ACCURACY SAMPLE PROBLEMS

- This is a set of sample problems selected to represent the difficulty range of the actual test.
- Ordering is based on difficulty, which increases as the question number increases.
- No aids are permitted other than pencils, scratch papers, graph papers, rulers, compasses, and erasers. No calculators, smartwatches, or other computing devices are allowed. No problems on the test will require the use of a calculator.
- Figures are not necessarily drawn to scale.

PROBLEMS

1 Problem 1

Jemele has 1 marker, while her friend, Jillian, has 12 markers. Every day Jemele doubles her markers, while Jillian gets 4 more markers. For instance, on the second day, Jemele would have 2 markers while Jillian would have 16. After how many days does Jemele first have more markers than Jillian?

2 Problem 2

Sam is thinking of a 3-digit number such that the hundreds and tens digits are divisible by 2 and the hundreds and units digits are divisible by 3. What is the maximum possible value of Sam's number?

3 Problem 3

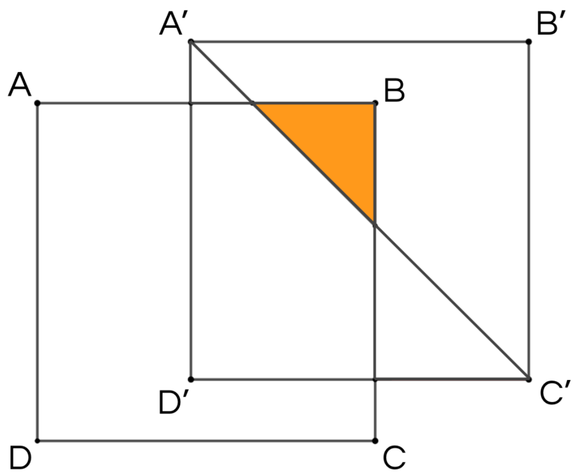
Find the number of possible values of

$$J \times M \times P \times S \times C$$

if all letter variables represent an integer and $0 < J < M < P < S < C < 7$.

4 Problem 4

Square $ABCD$ has side lengths of 25, and its sides are either horizontal or vertical. Square $ABCD$ is translated 11 units to the right and 4 units up to form square $A'B'C'D'$. Segment $A'C'$ is drawn. What is the area of the shaded region?



5 Problem 5

Christopher starts at the origin O of the coordinate plane. Every second, Christopher moves exactly one unit up, down, left, or right—each occurring with equal probability. After 7 seconds, the probability that Christopher is exactly five units away from point O can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.

SOLUTIONS

Solution to Problem 1

Answer 7: We can simply make a table, as follows:

Days Elapsed	Jemele	Jillian
1	1	12
2	2	16
3	4	20
4	8	24
5	16	28
6	32	32
7	64	36

Thus, Jemele first has more markers than Jillian after $\boxed{7}$ days. ■

Solution to Problem 2

Answer 689: Since the hundreds digit must be divisible by both 2 and 3 from the given information, it must be a nonzero multiple of 6. The only hundreds digit that satisfies our conditions is therefore 6. The only restriction on the tens digit is that it must be divisible by 2 - the maximum value is therefore 8. Similarly, the maximum value for the units digit is 9. Putting all three digits together, our answer is $\boxed{689}$. ■

Solution to Problem 3

Answer 6: If $C < 5$ then there cannot possibly exist a solution to the inequality because

$$J = 1 \qquad M = 2 \qquad P = 3 \qquad S = 4 \qquad C = 5$$

minimizes C. (Try finding a solution with $C = 4$) So,

$$JMPSC = 120$$

when $C = 5$. When $C = 6$, we have that

$$J, M, P, S \in 1, 2, 3, 4, 5$$

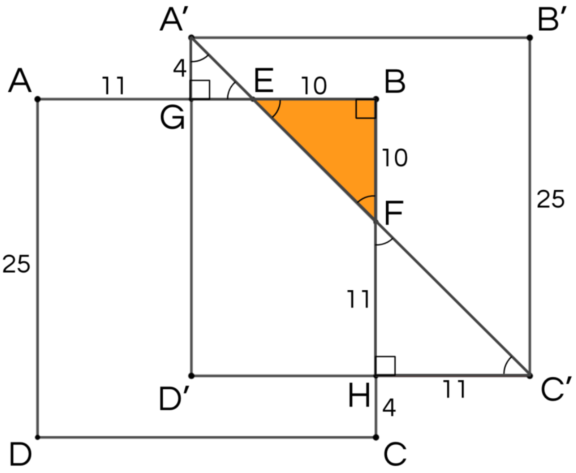
such that $J < M < P < S$. In each of the possible values of $JMPS$, we are leaving out exactly one of $1, 2, 3, 4, 5$, in the total product $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$ to get $120, 60, 40, 30$, and 24, respectively. Each of these are multiplied by $C = 6$ to get

$$720, 360, 240, 180, 144.$$

So, we have $\boxed{6}$ possible values for $JMPSC$. ■

Solution to Problem 4

Answer 50:



Since A is translated 11 units right and 4 units up, $GA = 11$ and $GA' = 4$. Similarly, we have $HC = 4$ and $HC' = 11$. In square $A'B'C'D'$, diagonal $A'C'$ bisects $\angle A'$ and $\angle C'$ into two 45° angles. Since all sides are horizontal or vertical, we have $GA' \perp GE$ and $HC' \perp HF$. We find

$$\angle HFC' = 180 - \angle FHC' - \angle HC'F = 180 - 90 - 45 = 45°.$$

Similarly, we have

$$\angle GEA' = 45°.$$

Thus, triangles GEA' and HFC' are 45-45-90 triangles. Since vertical angles are congruent, we know that

$$\angle EFB = \angle HFC' = 45°$$

and

$$\angle FEB = \angle GEA' = 45°.$$

Thus, $\triangle BEF$ is a 45-45-90 triangle. We proved earlier that $\triangle GEA'$ is 45-45-90, so $GE = GA' = 4$. Therefore,

$$BE = AB - AG - GE = 25 - 11 - 4 = 10.$$

So, $BF = 10$, and the area of $\triangle BEF$ is

$$\frac{10 \cdot 10}{2} = \boxed{50}.$$

Solution to Problem 5

Answer 4215: There are total 4^7 outcomes. The possible points are all permutations (orderings) of $(\pm 3, \pm 4)$, $(\pm 5, 0)$, and $(0, \pm 5)$. There are $\binom{7}{3}$ ways to get to one of the points under the first case. We take $(5,0)$ as an example for the two latter cases. We must either go right 6 times and left once, or right 5 times and up and down. Using the first letters of up, down, left, and right, what we want is all permutations (orderings) of RRRRRRL and RRRRRUD. This is equal to

$$\binom{7}{1} + \frac{7!}{5!} = 7 + 42 = 49.$$

However, we are still not done. There are 8 scenarios for the first case, and 4 for the second, meaning our probability is

$$\frac{8\binom{7}{3} + 4 \cdot 49}{4^7} = \frac{119}{4096},$$

which gives an answer of $119 + 4096 = \boxed{4215}$.