ACCURACY SAMPLE PROBLEMS

- This is a set of sample problems selected to represent the difficulty range of the actual test.
- Ordering is based on difficulty, which increases as the question number increases.
- No aids are permitted other than scratch paper, graph paper, rulers, compass, protractors, and erasers. No calculators, smartwatches, or computing devices are allowed. No problems on the test will require the use of a calculator.
- Figures are not necessarily drawn to scale.

PROBLEMS

1 Problem 1

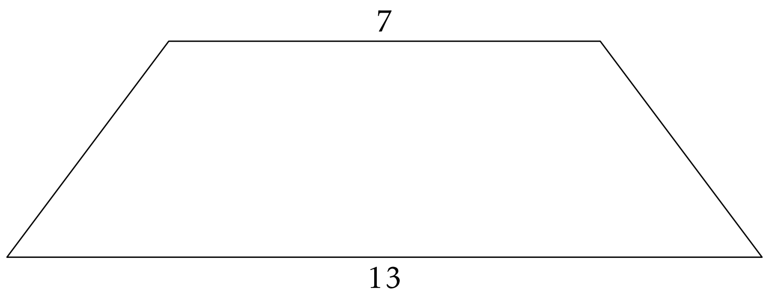
Jemele has 1 marker, while her friend, Jillian, has 12 markers. Every day Jemele doubles her markers, while Jillian gets 4 more markers. For instance, on the second day, Jemele would have 2 markers while Jillian would have 16. On what day does Jemele first have more markers than Jillian?

2 Problem 2

Sam is thinking of a 3-digit number such that the hundreds and tens digits are divisible by 2 and the hundreds and units digits are divisible by 3. What is the maximum possible value of Sam's number?

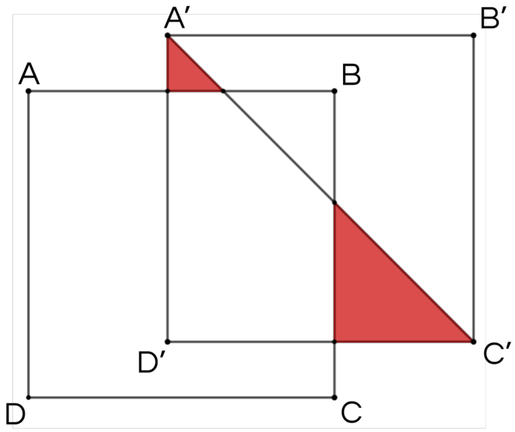
3 Problem 3

An isosceles trapezoid has bases of 7 and 13 and an area of 40. What is its perimeter?



4 Problem 4

Square $ABCD$ has side length 11, and its sides are either horizontal or vertical. Square $ABCD$ is translated 5 units to the right and 2 units up to form square $A'B'C'D'$. Segment $A'C'$ is drawn. What is the area of the shaded region?



5 Problem 5

Christopher starts at the origin O of the coordinate plane. Every second, Christopher moves exactly one unit up, down, left, or right—each occurring with equal probability. What is the probability that, after seven seconds, Christopher is exactly five units away from point O?

SOLUTIONS

Solution to Problem 1

Answer 7: We can simply make a table, as follows:

Day	Jemele	Jillian
1	1	12
2	2	16
3	4	20
4	8	24
5	16	28
6	32	32
7	64	36

Thus, the first day Jemele has more markers than Jillian is day $\boxed{7}$.

Solution to Problem 2

Answer 689: Since the hundreds digit must be divisible by both 2 and 3 from the given information, it must be a nonzero multiple of 6. The only hundreds digit that satisfies our conditions is therefore 6. The only restriction on the tens digit is that it must be divisible by 2 - the maximum value is therefore 8. Similarly, the maximum value for the units digit is 9. Putting all three digits together, our answer is $\boxed{689.}$

Solution to Problem 3

Answer 30: If we let h be the height of the trapezoid, then we know

$$h \times \frac{7+13}{2} = 40$$

by the formula for the area of a trapezoid. We find that $h = 4$ and seek to use this to find the congruent legs of our isosceles trapezoid.

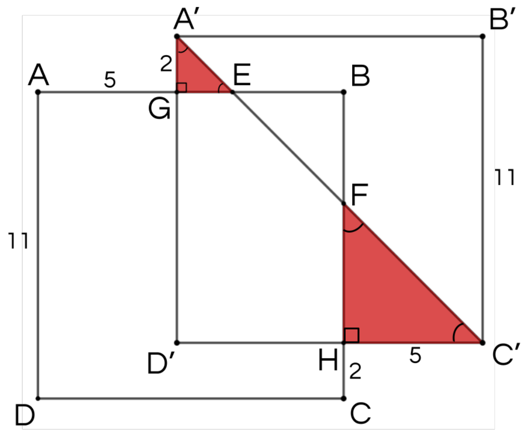
We drop two perpendiculars BE and CF as shown in the diagram, creating a rectangle and two right triangles which are congruent by the HL congruence theorem. Note that $BE = CF = h = 4$. Thus,

$$AE = FD = \frac{13-7}{2} = 3.$$

We use the Pythagorean Theorem to find that the legs of the trapezoid AB and CD are both 5. Thus, our answer is $7+13+5+5 = \boxed{30}$.

Solution to Problem 4

Answer $\frac{29}{2}$:



We note that since we translate A 5 units right and 2 units up, $AG = 5$ and $A'G = 2$. Similarly, $HC = 2$ and $HC' = 5$. Note that in square $A'B'C'D'$, diagonal $A'C'$ bisects $\angle A'$ and $\angle C'$ such that $D'A'C' = D'C'A' = 45°$. Since all side lengths are horizontal or vertical, $FH \perp HC'$ which implies $\angle FHC' = 90°$. In $\triangle FHC'$, we then must have

$$\angle HFC' = 45°.$$

Similarly,

$$\angle GEA' = 45°.$$

Therefore, $A'GE$ and FHC' are 45-45-90 triangles, which means that

$$GE = A'G = 2$$

and

$$FH = HC' = 5.$$

The area of $\triangle A'GE$ can be computed as

$$\frac{A'G \times GE}{2} = \frac{2 \times 2}{2} = 2.$$

The area of $\triangle FHC'$ can be computed as

$$\frac{FH \times HC'}{2} = \frac{5 \times 5}{2} = \frac{25}{2}.$$

So, our final answer is $2 + \frac{25}{2} = \boxed{\frac{29}{2}}$.

Solution to Problem 5

Answer $\frac{119}{4096}$: There are total 4^7 outcomes. The possible points are all permutations (orderings) of $(\pm 3, \pm 4)$, for a total of 8 possibilities, and all permutations of $(\pm 5, 0)$, for a total of 4 possibilities. There are $\binom{7}{3}$ to get to one of the points under the former case. For the latter case, take $(5, 0)$ as an example. We must either go right 6 times and left once, or right 5 times and up and down. Using the first letters of up, down, left, and right, what we want is all permutations (orderings) of RRRRRRL and RRRRRUD. This is

$$\binom{7}{1} + \frac{7!}{5!} = 7 + 42 = 49,$$

so our final probability is $\frac{8\binom{7}{3}+4\cdot 49}{4^7} = \boxed{\frac{119}{4096}}$.