

## SPRINT SAMPLE PROBLEMS

- This is a set of sample problems selected to represent the difficulty range of the actual test.
- Ordering is based on difficulty, which increases as the question number increases.
- No aids are permitted other than pencils, scratch papers, graph papers, rulers, compasses, and erasers. No calculators, smartwatches, or other computing devices are allowed. No problems on the test will require the use of a calculator.
- Figures are not necessarily drawn to scale.

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## PROBLEMS

### 1 Problem 1

Rich earns 12 dollars in 3 minutes and 36 seconds. If he continues to earn money at this rate, how many dollars will he earn in 336 minutes?

### 2 Problem 2

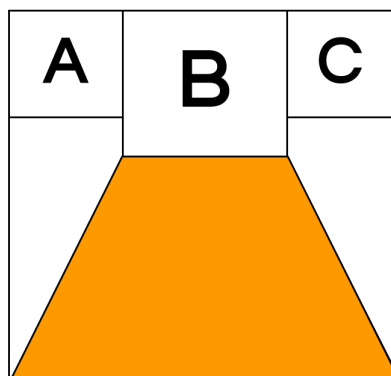
Captain Billy and his crew of 20 pirates discover a gold chest with 100 gold coins. If Billy wants to distribute the gold coins as evenly as possible among the 21 pirates such that everyone has either 4 or 5 coins, how many people will have 4 coins?

### 3 Problem 3

Jack has 15 balls that are all either blue, green, or red. He writes the labels BLUE, GREEN, and RED on corresponding balls. When counting the total number of *Es* written on all 15 balls, he finds that there are 20 *Es*. How many green balls are there?

### 4 Problem 4

A large square shown contains three smaller squares  $A$ ,  $B$ , and  $C$ . The areas are 9, 16, and 9, respectively. What is the area of the shaded region?



## 5 Problem 5

Let  $x$  and  $y$  be positive real numbers such that

$$2x^2 + xy = 7x + 9y$$

$$2y^2 + 3xy = 9x + 7y.$$

Find  $x + y$ .

## SOLUTIONS

### Solution to Problem 1

**Answer 1120:** Notice that 36 seconds is  $\frac{3}{5}$  of a minute, so Rich earns 12 dollars in  $3 + \frac{3}{5} = \frac{18}{5}$  minutes. So, he earns  $12 \cdot \frac{5}{18} = \frac{10}{3}$  dollars in 1 minute, which means he earns  $336 \cdot \frac{10}{3} = \boxed{1120}$  dollars in 336 minutes. ■

### Solution to Problem 2

**Answer 5:** If Billy gives 4 coins to each pirate, there would be  $100 - 21 \cdot 4 = 16$  coins left. So, he could give 16 pirates an extra coin, which means  $21 - 16 = \boxed{5}$  pirates will have 4 coins. ■

### Solution to Problem 3

**Answer 5:** Let there be  $b$  blue balls,  $g$  green balls, and  $r$  red balls. Since there are fifteen total balls, we know

$$b + g + r = 15 \quad (1).$$

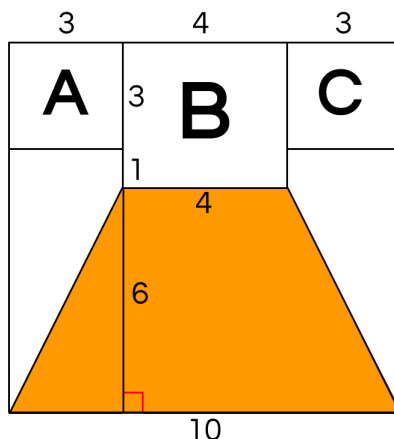
From the fact that 20  $E$ s are written, we can write the equation

$$b + 2g + r = 20 \quad (2).$$

Subtracting (1) from (2), we get  $g = \boxed{5}$ . ■

## Solution to Problem 4

**Answer 42:** Refer to the diagram below.



Because square A and C have an area of 9, their side lengths are both 3. Similarly, we conclude that square B has a side length of 4. Summing the side lengths on uppermost horizontal segment, we see that the side length of the larger square is

$$3 + 3 + 4 = 10.$$

This means the bases of the shaded trapezoid are 4 and 10, as shown. We also know that the altitude of the trapezoid is

$$10 - 4 = 6,$$

since the altitude plus the side length of B must add up to the side length of the large square. Using the area of a trapezoid formula, the area of the shaded region can be calculated as

$$h \times \frac{b_1 + b_2}{2} = 6 \times \frac{4 + 10}{2} = \boxed{42}.$$



## Solution to Problem 5

**Answer 8:** We add the two equations to get

$$2x^2 + 4xy + 2y^2 = 16(x + y),$$

then divide by two to get

$$x^2 + 2xy + y^2 = 8(x + y).$$

Note that  $(x + y)^2 = x^2 + 2xy + y^2$ , so we have

$$(x + y)^2 = 8(x + y).$$

Since  $x + y \neq 0$  as both variables represent positive real numbers, we divide by  $x + y$  on both sides to get

$$x + y = \boxed{8}.$$

