

ACM-template

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1 头文件

```
#include <bits/stdc++.h>
#define pb(x) push_back(x)
#define fir first
#define sec second
#define mem(a,x) memset(a,x,sizeof(a))
#define mkr make_pair
typedef long long ll;
using namespace std;
const int inf=0x3f3f3f3f;
const ll INF= 0x3f3f3f3f3f3f3f3f;
const double pi = acos(-1.0);
ll gcd(ll a,ll b) { return b?gcd(b,a%b):a;}
/*
    #ifndef ONLINE_JUDGE
        freopen("data.in", "r", stdin);
        freopen("data.out", "w", stdout);
    #endif
*/
int main(){
    //ios_base::sync_with_stdio(false);cin.tie(NULL);cout.tie(NULL);

    return 0;
}

/*
#include<cstdio>
#include<iostream>
#include<queue>
#include<stack>
#include<cmath>
#include<cstring>
#include<string>
#include<set>
#include<map>
#include<sstream>
#include<algorithm>
*/
```

2 图论

2.1 最短路

```

/* Dijkstra P0J - 2387 */
/* 复杂度  $O(E \log E)$  */
const int maxn = " ";
typedef pair<int,int> P;
int d[maxn];
int vis[maxn];
int n;
struct node{
    int to;int cost;
};
vector<node>g[maxn];
void dij(int s){
    memset(d,0x3f,sizeof(d));
    memset(vis,0,sizeof(vis));
    d[s]=0;
    priority_queue< P,vector<P>,greater<P> >que;
    que.push(P(0,1));
    while(!que.empty()){
        P a=que.top();que.pop();
        int u=a.second;
        if(vis[u]) continue;
        vis[u]=1; //or if(d[u]<a.first) continue; optimize in cf 938d
        for(int j=0;j<g[u].size();j++){
            node e=g[u][j];
            if(!vis[e.to]&&d[e.to]>e.cost+d[u]) {
                d[e.to]=e.cost+d[u];
                que.push(P(d[e.to],e.to));
            }
        }
    }
}

/****SPFA****/
/*若存在负环回路则返回1*/
const int maxn = " ";
struct node{
    int to,cost;
    node(int To,int Cost):to(To),cost(Cost){};
};
vector<node>edge[maxn];
int d[maxn];
int vis[maxn];//标记每个点是否在队列里
int cnt[maxn];//判断是否存在负环回路;若有点更新超过n次, 则存在负环
int spfa(int x)
{
    memset(d,0x3f,sizeof(d));
    memset(vis,0,sizeof(vis));
    memset(cnt,0,sizeof(cnt));
    d[x]=0;
    queue<int>que;
    que.push(x);
    vis[x]=1;cnt[x]=1;
    while(!que.empty())

```

```

{
    int t=que.top;
    que.pop();
    vis[t]=0;
    for(int i=0;i<edge[t].size;i++){
        node e=edge[t][i];
        if(d[e.to]>d[t]+e.cost){
            d[e.to]=d[t]+e.cost;
            if(!vis[e.to]){
                que.push(e.to);
                vis[e.to]=1;
                if(++cnt[e.to]>n) return 1;
            }
        }
    }
}
return 0;
}

/****floyd****/
for(int k=1;k<=n;k++)
    for(int i=1;i<=n;i++)
        for(int j=1;j<=n;j++)
            d[i][j]=min(d[i][j],d[i][k]+d[k][j]);

```

2.2 最小生成树

```

/****最小生成树Kruskal poj2421****/
/*时间复杂度取决于边数 o(E logE)*/
const int maxn=" ";
struct node{
    int u,v;ll w;
    bool operator<(const node &b)const{
        return w<b.w;
    }
};
vector<node>edge[maxn];
int p[maxn];
void init(int n)
{
    for(int i=0; i<=n; i++)
        p[i]=i;
}
int Find(int x)
{
    if(x==p[x])
        return p[x];
    int y=Find(p[x]);
    return p[x]=y;
}

int Union(int x,int y)
{
    int x1=Find(x);
    int y1=Find(y);
    if(x1==y1)
        return 0;
}

```

```

        p[x1]=y1;
        return 1;
    }

void kruskal(int n)
{
    ll sum=0;
    int num=0;//已选用的边的数目
    init(n);
    sort(edge.begin(),edge.end());
    for(int i=0;i<edge.size();i++)
    {
        int u=edge[i].u;
        int v=edge[i].v;
        ll w=edge[i].w;
        if(Union(u,v))
        {
            num++;sum+=w;
        }
        if(num==n-1) break;
    }
}

/*****最小生成树Prim *****/
/*时间复杂度取决于顶点数  $O(V^2)$ */
const int maxn = " ";
int cost[maxn][maxn];
int lowc[maxn];
int prim(int n){
    for(int i=2;i<=n;i++)
        lowc[i]=cost[0][i];
    lowc[1]=-1;//tag visited
    int sum=0;
    for(int i=1;i<n;i++){
        int min=inf;
        int u;
        for(int j=1;j<=n;j++){
            if(lowc[j]!=-1&&lowc[j]<min){
                min=lowc[j];u=j;
            }
        }
        if(min==inf) return -1;
        lowc[u]=-1;
        sum+=min;

        for(int j=1;j<=n;j++){
            if(lowc[j]!=-1&&lowc[j]>cost[u][j])
                lowc[j]=cost[u][j];
        }
    }
    return sum;
}

```

2.3 二分图

```

/****二分图最大匹配 匈牙利算法****/
/*o(VE)*/
int line[maxn][maxn];
int used[maxn]; //标记这条边有没有用过
int match[maxn]; //标记右侧的点是否被匹配, 以及匹配的是左侧哪个点
int nl;
int nr;
bool find(int x){
    for(int i=1; i<=nr; i++){
        if(line[x][i] && !used[i]){
            used[i]=1;
            if(match[i]==0 || find(match[i])){
                match[i]=x;
                return true;
            }
        }
    }
    return false;
}
int hungarian()
{
    int ans = 0;
    memset(match, 0, sizeof(match));
    for (int i=1; i<=nl; i++) {
        memset(used, 0, sizeof(used));
        if(find(i)) ans++;
    }
    return ans;
}

/* * 二分图匹配 (Hopcroft-Carp算法)
   复杂度  $O(\sqrt{n} * E)$  邻接表存图 vector实现
   vector先初始化, 然后假如边uN 为左端的顶点数, 使用前赋值(点编号0开始)
   */
const int MAXN = 3000;
vector<int> G[MAXN];
int uN;
int Mx[MAXN], My[MAXN];
int dx[MAXN], dy[MAXN];
int dis;
bool used[MAXN];
bool SearchP() {
    queue<int> Q;
    dis = inf;
    memset(dx, -1, sizeof(dx));
    memset(dy, -1, sizeof(dy));
    for(int i = 0 ; i < uN; i++)
        if(Mx[i] == -1) {
            Q.push(i);
            dx[i] = 0; }
    while(!Q.empty()) {
        int u = Q.front();
        Q.pop();

```



```

        if(dx[u] > dis)
            break;
        int sz = G[u].size();
        for(int i = 0;i < sz;i++) {
            int v = G[u][i];
            if(dy[v] == -1) {
                dy[v] = dx[u] + 1;
            }
            if(My[v] == -1)dis = dy[v];
            else {
                dx[My[v]] = dy[v] + 1;
                Q.push(My[v]);
            } } } }

return dis != inf;
}

bool DFS(int u) {
    int sz = G[u].size();
    for(int i = 0;i < sz;i++) {
        int v = G[u][i];
        if(!used[v] && dy[v] == dx[u] + 1) {
            used[v] = true;
            if(My[v] != -1 && dy[v] == dis)
                continue;
            if(My[v] == -1 || DFS(My[v])) {
                My[v] = u;
                Mx[u] = v;
            }
            return true;
        } } }
return false; }

int MaxMatch() {
    int res = 0;
    memset(Mx,-1,sizeof(Mx));
    memset(My,-1,sizeof(My));
    while(SearchP()) {
        memset(used,false,sizeof(used));
        for(int i = 0;i < uN;i++)
            if(Mx[i] == -1 && DFS(i))
                res++;
    }
    return res; }

/*KM算法*/
/**二分图最佳匹配**/
/*steal from csl*/
const int maxn = " ";
int n;
int cost[maxn][maxn];
int lx[maxn], ly[maxn], match[maxn], slack[maxn];
int prev[maxn];
bool vy[maxn];

void augment(int root)

```

```

{
    fill(vy + 1, vy + n + 1, false);
    fill(slack + 1, slack + n + 1, inf);
    int py;
    match[py = 0] = root;
    do
    {
        vy[py] = true;
        int x = match[py], yy;
        int delta = inf;
        for (int y = 1; y <= n; y++)
        {
            if (!vy[y])
            {
                if (lx[x] + ly[y] - cost[x][y] < slack[y])
                    slack[y] = lx[x] + ly[y] - cost[x][y], prev[y] = py;
                if (slack[y] < delta) delta = slack[y], yy = y;
            }
        }
        for (int y = 0; y <= n; y++)
        {
            if (vy[y])
                lx[match[y]] -= delta, ly[y] += delta;
            else
                slack[y] -= delta;
        }
        py = yy;
    } while (match[py] != -1);
    do
    {
        int pre = prev[py];
        match[py] = match[pre], py = pre;
    } while (py);
}

int KM()
{
    for (int i = 1; i <= n; i++)
    {
        lx[i] = ly[i] = 0;
        match[i] = -1;
        for (int j = 1; j <= n; j++) lx[i] = max(lx[i], cost[i][j]);
    }
    int answer = 0;
    for (int root = 1; root <= n; root++) augment(root);
    for (int i = 1; i <= n; i++) answer += lx[i], answer += ly[i];
    return answer;
}

```

2.4 2-SAT

```

/**2-SAT**/
vector<int> g[maxn*2];
bool mark[maxn*2];
int s[maxn*2], c;
bool dfs(int x){
    if(mark[x^1]) return false;

```

```

    if(mark[x]) return true;
    mark[x]=true;
    s[c++]=x;
    for(int i=0;i<g[x].size();i++)
        if(!dfs(g[x][i])) return false;
    return true;
}
void init(int n){
    for(int i=0;i<n*2;i++) g[i].clear();
    memset(mark,0,sizeof(mark));
}
void add_clause(int x,int y){
    g[x].push_back(y); //选了x就必须选y^1
}
bool solve(int n){
    for(int i=0;i<n*2;i+=2)
        if(!mark[i]&&!mark[i+1]){
            c=0;
            if(!dfs(i)){
                while(c>0) mark[s[--c]]=false;
                if(!dfs(i+1)) return false;
            }
        }
    return true;
}

```

2.5 割点与强连通

```

/**求割点*/
/*根节点的儿子数量>=2即为割点*/
vector<int>edge[maxn];
int low[maxn],dfn[maxn],tot;
bool iscut[maxn]; //判断是不是割点
void init(){
    for(int i = 1; i <= n; i++){
        edge[i].clear();
        mem(low);
        mem(dfn);
        mem(iscut);
        tot = 0;
    }
}
void dfs(int u,int fa){
    low[u] = dfn[u] = ++tot;
    int child=0;
    for(int i = 0; i < edge[u].size(); i++){
        int v = edge[u][i];
        if(!dfn[v]){
            dfs(v,u);
            child++;
            low[u] = min(low[u],low[v]);
            if(low[v] >= dfn[u])
                iscut[u] = true;
        }
        else if(v != fa){
            low[u] = min(low[u],dfn[v]);
        }
    }
}

```

```

    }
}
if (fa < 0 && child == 1) iscut[u] = 0; //根节点
}

```

```

/*强连通分量*/
/* Tarjan算法 * 复杂度O(N+M)*/
/*边双连通分量加一个!=pre即可*/
vector<int>edge[maxn];
stack<int>st;
int low[maxn], dfn[maxn];
int instack[maxn];
int tot;
int scc; //强连通分量个数
int belong[maxn]; //记录每个点属于哪个连通分量
void init(){
    mem(dfn, 0);
    mem(low, 0);
    mem(instack, 0);
    for (int i = 0; i < maxn; i++)
        edge[i].clear();
    while (!st.empty())
        st.pop();
    tot = scc = 0;
}

```

```

void tar(int u){
    dfn[u] = low[u] = ++tot;
    st.push(u);
    instack[u] = 1;
    for (int i = 0; i < edge[u].size(); i++){
        int v = edge[u][i];
        if (!dfn[v]){
            tar(v);
            if (low[u] > low[v])
                low[u] = low[v];
        }
        else if (instack[v] && low[u] > dfn[v])
            low[u] = dfn[v];
    }
    if (low[u] == dfn[u]) {
        int v;
        scc++;
        do{
            v = st.top();
            st.pop();
            belong[v] = scc;
            instack[v] = 0;
        } while (v != u);
    }
}

```

2.6 最大流

```

/**Dinic算法 HDU - 1532****/
/*复杂度 $O(n^2*m)$ */
struct E{
    int to, cap;
    int rev; //反向边的序号
};
vector<E> edge[maxn];
int level[maxn];

void addedge(int u, int v, int cap){
    edge[u].push_back((E){v, cap, edge[v].size()});
    edge[v].push_back((E){u, 0, edge[u].size()-1});
}

void bfs(int s){
    memset(level, -1, sizeof(level));
    level[s] = 0;
    queue<int> que;
    que.push(s);
    while(!que.empty()){
        int u = que.front(); que.pop();
        for(int i = 0; i < edge[u].size(); i++){
            E e = edge[u][i];
            if(e.cap > 0 && level[e.to] == -1){
                level[e.to] = level[u] + 1;
                que.push(e.to);
            }
        }
    }
}

int dfs(int u, int t, int f){
    if(u == t) return f;
    for(int i = 0; i < edge[u].size(); i++){
        E e = edge[u][i];
        if(level[e.to] == level[u] + 1 && e.cap > 0){
            int d = dfs(e.to, t, min(f, e.cap));
            if(d > 0){
                edge[u][i].cap -= d;
                edge[e.to][e.rev].cap += d;
                return d;
            }
        }
    }
    return 0;
}

int max_flow(int s, int t){
    int flow = 0;
    while(1){
        bfs(s);
        if(level[t] < 0) return flow;
        int f;
        while((f = dfs(s, t, inf)) > 0)
            flow += f;
    }
    return flow;
}

```

```

}

/*ISAP*/
const int maxn= " ";
struct Edge{
    int from,to,cap,flow;
    Edge(int u,int v,int c,int f):from(u),to(v),cap(c),flow(f){}
};
struct ISAP{
    int n,m,s,t;
    vector<Edge> edges;
    vector<int > G[maxn];
    bool vis[maxn];
    int d[maxn],cur[maxn];
    int p[maxn],num[maxn];
    void init(int n)
    {
        this->n=n;
        for(int i=0;i<=n;i++)
            G[i].clear();
        edges.clear();
        memset(d,0x3f3f3f3f,sizeof(d));
    }
    void addedge(int from,int to,int cap)
    {
        edges.push_back(Edge(from,to,cap,0));
        edges.push_back(Edge(to,from,0,0));
        m=edges.size();
        G[from].push_back(m-2);
        G[to].push_back(m-1);
    }
    bool BFS()
    {
        memset(vis,false,sizeof(vis));
        queue<int >q;
        q.push(t);
        d[t]=0;
        vis[t]=true;
        while(!q.empty())
        {
            int u=q.front();
            q.pop();
            for(int i=0;i<G[u].size();i++)
            {
                Edge &e=edges[G[u][i]^1];
                if(!vis[e.from]&&e.cap>e.flow)
                {
                    vis[e.from]=true;
                    d[e.from]=d[u]+1;
                    q.push(e.from);
                }
            }
        }
        return vis[s];
    }
}

```

```

int Augment()
{
    int flow=inf;
    for(int u=t;u!=s;u=edges[p[u]].from)
    {
        Edge &e=edges[p[u]];
        flow=min(flow,e.cap-e.flow);
    }
    for(int u=t;u!=s;u=edges[p[u]].from)
    {
        edges[p[u]].flow+=flow;
        edges[p[u]^1].flow-=flow;
    }
    return flow;
}

int Maxflow(int s,int t)//cal
{
    this->s=s;
    this->t=t;
    int flow=0;
    BFS();
    if(d[s]>=n)
        return 0;
    memset(num,0,sizeof(num));
    for(int i=0;i<n;i++)
        if(d[i]<INF)
            num[d[i]]++;

    int u=s;
    memset(cur,0,sizeof(cur));
    while(d[s]<n)
    {
        if(u==t)
        {
            flow+=Augment();
            u=s;
        }
        int ok=0;
        for(int i=cur[u];i<G[u].size();i++)
        {
            Edge &e=edges[G[u][i]];
            if(e.cap>e.flow && d[u]==d[e.to]+1)
            {
                ok=1;
                p[e.to]=G[u][i];
                cur[u]=i;
                u=e.to;
                break;
            }
        }
        if(!ok)
        {
            int m=n-1;
            for(int i=0;i<G[u].size();i++)
            {
                Edge &e=edges[G[u][i]];
                if(e.cap>e.flow)

```

```

                                m=min(m,d[e.to]);
                                }
                                if(--num[d[u]]==0)
                                    break;
                                ++num[d[u]=m+1];
                                cur[u]=0;
                                if(u!=s)
                                    u=edges[p[u]].from;
                                }
                                }
                                return flow;
                            }
    };
};

```

2.7 最小费用最大流

/*费用流*/

```

const int maxn = " ";
struct Edge
{
    int from, to, cap, flow, cost;
    Edge(int u, int v, int c, int f, int w)
        : from(u), to(v), cap(c), flow(f), cost(w) {}
};
struct MCMF
{
    int n, m;
    vector<Edge> edges;
    vector<int> G[maxn];
    int inq[maxn]; //是否在队列中
    int d[maxn];   //bellmanford
    int p[maxn];   //上一条弧
    int a[maxn];   //可改进量
    void init(int n)
    {
        this->n = n;
        for (int i = 0; i < n; i++) G[i].clear();
        edges.clear();
    }
    void AddEdge(int from, int to, int cap, int cost)
    {
        edges.pb(Edge(from, to, cap, 0, cost));
        edges.pb(Edge(to, from, 0, 0, -cost));
        m = edges.size();
        G[from].pb(m - 2);
        G[to].pb(m - 1);
    }
    bool BellmanFord(int s, int t, int& flow, ll& cost)
    {
        for (int i = 0; i < n; i++) d[i] = inf;
        mem(inq, 0);
        d[s] = 0;
        inq[s] = 1;
        p[s] = 0;
        a[s] = inf;
    }
};

```



```

queue<int> q;
q.push(s);
while (!q.empty())
{
    int u = q.front();
    q.pop();
    inq[u] = 0;
    for (int i = 0; i < G[u].size(); i++)
    {
        Edge& e = edges[G[u][i]];
        if (e.cap > e.flow && d[e.to] > d[u] + e.cost)
        {
            d[e.to] = d[u] + e.cost;
            p[e.to] = G[u][i];
            a[e.to] = min(a[u], e.cap - e.flow);
            if (!inq[e.to])
            {
                q.push(e.to);
                inq[e.to] = 1;
            }
        }
    }
}
if (d[t] == inf) return false; // 当没有可增广的路时退出
flow += a[t];
cost += (ll)d[t] * (ll)a[t];
for (int u = t; u != s; u = edges[p[u]].from)
{
    edges[p[u]].flow += a[t];
    edges[p[u] ^ 1].flow -= a[t];
}
return true;
}
int MincostMaxflow(int s, int t, ll& cost)
{
    int flow = 0;
    cost = 0;
    while (BellmanFord(s, t, flow, cost));
    return flow;
}
};

```

2.8 第k短路

```

/*第k短路*/
const int MAXM= "" ;
const int MAXN= "" ;
struct node
{
    int v, w, next;
}edge[MAXM], revedge[MAXM];
struct A
{
    ll f, g, v;
    bool operator <(const A a)const {
        if(a.f == f) return a.g < g;
    }
}

```

```

        return a.f < f;
    }
};
int e, vis[MAXN], q[MAXM * 5];
ll d[MAXN];
int head[MAXN], revhead[MAXN];
int n, m, s, t, k;
void init()
{
    e = 0;
    memset(head, -1, sizeof(head));
    memset(revhead, -1, sizeof(revhead));
}
void insert(int x, int y, int w) //插入边
{
    edge[e].v = y;
    edge[e].w = w;
    edge[e].next = head[x];
    head[x] = e;
    revedge[e].v = x;
    revedge[e].w = w;
    revedge[e].next = revhead[y];
    revhead[y] = e++;
}
void spfa(int src)
{
    for(int i = 1; i <= n; i++) d[i] = INF;
    memset(vis, 0, sizeof(vis));
    vis[src] = 0;
    int h = 0, t = 1;
    q[0] = src;
    d[src] = 0;
    while(h < t)
    {
        int u = q[h++];
        vis[u] = 0;
        for(int i = revhead[u] ; i != -1; i = revedge[i].next)
        {
            int v = revedge[i].v;
            int w = revedge[i].w;
            if(d[v] > d[u] + w)
            {
                d[v] = d[u] + w;
                if(!vis[v])
                {
                    q[t++] = v;
                    vis[v] = 1;
                }
            }
        }
    }
}
ll Astar(int src, int des)
{
    int cnt = 0;
    priority_queue<A>Q;

```

```

    if(src == des) k++;
    if(d[src] == INF) return -1;
    A t, tt;
    t.v = src, t.g = 0, t.f = t.g + d[src];
    Q.push(t);
    while(!Q.empty())
    {
        tt = Q.top();
        Q.pop();
        if(tt.v == des)
        {
            cnt++;
            if(cnt == k)
                return tt.g;
        }
        for(int i = head[tt.v]; i != -1; i = edge[i].next)
        {
            t.v = edge[i].v;
            t.g = tt.g + edge[i].w;
            t.f = t.g + d[t.v];
            Q.push(t);
        }
    }
    return -1;
}

int main()
{
    init();
    scanf("%d%d%d", &s, &t, &k);
    for(int i = 1; i <= m; i++)
    {
        int x,y,w;
        scanf("%d%d%d", &x, &y, &w);
        insert(x, y, w);
    }
    spfa(t);
    ll ans=Astar(s,t);
    return 0;
}

```

2.9 欧拉路径

/*找欧拉路径*/

```

#include <bits/stdc++.h>
using namespace std;
const int maxn=1e5+10;
struct edge
{
    int to;
    int id;
    edge(){}
    edge(int to,int id):to(to),id(id){}
};
int top;
vector<edge> G[maxn];

```

```

vector<int> J[maxn],ans[maxn];
int N,M;
int tot;
int cnt;
int s[maxn*5];
int vis[maxn*5];
int deg[maxn];
int f[maxn];
int Find(int x)
{
    if(f[x]==x)
        return f[x];
    return f[x]=Find(f[x]);
}
void dfs(int u)
{
    for(int i=0;i<G[u].size();i++)
    {
        edge e=G[u][i];
        if(vis[e.id>>1])
            continue;
        vis[e.id>>1]=1;
        dfs(e.to);
        if(e.id%2==1)
            s[++top]=(-(e.id>>1));
        else
            s[++top]=(e.id>>1);
    }
}
void init()
{
    for(int i=0;i<maxn;i++)
    {
        f[i]=i;
        G[i].clear();
        J[i].clear();
        ans[i].clear();
    }
    memset(vis,0,sizeof(vis));
    memset(deg,0,sizeof(deg));
}
int main()
{
    //freopen("1003.in","r",stdin);
    while(scanf("%d%d",&N,&M)!=EOF)
    {
        init();
        tot=1;
        for(int i=1;i<=M;i++)
        {
            int u,v;
            scanf("%d%d",&u,&v);
            G[u].push_back(edge(v,++tot));
            G[v].push_back(edge(u,++tot));
            deg[u]++;
            deg[v]++;
        }
    }
}

```

```

        f[Find(u)]=Find(v);
    }
    for(int i=1;i<=N;i++)
    {
        if(deg[i]%2==1)
            J[Find(i)].push_back(i);
    }
    cnt=0;
    for(int i=1;i<=N;i++)
    {
        if(f[i]==i)
        {
            if(!J[i].size())
            {
                top=0;
                dfs(i);
                cnt++;
                while(top)
                {
                    ans[cnt].push_back(s[top--]);
                }
            }
            else
            {
                top=0;
                for(int j=0;j<J[i].size();j+=2)
                {
                    int u=J[i][j],v=J[i][j+1];
                    G[u].push_back(edge(v,++tot));
                    G[v].push_back(edge(u,++tot));
                }
                dfs(i);
                vector<int> pos;
                for(int i=top;i-->0)
                {
                    if(s[i]>M||s[i]<-M)
                        pos.push_back(i);
                }
                for(int i=0;i<pos.size()-1;i++)
                {
                    cnt++;
                    for(int j=pos[i]-1;j>pos[i+1];j--)
                        ans[cnt].push_back(s[j]);
                }
                cnt++;
                for(int j=pos[pos.size()-1]-1;j-->0)
                    ans[cnt].push_back(s[j]);
                for(int j=top;j>pos[0];j--)
                    ans[cnt].push_back(s[j]);
            }
        }
    }
    int k=cnt;
    for(int i=1;i<=cnt;i++)
        if(ans[i].size()==0)
            k--;

```

```

    printf("%d\n",k);
    for(int i=1;i<=cnt;i++)
    {
        if(ans[i].size()!=0)
        {
            printf("%d",ans[i].size());
            for(int j=0;j<ans[i].size();j++)
                printf(" %d",ans[i][j]);
            printf("\n");
        }
    }
}
return 0;
}

```

2.10 次小生成树

/*次小生成数*/

```

#include <cstdio>
#include <iostream>
#include <cstring>
#include <algorithm>
#include <vector>
#include <cmath>
using namespace std;
const int maxn=1000+5;
int N,M;
int f[maxn];
int judge[maxn];
int depth[maxn];
int gra[maxn][18];
int maxd[maxn][18];
struct Edge
{
    int u,v,d;
    Edge(int from,int to,int cost):u(from),v(to),d(cost){}
    bool operator <(const Edge &a) const
    {
        return d<a.d;
    }
};
vector<Edge> edges;
vector<Edge> G[maxn];
void init()
{
    for(int i=0;i<maxn;i++)
        f[i]=i;
    memset(judge,0,sizeof(judge));
    memset(depth,0,sizeof(depth));
    depth[1]=1;
    edges.clear();
    for(int i=0;i<maxn;i++)
        G[i].clear();
}
int Find(int x)

```

```

{
    if(f[x]==x)
        return f[x];
    return f[x]=Find(f[x]);
}
void unit(int x,int y)
{
    x=Find(x);
    y=Find(y);
    if(x==y)
        return ;
    f[x]=y;
}
bool same(int x,int y)
{
    return Find(x)==Find(y);
}
int Kruskal()
{
    sort(edges.begin(),edges.end());
    int res=0;
    for(int i=0;i<edges.size();i++)
    {
        Edge e=edges[i];
        if(!same(e.u,e.v))
        {
            res+=e.d;
            G[e.u].push_back(Edge(e.u,e.v,e.d));
            G[e.v].push_back(Edge(e.v,e.u,e.d));
            judge[i]=1;
            unit(e.u,e.v);
        }
    }
    return res;
}
void dfs(int x,int fa)
{
    for(int i=0;i<G[x].size();i++)
    {
        Edge e=G[x][i];
        if(e.v==fa)
            continue;
        depth[e.v]=depth[x]+1;
        gra[e.v][0]=x;
        maxd[e.v][0]=e.d;
        dfs(e.v,x);
    }
}
void solve()
{
    for(int i=1;(1<<i)<=N;i++)
    {
        for(int u=1;u<=N;u++)
        {
            gra[u][i]=gra[gra[u][i-1]][i-1];
            maxd[u][i]=max(maxd[u][i-1],maxd[gra[u][i-1]][i-1]);
        }
    }
}

```

```

    }
}
int lca(int u,int v)
{
    if(depth[u]<depth[v])
        swap(u,v);
    int d=depth[u]-depth[v];
    for(int i=0;(1<<i)<=d;i++)
    {
        if((1<<i)&d)
            u=gra[u][i];
    }
    if(u==v)
        return u;
    for(int i=(int)log(N);i>=0;i--)
    {
        if(gra[u][i]!=gra[v][i])
            u=gra[u][i],v=gra[v][i];
    }
    return gra[u][0];
}
int qmax(int u,int v)
{
    int tmp=-0x3f3f3f3f;
    for(int i=0;(1<<i)<=N;i++)
    {
        if(depth[gra[u][i]]>=depth[v])
            tmp=max(tmp,maxd[u][i]);
    }
    //cout<<u<<" "<<v<<" "<<tmp<<endl;
    return tmp;
}
int main()
{
    ios::sync_with_stdio(0);
    int T;
    cin>>T;
    while(T--)
    {
        cin>>N>>M;
        int u,v,c;
        init();
        while(M--)
        {
            cin>>u>>v>>c;
            edges.push_back(Edge(u,v,c));
            //edges.push_back(Edge(v,u,c));
        }
        int MST=Kruskal();
        dfs(1,-1);
        solve();
        int ans=0x3f3f3f3f;
        for(int i=0;i<edges.size();i++)
        {
            if(judge[i]==1)

```



```
        continue;
    Edge e=edges[i];
    u=e.u,v=e.v,c=e.d;
    int LCA=lca(u,v);
    int maxu=qmax(u,LCA);
    int maxv=qmax(v,LCA);
    ans=min(ans,c-max(maxu,maxv));
}
cout<<MST<<" "<<ans+MST<<endl;
}
return 0;
}
```

3 数据结构

3.1 并查集

```
/*并查集（带路径压缩）*/
const int maxn= " ";
int p[maxn];
void init(int n)
{
    for(int i=0; i<=n; i++)
        p[i]=i;
}
int Find(int x)
{
    if(x==p[x])
        return p[x];
    int y=Find(p[x]);
    return p[x]=y;
}

int Union(int x,int y)
{
    int x1=Find(x);
    int y1=Find(y);
    if(x1==y1)
        return 0;
    p[x1]=y1;
    return 1;
}

/* 带权并查集 */
const int maxn=" ";
int p[maxn],ran[maxn];
void init(int n)
{
    for(int i=0; i<=n; i++)
    {
        p[i]=i;
        ran[i]=0;
    }
    return;
}
int Find(int x)
{
    if(x==p[x])
        return p[x];
    int y=Find(p[x]);
    ran[x]=(ran[x]+ran[p[x]])%3;
    return p[x]=y;
}
int Union(int x,int y,int typ)
{
    int x1=Find(x);
    int y1=Find(y);
    if(x1==y1)
    {
```

```

        if((ran[x]-ran[y]+3)%3==typ-1)//
            return 0;
        return 1;
    }
    p[x1]=y1;
    ran[x1]=(-ran[x]+typ-1+ran[y]+3)%3;//
    return 0;
}

```

3.2 LCA

```

/**倍增lca*/
const int maxn= " ";
const int N= "30";
int n;
int fa[maxn][N+5];
int deep[maxn];
vector<int>edge[maxn];

void dfs(int u,int pre){
    for(int i=0;i<edge[u].size();i++){
        int v=edge[u][i];
        if(v==pre) continue;
        fa[v][0]=u;//should give fa[v][0] value
        deep[v]=deep[u]+1; //also can preprocessing distance here
        dfs(v,u);
    }
}

void bz(){
    for(int j=1;j<=N;j++){
        for(int i=1;i<=n;i++){
            fa[i][j]=fa[fa[i][j-1]][j-1];
        }
    }

    int lca(int u,int v){
        if(deep[u]<deep[v]) swap(u,v);
        int dc=deep[u]-deep[v];
        for(int i=0;i<N;i++){
            if((1<=i)&dc)//move u to dc+u
                u=fa[u][i];
        }
        if(u==v) return u;
        for(int i=N-1;i>=0;i--){
            if(fa[u][i]!=fa[v][i]){
                u=fa[u][i];v=fa[v][i];
            }
        }
        u=fa[u][0];//on the next level of lca,just move up one
        return u;
    }

    /*ST表预处理lca o(nlogn+q) */
    vector<int> edge[maxn], sp;
    int dep[maxn], dfn[maxn];
    pair<int,int> dp[21][maxn << 1];

```

```

void init(int n)
{
    for (int i = 0; i < n; i++) edge[i].clear();
    sp.clear();
}
void dfs(int u, int fa)
{
    dep[u] = dep[fa] + 1;
    dfn[u] = sp.size();
    sp.push_back(u);
    for (auto& v : edge[u])
    {
        if (v == fa) continue;
        dfs(v, u);
        sp.push_back(u);
    }
}
void initrmq()
{
    int n = sp.size();
    for (int i = 0; i < n; i++) dp[0][i] = {dfn[sp[i]], sp[i]};
    for (int i = 1; (1 << i) <= n; i++)
        for (int j = 0; j + (1 << i) - 1 < n; j++)
            dp[i][j] = min(dp[i - 1][j], dp[i - 1][j + (1 << (i - 1))]);
}
int lca(int u, int v)
{
    int l = dfn[u], r = dfn[v];
    if (l > r) swap(l, r);
    int k = 31 - __builtin_clz(r - l + 1);
    return min(dp[k][l], dp[k][r - (1 << k) + 1]).sec;
}

```

3.3 RMQ

```

/*RMQ HDU-4123*/
void ST(int n) {
    for (int j = 1; (1 << j) <= n; j++) {
        for (int i = 1; i + (1 << j) - 1 <= n; i++) {
            dp[i][j] = max(dp[i][j - 1], dp[i + (1 << (j - 1))][j - 1]);
        }
    }
}

int RMQ(int l, int r) {
    // int k = 0;
    int k = 31 - __builtin_clz(r - l + 1);
    // while ((1 << (k + 1)) <= r - l + 1) k++;
    return max(dp[l][k], dp[r - (1 << k) + 1][k]);
}

```

3.4 线段树

```

/****单点更新 HDU - 1166****/
T tree[maxn<<2];

```

```

void pushup(int rt){
    tree[rt]=tree[rt*2]+tree[rt*2+1];
}

void build(int l,int r,int rt){
    if(l==r) {
        //scanf("%d",&tree[rt]);
        return;
    }
    int mid=(l+r)/2;
    build(l,mid,rt*2);
    build(mid+1,r,rt*2+1);
    pushup(rt);
}

T query(int l,int r,int L,int R,int rt){
    if(l>=L&&r<=R)
        return tree[rt];
    T ans=0;
    int mid=(l+r)/2;
    if(L<=mid){
        ans+=query(l,mid,L,R,rt*2);
    }
    if(R>mid){
        ans+=query(mid+1,r,L,R,rt*2+1);
    }
    return ans;
}

void update(int l,int r,int index,T add,int rt){
    if(l==r) {
        tree[rt]+=add;
        return;
    }
    int mid=(l+r)/2;
    if(index<=mid)
        update(l,mid,index,add,rt*2);

    else update(mid+1,r,index,add,rt*2+1);
    pushup(rt);
}

/****区间更新 poj-3468****/
T tree[maxn<<2];
T seg[maxn<<2];
void pushup(int rt){
    tree[rt]=tree[rt*2]+tree[rt*2+1];
}

void pushdown(int len,int rt){
    if(seg[rt]){
        seg[rt*2]+=seg[rt];
        seg[rt*2+1]+=seg[rt];
        tree[rt*2]+=(len-len/2)*seg[rt];
        tree[rt*2+1]+=len/2*seg[rt];
    }
}

```

```

    seg[rt]=0;
}
}
void build(int l,int r,int rt){
    seg[rt]=0;
    if(l==r) {
        //scanf("%d",&tree[rt]);
        return;}
    int mid=(l+r)/2;
    build(l,mid,rt*2);
    build(mid+1,r,rt*2+1);
    pushup(rt);
}
void update(int l,int r,int L,int R,T add,int rt){
    if(l>=L&&r<=R) {
        seg[rt]+=add;
        tree[rt]+=(r-l+1)*add;
        return;
    }
    pushdown(r-l+1,rt);
    int mid=(l+r)/2;
    if(L<=mid)
        update(l,mid,L,R,add,rt*2);

    if(R>mid)
        update(mid+1,r,L,R,add,rt*2+1);

    pushup(rt);
}

T query(int l,int r,int L,int R,int rt){
    if(l>=L&&r<=R){
        return tree[rt];
    }
    T ans=0;
    pushdown(r-l+1,rt);
    int mid=(l+r)/2;
    if(L<=mid) ans+=query(l,mid,L,R,rt*2);
    if(R>mid) ans+=query(mid+1,r,L,R,rt*2+1);
    return ans;
}

```

3.5 树状数组

/* 树状数组单点更新 */

```

int lowbit(int x){
    return x&(-x);
}
T sum(int x){
    T ret=0;
    while(x>0){
        ret+=bit[x];
        x-=lowbit(x);
    }
    return ret;
}

```

```

}

void add(int x,T d){
    if(x<0) return;
    while(x<=n){
        bit[x]+=d;
        x+=lowbit(x);
    }
}

/**区间更新区间查询 **/
int lowbit(int x){
    return x&(-x);
}

void add(int x,int y){
    for(int i=x;i<=n;i+=lowbit(i))
        for(int j=y;j<=n;j+=lowbit(j))
            bit[i][j]++;
}

T sum(int x,int y){
    T ret=0;
    for(int i=x;i>0;i-=lowbit(i))
        for(int j=y;j>0;j-=lowbit(j))
            ret+=bit[i][j];
    return ret;
}

```

3.6 主席树

```

/**主席树 区间第k小 POJ 2104*/
int n,m,cnt;
int root[maxn],a[maxn];
int x,y,k;
struct node{
    int l,r,sum;
}T[maxn*40];
vector<int>v;

int getid(int x){
    return lower_bound(v.begin(),v.end(),x)-v.begin()+1;
}

void init()
{
    cnt=0;
    root[0]=0;
    T[0].l = T[0].r = T[0].sum = 0;
    v.clear();
}

void update(int l,int r,int &x,int y,int pos){
    T[++cnt]=T[y];
    T[cnt].sum++;
    x=cnt;
}

```

```

    if(l==r) return ;
    int mid=(l+r)/2;
    if(mid>=pos) update(l,mid,T[x].l,T[y].l,pos);
    else update(mid+1,r,T[x].r,T[y].r,pos);
}
int query(int l,int r,int x,int y,int k){
    if(l==r) return l;
    int mid=(l+r)/2;
    int sum=T[T[y].l].sum-T[T[x].l].sum;
    if(sum>=k) return query(l,mid,T[x].l,T[y].l,k);
    else return query(mid+1,r,T[x].r,T[y].r,k-sum);
}
int main(){
    while(scanf("%d%d",&n,&m)==2){
        init();
        //cnt=0;
        for(int i=1;i<=n;i++){
            scanf("%d",&a[i]),v.push_back(a[i]);
            sort(v.begin(),v.end()),v.erase(unique(v.begin(),v.end()),v.end());

            for(int i=1;i<=n;i++){
                update(1,n,root[i],root[i-1],getid(a[i]));
            }
            while(m--){
                int l,r,k;
                scanf("%d%d%d",&x,&y,&k);
                printf("%d\n",v[query(1,n,root[x-1],root[y],k)-1]);
            }
        }
    }
    return 0;
}

```

```

/**主席树区间更新 HDU 4348*/
const int maxn=1e5+100;
struct node{
    int l,int r;
    ll lazy;
    ll sum;
}T[maxn*40];
int cnt;
int root[maxn];
void pushup(int x,int len){
    T[x].sum=T[T[x].l].sum+T[T[x].r].sum+T[x].lazy*len;
}
void build(int l,int r,int &x){
    x++cnt;
    if(l==r) {
        T[x].lazy=0;
        scanf("%lld",&T[x].sum);
        return ;
    }
    int mid=(l+r)/2;
    build(l,mid,T[x].l);
    build(mid+1,r,T[x].r);
}

```



```

    pushup(x,r-l+1);
}

void update(int l,int r,int L,int R,int &x,int y,int val){
    T[++cnt]=T[y];
    x=cnt;
    if(l>=L&&r<=R){
        T[x].lazy+=val;
        T[x].sum+=(r-l+1)*val;
        return;
    }
    int mid=(l+r)/2;
    if(mid>=L)    update(l,mid,L,R,T[x].l,T[y].l,val);
    if(mid<R)    update(mid+1,r,L,R,T[x].r,T[y].r,val);
    pushup(x,r-l+1);
}

ll query(int l,int r,int L,int R,ll adv,int x){
    if(l>=L&&r<=R){
        return T[x].sum+adv*(r-l+1);
    }
    adv+=T[x].lazy;
    int mid=(l+r)/2;
    ll sum=0;
    if(L<=mid)    sum+=query(l,mid,L,R,adv,T[x].l);
    if(R>mid)    sum+=query(mid+1,r,L,R,adv,T[x].r);
    return sum;
}

```

3.7 树链剖分

```

#include <bits/stdc++.h>
using namespace std;
#define lson rt<<1
#define rson rt<<1|1
#define Lson L,mid,lson
#define Rson mid+1,R,rson
const int maxn=1e5+10;
typedef unsigned long long ull;
ull INF=0xffffffffffffffff;
int top[maxn],son[maxn],dep[maxn],f[maxn];
int sz[maxn],key[maxn];
int id[maxn];
vector<int> G[maxn];
int N;
int tot;
ull sum[maxn*4];
ull add[maxn*4];
ull mul[maxn*4];
void pushup(int rt)
{
    sum[rt]=sum[lson]+sum[rson];
}
void pushdown(int rt,int len)
{

```

```

    if(add[rt]!=0||mul[rt]!=1)
    {
        add[rt<<1]=(add[rt<<1]*mul[rt]+add[rt]);
        add[rt<<1|1]=(add[rt<<1|1]*mul[rt]+add[rt]);
        mul[rt<<1]=(mul[rt<<1]*mul[rt]);
        mul[rt<<1|1]=(mul[rt<<1|1]*mul[rt]);
        sum[rt<<1]=(add[rt]*(len-(len>>1))+sum[rt<<1]*mul[rt]);
        sum[rt<<1|1]=((add[rt]*(len>>1))+sum[rt<<1|1]*mul[rt]);
        add[rt]=0;
        mul[rt]=1;
    }
}
void init()
{
    memset(son,0,sizeof(son));
    memset(sz,0,sizeof(sz));
    for(int i=0;i<maxn;i++)
        G[i].clear();
    tot=0;
    dep[1]=0;
}
void build(int L,int R,int rt)
{
    add[rt]=0;
    mul[rt]=1;
    if(L==R)
    {
        sum[rt]=0;
        return ;
    }
    int mid=(L+R)>>1;
    build(Lson);
    build(Rson);
    pushup(rt);
}
void dfs1(int u,int fa)
{
    sz[u]=1;
    f[u]=fa;
    for(int i=0;i<G[u].size();i++)
    {
        int v=G[u][i];
        if(v==fa)
            continue;
        dep[v]=dep[u]+1;
        dfs1(v,u);
        sz[u]+=sz[v];
        if(son[u]==0||sz[v]>sz[son[u]])
        {
            son[u]=v;
        }
    }
}
void dfs2(int u,int fa)
{
    top[u]=fa;

```

```

    id[u]=++tot;
    if(son[u])
        dfs2(son[u],fa);
    for(int i=0;i<G[u].size();i++)
    {
        int v=G[u][i];
        if(v==f[u])
            continue;
        if(v!=son[u])
            dfs2(v,v);
    }
}
void updateplus(int l,int r,ull val,int L,int R,int rt)
{
    if(l<=L&&r>=R)
    {
        add[rt]+=val;
        sum[rt]+=val*(R-L+1);
        return ;
    }
    pushdown(rt,R-L+1);
    int mid=(L+R)>>1;
    if(l<=mid)
        updateplus(l,r,val,Lson);
    if(r>mid)
        updateplus(l,r,val,Rson);
    pushup(rt);
}
void updatemul(int l,int r,ull val,int L,int R,int rt)
{
    if(l<=L&&r>=R)
    {
        add[rt]*=val;
        mul[rt]*=val;
        sum[rt]*=val;
        return ;
    }
    pushdown(rt,R-L+1);
    int mid=(L+R)>>1;
    if(l<=mid)
        updatemul(l,r,val,Lson);
    if(r>mid)
        updatemul(l,r,val,Rson);
    pushup(rt);
}
void changeadd(int x,int y,ull val)
{
    while(top[x]!=top[y])
    {
        if(dep[top[x]]<dep[top[y]])
            swap(x,y);
        updateplus(id[top[x]],id[x],val,1,N,1);
        x=f[top[x]];
    }
    if(dep[x]>dep[y])
        swap(x,y);

```

```

        updateplus(id[x],id[y],val,1,N,1);
    }
    void changemul(int x,int y,ull val)
    {
        //cout<<x<<" "<<y<<" "<<val<<endl;
        while(top[x]!=top[y])
        {
            if(dep[top[x]]<dep[top[y]])
                swap(x,y);
            updatemul(id[top[x]],id[x],val,1,N,1);
            x=f[top[x]];
        }
        if(dep[x]>dep[y])
            swap(x,y);
        updatemul(id[x],id[y],val,1,N,1);
    }
    ull query(int l,int r,int L,int R,int rt)
    {
        if(l<=L&&r>=R)
        {
            return sum[rt];
        }
        pushdown(rt,R-L+1);
        int mid=(L+R)>>1;
        ull res=0;
        if(l<=mid)
            res+=query(l,r,Lson);
        if(r>mid)
            res+=query(l,r,Rson);
        return res;
    }
    ull get(int x, int y)
    {
        ull res=0;
        while(top[x] != top[y])
        {
            if(dep[top[x]] < dep[top[y]])
                swap(x, y);
            res+=query(id[top[x]],id[x],1,N,1);
            x = f[top[x]];
        }
        if(dep[x] > dep[y])
            swap(x, y);
        res+=query(id[x],id[y],1,N,1);
        return res;
    }
}

```

3.8 LCT

//维护点权

```

struct LCT
{
    int val[maxn],sum[maxn];
    int rev[maxn],ch[maxn][2],fa[maxn];
    int nxt[maxn];
    int stk[maxn];
}

```

```

void init(int n)
{
    for(int i=1;i<=n;i++)
        val[i]=1,fa[i]=0,rev[i]=0,ch[i][0]=ch[i][1]=0;
}
bool isroot(int x)
{
    return ch[fa[x]][0]!=x&&ch[fa[x]][1]!=x;
}
bool get(int x)
{
    return ch[fa[x]][1]==x;
}
void pushdown(int x)
{
    if(!rev[x])
        return ;
    swap(ch[x][0],ch[x][1]);
    if(ch[x][0])
        rev[ch[x][0]]^=1;
    if(ch[x][1])
        rev[ch[x][1]]^=1;
    rev[x]^=1;
}
void pushup(int x)
{
    sum[x]=val[x]+sum[ch[x][0]]+sum[ch[x][1]];
}
void rotate(int x)
{
    int y=fa[x],z=fa[fa[x]],d=get(x);
    if(!isroot(y))
        ch[z][get(y)]=x;
    fa[x]=z;
    ch[y][d]=ch[x][d^1],fa[ch[y][d]]=y;
    ch[x][d^1]=y,fa[y]=x;
    pushup(y),pushup(x);
}
void splay(int x)
{
    int top=0;
    stk[++top]=x;
    for(int i=x;!isroot(i);i=fa[i]) stk[++top]=fa[i];
    for(int i=top;i;i--) pushdown(stk[i]);
    for(int f;!isroot(x);rotate(x))
        if(!isroot(f=fa[x]))
            rotate(get(x)==get(f)?f:x);
}
void access(int x)
{
    for(int y=0;x;y=x,x=fa[x])
    {
        splay(x);
        ch[x][1]=y;
        pushup(x);
    }
}

```

```

    }
    int find(int x)
    {
        access(x), splay(x);
        while(ch[x][0])
            x = ch[x][0];
        return x;
    }
    void makeroot(int x) {access(x), splay(x), rev[x]^=1;}
    void link(int x, int y) {makeroot(x), fa[x]=y, splay(x);}
    void cut(int x, int y) {makeroot(x), access(y), splay(y), fa[x]=ch[y][0]=0;}
    void update(int x, int v) {val[x]=v, access(x), splay(x);}
    int query(int x, int y)
    {
        makeroot(y), access(x), splay(x);
        return sum[ch[x][0]];
    }
}lct;

//维护子树
#include <cstdio>
#include <algorithm>
#define N 100010
using namespace std;
int fa[N], c[2][N], si[N], sum[N], rev[N];
char str[5];
void pushup(int x)
{
    sum[x] = sum[c[0][x]] + sum[c[1][x]] + si[x] + 1;
}
void pushdown(int x)
{
    if(rev[x])
    {
        int l = c[0][x], r = c[1][x];
        swap(c[0][l], c[1][l]), swap(c[0][r], c[1][r]);
        rev[l]^=1, rev[r]^=1, rev[x]=0;
    }
}
bool isroot(int x)
{
    return c[0][fa[x]] != x && c[1][fa[x]] != x;
}
void update(int x)
{
    if(!isroot(x)) update(fa[x]);
    pushdown(x);
}
void rotate(int x)
{
    int y = fa[x], z = fa[y], l = (c[1][y] == x), r = l^1;
    if(!isroot(y)) c[c[1][z] == y][z] = x;
    fa[x] = z, fa[y] = x, fa[c[r][x]] = y, c[1][y] = c[r][x], c[r][x] = y;
    pushup(y), pushup(x);
}
void splay(int x)

```

```

{
    update(x);
    while(!isroot(x))
    {
        int y = fa[x] , z = fa[y];
        if(!isroot(y))
        {
            if((c[0][y] == x) ^ (c[0][z] == y)) rotate(x);
            else rotate(y);
        }
        rotate(x);
    }
}
void access(int x)
{
    int t = 0;
    while(x)
    {
        splay(x) ;
        si[x] += sum[c[1][x]] - sum[t] , c[1][x] = t , pushup(x) , t = x , x = fa[x];
    }
}
void makeroot(int x)
{
    access(x) , splay(x) , swap(c[0][x] , c[1][x]) , rev[x] = 1;
}
void split(int x , int y)
{
    makeroot(x) , makeroot(y);
}
void link(int x , int y)
{
    split(x , y) , fa[x] = y , si[y] += sum[x] , pushup(y);
}

```

3.9 Splay

```

#define key_value ch[ch[root][1]][0]
const int maxn = 1 << 19;

struct Splay
{
    int a[maxn];
    int sz[maxn], ch[maxn][2], fa[maxn];
    int key[maxn], rev[maxn];
    int root, tot;
    int stk[maxn], top;

#ifdef ONLINE_JUDGE
    void Treavel(int x)
    {
        if (x)
        {
            Treavel(ch[x][0]);
            printf("结点:%2d: 左儿子 %2d 右儿子 %2d 父结点 %2d size= %2d key= %2d\n",
                x, ch[x][0], ch[x][1], fa[x], sz[x], key[x]);
        }
    }

```

```

        Treavel(ch[x][1]);
    }
}

void debug()
{
    printf("root:%d\n", root);
    Treavel(root);
}
#endif

void init(int n)
{
    tot = 0, top = 0;
    root = newnode(0, -1);
    ch[root][1] = newnode(root, -1);
    for (int i = 0; i < n; i++) a[i] = i + 1;
    key_value = build(0, n - 1, ch[root][1]);
    pushup(ch[root][1]);
    pushup(root);
}

int newnode(int p = 0, int k = 0)
{
    int x = top ? stk[top--] : ++tot;
    fa[x] = p;
    sz[x] = 1;
    ch[x][0] = ch[x][1] = 0;
    key[x] = k;
    rev[x] = 0;
    return x;
}

void pushdown(int x)
{
    if (rev[x])
    {
        swap(ch[x][0], ch[x][1]);
        if (ch[x][0]) rev[ch[x][0]] ^= 1;
        if (ch[x][1]) rev[ch[x][1]] ^= 1;
        rev[x] = 0;
    }
}

void pushup(int x)
{
    sz[x] = sz[ch[x][0]] + sz[ch[x][1]] + 1;
}

void rotate(int x, int d)
{
    int y = fa[x];
    pushdown(y), pushdown(x);
    ch[y][d ^ 1] = ch[x][d];
    fa[ch[x][d]] = y;
    if (fa[y]) ch[fa[y]][ch[fa[y]][1] == y] = x;
}

```



```

    fa[x] = fa[y];
    ch[x][d] = y;
    fa[y] = x;
    pushup(y);
}

void splay(int x, int goal = 0)
{
    pushdown(x);
    while (fa[x] != goal)
    {
        if (fa[fa[x]] == goal)
            rotate(x, ch[fa[x]][0] == x);
        else
        {
            int y = fa[x];
            int d = ch[fa[y]][0] == y;
            ch[y][d] == x ? rotate(x, d ^ 1) : rotate(y, d);
            rotate(x, d);
        }
    }
    pushup(x);
    if (goal == 0) root = x;
}

int kth(int r, int k)
{
    pushdown(r);
    int t = sz[ch[r][0]] + 1;
    if (t == k) return r;
    return t > k ? kth(ch[r][0], k) : kth(ch[r][1], k - t);
}

int build(int l, int r, int p)
{
    if (l > r) return 0;
    int mid = l + r >> 1;
    int x = newnode(p, a[mid]);
    ch[x][0] = build(l, mid - 1, x);
    ch[x][1] = build(mid + 1, r, x);
    pushup(x);
    return x;
}

void select(int l, int r)
{
    splay(kth(root, l), 0);
    splay(kth(ch[root][1], r - l + 2), root);
}

void filp(int l, int r)
{
    select(l, r);
    rev[key_value] ^= 1;
}

```

```

void cut(int l, int r, int c)
{
    select(l, r);
    int tmp = key_value;
    key_value = 0;
    pushup(ch[root][1]), pushup(root);
    select(c + 1, c);
    key_value = tmp, fa[key_value] = ch[root][1];
    pushup(ch[root][1]), pushup(root);
    splay(tmp);
}

int ans[maxn], pos;

void dfs(int x)
{
    if (x)
    {
        pushdown(x);
        dfs(ch[x][0]);
        if (~key[x]) ans[pos++] = key[x];
        dfs(ch[x][1]);
    }
}

void print()
{
    pos = 0;
    dfs(root);
    for (int i = 0; i < pos; i++) printf("%d%c", ans[i], " \n"[i == pos - 1]);
}
} gao;

```

3.10 区间不同数

```

/* 区间不同数 */
/*主席树*/
#include <bits/stdc++.h>
using namespace std;
const int maxn=2*1e5+100;
int data[maxn];
int a[maxn];
int ans[maxn];
map<int,int>mp;
void init()
{
    for(int i=0;i<maxn;i++)
        data[i]=0;
    mp.clear();
}
struct node{
    int l,r,id;
    bool operator<(const node &t)const{
        return l<t.l;
    }
}q[maxn];

```

```

int sum(int i){
    int ans=0;
    while(i>0){
        ans+=data[i];
        i-=i&-i;
    }
    return ans;
}

void add(int i,int x){
    while(i<maxn){
        data[i]+=x;
        i+=i&-i;
    }
}

int main()
{
    // ios_base::sync_with_stdio(false);cin.tie(NULL);cout.tie(NULL);
    int n,Q;
    while(scanf("%d%d",&n,&Q)==2)
    {
        init();
        for(int i=1;i<=n;i++)
            scanf("%d",&a[i]),a[i+n]=a[i];

        for(int i=1;i<=Q;i++)
        {
            scanf("%d%d",&q[i].l,&q[i].r);
            q[i].l=q[i].l+n;
            q[i].id=i;
        }

        sort(q+1,q+1+Q);
        int pre=1;
        for(int i=1;i<=Q;i++)
        {
            for(int j=pre;j<=q[i].l;j++)
            {
                if(mp[a[j]])
                {
                    add(mp[a[j]],-1);
                }
                add(j,1);
                mp[a[j]]=j;
            }
            pre=q[i].l+1;

            ans[q[i].id]=sum(q[i].l)-sum(q[i].r-1);
        }

        for(int i=1;i<=Q;i++)
            printf("%d\n",ans[i]);
        //cout<<ans[i]<<endl;
    }
}
/*

```

树状数组

```

*/
#include <cstdio>
#include <iostream>
#include <queue>
#include <stack>
#include <cmath>
#include <cstring>
#include <string>
#include <map>
#include <sstream>
#include <algorithm>
#define pb(x) push_back(x)
#define fir first
#define sec second
#define mem(a,x) memset(a,x,sizeof(a))
typedef long long ll;
using namespace std;
const int inf=0x3f3f3f3f;
const ll INF= 0x3f3f3f3f3f3f3f3f;
/*
#ifdef ONLINE_JUDGE
    freopen("data.in", "r", stdin);
    freopen("data.out", "w", stdout);
#endif
*/
const int maxn=2*1e5+100;
int n,m,cnt;
int root[maxn],a[maxn];
int x,y,k;
map<int,int>mp;
struct node{
    int l,r,sum;
}T[maxn*40];
void init()
{
    cnt=0;
    mp.clear();
    root[0]=0;
    T[0].l = T[0].r = T[0].sum = 0;
}

void update(int l,int r,int &x,int y,int pos,int val){
    T[++cnt]=T[y];
    T[cnt].sum+=val;
    x=cnt;
    if(l==r) return ;
    int mid=(l+r)/2;
    if(mid>=pos) update(l,mid,T[x].l,T[y].l,pos,val);
    else update(mid+1,r,T[x].r,T[y].r,pos,val);
}

int query(int l,int r,int pos,int y){
    if(l==r) return T[y].sum;

```

```

    int mid=(l+r)/2;
    if(pos <= mid)
        return query(1,mid,pos,T[y].l) + T[T[y].r].sum;
    else
        return query(mid+1,r,pos, T[y].r);
}

int main(){
    //ios_base::sync_with_stdio(false);cin.tie(NULL);cout.tie(NULL);
    int q;
    while(scanf("%d%d",&n,&q)==2)
    {
        init();
        for(int i=1;i<=n;i++)
            scanf("%d",&a[i]),a[i+n]=a[i];
        int m=n;
        n*=2;
        int tmp;
        for(int i=1;i<=n;i++){
            if(mp[a[i]]==0){
                update(1,n,root[i],root[i-1],i,1);
                mp[a[i]]=i;
            }
            else{
                update(1,n,tmp,root[i-1],mp[a[i]],-1);
                update(1,n,root[i],tmp,i,1);
                mp[a[i]]=i;
            }
        }

        for(int i=1;i<=q;i++)
        {
            int l,r;
            scanf("%d%d",&l,&r);
            l+=m;
            printf("%d\n",query(1,n,r,root[l]));
        }
    }

    return 0;
}

```

3.11 矩形面积并

```

/*矩形面积并*/
#include <bits/stdc++.h>
using namespace std;
const int maxn=2010;
struct seg{
    double l,r,h;
    int s;
}s[maxn];
int res;
int col[maxn<<2];
double sum[maxn<<2];
double x[maxn<<2];

```

```

int cmp(seg a,seg b){
return a.h<b.h;
}
void pushup(int rt,int l,int r){
    if(col[rt]) sum[rt]=x[r+1]-x[l];//[ )
    else if(l==r) sum[rt]=0;
    else sum[rt]=sum[rt<<1]+sum[rt<<1|1];
}

void update(int l,int r,int c,int rt,int ll,int rr){//l,r is fresh area
    if(ll>=l&&rr<=r){
        col[rt]+=c;
        pushup(rt,ll,rr);
        return;
    }

    int mid=(ll+rr)/2;
    if(l<=mid) update(l,r,c,rt*2,ll,mid);
    if(r>mid) update(l,r,c,rt*2+1,mid+1,rr);
    pushup(rt,ll,rr);
}

int bfind(double t){
    /*int lb=0,rb=res;
    while(lb<=rb){
        int mid=(lb+rb)/2;
        if(x[mid]>t) rb=mid-1;
        else lb=mid+1;
    }
    return lb;*/
    int lb=-1,ub=res;
    while(ub-lb>1){
        int mid=(lb+ub)/2;
        if(x[mid]>=t) ub=mid;
        else lb=mid;
    }
    return ub;
}

int main(){
    int n;int k=0;
    while(cin>>n&&n){
        k++;
        int cnt=0;
        for(int i=0;i<n;i++){
            double a,b,c,d;
            cin>>a>>b>>c>>d;
            s[cnt]=seg{a,c,b,1};//bottom line
            x[cnt++]=a;
            s[cnt]=seg{a,c,d,-1};//top line
            x[cnt++]=c;
        }
        sort(x,x+cnt);
        sort(s,s+cnt,cmp);
        // int
        res=0;
    }
}

```

```

        for(int i=1;i<cnt;i++){
            if(x[i]!=x[i-1]){
                x[++res]=x[i];
            }
        }
        memset(col,0,sizeof(col));
        memset(sum,0,sizeof(sum));
        double ans=0;
        for(int i=0;i<cnt-1;i++){
            //cout<<1<<endl;
            int l=bfind(s[i].l);
            int r=bfind(s[i].r)-1;
            update(l,r,s[i].s,1,0,res);
            ans+=sum[1]*(s[i+1].h-s[i].h);
        }
        printf("Test case #%d\nTotal explored area: %.21f\n\n",k,ans);
    }
}

```

3.12 矩形面积交

/*矩形面积交*/

```

#include <cstdio>
#include <iostream>
#include <cstring>
#include <algorithm>
using namespace std;
#define lson rt<<1
#define rson rt<<1|1
#define Lson L,mid,lson
#define Rson mid+1,R,rson
const int maxn=2005;
struct segment
{
    int l;
    int r;
    int h;
    int type;
    segment(){}
    segment(int l,int r,int h,int type):l(l),r(r),h(h),type(type){}
    bool operator <(const segment & ryh) const
    {
        return h<ryh.h;
    }
}a[maxn];
struct point
{
    int x1,y1,x2,y2,z1,z2;
    point(){}
    point(int x1,int y1,int z1,int x2,int y2,int z2):x1(x1),y1(y1),z1(z1),x2(x2),y2(y2),z2(z2){}
}cube[maxn];
int cnt[maxn<<2];
int allx[maxn];

```

```

int allz[maxn];
int one[maxn<<2],two[maxn<<2],three[maxn<<2];
void pushup(int L,int R,int rt)
{
    if(cnt[rt]>=3)
    {
        one[rt]=two[rt]=three[rt]=allx[R+1]-allx[L];
    }
    else if(cnt[rt]==2)
    {
        one[rt]=two[rt]=allx[R+1]-allx[L];
        if(L==R)
            three[rt]=0;
        else
            three[rt]=one[lson]+one[rson];
    }
    else if(cnt[rt]==1)
    {
        one[rt]=allx[R+1]-allx[L];
        if(L==R)
        {
            two[rt]=three[rt]=0;
        }
        else
        {
            three[rt]=two[lson]+two[rson];
            two[rt]=one[lson]+one[rson];
        }
    }
    else
    {
        if(L==R)
        {
            one[rt]=two[rt]=three[rt]=0;
        }
        else
        {
            one[rt]=one[lson]+one[rson];
            two[rt]=two[lson]+two[rson];
            three[rt]=three[lson]+three[rson];
        }
    }
}

void update(int l,int r,int val,int L,int R,int rt)
{
    if(l<=L&&r>=R)
    {
        cnt[rt]+=val;
        pushup(L,R,rt);
        return ;
    }
    int mid=(L+R)>>1;
    if(l<=mid)
        update(l,r,val,Lson);
    if(r>mid)
        update(l,r,val,Rson);
}

```



```

    pushup(L,R,rt);
}
int main()
{
    int T;
    scanf("%d",&T);
    int kase=0;
    while(T--)
    {
        int N;
        scanf("%d",&N);
        for(int i=1;i<=N;i++)
        {
            int x1,x2,y1,y2,z1,z2;
            scanf("%d%d%d%d%d%d",&x1,&y1,&z1,&x2,&y2,&z2);
            cube[i]=point(x1,y1,z1,x2,y2,z2);
            allz[i]=z1,allz[i+N]=z2;
        }
        sort(allz+1,allz+1+N*2);
        int cntz=unique(allz+1,allz+1+N*2)-allz-1;
        long long ans=0;
        for(int i=1;i<cntz;i++)
        {
            int tot=0;
            memset(cnt,0,sizeof(cnt));
            memset(one,0,sizeof(one));
            memset(two,0,sizeof(two));
            memset(three,0,sizeof(three));
            for(int j=1;j<=N;j++)
            {
                if(cube[j].z1<=allz[i]&&cube[j].z2>=allz[i+1])
                {
                    a[++tot]=segment(cube[j].x1,cube[j].x2,cube[j].y1,1);
                    allx[tot]=cube[j].x1;
                    a[++tot]=segment(cube[j].x1,cube[j].x2,cube[j].y2,-1);
                    allx[tot]=cube[j].x2;
                }
            }
            sort(allx+1,allx+1+tot);
            int m=unique(allx+1,allx+1+tot)-allx-1;
            sort(a+1,a+tot+1);
            for(int j=1;j<tot;j++)
            {
                int l=lower_bound(allx+1,allx+1+m,a[j].l)-allx;
                int r=lower_bound(allx+1,allx+1+m,a[j].r)-allx;
                if(l<r)
                    update(l,r-1,a[j].type,1,m,1);
                ans+=(long long)(three[1])*(a[j+1].h-a[j].h)*(allz[i+1]-allz[i]);
            }
        }
        printf("Case %d: %lld\n",++kase,ans);
    }
    return 0;
}

```

3.13 矩形周长并

```

#include <cstdio>
#include <iostream>
#include <queue>
#include <cmath>
#include <cstring>
#include <algorithm>
// #define ll long long
#define pb(x) push_back(x)
#define fir first
#define sec second
using namespace std;

// freopen("data.in", "r", stdin);
// freopen("data.out", "w", stdout);
// ios_base::sync_with_stdio(false); cin.tie(NULL); cout.tie(NULL);

const int INF=0x3f3f3f3f;
const int maxn=5010;
struct seg{
    int l,r,h;
    int s;
}s[maxn<<1];
int cn[maxn*3];
unsigned char rnum[maxn*3],lnum[maxn*3];
int col[maxn*3];
int sum[maxn*3];

int cmp(seg a,seg b){
return a.h<b.h;
}

void pushup(int rt,int l,int r){
    if(col[rt]) {sum[rt]=r-l+1; //[ )
                cn[rt]=1;rnum[rt]=lnum[rt]='1';}
    else if(l==r) {sum[rt]=0;cn[rt]=0;rnum[rt]=lnum[rt]='0';}
    else {sum[rt]=sum[rt<<1]+sum[rt<<1|1];
          lnum[rt]=lnum[rt<<1];rnum[rt]=rnum[rt<<1|1];
          cn[rt]=cn[rt<<1]+cn[rt<<1|1]-(rnum[rt<<1]-'0')&&(lnum[rt<<1|1]-'0');
          }
}

void update(int l,int r,int c,int rt,int ll,int rr){//l,r is fresh area
    if(ll>=l&&rr<=r){
        col[rt]+=c;
        pushup(rt,ll,rr);
        return;
    }
    int mid=(ll+rr)/2;
    if(l<=mid) update(l,r,c,rt*2,ll,mid);
    if(r>mid) update(l,r,c,rt*2+1,mid+1,rr);
    pushup(rt,ll,rr);
}

```

```

int main(){
    int n;int k=0;
    while(scanf("%d",&n)==1&&n){
        k++;
        int cnt=0;int ll=INF;int rr=-INF;
        for(int i=0;i<n;i++){
            int a,b,c,d;
            scanf("%d%d%d%d",&a,&b,&c,&d);
            //cin>>a>>b>>c>>d;
            s[cnt++]=seg{a,c,b,1};//bottom line

            s[cnt++]=seg{a,c,d,-1};//top line
            ll=min(ll,a);
            rr=max(rr,c);
            // x[cnt++]=c;
        }
        // sort(x,x+cnt);
        sort(s,s+cnt,cmp);

        memset(col,0,sizeof(col));
        memset(sum,0,sizeof(sum));
        //memset(rnum,'0',sizeof(rnum));
        //memset(lnum,'0',sizeof(lnum));
        memset(cn,0,sizeof(cn));
        for(int i=0;i<maxn*3;i++){
            rnum[i]=lnum[i]='0';
        }
        int ans=0;int pre=0;
        for(int i=0;i<cnt-1;i++){
            update(s[i].l,s[i].r-1,s[i].s,1,ll,rr);
            ans+=abs(sum[1]-pre)+cn[1]*2*(s[i+1].h-s[i].h);
            pre=sum[1];
        }
        ans+=sum[1];
        printf("%d\n",ans);
        //cout<<ans<<endl;
    }
    return 0;
}

```

4 字符串

4.1 哈希

```

/* hash */
/**备选素数 1572869, 3145739, 6291469, 12582917, 25165843, 50331653*/
const int maxn = "1e5";
const int seed=31;
ull h[maxn];
ull base[maxn];
typedef pair<int,int> P;
void init()
{
    base[0]=1;
    for(int i=1;i<maxn;i++)
        base[i]=base[i-1]*seed;
}
ull str_hash(int l,int r){
    return h[r]-h[l-1]*base[r-l+1];
}
void Hash()
{
    for(int i=0;i<len;i++)
        h[i+1]=h[i]*seed+s[i]-'a'+1;
}

/* 随机数双哈希 (csl) Gym101808B */
map<pair<int,int>,pair<ull,ull>> dic;

inline pair<ull,ull>gethash(int x,int y)
{
    if(x>y) swap(x,y);
    if(dic.find({x,y})!=dic.end())
        return dic[{x,y}];
    ull h1=1;
    ull h2=1;
    for(int i=0;i<5;i++) h1*=rand();
    for(int i=0;i<5;i++) h2*=rand(); //用随机数hash
    return dic[{x,y}]={h1,h2};
}

map<pair<ull,ull>,int> cnt[maxn];

dic.clear();
for(int i=0;i<n;i++)
    a[i]=gethash(x,y);

for(int i=0;i<n;i++)
{
    ull hash1=0,hash2=0;
    for(int j=i;j<n;j++)
    {
        hash1+=a[j].first;hash2+=a[j].second;
        ans+=cnt[j-i][{hash1,hash2}];
        cnt[j-i][{hash1,hash2}]++; //长度相同的放在一个map
    }
}

```

```

}

/* 双哈希 (csl) */

const int seed1= "19260817" ;
const int mod1= "1e9+7";
const int seed2= "23333333";
const int mod2= "1e9+9" ;
int base1[maxn],base2[maxn];

map<pair<int,int>,int> dic;
inline int getid(int x,int y)
{
    if(x>y) swap(x,y);
    if(dic[{x,y}]) return dic[{x,y}];
    return dic[{x,y}]=dic.size();
}

void init(int n)
{
    base1[0]=1,base2[0]=1;
    for(int i=1;i<=n;i++)
        base1[i]=(1LL*base1[i-1]*seed1);
        base2[i]=(1LL*base2[i-1]*seed2);
}

dic.clear();
for(int i=0;i<n;i++)
    a[i]=getid(x,y);

map<pair<int,int>,int> cnt[maxn];
for(int i=0;i<n;i++)
{
    int sum1=0,sum2=0;
    for(int j=i;j<n;j++)
    {
        sum1=(sum1+base1[a[j]])%mod1;
        sum2=(sum2+base2[a[j]])%mod2;
        ans+=cnt[j-i][{sum1,sum2}];
        cnt[j-i][{sum1,sum2}]++; //长度相同的放在一个map
    }
}

/*PROVIDE BY CSL*/
typedef unsigned long long ull;
const ull Seed_Pool[] = {146527, 19260817};
const ull Mod_Pool[] = {1000000009, 998244353};
struct Hash
{
    ull SEED, MOD;
    vector<ull> p, h;
    Hash() {}
    Hash(const string& s, const int& seed_index, const int& mod_index)
    {
        SEED = Seed_Pool[seed_index];
        MOD = Mod_Pool[mod_index];
    }
}

```

```

    int n = s.length();
    p.resize(n + 1), h.resize(n + 1);
    p[0] = 1;
    for (int i = 1; i <= n; i++) p[i] = p[i - 1] * SEED % MOD;
    for (int i = 1; i <= n; i++) h[i] = (h[i - 1] * SEED % MOD + s[i - 1]) % MOD;
}
ull get(int l, int r) { return (h[r] - h[l] * p[r - l] % MOD + MOD) % MOD; }
ull substr(int l, int m) { return get(l, l + m); }
};

```

4.2 KMP

```

const int maxn = " ";
int fail[maxn];
void getfail(char *x)
{
    int m=strlen(x);
    int i = 0, j = fail[0] = -1;
    while (i < m)
    {
        while (j != -1 && x[i] != x[j]) j = fail[j];
        fail[++i] = ++j;
    }
}
//x是模式串, y是主串
// 返回y中x的个数
int kmp(char *x, char *y)
{
    int i, j, ans;
    i = j = ans = 0;
    getfail(x);
    int m=strlen(x);
    int n=strlen(y);
    while (i < n)
    {
        while (j != -1 && y[i] != x[j]) j = fail[j];
        i++, j++;
        if (j >= m) ans++, j = fail[j];
    }
    return ans;
}

```

4.3 扩展KMP

```

#include <cstdio>
#include <cstring>
#include <algorithm>
#include <iostream>
#include <map>
using namespace std;
const int maxn=1e5+5;
struct exKMP
{
    char t[maxn];
    char p[maxn];
    int f[maxn];

```

```

int extend[maxn];
void getfail(char *p,int *f)
{
    int m=strlen(p);
    f[0]=m;
    int i=0;
    while(i<m-1&&p[i]==p[i+1])
        i++;
    f[1]=i;
    int po=1;
    for(i=2;i<m;i++)
    {
        if(f[i-po]+i<po+f[po])
            f[i]=f[i-po];
        else
        {
            int j=po+f[po]-i;
            if(j<0)
                j=0;
            while((i+j<m)&&p[i+j]==p[j])
                j++;
            f[i]=j;
            po=i;
        }
    }
}

void getextend(char *t,char *p,int *f,int *extend)
{
    int n=strlen(t);
    int m=strlen(p);
    getfail(p,f);
    int i=0;
    while(t[i]==p[i]&&i<n&&i<m)
        i++;
    extend[0]=i;
    int po=0;
    for(int i=1;i<n;i++)
    {
        if(f[i-po]+i<extend[po]+po)
            extend[i]=f[i-po];
        else
        {
            int j=extend[po]+po-i;
            if(j<0)
                j=0;
            while(i+j<n&&j<m&&t[i+j]==p[j])
                j++;
            extend[i]=j;
            po=i;
        }
    }
}

}ans;
map<char,char> m;
int main()
{

```

```

int T;
scanf("%d",&T);
char code[30];
while(T--)
{
    scanf("%s",code);
    scanf("%s",ans.t);
    int n=strlen(ans.t);
    for(int i=0;i<26;i++)
        m[code[i]]=i+'a';
    for(int i=0;i<n;i++)
    {
        ans.p[i]=m[ans.t[i]];
    }
    ans.p[n]='\0';
    ans.getfail(ans.p,ans.f);
    ans.getextend(ans.t,ans.p,ans.f,ans.extend);
    int i;
    for( i=0;i<n;i++)
        if(i+ans.extend[i]>=n&&i>=ans.extend[i])
        {
            break;
        }
    for(int j=0;j<i;j++)
        printf("%c",ans.t[j]);
    for(int j=0;j<i;j++)
        printf("%c",ans.p[j]);
    printf("\n");
}
return 0;
}

```

4.4 Manacher

```

/*
复杂度线性
加完特殊字符后，最长子串的长度是半径减1，起始位置是中间位置减去半径再除以2。
*/
#include <bits/stdc++.h>
using namespace std;
const int maxn=1e6+5;
struct Manacher
{
    char p[maxn];
    char temp[maxn<<1];
    int f[maxn<<1];
    void init(char *p,char *temp)
    {
        int n=strlen(p);
        temp[0]='*';
        for(int i=0;i<=n;i++)
        {
            temp[i*2+1]='#';
            temp[i*2+2]=p[i];
        }
        temp[2*n+2]='\0';
    }

```



```

    }
    void getlen(char *p,int *f)
    {
        int mx=0,po=0;
        int n=strlen(p);
        f[0]=0;
        for(int i=2;i<n;i++)
        {
            if(mx>i)
                f[i]=min(mx-i,f[2*po-i]);
            else
                f[i]=1;
            while(p[i-f[i]]==p[i+f[i]])
                f[i]++;
            if(f[i]+i>mx)
            {
                po=i;
                mx=f[i]+i;
            }
        }
    }
}ans;
int main()
{
    int kase=0;
    while(scanf("%s",ans.p)==1&&ans.p[0]!='E')
    {
        ans.init(ans.p,ans.temp);
        ans.getlen(ans.temp,ans.f);
        int n=strlen(ans.temp);
        int res=1;
        for(int i=2;i<n;i++)
        {
            res=max(res,ans.f[i]-1);
        }
        printf("Case %d: %d\n",++kase,res);
    }
    return 0;
}

```

4.5 01字典树

```

//HDU4835
struct Trie{
    int next[maxnode][2];
    ll val[maxnode];
    int root,cnt;
    int newnode()
    {
        next[cnt][0]=next[cnt][1]=-1;
        val[cnt++]=0;
        return cnt-1;
    }
    void init()
    {
        cnt=0;
    }
}

```

```

        root=newnode();
    }
    void insert(ll x)
    {
        int now=root;
        for(int i=32;i>=0;i--)//attention
        {
            int id=((x>>i)&1);
            if(next[now][id]==-1)
                next[now][id]=newnode();

            now=next[now][id];
        }
        val[now]=x;
    }
    ll query(ll x)
    {
        int now=root;
        for(int i=32;i>=0;i--)
        {
            int id=((x>>i)&1);
            if(next[now][id^1]!=-1)
            {
                now=next[now][id^1];
            }
            else
            {
                now=next[now][id];
            }
        }
        return val[now];
    }
};

```

4.6 ac自动机

```

/****AC自动机 HUD-2222****/
/*以now节点结尾的后缀 与 root-fail[now]所表示的字符串 相同*/
const int NUMA=26;
struct trie{

    int next[maxnode][NUMA],fail[maxnode],ed[maxnode];//attention
    int root,cnt;
    int newnode(){
        for(int i=0;i<NUMA;i++)
            next[cnt][i]=-1;
        ed[cnt++]=0;
        return cnt-1;
    }

    void init(){
        cnt=0;
        root=newnode();
    }
}

```

```

void inser(char* buf){
    int len=strlen(buf);
    int now=root;
    for(int i=0;i<len;i++){
        if(next[now][buf[i]-'a']==-1)
            next[now][buf[i]-'a']=newnode();
        now=next[now][buf[i]-'a'];
    }
    ed[now]++;
}

void build(){
    queue<int>que;
    fail[root]=root;
    for(int i=0;i<NUMA;i++){
        if(next[root][i]==-1)
            next[root][i]=root;
        else{
            fail[next[root][i]]=root;
            que.push(next[root][i]);
        }
    }
    while(!que.empty()){
        int now=que.front();
        que.pop();
        for(int i=0;i<NUMA;i++){
            if(next[now][i]==-1)
                next[now][i]=next[fail[now]][i];
            else{
                fail[next[now][i]]=next[fail[now]][i];
                que.push(next[now][i]);
            }
        }
    }
}

int query(char* buf){
    int len=strlen(buf);
    int now=root;
    int res=0;
    for(int i=0;i<len;i++){
        now=next[now][buf[i]-'a'];
        int temp=now;
        while(temp!=root){
            res+=ed[temp];
            ed[temp]=0;
            temp=fail[temp];
        }
    }
    return res;
}

};
char buf[maxn];
trie ac;
int main(){
    int n;
    cin>>n;

```

```

    ac.init();
    for(int i=0;i<n;i++)
        cin>>buf,ac.inser(buf);
    ac.build();
    cin>>buf;
    cout<<ac.query(buf)<<endl;
return 0;
}

```

4.7 后缀数组

```

/*后缀数组 DA倍增算法  $O(n \log(n))$ */
const int maxn = "Edit";
char s[maxn];
int sa[maxn], t[maxn], t2[maxn], c[maxn], ran[maxn], height[maxn];
/*
sa为后缀数组, 保存sa[第i个名次]=是i开头后缀
rank为名次数组rank[i开头的后缀]=的名次
height为相邻两个后缀的最长公共前缀
*/
//n为字符串的长度, 字符集的值0~m-1
/*
build(128,s.size())
height[ 2-s.size() ] 有效
height[i]为以sa[i-1]和sa[i]开头的后缀 的最长公共前缀
*/
void build_sa(int m, int n)
{
    n++;
    int *x = t, *y = t2;
    //基数排序
    for (int i = 0; i < m; i++) c[i] = 0;
    for (int i = 0; i < n; i++) c[x[i]] = s[i]++;
    for (int i = 1; i < m; i++) c[i] += c[i - 1];
    for (int i = n - 1; ~i; i--) sa[--c[x[i]]] = i;
    for (int k = 1; k <= n; k <= 1)
    {
        //直接利用sa数组排序第二关键字
        int p = 0;
        for (int i = n - k; i < n; i++) y[p++] = i;
        for (int i = 0; i < n; i++)
            if (sa[i] >= k) y[p++] = sa[i] - k;
        //基数排序第一关键字
        for (int i = 0; i < m; i++) c[i] = 0;
        for (int i = 0; i < n; i++) c[x[y[i]]]++;
        for (int i = 0; i < m; i++) c[i] += c[i - 1];
        for (int i = n - 1; ~i; i--) sa[--c[x[y[i]]]] = y[i];
        //根据sa和y数组计算新的x数组
        swap(x, y);
        p = 1;
        x[sa[0]] = 0;
        for (int i = 1; i < n; i++)
            x[sa[i]] = y[sa[i - 1]] == y[sa[i]] &&
                y[sa[i - 1] + k] == y[sa[i] + k] ? p - 1 : p++;
        if (p >= n) break; //以后即使继续倍增, sa也不会改变, 推出
        m = p; //下次基数排序的最大值
    }
}

```

```

    }
    n--;
    int k = 0;
    for (int i = 0; i <= n; i++) ran[sa[i]] = i;
    for (int i = 0; i < n; i++)
    {
        if (k) k--;
        int j = sa[ran[i] - 1];
        while (s[i + k] == s[j + k]) k++;
        height[ran[i]] = k;
    }
}

int dp[maxn][30];
void initrmq(int n)
{
    for (int i = 1; i <= n; i++)
        dp[i][0] = height[i];
    for (int j = 1; (1 << j) <= n; j++)
        for (int i = 1; i + (1 << j) - 1 <= n; i++)
            dp[i][j] = min(dp[i][j - 1], dp[i + (1 << (j - 1))][j - 1]);
}

int rmq(int l, int r)
{
    int k = 31 - __builtin_clz(r - l + 1); // __builtin_clz 二进制中前导零的个数
    return min(dp[l][k], dp[r - (1 << k) + 1][k]);
}

int lcp(int a, int b)
{
    // 求两个后缀的最长公共前缀
    a = ran[a], b = ran[b];
    if (a > b) swap(a, b);
    return rmq(a + 1, b);
}

```

4.8 后缀自动机

```

const int maxn = "";
struct SAM
{
    int len[maxn << 1], link[maxn << 1], ch[maxn << 1][26];
    int sz, rt, last;
    int newnode(int x = 0)
    {
        len[sz] = x;
        link[sz] = -1;
        mem(ch[sz], -1);
        return sz++;
    }
    void init() { sz = last = 0, rt = newnode(); }
    void extend(int c)
    {
        int np = newnode(len[last] + 1);
        int p;
        for (p = last; ~p && ch[p][c] == -1; p = link[p]) ch[p][c] = np;
        if (p == -1)
            link[np] = rt;
    }
}

```

```

        else
        {
            int q = ch[p][c];
            if (len[p] + 1 == len[q])
                link[np] = q;
            else
            {
                int nq = newnode(len[p] + 1);
                memcpy(ch[nq], ch[q], sizeof(ch[q]));
                link[nq] = link[q], link[q] = link[np] = nq;
                for (; ~p && ch[p][c] == q; p = link[p]) ch[p][c] = nq;
            }
        }
        last = np;
    }

    int topcnt[maxn], topsam[maxn << 1];
    void sort()
    { // 加入串后拓扑排序
        mem(topcnt, 0);
        for (int i = 0; i < sz; i++) topcnt[len[i]]++;
        for (int i = 0; i < maxn - 1; i++) topcnt[i + 1] += topcnt[i];
        for (int i = 0; i < sz; i++) topsam[--topcnt[len[i]]] = i;
    }
};

```

4.9 回文自动机

```

/*回文自动机*/
const int N = 100005;
struct Palindromic_Tree
{
    int ch[N][26], f[N], cnt[N], len[N], s[N];
    int last, sz, n;
    int newnode(int x)
    {
        clr(ch[sz], 0);
        cnt[sz] = 0, len[sz] = x;
        return sz++;
    }
    void init()
    {
        sz = 0;
        newnode(0), newnode(-1);
        last = 0, n = 0, s[0] = -1, f[0] = 1;
    }
    int get_fail(int u)
    {
        while (s[n - len[u] - 1] != s[n]) u = f[u];
        return u;
    }
    void add(int c)
    {
        s[++n] = c;
        int u = get_fail(last);
    }
}

```

```
    if (!ch[u][c])
    {
        int np = newnode(len[u] + 2);
        f[np] = ch[get_fail(f[u])][c];
        ch[u][c] = np;
    }
    last = ch[u][c];
    cnt[last]++;
}
void count()
{
    for (int i = sz - 1; ~i; i--) cnt[f[i]] += cnt[i];
}
};
```

5 优化算法

5.1 二分

```
/**upper_bound **/  
  
while(l<=r)  
{  
    int mid=(l+r)/2;  
    if(ok) l=mid+1;  
    else r=mid-1;  
}  
return l;  
  
/**lower_bound **/  
int lb=-1,ub=res;  
while(ub-lb>1)  
{  
    int mid=(lb+ub)/2;  
    if(ok) ub=mid;  
    else lb=mid;  
}  
return ub;  
  
/*另一种好用的lower_bound*/  
int ans=0;  
while(l<=r)  
{  
    int mid=(l+r)/2;  
    if(ok) l=mid+1,ans=mid;  
    else r=mid-1;  
}  
return ans;  
  
/**浮点数二分*/  
for(int i = 0; i < 100; i++)  
{  
    double mid = (l + r) / 2.0;  
    if(check(mid)) r = mid;  
    else l = mid;  
}
```

5.2 数位DP

```
/* 数位DP HDU-2089 不要62*/  
int bit[30];  
ll dp[30][2];  
ll dfs(int pos,int st,int flag)  
{  
    if(pos==0) return 1;  
    if(flag&&dp[pos][st]!=-1)  
        return dp[pos][st];  
    int u=flag?9:bit[pos];  
    ll ans=0;  
    for(int i=0;i<=u;i++){
```



```

        if(i==4) continue;
        else if(st==0&&i==2) continue;
        else if(i!=6) ans+=dfs(pos-1,1,flag||i<u);
        else if(i==6) ans+=dfs(pos-1,0,flag||i<u);
    }
    if(flag) dp[pos][st]=ans;
return ans;
}

ll solve(int n){
    int len=0;
    while(n){
        bit[++len]=n%10;//len=1为最低位
        n/=10;
    }
    return dfs(len,1,0);
}

```

5.3 树上启发式合并

```

//CF 600E
#include <bits/stdc++.h>
using namespace std;
const int maxn=1e5+10;
vector<int> G[maxn];
int v[maxn];
int sz[maxn];
int f[maxn];
int son[maxn];
int vis[maxn];
int cnt[maxn];
typedef long long ll;
ll ans[maxn];
ll sum;
ll mx;
void lh(int u,int fa)
{
    sz[u]=1;
    for(int i=0;i<G[u].size();i++)
    {
        int v=G[u][i];
        if(v==fa)
            continue;
        lh(v,u);
        sz[u]+=sz[v];
        if(son[u]==0||sz[v]>sz[son[u]])
        {
            son[u]=v;
        }
    }
}
void add(int u,int fa,int val)
{
    cnt[v[u]]+=val;
    if(cnt[v[u]]>mx)
    {

```

```

        sum=v[u],mx=cnt[v[u]];
    }
    else if(cnt[v[u]]==mx)
    {
        sum+=v[u];
    }
    for(int i=0;i<G[u].size();i++)
    {
        int v=G[u][i];
        if(v!=fa&&vis[v]==0)
            add(v,u,val);
    }
}
void dfs(int u,int fa,int flag)
{
    for(int i=0;i<G[u].size();i++)
    {
        int v=G[u][i];
        if(v!=fa&&v!=son[u])
            dfs(v,u,0);
    }
    if(son[u])
        dfs(son[u],u,1),vis[son[u]]=1;
    add(u,fa,1);
    ans[u]=sum;
    if(son[u])
        vis[son[u]]=0;
    if(flag==0)
        add(u,fa,-1),sum=0,mx=-1;
}
int main()
{
    int N;
    scanf("%d",&N);
    for(int i=1;i<=N;i++)
        scanf("%d",&v[i]);
    for(int i=1;i<=N-1;i++)
    {
        int u,v;
        scanf("%d%d",&u,&v);
        G[u].push_back(v);
        G[v].push_back(u);
    }
    lh(1,0);
    dfs(1,0,1);
    for(int i=1;i<=N;i++)
        printf("%lld ",ans[i]);
    printf("\n");
    //scanf("%d",&N);
    return 0;
}

```

5.4 莫队算法

/*
莫队算法复杂度 $O(n*\sqrt{n})$

```

NBUT-1457
*/
ll lastans;
int block;

struct node{
    int l,r,id;
    int pos;//分块
    void init(){
        pos=l/block;
    }
    bool operator<(const node &a)const{
        if(pos==a.pos) return r<a.r;
        return pos<a.pos;
    }
}q[maxn];

void addl(){

}
void dell(){

}
void addr(){

}
void delr(){

}

block=sqrt(n+0.5);//
for(int i=1;i<=m;i++)
{
    scanf("%d%d",&q[i].l,&q[i].r);
    q[i].id=i;
    q[i].init();
}
sort(q+1,q+m+1);
int lastl=2,lastr=1;
lastans=0;
for(int i=1;i<=m;i++)
{
    while(lastl>q[i].l) dell(--lastl);
    while(lastr<q[i].r) addr(++lastr);
    while(lastl<q[i].l) addl(lastl++);
    while(lastr>q[i].r) delr(lastr--);
    ans[q[i].id]=lastans;
}
for(int i=1;i<=m;i++)
    printf("%lld\n",ans[i]);

```

5.5 单调栈单调队列笛卡尔树

```

/*单调栈单调队列和笛卡尔树*/
/** 单调栈 cf602D **/
/*找出左侧第一个大于它的数*/

```

```

int top=-1;
for(int i=1;i<=n;i++)
{
    while(top>=0&&h[i]>=h[st[top]]) //delete the elem in stack no more larger than i
        top--;
    if(top==-1)
        //all the elem in the left no more larger than i
    else
        //the nearest left elem larger than i

    st[++top]=i;//add i to the stack
}

/*单调队列 POJ - 2823 */
/*求长度为k的区间最小（大）值，或长度不超过k的区间最小（大）值*/
/*求长度为k的区间最小值*/
int top=-1;
for(int i=1;i<=k;i++)
{
    while(top>=0&&a[i]<=a[que[top]])
        top--;

    que[++top]=i;
    b[0]=a[que[0]];
}

int s=0;
for(int i=k+1;i<=n;i++)
{
    if(que[s]==i-k) s++;
    while(top>=s&&a[i]<=a[que[top]])
        top--;

    que[++top]=i;
    b[i-k]=a[que[s]];
}

/**笛卡尔树**/
/**
 * 中序遍历得到的序列为原数组序列
 * 节点的key值要大于其左右子节点的key值
 * 利用单调栈建树
 */
void build() {
    int top=0;
    for(int i=1;i<=n;i++)
        l[i]=0,r[i]=0,vis[i]=0;
    for(int i=1;i<=n;i++)
    {
        int k=top;
        while (k>0&&a[stk[k-1]]<a[i]) --k;
        if (k) r[stk[k-1]]=i;//找出i左边第一个比它大的数，把i连到它的右子树
        if (k<top) l[i]=stk[k];//将该数字原来的右子树连到i的左子树
        stk[k++]=i;
        top=k;
    }
}

```

```
    }
    for(int i=1;i<=n;i++)
        vis[l[i]]=vis[r[i]]=1;
    int rt=0;
    for(int i=1;i<=n;i++)
        if (vis[i]==0) rt=i;//find the root
}
```

5.6 最长上升子序列

```
/*最长上升子序列  $O(n\log n)$  */
const int maxn= " ";
int a[maxn],b[maxn];

int lis(int n) {
    b[1]=a[1];
    int len=1;
    for(int i=2;i<=n;i++)
    {
        if(a[i]>=b[len])
        {
            len=len+1;
            b[len]=a[i];
        }
        else
        {
            int pos=upper_bound(b+1,b+1+len,a[i])-b;
            b[pos]=a[i];
        }
    }
    return len;
}
```

6 不会数学

6.1 快速幂与矩阵快速幂

```

/* 快速幂 */
ll quickmod(ll a,ll b,ll c){
    ll res=1;
    while(b){
        if(b&1)
            res=res*a%c;
        a=a*a%c;
        b>>=1;
    }
    return res;
}

****矩阵快速幂****
typedef vector<ll> vec;
typedef vector<vec> mat;
const ll mod=" ";
//A*B
mat mul(mat &A,mat &B){
    mat C(A.size(),vec(B[0].size()));
    for(int i=0;i<A.size();i++)
        for(int k=0;k<B.size();k++)
            for(int j=0;j<B[0].size();j++)
                C[i][j]=(C[i][j]+A[i][k]*B[k][j])%mod;
    return C;
}
//A^n
mat pow(mat A,ll n){
    mat B(A.size(),vec(A.size()));
    for(int i=0;i<A.size();i++)
        B[i][i]=1;
    while(n>0){
        if(n&1) B=mul(B,A);
        A=mul(A,A);
        n>>=1;
    }
    return B;
}
ll n;
void solve(){
    mat A(2,vec(2)); //行数, 列数
    A[0][0]=1;A[0][1]=1;
    A[1][0]=1;A[1][1]=0;
    A=pow(A,n);
    cout<<A[1][0]<<endl;
}

```

6.2 欧几里得与逆元

```

ll gcd(ll a,ll b){
    if(a < b)
        swap(a,b);
    while(a%b){

```

```

        ll r = a % b;
        a = b;
        b = r;
    }
    return b;
}

/*求ax+by=c的xy 其中c%gcd(a,b)==0时才有解
解出的为特解x0,y0 */
ll ex_gcd(ll a,ll b,ll &x,ll &y)
{
    if(a==0&&b==0) return -1;
    if(b==0){x=1;y=0;return a;}
    ll d=ex_gcd(b,a%b,y,x);
    y-=a/b*x;
    return d;
}

/*求最小正整数解x;若无解返回false*/
/*已求出特解 x0,y0 设t=c/d,则通解为 x=t*x0+b/d*k; y=t*y0-a/d*k; k为任意整数
最小正整数解 x=x0*t; x=(x% s+s)%s;*/
bool min_x(int a,int b,int &x,int &y,int c)
{
    int d=ex_gcd(a,b,x,y);
    if(c%d) return false;
    x=x*c/d;
    int s=b/d;
    s=s>0?s:-1*s;
    x=(x%s+s)%s;
    y=(c-a*x)/b;
    return true;
}

/****费马小定理求逆元****/
/*mod为素数,而且a和m互质 a^(p-1)=1(mod p) */
ll inv(ll a,ll mod) {
    return quickmod(a,mod-2,mod);
}

/*线性递推求逆元*/
void init()
{
    inv[1]=1;
    for(int i=2;i<=1e6;i++)
        inv[i]=inv[mod%i]*(mod-mod/i)%mod;
}

```

6.3 欧拉函数

```

/****筛法欧拉函数****/
/*
1. i mod p==0 phi(i * p) == p * phi(i)
2. i mod p!=0 phi(i * p) == phi(i) * (p-1)
算法就是将 x 中的素数因子pi减1, 并且相同的素数因子只有1个减1
*/

```

```

const int maxn= "3e5+10";
int euler[maxn];
void getEuler() {
    memset(euler,0,sizeof(euler));
    euler[1] = 1;
    for(int i = 2;i < maxn;i++)
        if(!euler[i])
            for(int j = i;j < maxn; j += i){
                if(!euler[j])
                    euler[j] = j;
                euler[j] = euler[j]/i*(i-1);
            }
}

/*求单个数的欧拉函数*/
/*
 $\phi(n)=n*((u(d)/d)\text{-之和}d/n)$ 
*/
ll euler(ll n) {
    ll ans = n;
    for(int i = 2;i*i <= n;i++)
    {
        if(n % i == 0)
        {
            ans -= ans/i;
            while(n % i == 0)
                n /= i;
        }
    }
    if(n > 1) ans -= ans/n;
    return ans;
}

/*
其他结论:
 $\phi(mn)=d*\phi(m)*\phi(n)/\phi(d)$      $d=(m,n)$ 
*/

```

6.4 素数与质因子

```

/* 素数判断 */
ull mul(ull x,ull y,ull Z)
{
    ull tmp=x/(long double)Z*y+1e-3;
    return (x*y+Z-tmp*Z)%Z;
}
ull MUL(ull x,ull p,ull Z)
{
    ull y=1;
    while(p)
    {
        if(p&1)y=mul(y,x,Z);
        x=mul(x,x,Z);
        p>>=1;
    }
    return y;
}

```



```

}
bool miuller_rabin(ull n)
{
    if(n<=1)return 0;
    if(n==2)return 1;
    if(n%2==0)return 0;
    ull p=n-1;
    srand(time(NULL));
    int TIMES=8;
    for(int i=1;i<=TIMES;i++)
    {
        ull x=rand()%(n-1)+1;
        if(MUL(x,p,n)!=1)return 0;
    }
    return 1;
}

/* 埃式筛 */
/*值为false表示素数, 值为true表示非素数 */
/* 复杂度O(n*log(logn)) */

const int MAXN = " 1e6 ";
bool notprime[MAXN];
void init()
{
    memset(notprime,false,sizeof(notprime));
    notprime[0]=notprime[1]=true;
    for(int i=2;i<MAXN;i++)
        if(!notprime[i])
        {
            if(i>MAXN/i) continue;//防爆 long long
            for(int j=i*i;j<MAXN;j+=i)
                notprime[j]=true;
        }
}

/* 区间内素数筛选 埃式筛 */
/*prime[0]存素数个数, prime[i]为1-MAXN间第i个素数*/
/* hash[i]=false 表示i为素数 */
/* 每个合数都只会被其最小质因子筛到 复杂度线性 */
const int MAXN = "1e7";
bool has[MAXN+1];
int prime[MAXN/10+1];
void getPrime()
{
    memset(prime,0,sizeof(prime));
    memset(has,false,sizeof(has));
    for(int i=2;i<=MAXN;i++)
    {
        if(!has[i]) prime[++prime[0]]=i;
        for(int j=1;j<=prime[0]&&prime[j]<=MAXN/i;j++)
        {
            has[prime[j]*i]=true;
            if(i%prime[j]==0)
                break;
        }
    }
}

```

```

    }
}

/*合数分解*/
/* fat[i][0]表示第i个质因子, fat[i][1]表示该质因子的个数 */
ll fat[100][2];
int fcnt;
int getfats(ll x)
{
    fcnt=0;
    ll tmp=x;
    for(int i=1;prime[i]<=tmp/prime[i];i++)
        // if x is larger than 1e7,may need add condition i<prime[0]
    {
        fat[fcnt][1]=0;
        if(tmp%prime[i]==0)
        {
            fat[fcnt][0]=prime[i];
            while(tmp%prime[i]==0)
            {
                fat[fcnt][1]++;
                tmp/=prime[i];
            }
            fcnt++;
        }
    }
    if(tmp!=1)
    {
        fat[fcnt][0]=tmp;
        fat[fcnt++][1]=1;
    }
    return fcnt;
}

/* 预处理1-n所有数的质因子 接近o(n) */
const int maxn = " ";
vector<int>d[maxn];
int vis[maxn];
void init(){
    memset(vis,0,sizeof(vis));
    for(int i=2;i<maxn;i++){
        if(!vis[i]){
            for(int j=i;j<maxn;j+=i)
            {
                d[j].push_back(i);
                vis[j]=1;
            }
        }
    }
}
}

```

6.5 组合数学与容斥原理

```

/* 容斥原理 UVA-10325 */
for(int i=1;i<(1<m);i++){
    ll ans=1;int ant=0;
    for(int j=0;j<m;j++){

```

```

        if(i&&(1<<j)){
            ans=lcm(ans,a[j]);
            ant++;
        }
    }
    if((ant-1)%2) num-=(n/ans);
    else num+=(n/ans);
}

/* 组合数预处理 */
/* c(n,m) n>m */
const int N=50;
for(int j=0;j<N;j++)
    c[j][0]=1;
for(int i=0;i<N;i++)
    for(int j=1;j<N;j++)
    {
        if(i==j) c[i][j]=1;
        else if(i<j) c[i][j]=0;
        else c[i][j]=(c[i-1][j-1]+c[i-1][j])%mod;
    }

/*Lucas 组合数取模*/
/* 模数p<=1e5 */
const int maxn="1e5";
ll fac[maxn];
void getf(ll p)
{
    fac[0]=1;
    for(int i=1;i<=p;i++)
        fac[i]=fac[i-1]*i%p;
}

ll quickmod(ll a,ll b,ll c){
    ll res=1;
    while(b){
        if(b&1)
            res=res*a%c;
        a=a*a%c;
        b>>=1;
    }
    return res;
}

ll lucas(ll n,ll m,ll p)
{
    ll ans=1;
    while(n&& m)
    {
        ll a=n%p;
        ll b=m%p;
        if(a<b) return 0;
        ans=(ans*fac[a]*quickmod(fac[b]*fac[a-b]%p,p-2,p))%p;
        n/=p;
        m/=p;
    }
}

```

```

    return ans;
}

```

6.6 中国剩余定理

```

/*中国剩余定理*/
/*x=bi(mod m0i)*/
/*x0+=biM0iyi*/
ll m0[maxn];
ll b[maxn];
ll ChinaRemainder(int n){
    ll m=1,a=0;
    for (int i=1; i<=n; i++) m=m*m0[i];
    for (int i=1; i<=n; i++) {
        ll MM=m/m0[i];
        ll x=inv(MM,m0[i]);
        a=(a+MM*x*b[i]) % m;
    }
    return a;
}

/*不互质的中国剩余定理*/
ll m0[maxn];
ll b[maxn];

ll china(ll n)
{
    ll a,bb,d,x,y,dm;
    ll c,c1,c2;
    ll dg;//lcm
    a=m0[1]; c1=b[1];
    for (int i=2; i<=n; i++)
    {
        bb=m0[i]; c2=b[i];
        d=ex_gcd(a, bb,x, y);
        dm=bb/d;
        c=c2-c1;
        if (c%d) return -1;//无解
        x=((x*c/d)%dm+dm)%dm;//x可能为负
        c1=a*x+c1;
        a=a*bb/d;
    }
    dg=a;//dg是最大公约数
    if (!c1)//考虑c1为0的情况
    {
        c1=1;
        for (int i=1; i<=n; i++)
        {
            c1=c1*m0[i]/__gcd(c1, m0[i]);
        }
        dg=c1;//此时dg为最小公倍数
    }
    return c1;//c1为最小的x
}

```

6.7 FFT

```

typedef complex<double> cd;
void fft(cd *a, int n, int f) {
    for(int i = 0; i < n; i++)
        if(i < rev[i])
            swap(a[i], a[rev[i]]);
    for(int i = 1; i < n; i <= 1) {
        cd wn(cos(pi / i), sin(f * pi / i)), x, y;
        for(int j = 0; j < n; j += (i < 1)) {
            cd w(1, 0);
            for(int k = 0; k < i; k++, w *= wn) {
                x = a[j + k], y = w * a[i + j + k];
                a[j + k] = x + y;
                a[i + j + k] = x - y;
            }
        }
    }
}

for(N = 1; N <= (M + 1) * 2; N <= 1) bit++; //bit为数组的大小

for(int i = 0; i < N; i++)
    rev[i] = (rev[i >> 1] >> 1) | ((i & 1) << (bit - 1));

for(int i = 0; i < N; i++)
    d[i] = (ll)(c[i].real() / N + 0.5); //四舍五入

```

7 计算几何

Thanks for csl.

7.1 点的定义

```
#define zero(x) ((fabs(x) < eps ? 1 : 0))
#define sgn(x) (fabs(x) < eps ? 0 : ((x) < 0 ? -1 : 1))

struct point
{
    double x, y;
    point(double a = 0, double b = 0) { x = a, y = b; }
    point operator-(const point& b) const { return point(x - b.x, y - b.y); }
    point operator+(const point& b) const { return point(x + b.x, y + b.y); }
    // 两点是否重合
    bool operator==(point& b) { return zero(x - b.x) && zero(y - b.y); }
    // 点积(以原点为基准)
    double operator*(const point& b) const { return x * b.x + y * b.y; }
    // 叉积(以原点为基准)
    double operator^(const point& b) const { return x * b.y - y * b.x; }
    // 绕P点逆时针旋转a弧度后的点
    point rotate(point b, double a)
    {
        double dx, dy;
        (*this - b).split(dx, dy);
        double tx = dx * cos(a) - dy * sin(a);
        double ty = dx * sin(a) + dy * cos(a);
        return point(tx, ty) + b;
    }
    // 点坐标分别赋值到a和b
    void split(double& a, double& b) { a = x, b = y; }
};

struct line
{
    point s, e;
    line() {}
    line(point ss, point ee) { s = ss, e = ee; }
};
```

7.2 点直线与线段之间的位置关系

```
/**点直线与线段之间的位置关系*/

/*两点间距离*/
double dist(point a, point b) { return sqrt((a - b) * (a - b)); }

/*两条直线之间的关系*/
// <0, *> 表示重合; <1, *> 表示平行; <2, P> 表示交点是P;
pair<int, point> spoint(line l1, line l2)
{
    point res = l1.s;
    if (sgn((l1.s - l1.e) ^ (l2.s - l2.e)) == 0)
        return {sgn((l1.s - l2.e) ^ (l2.s - l2.e)) != 0, res};
    double t = ((l1.s - l2.s) ^ (l2.s - l2.e)) / ((l1.s - l1.e) ^ (l2.s - l2.e));
```

```

    res.x += (l1.e.x - l1.s.x) * t;
    res.y += (l1.e.y - l1.s.y) * t;
    return {2, res};
}

/**判断线段相交*/
bool segxseg(line l1, line l2)
{
    return
        max(l1.s.x, l1.e.x) >= min(l2.s.x, l2.e.x) &&
        max(l2.s.x, l2.e.x) >= min(l1.s.x, l1.e.x) &&
        max(l1.s.y, l1.e.y) >= min(l2.s.y, l2.e.y) &&
        max(l2.s.y, l2.e.y) >= min(l1.s.y, l1.e.y) &&
        sgn((l2.s - l1.e) ^ (l1.s - l1.e)) * sgn((l2.e - l1.e) ^ (l1.s - l1.e)) <= 0 &&
        sgn((l1.s - l2.e) ^ (l2.s - l2.e)) * sgn((l1.e - l2.e) ^ (l2.s - l2.e)) <= 0;
}

/**直线与线段相交*/
//l1是直线, l2是线段
bool segxline(line l1, line l2)
{
    return sgn((l2.s - l1.e) ^ (l1.s - l1.e)) * sgn((l2.e - l1.e) ^ (l1.s - l1.e)) <= 0;
}

/**点与直线相交*/
double pointtoline(point p, line l)
{
    point res;
    double t = ((p - l.s) * (l.e - l.s)) / ((l.e - l.s) * (l.e - l.s));
    res.x = l.s.x + (l.e.x - l.s.x) * t, res.y = l.s.y + (l.e.y - l.s.y) * t;
    return dist(p, res);
}

/**点与线段相交*/
double pointtosegment(point p, line l)
{
    point res;
    double t = ((p - l.s) * (l.e - l.s)) / ((l.e - l.s) * (l.e - l.s));
    if (t >= 0 && t <= 1)
        res.x = l.s.x + (l.e.x - l.s.x) * t, res.y = l.s.y + (l.e.y - l.s.y) * t;
    else
        res = dist(p, l.s) < dist(p, l.e) ? l.s : l.e;
    return dist(p, res);
}

/**点是否在线段上*/
bool PointOnSeg(point p, line l)
{
    return
        sgn((l.s - p) ^ (l.e - p)) == 0 &&
        sgn((p.x - l.s.x) * (p.x - l.e.x)) <= 0 &&
        sgn((p.y - l.s.y) * (p.y - l.e.y)) <= 0;
}

```

7.3 多边形

```

/*多边形面积计算*/
double area(point p[], int n)
{
    double res = 0;
    for (int i = 0; i < n; i++) res += (p[i] ^ p[(i + 1) % n]) / 2;
    return fabs(res);
}

/*判断点是否在凸多边形内*/
// 点形成一个凸包, 而且按逆时针排序 (如果是顺时针把里面的<0改为>0)
// 点的编号 : [0,n)
// -1 : 点在凸多边形外
// 0 : 点在凸多边形边界上
// 1 : 点在凸多边形内
int PointInConvex(point a, point p[], int n)
{
    for (int i = 0; i < n; i++)
        if (sgn((p[i] - a) ^ (p[(i + 1) % n] - a)) < 0)
            return -1;
        else if (PointOnSeg(a, line(p[i], p[(i + 1) % n])))
            return 0;
    return 1;
}

/*判断凸多边形*/
//点可以是顺时针给出也可以是逆时针给出
//点的编号1~n-1
bool isconvex(point poly[], int n)
{
    bool s[3];
    memset(s, 0, sizeof(s));
    for (int i = 0; i < n; i++)
    {
        s[sgn((poly[(i + 1) % n] - poly[i]) ^ (poly[(i + 2) % n] - poly[i])) + 1] = 1;
        if (s[0] && s[2]) return 0;
    }
    return 1;
}

```

7.4 求圆的外心

```

/*求圆的外心*/

point waixin(point a, point b, point c)
{
    double a1 = b.x - a.x, b1 = b.y - a.y, c1 = (a1 * a1 + b1 * b1) / 2;
    double a2 = c.x - a.x, b2 = c.y - a.y, c2 = (a2 * a2 + b2 * b2) / 2;
    double d = a1 * b2 - a2 * b1;
    return point(a.x + (c1 * b2 - c2 * b1) / d, a.y + (a1 * c2 - a2 * c1) / d);
}

```

7.5 求整点数

```

/*求整点数*/

```



```

/*线段上整点数*/
int OnSegment(line l) { return __gcd(fabs(l.s.x - l.e.x), fabs(l.s.y - l.e.y)) + 1; }

/*多边形边上整点数*/
int OnEdge(point p[], int n)
{
    int i, ret = 0;
    for (i = 0; i < n; i++)
        ret += __gcd(fabs(p[i].x - p[(i + 1) % n].x), fabs(p[i].y - p[(i + 1) % n].y));
    return ret;
}

/*多边形内部整点数*/
int InSide(point p[], int n)
{
    int i, area = 0;
    for (i = 0; i < n; i++)
        area += p[(i + 1) % n].y * (p[i].x - p[(i + 2) % n].x);
    return (fabs(area) - OnEdge(p, n)) / 2 + 1;
}

```

刘汝佳版

7.6 刘汝佳版的定义

```

struct Point
{
    double x, y;
    Point(double x = 0, double y = 0) : x(x), y(y) {}
};

typedef Point Vector;

//向量+向量=向量, 点+向量=点
Vector operator+(Vector A, Vector B) { return Vector(A.x + B.x, A.y + B.y); }
//点-点=向量
Vector operator-(Point A, Point B) { return Vector(A.x - B.x, A.y - B.y); }
//向量*数=向量
Vector operator*(Vector A, double p) { return Vector(A.x * p, A.y * p); }
//向量/数=向量
Vector operator/(Vector A, double p) { return Vector(A.x / p, A.y / p); }

bool operator<(const Point& a, const Point& b)
{
    return a.x < b.x || (a.x == b.x && a.y < b.y);
}

const double eps = 1e-10;
double dcmp(double x)
{
    if (fabs(x) < eps)
        return 0;
    else
        return x < 0 ? -1 : 1;
}

```

```

bool operator==(const Point& a, const Point& b)
{
    return dcmp(a.x - b.x) == 0 && dcmp(a.y - b.y) == 0;
}

/*
 * 基本运算:
 * 点积
 * 叉积
 * 向量旋转
 */
double Dot(Vector A, Vector B) { return A.x * B.x + A.y * B.y; }
double Length(Vector A) { return sqrt(Dot(A, A)); }
double Angle(Vector A, Vector B) { return acos(Dot(A, B) / Length(A) / Length(B)); }

double Cross(Vector A, Vector B) { return A.x * B.y - A.y * B.x; }
double Area2(Point A, Point B, Point C) { return Cross(B - A, C - A); }

//rad是弧度
Vector Rotate(Vector A, double rad)
{
    return Vector(A.x * cos(rad) - A.y * sin(rad),
                  A.x * sin(rad) + A.y * cos(rad));
}

//调用前请确保A不是零向量
Vector Normal(Vector A)
{
    double L = Length(A);
    return Vector(-A.y / L, A.x / L);
}

/*
 * 点和直线:
 * 两直线交点
 * 点到直线的距离
 * 点到线段的距离
 * 点在直线上的投影
 * 线段相交判定
 * 点在线段上判定
 */

//调用前保证两条直线P+tv和Q+tw有唯一交点。当且仅当Cross(v, w)非0
Point GetLineIntersection(Point P, Vector v, Point Q, Vector w)
{
    Vector u = P - Q;
    double t = Cross(w, u) / Cross(v, w);
    return P + v * t;
}

double DistanceToLine(Point P, Point A, Point B)
{
    Vector v1 = B - A, v2 = P - A;
    return fabs(Cross(v1, v2)) / Length(v1); //如果不取绝对值, 得到的是有向距离
}

```

```

double DistanceToSegment(Point P, Point A, Point B)
{
    if (A == B) return Length(P - A);
    Vector v1 = B - A, v2 = P - A, v3 = P - B;
    if (dcmp(Dot(v1, v2)) < 0) return Length(v2);
    if (dcmp(Dot(v1, v3)) > 0) return Length(v3);
    return fabs(Cross(v1, v2)) / Length(v1);
}

Point GetLineProjection(Point P, Point A, Point B)
{
    Vector v = B - A;
    return A + v * (Dot(v, P - A) / Dot(v, v));
}

bool SegmentProperIntersection(Point a1, Point a2, Point b1, Point b2)
{
    double c1 = Cross(a2 - a1, b1 - a1), c2 = Cross(a2 - a1, b2 - b1),
           c3 = Cross(b2 - b1, a1 - b1), c4 = Cross(b2 - b1, a2 - b1);
    return dcmp(c1) * dcmp(c2) < 0 && dcmp(c3) * dcmp(c4) < 0;
}

bool OnSegment(Point p, Point a1, Point a2)
{
    return dcmp(Cross(a1 - p, a2 - p)) == 0 && dcmp(Dot(a1 - p, a2 - p)) < 0;
}

```

7.7 刘汝佳版多边形

多边形面积，点在多边形内，凸包，半平面交

```

typedef vector<Point> Polygon;
//多边形的有向面积
double PolygonArea(Polygon po)
{
    int n = po.size();
    double area = 0.0;
    for (int i = 1; i < n - 1; i++)
        area += Cross(po[i] - po[0], po[i + 1] - po[0]);
    return area / 2;
}

//点在多边形内判定
int isPointInPolygon(Point p, Polygon poly)
{
    int wn = 0; //绕数
    int n = poly.size();
    for (int i = 0; i < n; i++)
    {
        if (OnSegment(p, poly[i], poly[(i + 1) % n])) return -1; //边界上
        int k = dcmp(Cross(poly[(i + 1) % n] - poly[i], p - poly[i]));
        int d1 = dcmp(poly[i].y - p.y);
        int d2 = dcmp(poly[(i + 1) % n].y - p.y);
        if (k > 0 && d1 <= 0 && d2 > 0) wn++;
        if (k < 0 && d2 <= 0 && d1 > 0) wn--;
    }
}

```

```

    }
    if (wn != 0) return 1; //内部
    return 0;             //外部
}

//凸包 (Andrew算法)
//如果不希望在凸包的边上有输入点, 把两个 <= 改成 <
//如果不介意点集被修改, 可以改成传递引用
Polygon ConvexHull(vector<Point> p)
{
    sort(p.begin(), p.end());
    p.erase(unique(p.begin(), p.end()), p.end());
    int n = p.size(), m = 0;
    Polygon res(n + 1);
    for (int i = 0; i < n; i++)
    {
        while (m > 1 && Cross(res[m - 1] - res[m - 2], p[i] - res[m - 2]) <= 0) m--;
        res[m++] = p[i];
    }
    int k = m;
    for (int i = n - 2; i >= 0; i--)
    {
        while (m > k && Cross(res[m - 1] - res[m - 2], p[i] - res[m - 2]) <= 0) m--;
        res[m++] = p[i];
    }
    m -= n > 1;
    res.resize(m);
    return res;
}

```

//半平面交

```

vector<Point> HalfplaneIntersection(vector<Line>& L)
{
    int n = L.size();
    sort(L.begin(), L.end()); // 按极角排序

    int first, last; // 双端队列的第一个元素和最后一个元素的下标
    vector<Point> p(n); // p[i]为q[i]和q[i+1]的交点
    vector<Line> q(n); // 双端队列
    vector<Point> ans; // 结果

    q[first = last = 0] = L[0]; // 双端队列初始化为只有一个半平面L[0]
    for (int i = 1; i < n; i++)
    {
        while (first < last && !OnLeft(L[i], p[last - 1])) last--;
        while (first < last && !OnLeft(L[i], p[first])) first++;
        q[++last] = L[i];
        if (fabs(Cross(q[last].v, q[last - 1].v)) < eps)
        { // 两向量平行且同向, 取内侧的一个
            last--;
            if (OnLeft(q[last], L[i].p)) q[last] = L[i];
        }
        if (first < last) p[last - 1] = GetLineIntersection(q[last - 1], q[last]);
    }
    while (first < last && !OnLeft(q[first], p[last - 1])) last--; // 删除无用平面
    if (last - first <= 1) return vector<Point>(); // 空集
}

```

```

    p[last] = GetLineIntersection(q[last], q[first]); // 计算首尾两个半平面的交点

    return vector<Point>(q.begin() + first, q.begin() + last + 1);
}

```

7.8 刘汝佳版直线和圆

```

struct Line
{
    Point p; //直线上任意一点
    Vector v; //方向向量。它的左边就是对应的半平面
    double ang; //极角。即从x正半轴旋转到向量v所需要的角（弧度）
    Line() {}
    Line(Point p, Vector v) : p(p), v(v) { ang = atan2(v.y, v.x); }
    bool operator<(const Line& L) const // 排序用的比较运算符
    {
        return ang < L.ang;
    }
    Point point(double t) { return p + v * t; }
};

struct Circle
{
    Point c;
    double r;
    Circle(Point c, double r) : c(c), r(r) {}
    Point point(double a) { return c.x + cos(a) * r, c.y + sin(a) * r; }
};

int getLineCircleIntersection(Line L, Circle C, double& t1, double& t2, vector<Point>& sol)
{
    double a = L.v.x, b = L.p.x - C.c.x, c = L.v.y, d = L.p.y - C.c.y;
    double e = a * a + c * c, f = 2 * (a * b + c * d), g = b * b + d * d - C.r * C.r;
    double delta = f * f - 4 * e * g; //判别式
    if (dcmp(delta) < 0) return 0; //相离
    if (dcmp(delta) == 0) //相切
    {
        t1 = t2 = -f / (2 * e);
        sol.push_back(L.point(t1));
        return 1;
    }
    //相交
    t1 = (-f - sqrt(delta)) / (2 * e);
    t2 = (-f + sqrt(delta)) / (2 * e);
    sol.push_back(t1);
    sol.push_back(t2);
    return 2;
}

double angle(Vector v) { return atan2(v.y, v.x); }

int getCircleCircleIntersection(Circle C1, Circle C2, vector<Point>& sol)
{
    double d = Length(C1.c - C2.c);
    if (dcmp(d) == 0)
    {

```

```

        if (dcmp(C1.r - C2.r) == 0) return -1; //两圆重合
        return 0;
    }
    if (dcmp(C1.r + C2.r - d) < 0) return 0; //内含
    if (dcmp(fabs(C1.r - C2.r) - d) > 0) return 0; //外离

    double a = angle(C2.c - C1.c); //向量C1C2的极角
    double da = acos((C1.r * C1.r + d * d - C2.r * C2.r) / (2 * C1.r * d));
    //C1C2到C1P1的角
    Point p1 = C1.point(a - da), p2 = C1.point(a + da);

    sol.push_back(p1);
    if (p1 == p2) return 1;
    sol.push_back(p2);
    return 2;
}

//过点p到圆C的切线, v[i]是第i条切线的向量, 返回切线条数
int getTangents(Point p, Circle C, Vector* v)
{
    Vector u = C.c - p;
    double dist = Length(u);
    if (dist < C.r)
        return 0;
    else if (dcmp(dist - C.r) == 0)
    { //p在圆上, 只有一条切线
        v[0] = Rotate(u, M_PI / 2);
        return 1;
    }
    else
    {
        double ang = asin(C.r / dist);
        v[0] = Rotate(u, -ang);
        v[1] = Rotate(u, +ang);
        return 2;
    }
}

//两圆的公切线
//返回切线的条数。-1表示无穷条切线。
//a[i]和b[i]分别是第i条切线在圆A和圆B上的切点
int getTangents(Circle A, Circle B, Point* a, Point* b)
{
    int cnt = 0;
    if (A.r < B.r)
    {
        swap(A, B);
        swap(a, b);
    }
    int d2 = (A.c.x - B.c.x) * (A.c.x - B.c.x) + (A.c.y - B.c.y) * (A.c.y - B.c.y);
    int rdif = A.r - B.r;
    int rsum = A.r + B.r;
    if (d2 < rdif * rdif) return 0; //内含
    double base = atan2(B.c.y - A.c.y, B.c.x - A.c.x);
    if (d2 == 0 && A.r == B.r) return -1; //无限多条切线
    if (d2 == rdif * rdif)

```

```

{ //内切, 一条切线
    a[cnt] = A.point(base);
    b[cnt] = B.point(base);
    cnt++;
    return 1;
}
//有外共切线
double ang = acos(A.r - B.r) / sqrt(d2);
a[cnt] = A.point(base + ang);
b[cnt] = B.point(base + ang);
cnt++;
a[cnt] = A.point(base + ang);
b[cnt] = B.point(base - ang);
cnt++;
if (d2 == rsum * rsum)
{
    a[cnt] = A.point(base);
    b[cnt] = B.point(M_PI + base);
    cnt++;
}
else if (d2 > rsum * rsum)
{
    double ang = acos((A.r + B.r) / sqrt(d2));
    a[cnt] = A.point(base + ang);
    b[cnt] = B.point(M_PI + base + ang);
    cnt++;
    a[cnt] = A.point(base - ang);
    b[cnt] = B.point(M_PI + base - ang);
    cnt++;
}
return cnt;
}

//三角形外接圆 (三点保证不共线)
Circle CircumscribedCircle(Point p1, Point p2, Point p3)
{
    double Bx = p2.x - p1.x, By = p2.y - p1.y;
    double Cx = p3.x - p1.x, Cy = p3.y - p1.y;
    double D = 2 * (Bx * Cy - By * Cx);
    double cx = (Cy * (Bx * Bx + By * By) - By * (Cx * Cx + Cy * Cy)) / D + p1.x;
    double cy = (Bx * (Cx * Cx + Cy * Cy) - Cx * (Bx * Bx + By * By)) / D + p1.y;
    Point p = Point(cx, cy);
    return Circle(p, Length(p1 - p));
}

//三角形内切圆
Circle InscribedCircle(Point p1, Point p2, Point p3)
{
    double a = Length(p2 - p3);
    double b = Length(p3 - p1);
    double c = Length(p1 - p2);
    Point p = (p1 * a + p2 * b + p3 * c) / (a + b + c);
    return Circle(p, DistanceToLine(p, p1, p2));
}

```

8 各种操作

8.1 二进制位操作

- `__builtin_ffs(x)` 返回x中最后一个为1的位是从后向前的第几位，从1开始计数
 - `__builtin_popcount(x)` x中1的个数。
 - `__builtin_ctz(x)` x末尾0的个数。x=0时结果未定义。
 - `__builtin_clz(x)` x前导0的个数。x=0时结果未定义。
- 上面的x都是unsigned int型的，如果传入signed或者是char型，会被强制转换成unsigned int
- `__builtin_parity(x)` x中1的个数的奇偶性,返回1为奇。

8.2 bitset

- `bitset<100000>bt;`
- `bt<<1;` //整体移位
- `bt|=10;`
- `bt.count();` //b中置为1的二进制位的个数
- `bt.size();` //b中二进制位的个数
- `bt[pos];` //访问b中在pos处的二进制位
- `bt.test(pos);` //b中在pos处的二进制位是否为1
- `bt.set();` //把b中所有二进制位都置为1
- `bt.set(pos);` //把b中在pos处的二进制位置为1
- `bt.reset();` //把b中所有二进制位都置为0
- `bt.reset(pos);` //把b中在pos处的二进制位置为0

8.3 nth_element

- `nth_element(first,nth,last)`
- first, last 第一个和最后一个元素位置
- nth:要定位的第n个元素
- `nth_element`会将第n_th 元素放到它该放的位置上，左边元素都小于它，右边元素都大于它
- 期望复杂度 $O(n)$
- `nth_element(a,a+6,a+10)`

8.4 Rope

```

/*
专用于块状链表计算的rope容器
平衡树实现，各种操作的复杂度都是 $O(\log n)$ 
*/
//头文件
#include <ext/rope>
using namespace __gnu_cxx;

rope<int> T;

T.push_back(x); //在末尾添加x

T.insert(pos,x); //在pos插入x

T.erase(pos,x); //从pos开始删除x个

T.copy(pos,len,x); //从pos开始到pos+len为止用x代替

T.replace(pos,x); //从pos开始换成x

T.substr(pos,x); //提取pos开始x个

T.at(x)/[x]; //访问第x个元素

printf("%d\n",T[i]) //输出T[i]
cout<<T<<endl; //输出T

/* 2018nowcoder多校3 https://www.nowcoder.com/acm/contest/141/C */
#include <bits/stdc++.h>
#include <ext/rope> //函数头文件
using namespace __gnu_cxx;
using namespace std;
const int maxn=1e5+10;
rope<int> T;
int n,m;
int main()
{
    scanf("%d%d",&n,&m);
    for(int i=1;i<=n;i++)
        T.push_back(i);
    while(m--)
    {
        int p,s;
        scanf("%d%d",&p,&s);
        p--;
        T=T.substr(p,s)+T.substr(0,p)+T.substr(p+s,n-p-s);
    }
    for(int i=0;i<n;i++)
        printf("%d ",T[i]);
    return 0;
}

```

8.5 pb_ds

参考文献: [C++的pb_ds库在OI中的应用](#)

__gnu_pbds::priority_queue 可合并堆

- 头文件 ext/pb_ds/priority_queue.hpp
__gnu_pbds::priority_queue<T,greater,TAG>
- 函数: size(),empty(),push(T),top(),pop(),clear()
- 新增功能
 - begin(),end()获取iterator遍历
 - increase_key,decrease_key
 - 删除单个元素 erase(point_iterator)
 - point_iterator push(T)
 - 修改元素 modify(point_iterator,T)
 - 合并堆: q1.join(q2) 将q2合并到q1, q2被清空
- TAG:

五种操作:push,pop,modify,erase,join

 - pairing_heap_tag(配对堆): push,join $O(1)$, 其余均摊 $O(\log n)$ (默认)
 - binary_heap_tag(二叉堆): 只支持push,pop 均摊 $O(\log n)$
 - binomial_heap_tag(二项堆): push均摊 $O(1)$,其余 $O(\log n)$
 - rc_binomial_heap_tag: push $O(1)$, 其余 $O(\log n)$
 - thin_heap_tag(斐波那契堆): push $O(1)$,不支持join,其余 $O(\log n)$,只有increase_key的话 $O(1)$
- 合并,dij均使用: pairing_heap_tag
- 只有push,pop,join: binary_heap_tag

__gnu_pbds::tree

- 头文件 ext/pb_ds/assoc_container.hpp
ext/pb_ds/tree_policy.hpp
__gnu_pbds::tree<key,T,TAG,Node_Update>
- 函数类似于map: begin(),end(),size(),empty(),clear(),
find(key),lower_bound(key),upper_bound(key),
erase(iterator),erase(key),insert(<key,T>),operator
- 第二个参数改为null_type(null_mapped_type)即为set
- TAG:
 - rb_tree_tag
 - splay_tree_tag
- 寻找第order+1小的元素,order过大返回end(): iterator find_by_order(order)
- 询问有多少个比key小的元素: order_of_key(key)
- t1.join(t2)将t2所有元素移动到t1,t1、t2值域不能相交
- t1.split(key,t2)清空t2, 把所有大于key的元素移动到other
- 自带的Node_Update:tree_order_statistics_node_update统计子树大小

- 自定义Node_Update

```
template<class Node_CItr,class Node_Itr,class Cmp_Fn,class _Alloc>
struct my_node_update{
    virtual Node_CItr node_begin() const =0;
    virtual Node_CItr node_end() const =0;
    typedef int metadata_typde;//节点记录的额外信息的类型
}
/*
将系数但it的信息更新为其左右孩子的信息
传入end_it表示空节点
*/
inline void operator()(Node_Itr it,Node_CItr end_it)
{
    Node_Itr l=it.get_l_child(),r=it.get_r_child();
    int left=0,right=0;
    if(l!=end_it) left=l.get_metadata();
    if(r!=end_it) right=r.get_metadata();
    const_cast<metadata_type &>(it.get_metadata())
        =left+right+(*it)->second;
}
inline int prefix_sum(int x)
{
    int ans=0;
    Node_CItr it=node_begin();
    while(it!=node_end())
    {
        Node_CItr l= it.get_l_child(),r=it.get_r_child();
        if(Cmp_Fn()(x,(*it)->first)) it=l;
        else{
            ans+=(*it)->second;
            if(l!=node_end())
                ans+=l.get_metadata();
            it=r;
        }
    }
    return ans;
}
inline int interval_sum(int l,int r)
{
    return prefix_sum(r)-prefix_sum(l-1);
}
```

get_l_child,get_r_child获取左右孩子,(*it)获取节点信息, get_metadata获取节点额外信息

hash_table

- 头文件 ext/pb_ds_assoc_container.hpp
ext/pb_ds/hash_policy.hpp
__gnu_pbds::cc_hash_table<key,mapped>(拉链法)
__gun_pbds::gp_hash_table<key,mpped> (查探法较快)

8.6 String And Char

```

/*大小写转换函数*/
a=tolower(a);
b=toupper(b);

/**sstream**/
string sstream
string str; getline(cin,str);
stringstream ss(str); //对string对象进行读写
while(ss>>x)
    s.compare(b);

ss.clear(); //多次使用stringstream, 要先清空下
/**stringstream可以用来把string类型的字符串转换成int **/
string s("12345");
int x;
stringstream ss(s);
ss>>x;
/**法二**/
string s("12345");
int x;
stringstream ss;
ss<<s;
ss>>x;

/**将多种数值转换成字符串**/
typename x=5.222;
cin>>x;
stringstream ss;
ss<<x;
string s;
s=ss.str(); //s="5.222"

/**字符串处理*/
char s[] = "a,b*c,d";
const char *sep = ",*"; //可按多个字符来分割
char *p;
p = strtok(s, sep);
//在第一次被调用的时间str是传入需要被切割字符串的首地址; 在后面调用的时间传入NULL
while(p){
    printf("%s ", p);
    p = strtok(NULL, sep);
}

/*取子串*/
string sub1 = s.substr(5); //从下标为5开始一直到结尾
string sub2 = s.substr(5, 3); //从下标为5开始截取长度为3位

/*char 数组操作*/
strcpy
/*https://blog.csdn.net/ncabhd/article/details/72903123*/
strcat(charr5, " juice"); //添加到末尾

```

8.7 String To Int

int/float to string/array

- itoa(): 将整型值转换为字符串。
- ltoa(): 将长整型值转换为字符串。
- ultoa(): 将无符号长整型值转换为字符串。
- gcvt(): 将浮点型数转换为字符串，取四舍五入。
- ecvt(): 将双精度浮点型值转换为字符串，转换结果中不包含十进制小数点。
- fcvt(): 指定位数为转换精度，其余同ecvt()。

string/array to int/float

- atof(): 将字符串转换为双精度浮点型值。
- atoi(): 将字符串转换为整型值。
- atol(): 将字符串转换为长整型值。
- strtod(): 将字符串转换为双精度浮点型值，并报告不能被转换的所有剩余数字。
- strtol(): 将字符串转换为长整值，并报告不能被转换的所有剩余数字。
- strtoul(): 将字符串转换为无符号长整型值，并报告不能被转换的所有剩余数字。

8.8 IO

```
template <class T> inline bool scan_d(T &ret) {
    char c; int sgn;
    if(c=getchar(),c==EOF) return 0; //EOF
    while(c!='-'&&(c<'0' || c>'9'))
        c=getchar();
    sgn=(c=='-')?-1:1;
    ret=(c=='-')?0:(c-'0');
    while(c=getchar(),c>='0'&&c<='9')
        ret=ret*10+(c-'0');
    ret*=sgn;
    return 1; }

inline void out(ll x) {
    if(x>9) out(x/10);
    putchar(x%10+'0');
}

/*steal from zerol*/
inline char next_char() {
    static char buf[100000], *p1 = buf, *p2 = buf;
    return p1 == p2 &&
        (p2 = (p1 = buf) + fread(buf, 1, 100000, stdin), p1 == p2) ? EOF : *p1++;
}

inline bool maybe_digit(char c) {
    return c >= '0' && c <= '9';
}

template <typename T>
void rn(T& _v) {
    static char ch;
```

```

    static bool negative = false;
    _v = 0;
    while (!maybe_digit(ch)) {
        negative = ch == '-';
        ch = next_char();
    }
    do _v = (_v << 1) + (_v << 3) + ch - '0';
    while (maybe_digit(ch = next_char()));
    if (negative) _v = -_v;
}

template <typename T>
void o(T p) {
    static int stk[70], tp;
    if (p == 0) {
        putchar('0');
        return;
    }
    if (p < 0) { p = -p; putchar('-'); }
    while (p) stk[++tp] = p % 10, p /= 10;
    while (tp) putchar(stk[tp--] + '0');
}

```

8.9 Other Tips

```
printf("%04d\n",x); //输出4位,不足则前面填充0
```

```
/*c++格式化 long double的输出*/
```

```
long double t1 = (1 - t2 * v2) / v1;
cout << fixed << setprecision(10) << t1 << endl; //保留10位小数
```

9 Java

9.1 高精度

```

/*
四则运算: add (加) subtract (减) multiply (乘) divide (除)
remainder (取余)
*/

import java.util.Scanner;
import java.math.*;

public class Main {
    public static class point {
        BigDecimal a, b;
    }

    public static point tempx = new point();

    public static point find(point x, point y, point z) {
        BigDecimal a, b, a1, a2, b1, b2, c1, c2, t, d;
        t = BigDecimal.valueOf(2);
        a1 = y.a.subtract(x.a);
        b1 = y.b.subtract(x.b);
        tempx.a = a1;
        tempx.b = b1;
        c1 = ((a1.multiply(a1)).add(b1.multiply(b1))).divide(t);
        a2 = z.a.subtract(x.a);
        b2 = z.b.subtract(x.b);
        c2 = ((a2.multiply(a2)).add(b2.multiply(b2))).divide(t);
        d = (a1.multiply(b2)).subtract(a2.multiply(b1));
        tempx.a = x.a.add((c1.multiply(b2).subtract(c2.multiply(b1))).divide(d, 20, 0));
        tempx.b = x.b.add((a1.multiply(c2).subtract(a2.multiply(c1))).divide(d, 20, 0));
        return tempx;
    }

    public static BigDecimal distance(point x, point y) // 没有开根
    {
        BigDecimal temp;
        temp = ((x.a.subtract(y.a)).multiply(x.a.subtract(y.a))).add((x.b.subtract(y.b)).
multiply(x.b.subtract(y.b)));
        return temp;
    }

    static point p[] = new point[5], temp = null;

    public static void main(String[] args) {
        Scanner s = new Scanner(System.in);
        int T, i;
        T = s.nextInt();
        for (i = 0; i < 4; i++)
            p[i] = new point();
        while (T-- > 0) {

            for (i = 0; i < 4; i++) {
                p[i].a = s.nextBigDecimal();
            }
        }
    }
}

```

```

        p[i].b = s.nextBigDecimal();
    }
    temp = find(p[0], p[1], p[2]);
    if (distance(p[0], temp).compareTo(distance(p[3], temp)) < 0)
        System.out.println("Accepted");
    else
        System.out.println("Rejected");
    }
}
}

```

9.2 Java输入输出

/*Java大数高精度亲情讲义

java输入输出架构（注意主类为Main）

```

*/
import java.util.Scanner; //输入架构
public class Main {
    public static void main(String[] args) {
        Scanner s=new Scanner(System.in);
        int a,b;
        a=s.nextInt();
        b=s.nextInt();
        System.out.println((a+b));
    }
}

```

/*java函数调用*/

```

import java.util.Scanner;
public class Main {
    public static int gcd(int a,int b)
    {
        if(b==0)
            return a;
        else
            return gcd(b,a%b);
    }
    public static void main(String[] args) {
        Scanner s=new Scanner(System.in);
        int a,b,sum,i;
        while(s.hasNext())
        {
            a=s.nextInt();
            b=s.nextInt();
            sum=1;
            i=b;
            while(i!=0)
            {
                int temp=gcd(a,b);
                if(temp==1)
                    break;
                else
                {
                    sum*=temp;
                    a/=temp;
                }
            }
        }
    }
}

```



```

        }
        i--;
    }
    System.out.print(sum+"\n");
}
}
}

```

9.3 Java快速读入

```

import java.io.BufferedReader;
import java.io.InputStreamReader;
import java.io.PrintWriter;
import java.math.*;
import java.nio.Buffer;
import java.util.*;
public class Main {

    public static void main(String[] args) {

        int n;
        FastScanner sc=new FastScanner();
        PrintWriter pw=new PrintWriter(System.out);

        n = sc.nextInt();
        int ans;
        pw.println(ans);

        pw.flush();
    }
}

class FastScanner{
    BufferedReader br;
    StringTokenizer st;

    public FastScanner()
    {
        try{
            br=new BufferedReader(new InputStreamReader(System.in),32768);
            st=new StringTokenizer("");
        }
        catch (Exception e) {
            // TODO: handle exception
            e.printStackTrace();
        }
    }

    public boolean hasNext() {
        while(!st.hasMoreTokens()){
            String line=nextLine();
            if(line==null)
                return false;
            st=new StringTokenizer(line);
        }
        return true;
    }
}

```

```
}
public String next()
{
    while(!st.hasMoreTokens()){
        st=new StringTokenizer(nextLine());
    }
    return st.nextToken();
}
public int nextInt(){
    return Integer.parseInt(next());
}
public long nextLong(){
    return Long.parseLong(next());
}
public double nextDouble(){
    return Double.parseDouble(next());
}
public String nextLine(){
    String line="";
    try{
        line=br.readLine();
    }catch (Exception e) {
        e.printStackTrace();
        // TODO: handle exception
    }
    return line;
}
}
```

呜呜呜呜