Pipe Wrapper

Designing wrappers for pipes with arbitrary bends.

Part 1: Defining the Pipe Centerline

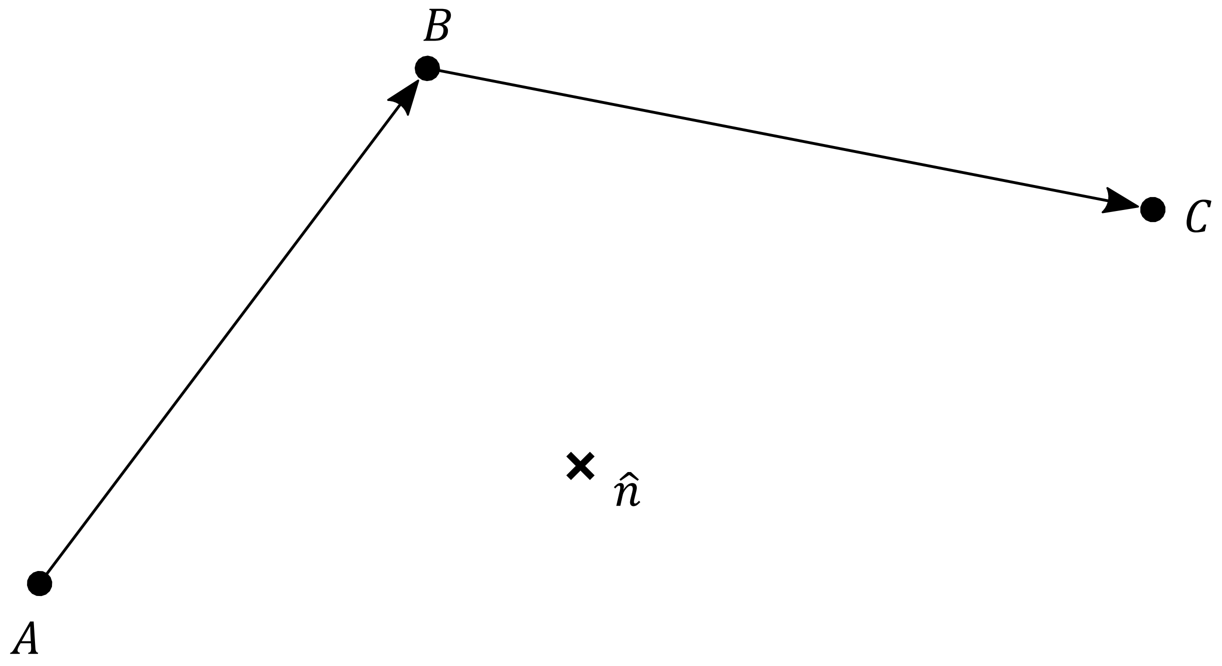
Rounding Corners with Specified Bend Radii.

Step 1: Orientation of the Bend Axis

The bend axis (say along $\hat{n}$) is normal to the plane of the bend.

i.e., $\hat{n}$ is perpendicular to $\overrightarrow{AB}$ and $\overrightarrow{BC}$.

$$\hat{\boldsymbol{n}} = \frac{\overrightarrow{\boldsymbol{AB}} \times \overrightarrow{\boldsymbol{BC}}}{|\overrightarrow{\boldsymbol{AB}} \times \overrightarrow{\boldsymbol{BC}}|}$$

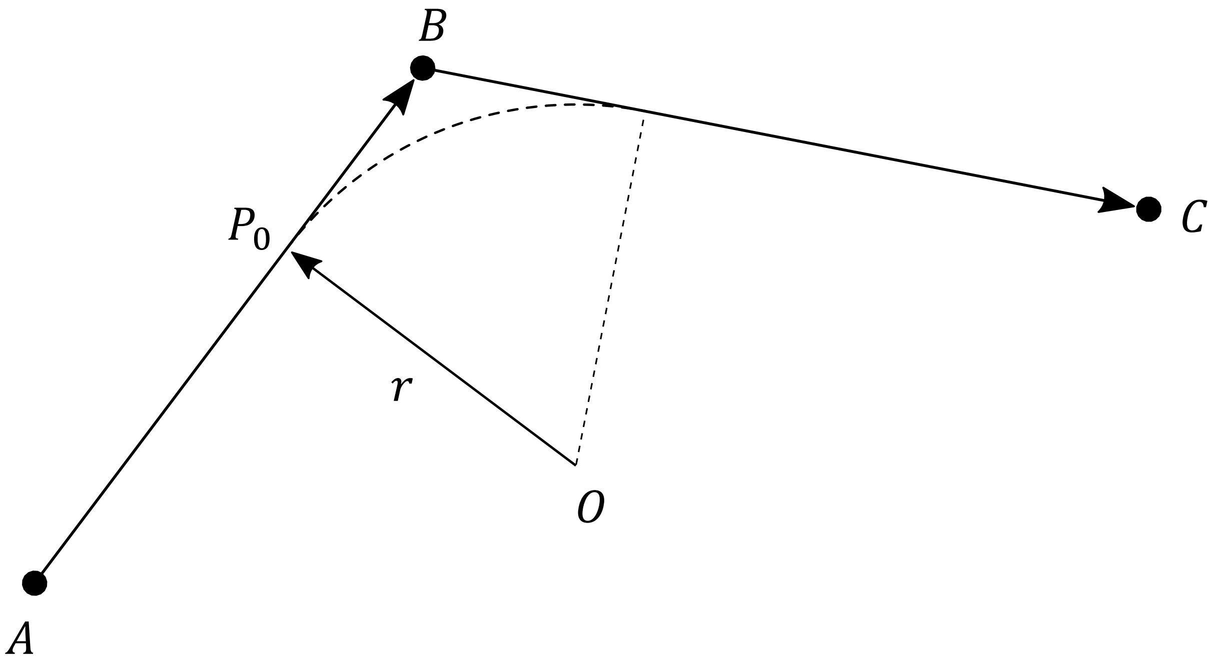


Step 2: Initial Radial Vector

Radii are in-plane with and perpendicular to arcs, so initial radial vector $\overrightarrow{OP_0}$ is perpendicular to $\hat{n}$ and to $\overrightarrow{AB}$.

Its magnitude is r, the bend radius.

$\overrightarrow{\boldsymbol{OP_0}} = \boldsymbol{r}\left(\widehat{\boldsymbol{AB}} \times \hat{\boldsymbol{n}}\right)$



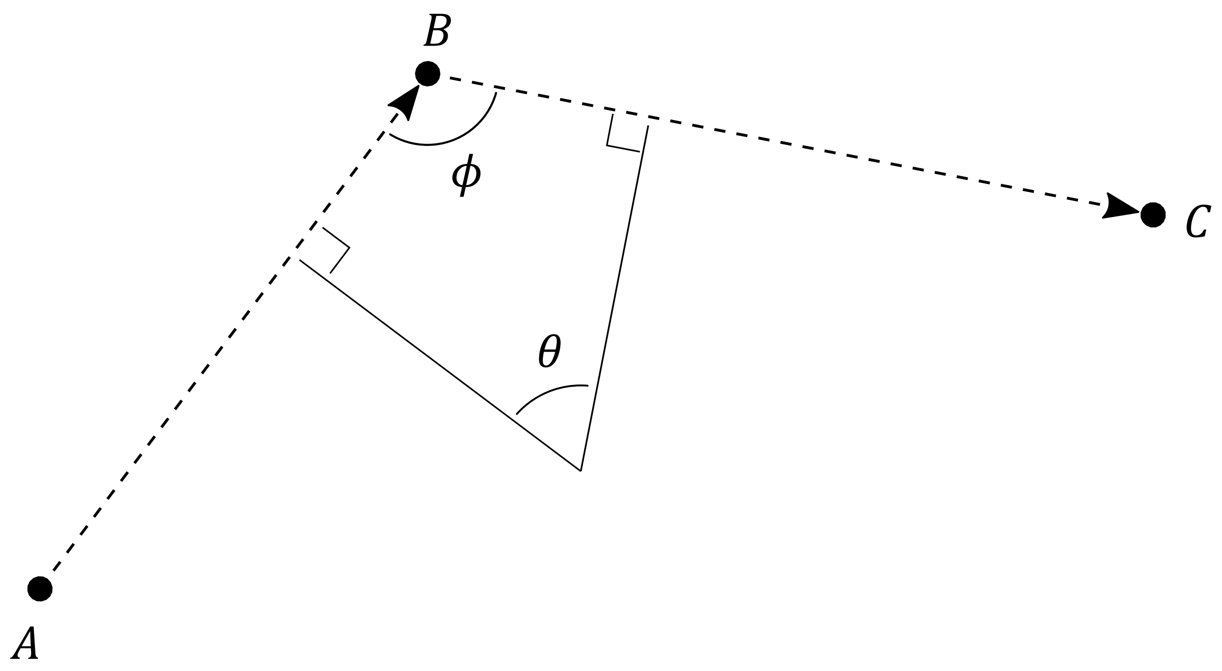
Step 3: Bend Angle

The angle between tail-to-tail unit vectors is the inverse cosine of their dot product.

θ is the angle through which an arc will be drawn.

$$\boldsymbol{\phi = cos^{-1}\left(-\widehat{AB} \cdot \widehat{BC}\right)}$$

$$\boldsymbol{\theta = \pi - \phi}$$

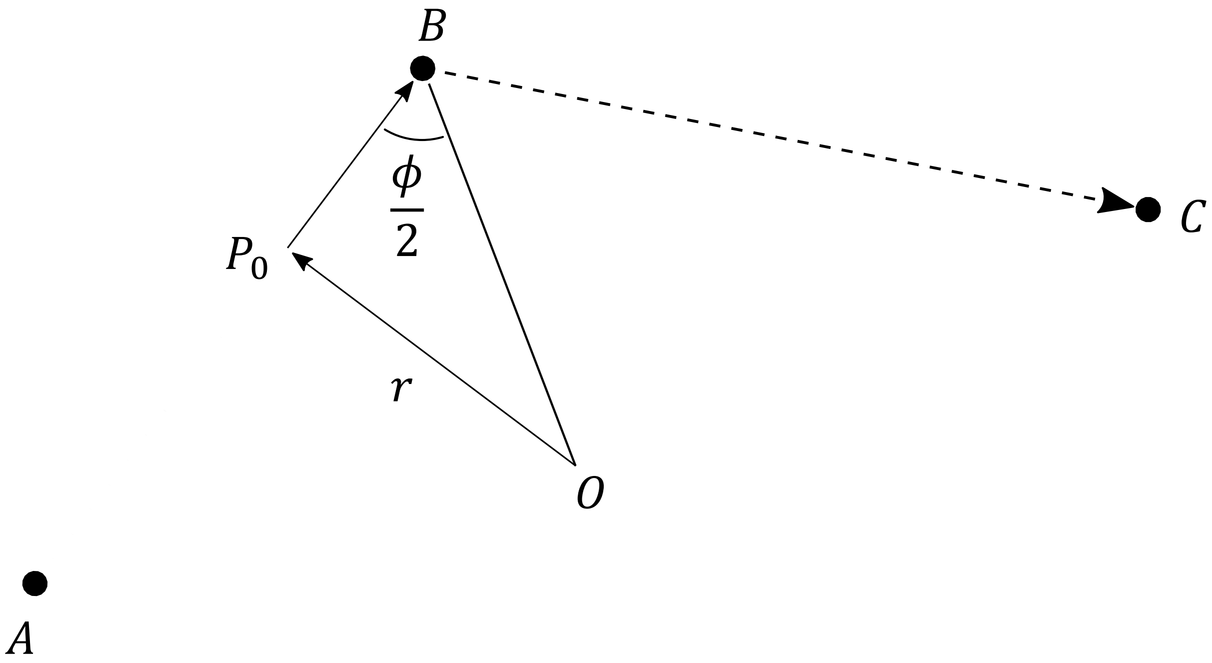


Step 4: Bend Center

$\overrightarrow{P_0B}$ is along $\widehat{AB}$ and has magnitude $\frac{r}{\tan\frac{\phi}{2}}$

$\vec{O}$ can be found by subtracting $\overrightarrow{P_0B}$ and $\overrightarrow{OP_0}$ from $\vec{B}$

$$\boldsymbol{\vec{O} = \vec{B} - \frac{r\widehat{AB}}{tan\frac{\phi}{2}} - \overrightarrow{OP_0}}$$



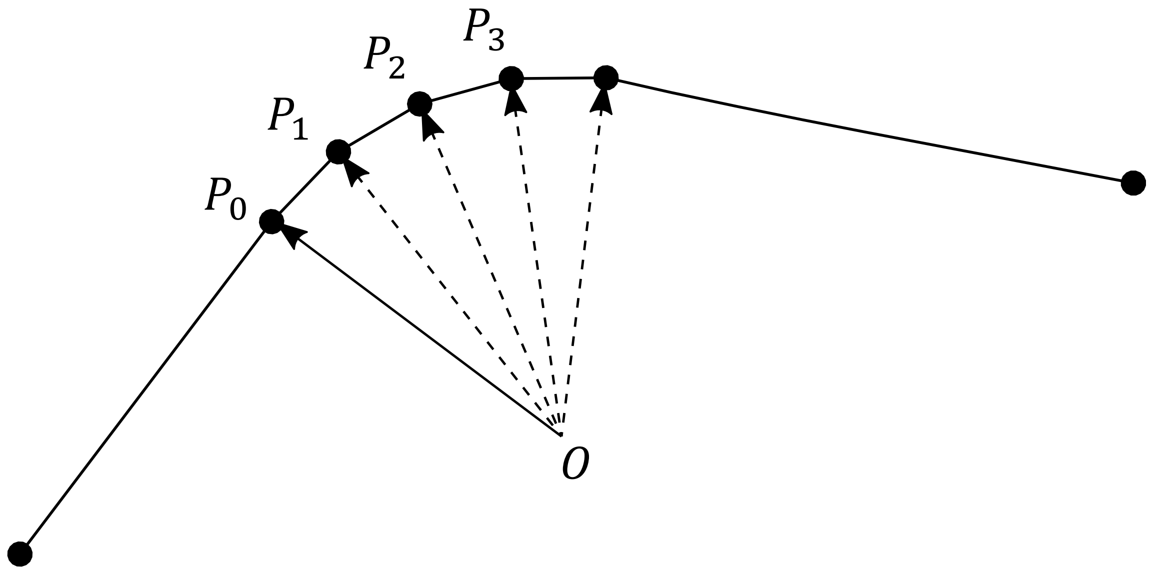
Step 5: Intermediate Points

Radial vectors $\overrightarrow{OP_1}$, $\overrightarrow{OP_2}$, $\overrightarrow{OP_3}$, etc. can be generated by rotating $\overrightarrow{OP_0}$

For an N-point bend, $\overrightarrow{OP_k}$ is $\overrightarrow{OP_0}$ rotated by $\frac{k\theta}{N}$

All rotations are about $\hat{n}$ and can be achieved using rotation matrices $R\left(\hat{n}, \frac{k\theta}{N}\right)$

$\boldsymbol{\overrightarrow{P_k} = \overrightarrow{O} + R\left(\hat{n}, \frac{k\theta}{N}\right)\overrightarrow{OP_0}}$



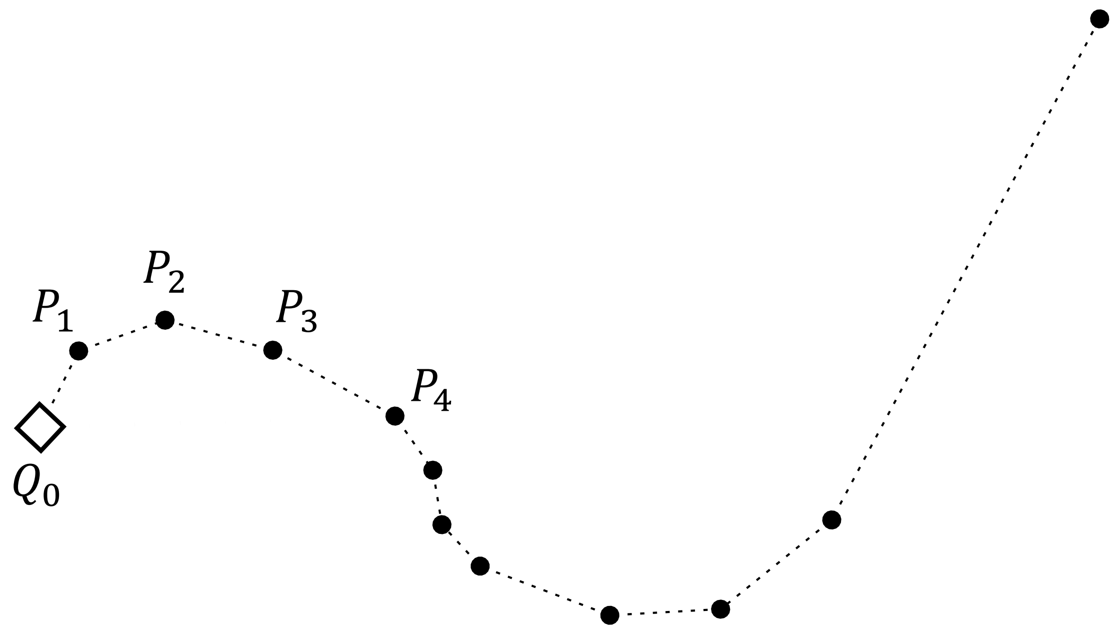
Part 2: Interpolation

Generating an evenly spaced interpolation of the pipe centerline.

Step 1: Start with the First Point

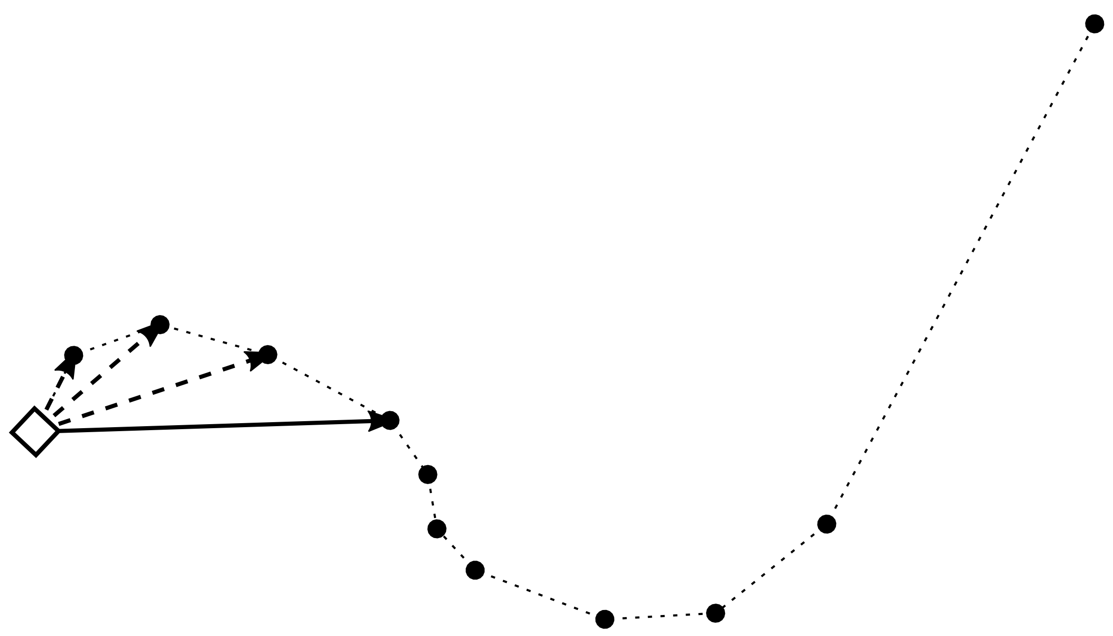
Add the first input point P_0 to the output, as-is.

$\boldsymbol{Q_0 = P_0}$



Step 2: Find Potential Increment > Desired Spacing

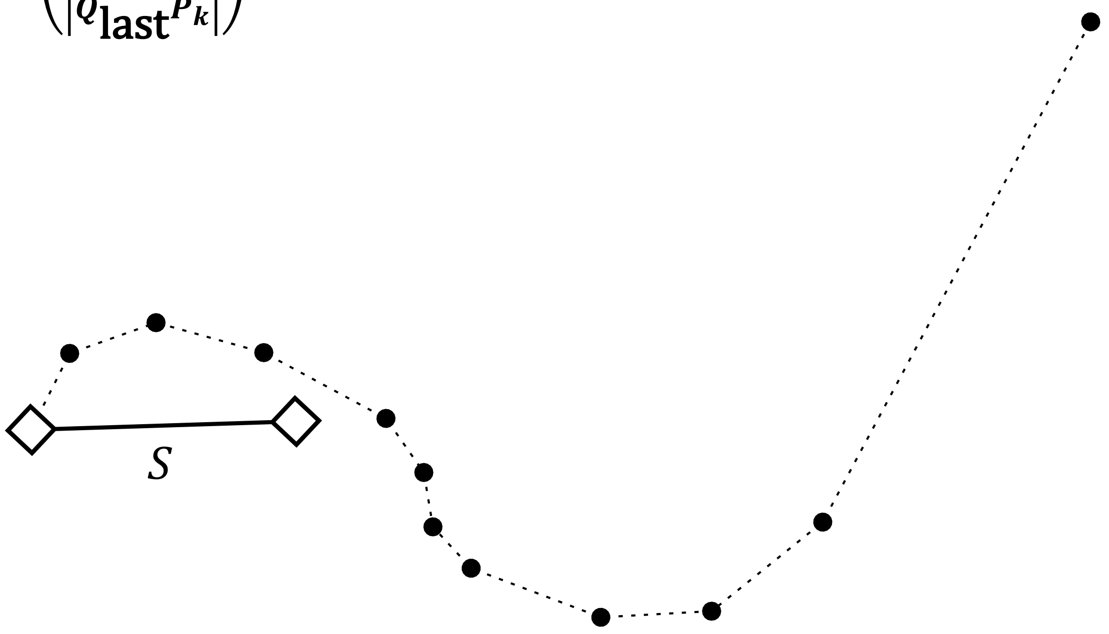
If the desired spacing is S, iterate through input points P_k until $\left|\overrightarrow{Q_{\text{last}} P_k}\right| > S$



Step 3: Add an Increment of the Correct Length

Add a new point to the output that is S distance from the last added point along $\overrightarrow{Q_{\text{last}}P_k}$

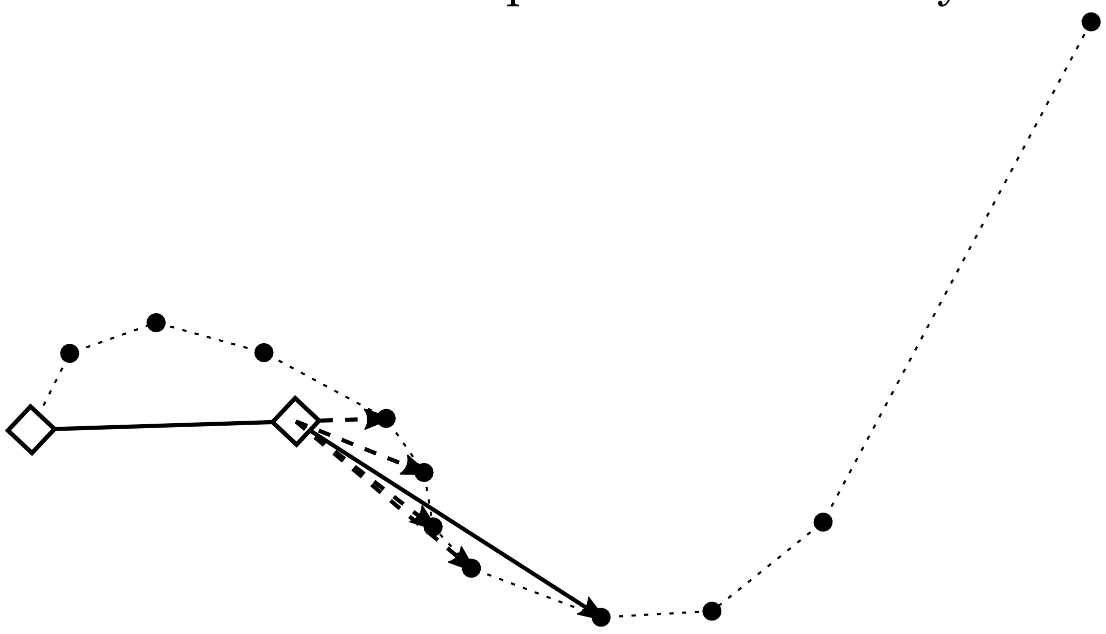
$$\overrightarrow{\boldsymbol{Q}_{\textbf{new}}} = \overrightarrow{\boldsymbol{Q}_{\textbf{last}}} + \boldsymbol{S}\left(\frac{\overrightarrow{\boldsymbol{Q}_{\textbf{last}}\boldsymbol{P}_{\boldsymbol{k}}}}{\left|\overrightarrow{\boldsymbol{Q}_{\textbf{last}}\boldsymbol{P}_{\boldsymbol{k}}}\right|}\right)$$



Step 4: Repeat

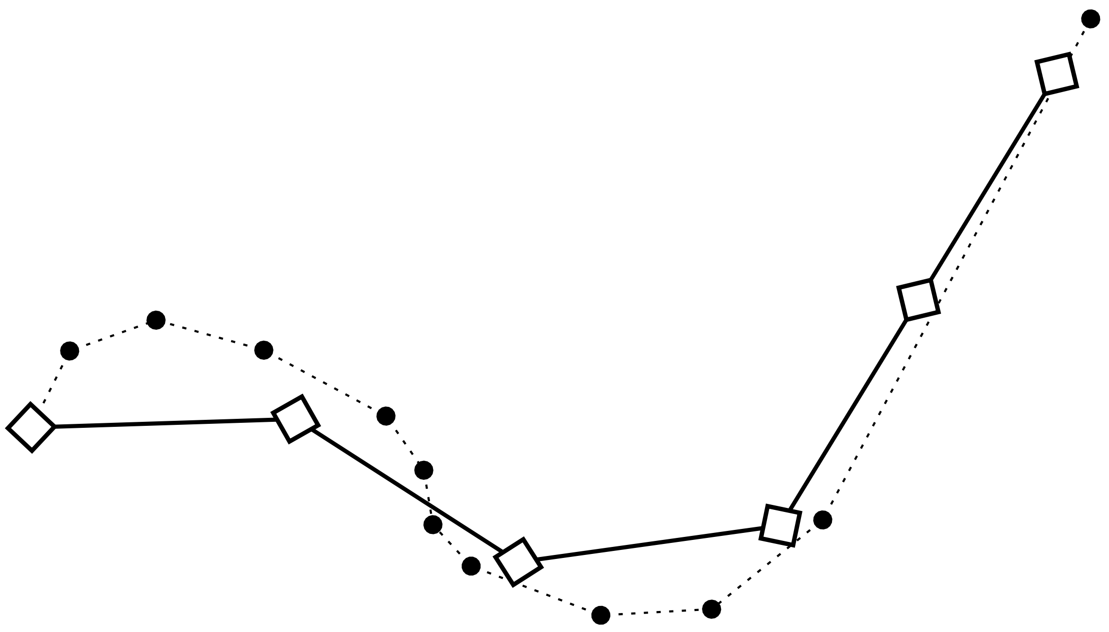
Repeat search for potential increment

Start by re-checking the last-checked input point, so that multiple increments in the same direction are possible if necessary.



Step 5: Stop when no Further Increment is Possible

Stop adding points when the last input point is less that S distance away from the last output point



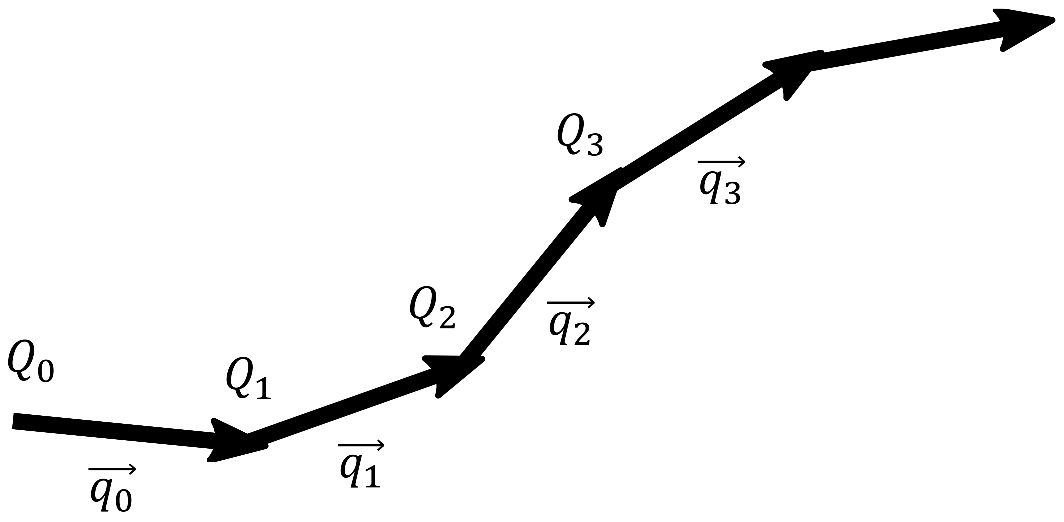
Part 3: Creating the "Helix"

Generating points along a curve that winds around the pipe centerline.

Step 1: Pipeline Direction Vectors

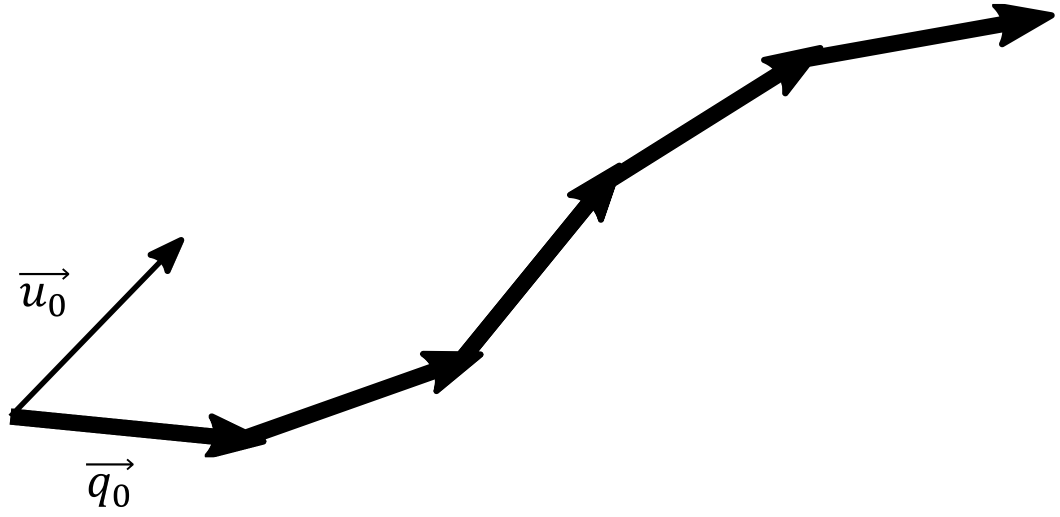
Generate vectors joining consecutive pairs of evenly spaced pipe centerline points

$$\overrightarrow{\boldsymbol{q_k}} = \overrightarrow{\boldsymbol{Q_{k+1}}} - \overrightarrow{\boldsymbol{Q_k}}$$



Step 2: Choose Initial Non-Parallel Vector

Choose any vector $\overrightarrow{u_0}$ that is not parallel to the first direction vector $\overrightarrow{q_0}$



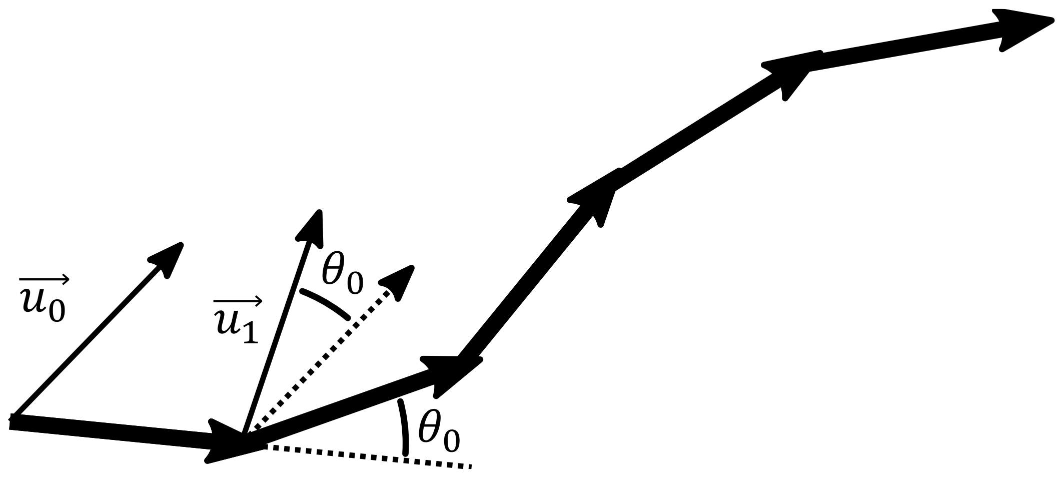
Step 3: Find Each Non-Parallel Vector by Rotating the Previous One

Find the rotation axis and angle between one pipe segment and the next

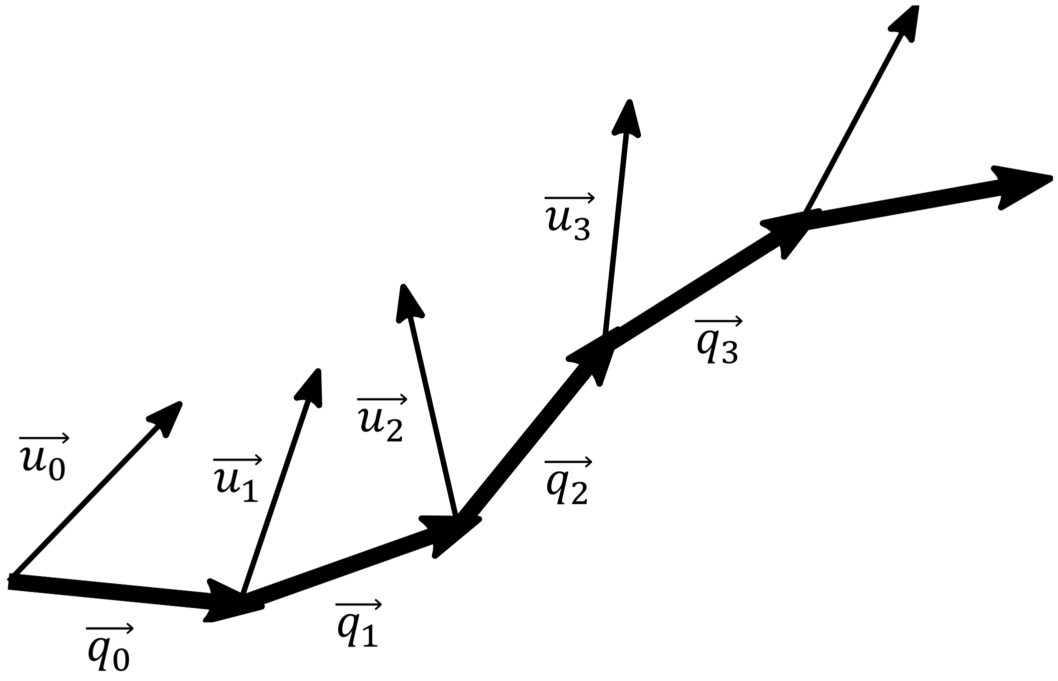
$\widehat{n_k} = \frac{\overrightarrow{q_k} \times \overrightarrow{q_{k+1}}}{|\overrightarrow{q_k} \times \overrightarrow{q_{k+1}}|}$, $\theta_k = \cos^{-1}(\widehat{q_k} \cdot \widehat{q_{k+1}})$

Find current non-parallel vector by rotating the previous one

$\boldsymbol{\overrightarrow{u_{k+1}} = R(\widehat{n_k}, \theta_k)\overrightarrow{u_k}}$ **if** $\boldsymbol{\theta \neq 0}$ **, or** $\boldsymbol{\overrightarrow{u_{k+1}} = \overrightarrow{u_k}}$ **if** $\boldsymbol{\theta = 0}$



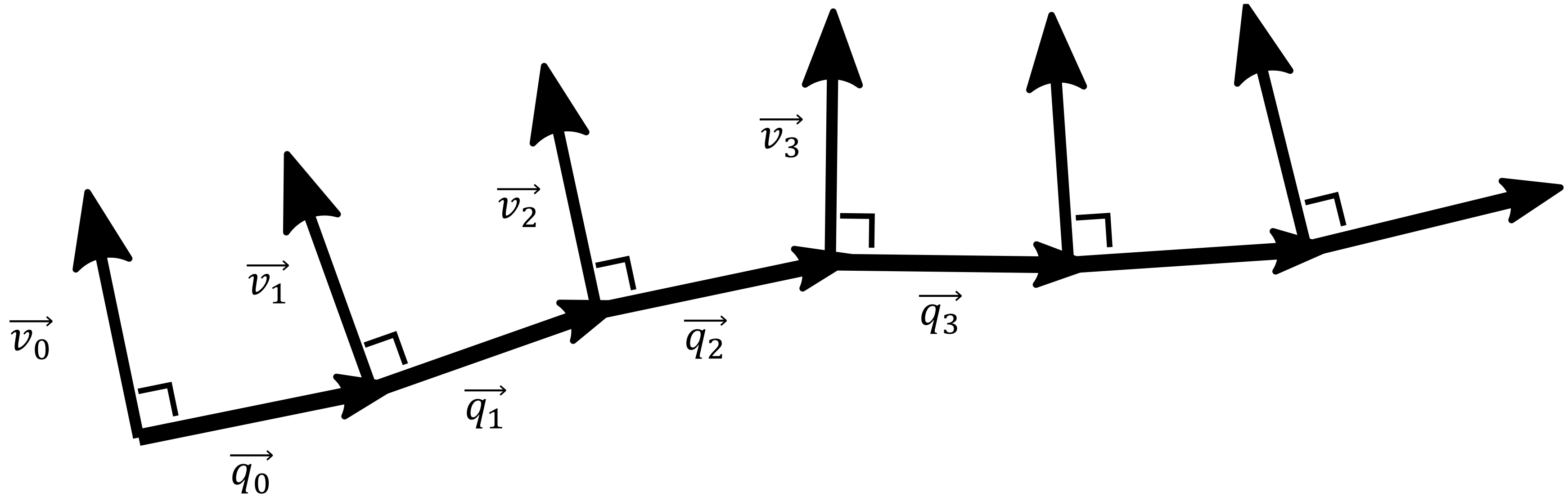
Non-Parallel Vectors



Step 4: Generate Perpendicular Vectors

$\overrightarrow{\boldsymbol{v_k}} = \overrightarrow{\boldsymbol{q_k}} \times \overrightarrow{\boldsymbol{u_k}}$

Since $\overrightarrow{u_0}, \overrightarrow{u_1}, \overrightarrow{u_2} \cdots$ follow the bending of the pipe, $\overrightarrow{v_0}, \overrightarrow{v_1}, \overrightarrow{v_2} \cdots$ will as well.

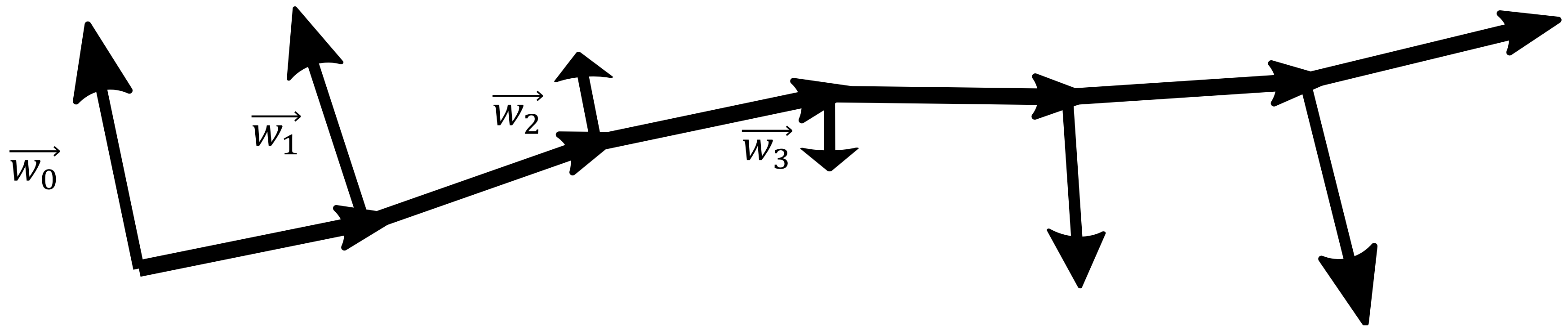


Step 5: Rotate the Perpendicular Vectors

If Ω is the helix wrap rate in radians per unit length and S is the spacing,

Rotate each perpendicular vector $\overrightarrow{v_k}$ about $\widehat{q_k}$ by $\Omega S k$

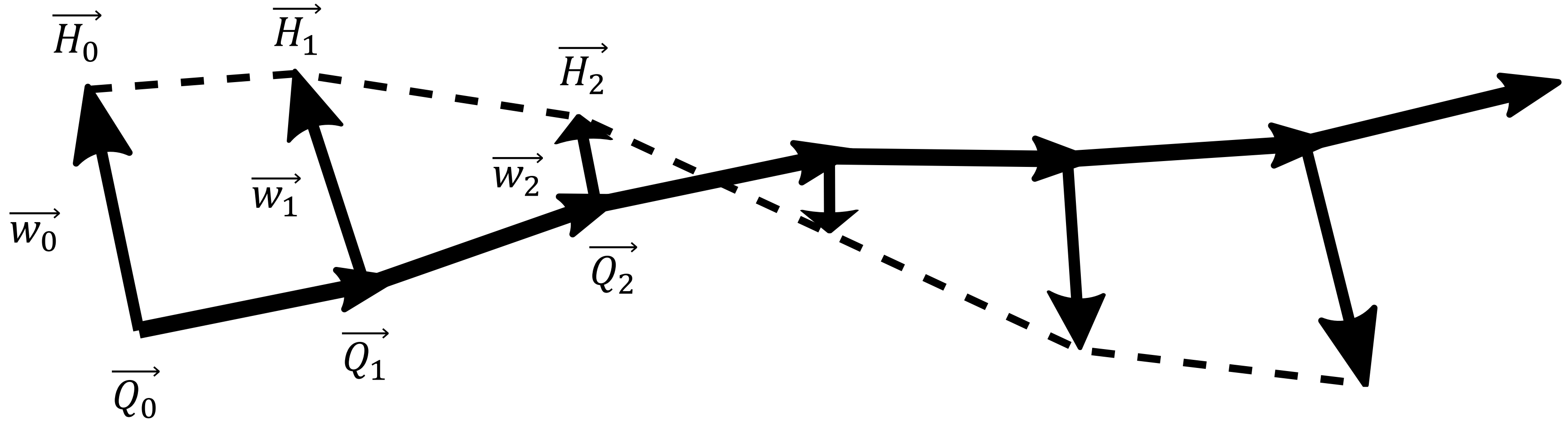
$\boldsymbol{\overrightarrow{w_k} = R(\widehat{q_k}, \Omega S k)\overrightarrow{v_k}}$



Step 6: Find and Connect the Dots

If the wrap radius is L, then each point H_k on the "helix" is L distance away along rotated perpendicular vector $\overrightarrow{w_k}$ from the centerline.

$$\overrightarrow{\boldsymbol{H_k}} = \overrightarrow{\boldsymbol{Q_k}} + \boldsymbol{L}\widehat{\boldsymbol{w_k}}$$



Part 4: Unwrapping

Calculating a 2D shape that wraps along the “Helix” to create a “Pipe”.

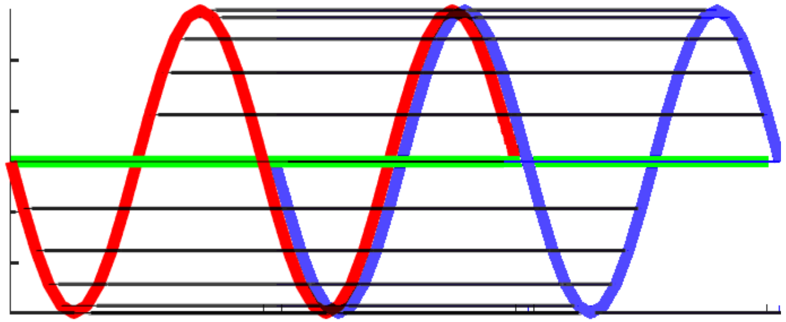
Background: Defining The Wrapper in 3D

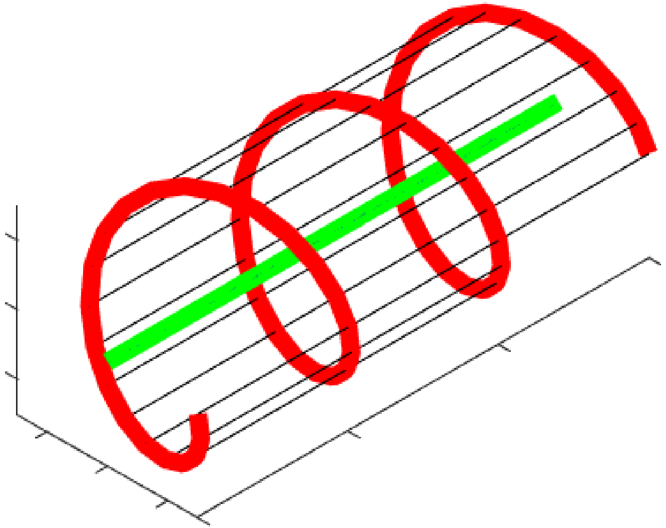
Two copies of the generated Helix offset by one turn are the left and right edges of the “wrapper”.

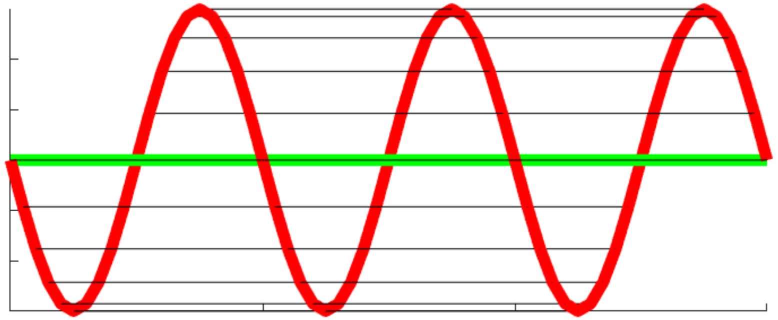
Or equivalently, given wrap rate Ω and spacing S :

Connect $\boldsymbol{H_k}$ to $\boldsymbol{H_{k+\frac{2\pi}{\Omega S}}}$

to form **“ladder rungs”** across the wrap.







Background: Transferring a Triangle in 3D Space to the xy-Plane

The goal is to find C_{2D} given A_{3D} , B_{3D} , C_{3D} , A_{2D} , B_{2D}

Drop a perpendicular CD onto AB (in 3D and 2D)

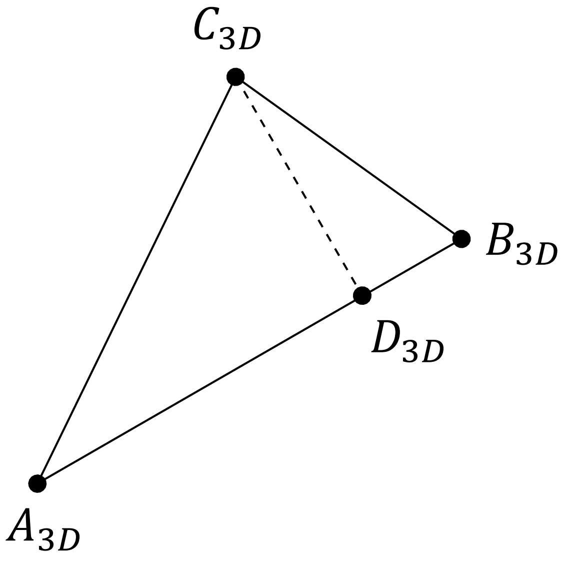
$$\overrightarrow{\boldsymbol{A_{3D}D_{3D}}} = \left(\overrightarrow{\boldsymbol{A_{3D}C_{3D}}} \cdot \widehat{\boldsymbol{A_{3D}B_{3D}}}\right)\widehat{\boldsymbol{A_{3D}B_{3D}}}$$

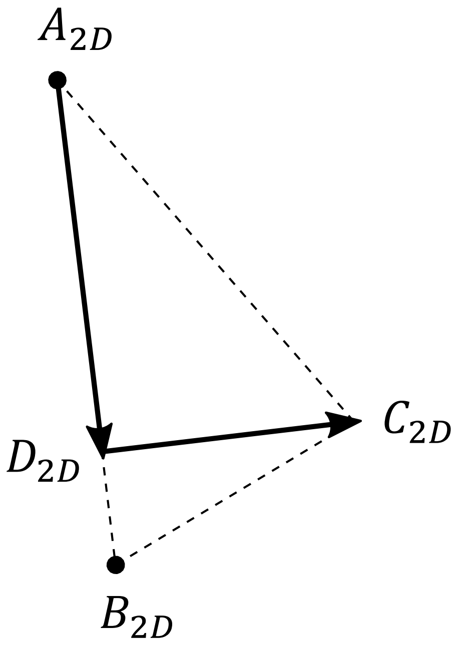
$$\overrightarrow{\boldsymbol{D_{3D}C_{3D}}} = \overrightarrow{\boldsymbol{A_{3D}C_{3D}}} - \overrightarrow{\boldsymbol{A_{3D}D_{3D}}}$$

$$\overrightarrow{\boldsymbol{A_{2D}D_{2D}}} = \left|\overrightarrow{\boldsymbol{A_{3D}D_{3D}}}\right|\widehat{\boldsymbol{A_{2D}B_{2D}}}$$

$$\overrightarrow{\boldsymbol{D_{2D}C_{2D}}} = \left|\overrightarrow{\boldsymbol{D_{3D}C_{3D}}}\right|\left(\widehat{\boldsymbol{A_{2D}B_{2D}}} \times \hat{\boldsymbol{z}}\right)$$

$$\overrightarrow{\boldsymbol{C_{2D}}} = \overrightarrow{\boldsymbol{A_{2D}}} + \overrightarrow{\boldsymbol{A_{2D}D_{2D}}} + \overrightarrow{\boldsymbol{D_{2D}C_{2D}}}$$



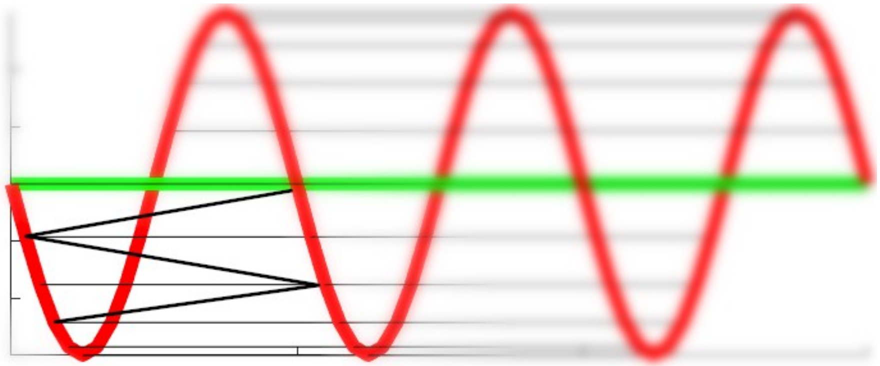


Step 1: Divide the Wrapper into Triangles about Alternate Diagonals

Join the left end of the current rung to the right end of next rung

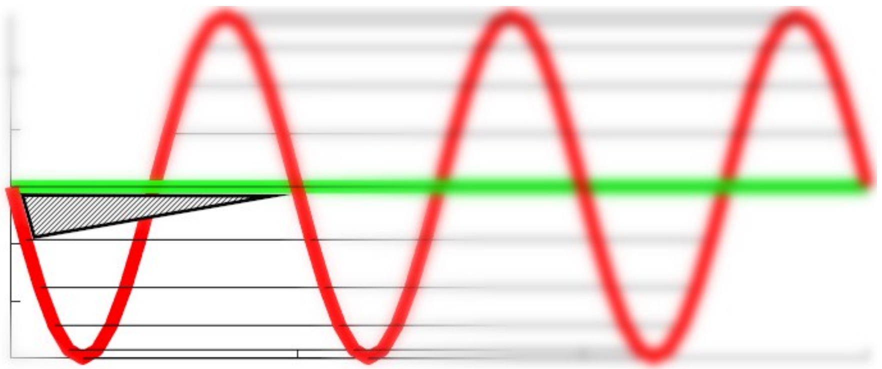
Then join the right end of that rung to the left end of the rung after it

etc.



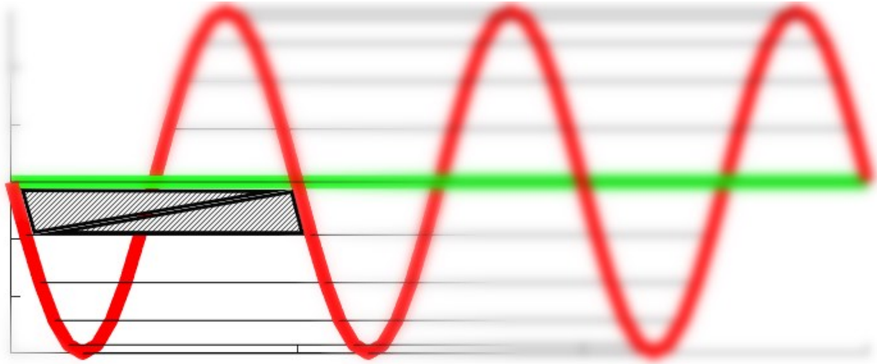
Step 2: Transfer the First Triangle to 2D

Recreate the first triangle in 2D with any arbitrary base orientation



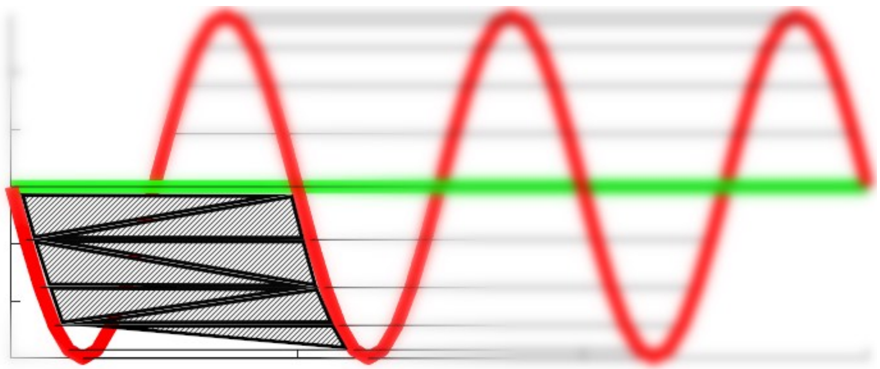
Step 2: Transfer the Next Triangle to 2D

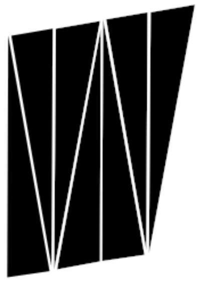
Recreate the next triangle in 2D using the last edge of the last 2D triangle as the base.



Step 3: Repeat

Continue unwrapping triangle-by-triangle



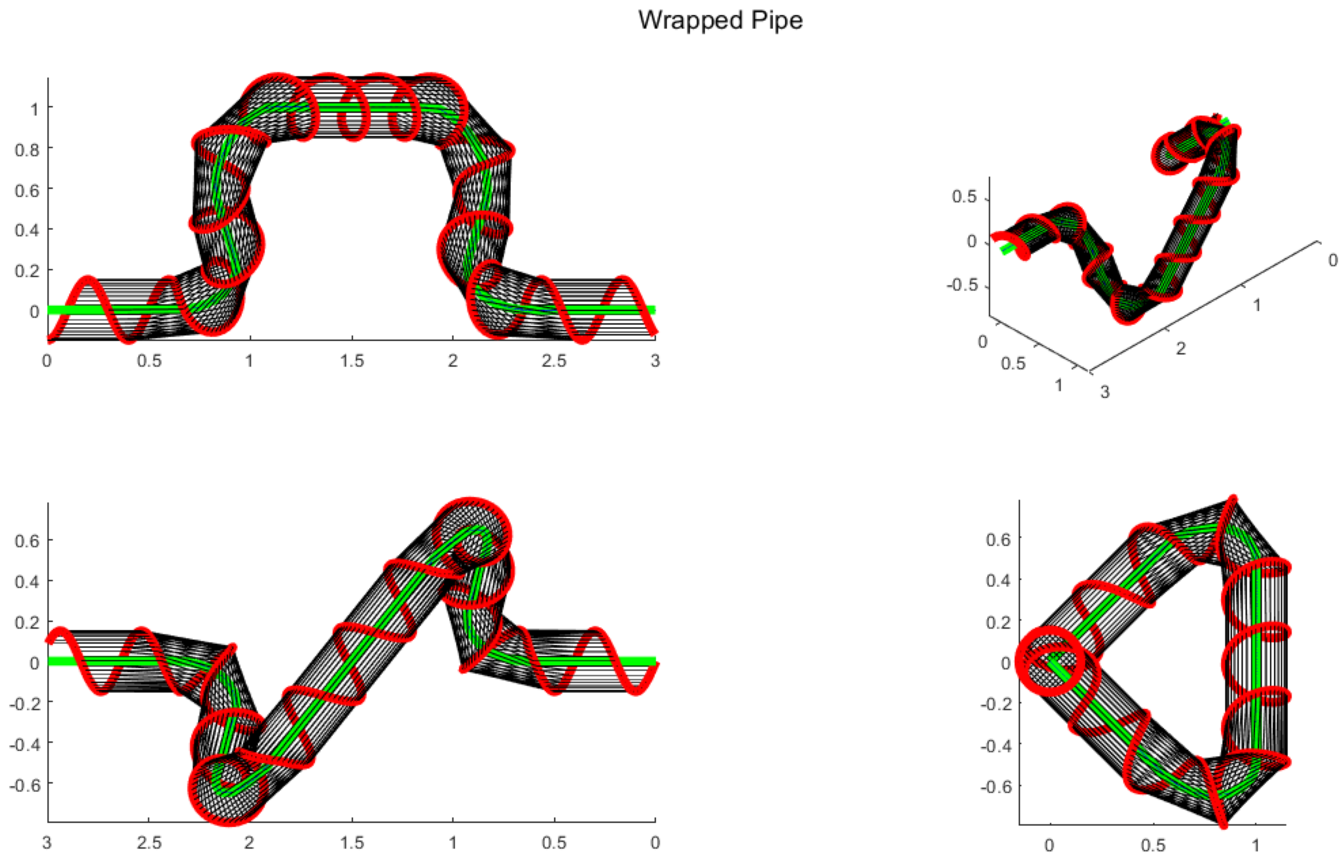


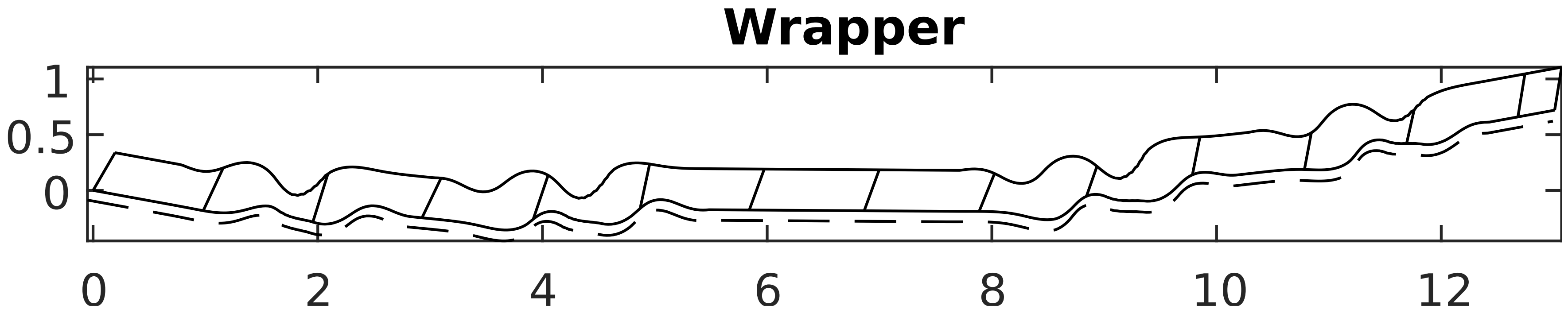
Part 5: Sample Output

Output from MATLAB and Python code for example pipes.

```
cornersx = [0 1 0.7 2.3 2 3];
cornersy = [0 0 1 1 0 0];
cornersz = [0 0 1 -1 0 0];
bendradii = 0.3*[1 1 1 1];
bendpoints = 200;

PipeRadius = 0.15;
TurnsPerMeter = 2.5;
Overlap = 0.1;
Resolution = 100;
PlotAngle = pi/3;
RefVector = [0; 0; 1];
```





Insulation Radius: 0.1
Turns per Unit Length: 3
Overlap: 0.01
Resolution: 100
Plot Angle: 90
Bend Segments: 200
Output Excel File: out.xlsx
Ref Vector: 1 0 0

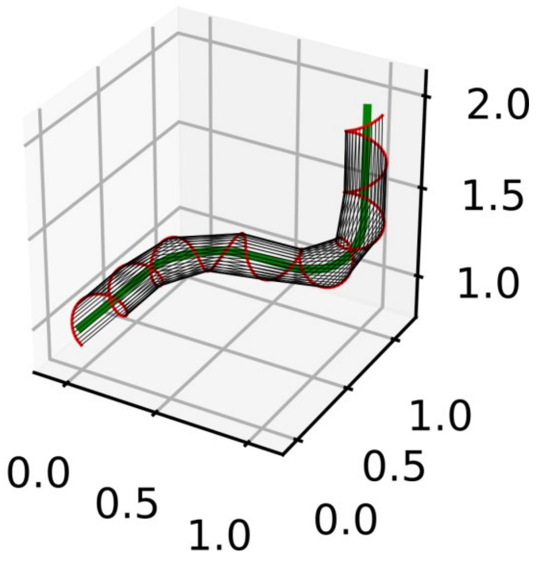
Pipe Centerline Data

x	y	z	Bend Radius
0.0	0.0	1.0	0.0
0.0	1.0	1.0	0.4
1.0	1.0	1.0	0.4
1.0	1.0	2.0	0.0

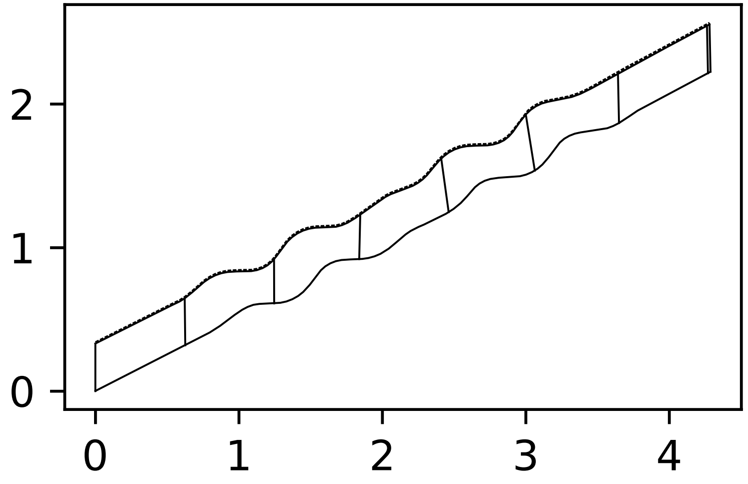
Bend Radius: 0.4
x: 1 y: 1 z: 2

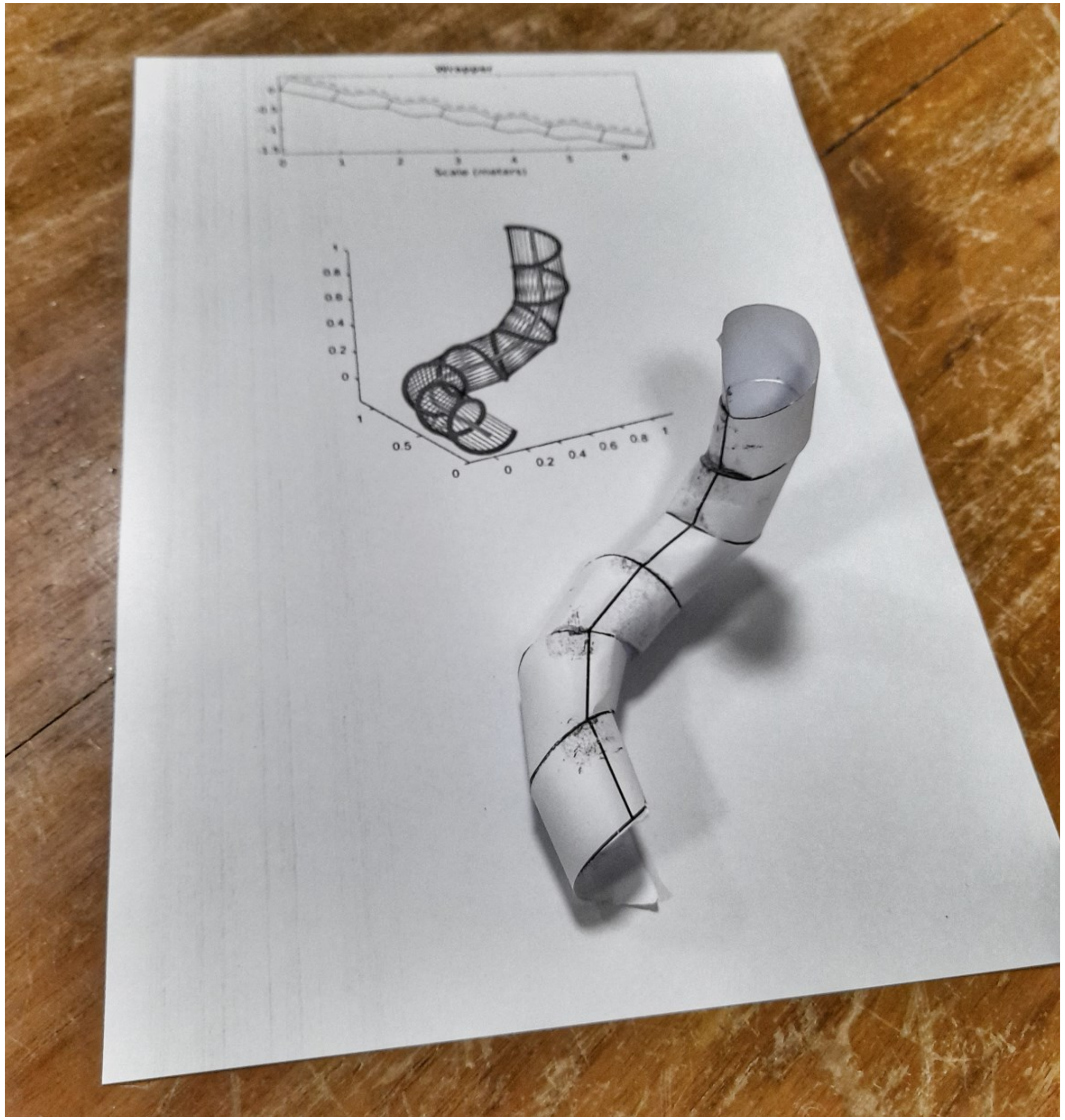
GEN | GEN (New Fig) | Add Point | Delete Point

Wrapped Pipe



Wrapper




Wrapper
Scale (meters)

End.