Topology

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1. Prove that if X is finite then the cofinite topology on X =discrete topology.

   Solution: since every subset of X is finite then their complement is also finite, so every subset of $X \in \mathfrak{F} \Rightarrow$ the finite topology is the discrete topology.

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2. Prove that the union of a finite number of closed sets is closed.

   Solution: Let $F_1, F_2, F_3, ..., F_n$ be closed$\Rightarrow$ $F_1^c, F_2^c, F_3^c, ..., F_n^c$ are open and their was in $\mathfrak{F}$, that's mean $F_1^c \cap F_2^c \cap ... \cap F_n^c$ is open in $\mathfrak{F} \Rightarrow$ $(F_1^c \cap F_2^c \cap ... \cap F_n^c)^c = F_1 \cup F_2 \cup ... \cup F_n$ is closed.

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3. Let $\mathfrak{F}$ be a topology on X; $A, B \subset X$, Prove that:

   - $A^\circ \cup B^\circ \subseteq (A \cup B)^\circ$

     Solution: $A \subseteq A \cup B \Rightarrow A^\circ \subseteq (A \cup B)^\circ$

     $B \subseteq A \cup B \Rightarrow B^\circ \subseteq (A \cup B)^\circ$

     $\therefore A^\circ \cup B^\circ \subseteq (A \cup B)^\circ$

   - $A^\circ \cap B^\circ = (A \cap B)^\circ$

     Solution: $A \cap B \subseteq A \Rightarrow (A \cap B)^\circ \subseteq A^\circ$

     $A \cap B \subseteq B \Rightarrow (A \cap B)^\circ \subseteq B^\circ$

     $\therefore (A \cap B)^\circ \subseteq A^\circ \cap B^\circ$

     Let $x \in A^\circ \cap B^\circ \Rightarrow x \in A^\circ$ & B°

     $\Rightarrow \exists u_x$ nbd of x such that $u_x \subseteq A$ & $u_x \subseteq B$

     $\Rightarrow u_x \subseteq A \cap B \Rightarrow x \in (A \cap B)^\circ$

     $\therefore A^\circ \cap B^\circ \subseteq (A \cap B)^\circ$

- Give an example where $(A \cup B)^\circ \neq A^\circ \cup B^\circ$

  Solution: Let $A = \mathbb{Q} \Rightarrow A^\circ = \phi$

  Let $B = \mathbb{Q}^c \Rightarrow B^\circ = \phi \Rightarrow A^\circ \cup B^\circ = \phi$

  But $(A \cup B)^\circ = (\mathbb{R})^\circ = \mathbb{R}$

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4. Prove that $\bar{A} = \cap\{F : \ F \text{ is closed }, A \subseteq F\}$

   Solution:$\bar{A}$ is one of the closed sets containing A so $\cap\{F : \ F \text{ closed}, \ A \subseteq F\} \subseteq \bar{A}$. R.H.S$\subseteq$ L.H.S

   Since $A \subseteq F \Rightarrow \bar{A} \subseteq \bar{F}$ but F is closed $\Rightarrow \bar{F} = F \Rightarrow \bar{A} \subseteq F \Rightarrow \bar{A} \subseteq \cap\{F : F \ closed \ A \subseteq F\}$.

   L.H.S$\subseteq$ R.H.S

   $$so \ \bar{A} = \{F : F \ closed \ A \subseteq F\}$$

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5. Let $\mathfrak{F}_1, \mathfrak{F}_2$ be two topology on X prove that $\mathfrak{F}_1 \cap \mathfrak{F}_2$ is a topology on X.

   Solution:

   - since $X, \phi \in \mathfrak{F}_1$ and $\mathfrak{F}_2 \Rightarrow X, \phi \in \mathfrak{F}_1 \cap \mathfrak{F}_2$.
   - Let $\mathfrak{F} = \mathfrak{F}_1 \cap \mathfrak{F}_2$

     if $A_1, A_2, A_3, ... \in \mathfrak{F} \Rightarrow A_1, A_2, A_3, ... \in \mathfrak{F}_1 \ and \ A_1, A_2, A_3, ... \in \mathfrak{F}_2$

     $\Rightarrow A_1 \cup A_2 \cup A_3 \cup ... \in \mathfrak{F}_1 \ \& \ \mathfrak{F}_2 \Rightarrow A_1 \cup A_2 \cup A_3 \cup ... \in \mathfrak{F}$.
   - if $A_1, A_2, ..., A_n \in \mathfrak{F} \Rightarrow A_1, A_2, ..., A_n \in \mathfrak{F}_1 \ and \ A_1, A_2, ..., A_n \in \mathfrak{F}_2$

     $\Rightarrow A_1 \cap A_2 \cap ... \cap A_n \in \mathfrak{F}_1 \ \& \ \mathfrak{F}_2 \Rightarrow A_1 \cap A_2 \cap ... \cap A_n \in \mathfrak{F}$.