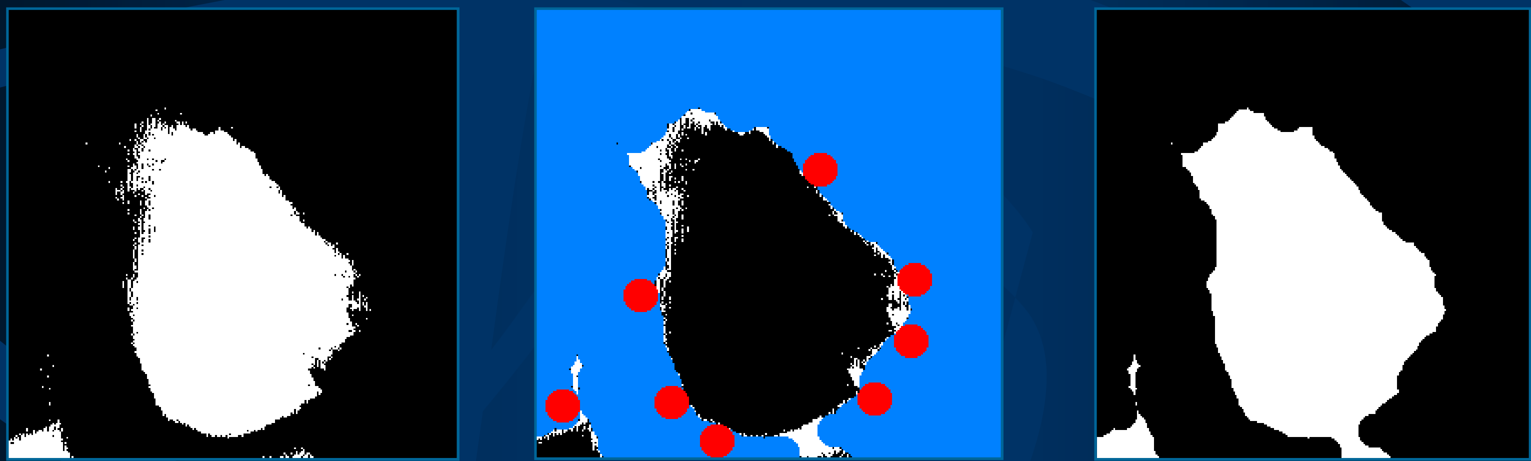
- Idempotence:

$$(A \circ B) \circ B = A \circ B$$

Binary Closing

- Binary Closing: An image F closed by a SE K is defined as:

$$C(F,K) = F \bullet K = (F \oplus K) \ominus K$$

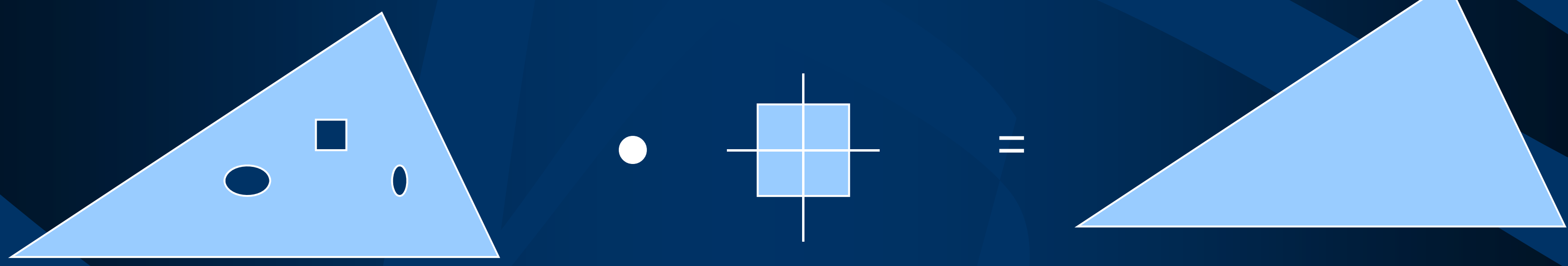


Binary Closing

- Example:

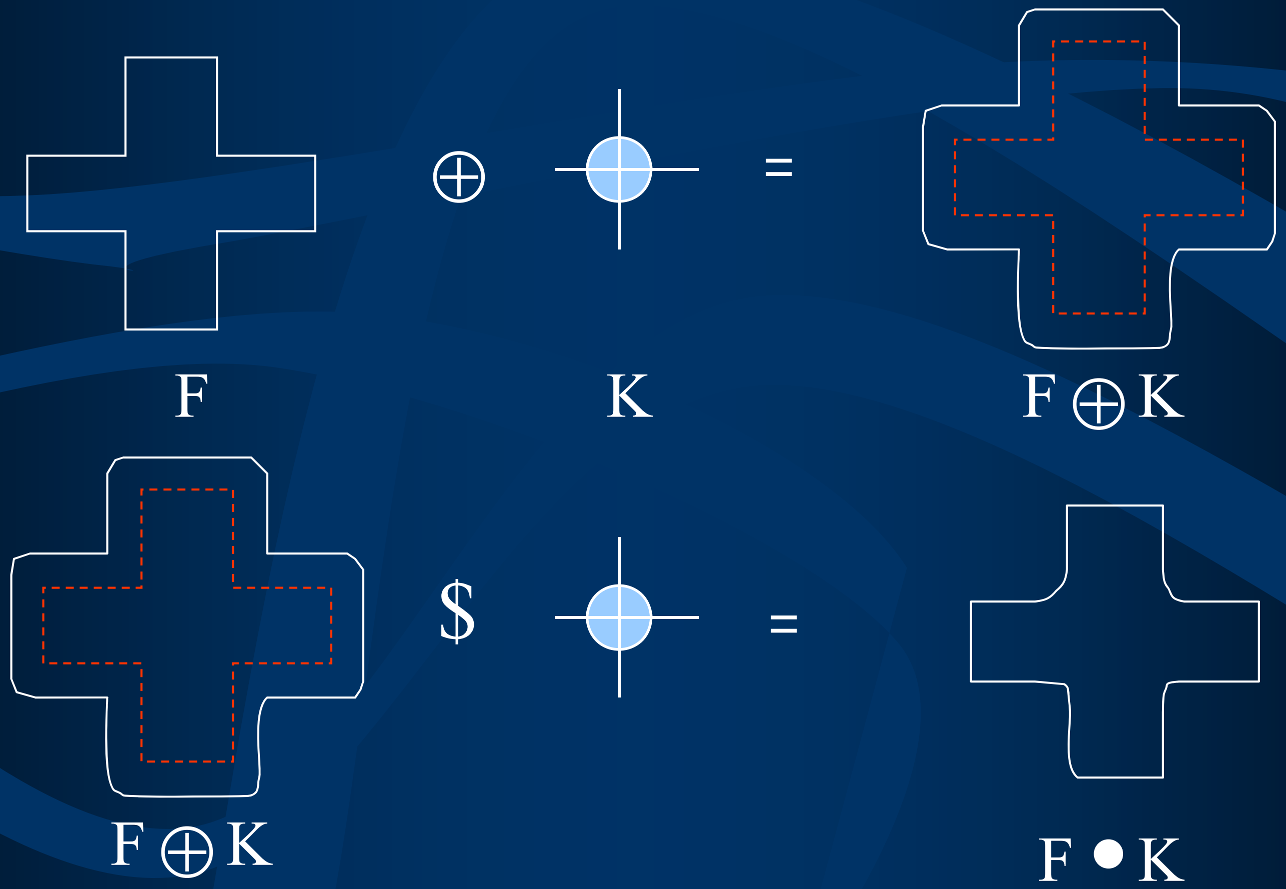
$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix} \bullet \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

F K F•K



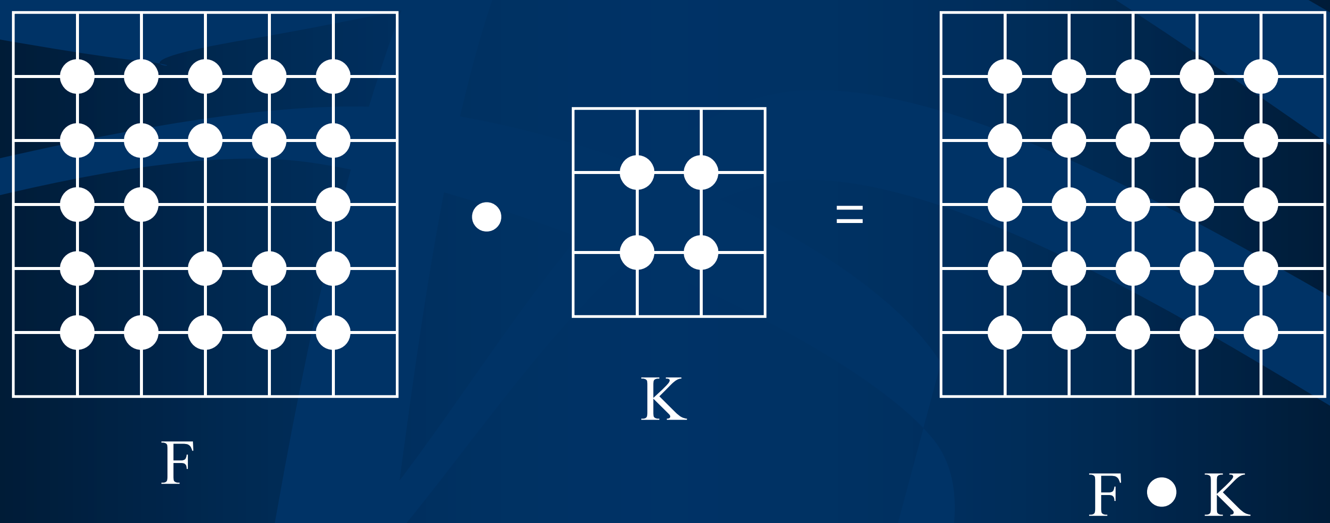
Binary Closing

- Example:

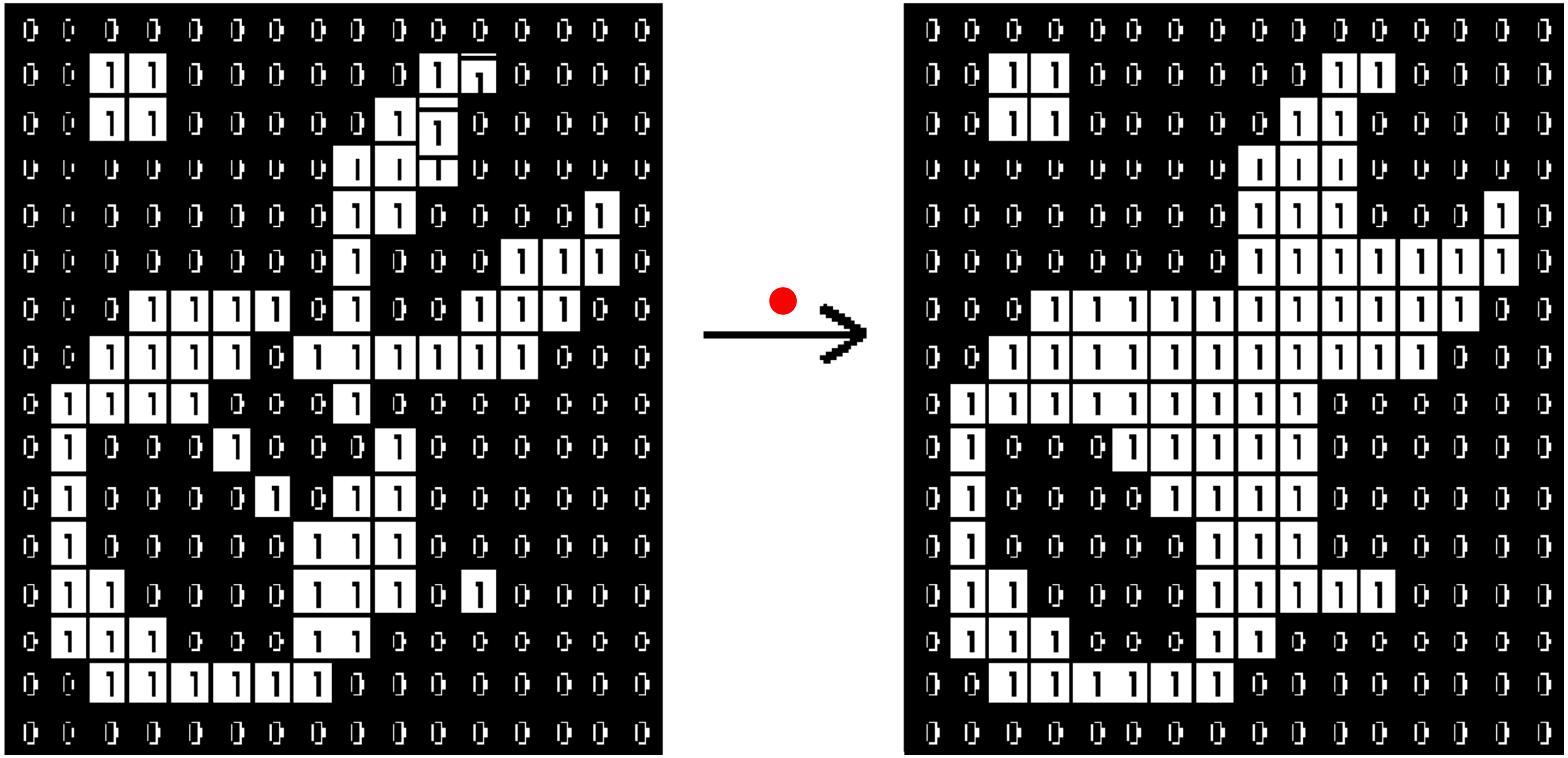


Binary Closing

- Closing can fill the small holes which are smaller than the structuring element



Binary Closing



Binary Closing

- Translation: $C(A+x, B) = C(A, B) + x$
- Extensivity: $A \subseteq C(A, B)$
- Increasing monotonicity:

$$A_1 \subseteq A_2 \Rightarrow (A_1 \bullet B) \subseteq (A_2 \bullet B)$$

- Idempotence:

$$(A \bullet B) \bullet B = A \bullet B$$

Opening and Closing

- Opening and closing are dual operations:

$$A \circ B = (A^C \bullet B)^C$$

$$A \bullet B = (A^C \circ B)^C$$

- They are all have Translation, Increasing monotonicity and Idempotence features, but open operation is Anti-extensive, but closing operation is extensive.

Discussion