Question 4 - PAC, VC dimension, Bias vs Variance

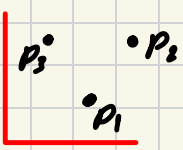
section 1:

A circle (r, C) is defined by its center C and its radius r. Look at the following classifiers family: $H = \{h_{r,c} : r \in \mathbb{R}, C \in \mathbb{R}^2\}$ where $h_{r,c}(x) = 1$ iff x inside the circle (r, C).

Find the VCdim of this class with full proof.

The VC-dim of this class is 3, first let's prove that we can shatter 3 points then we will show that we can't shatter 4 points.

given the following points p_1, p_2, p_3 as seen in the visual representation:



we have the following possibilities:

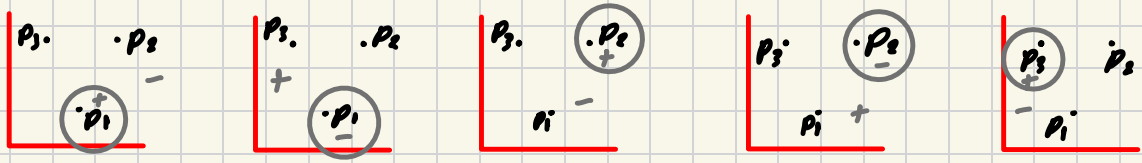
p_1	1	1	1	1	0	0	0	0
p_2	1	1	0	0	0	1	0	1
p_3	1	0	1	0	0	0	1	1

now in order to show how can we shatter the 3 points we will visualize 2 cases that can be generalized to include all the 8 possibilities that we listed before.

Case 1: cover all 3 points

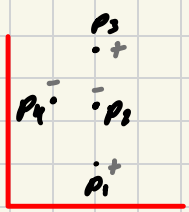
this case covers two possibilities from the grid.

Case 2: cover only 1 point



as we can see in the graphs, we managed to shatter 3 points, and this applies to any 3 points

now we will show a case wher we can't shatter 4 points given the points p_1, p_2, p_3, p_4 and the labels 1,0,1,0 :



now as we can see in the graph, if we draw a circle with radius r that will include p_1 and p_3 then it will have to include p_2, because any 2 point on a circle the line between them will be inside the circle, and since point p_2 is on the line from p_1 to p_3, and p_1, p_2 are inside the circle then p_2 is also inside the circle and therefore the combination $p_1=1, p_2=0, p_3=1, p_4=0$ is impossible to shatter with a circle

hence **VC-dim = 3**

Section 2:

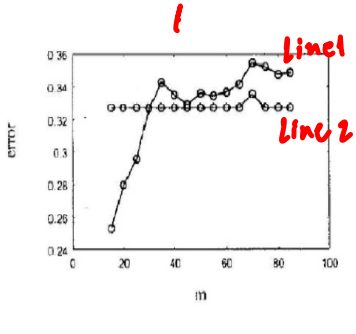
Consider a training set $S=\{(x_1,y_1),\dots,(x_n,y_n)\}$ where $x_i \in \{0,1\}^3$. in other words, each sample has 3 Boolean features $\{x_1,x_2,x_3\}$. You are also given the classification rule: $Y=(x_1 \wedge x_2) \vee (\neg x_1 \wedge \neg x_2)$. we try to learn the function $f: X \to Y$ using a "depth 1 decision trees" A "depth-1 decision tree" is a tree with two leaves, all distance 1 from the root. Analyze this problem and decide the appropriate sample complexity formula. Justify your answer.

We are given a depth-1 decision tree which can only check one feature at a time, now let's analize the function Y. According to its definition, the function returns true only if X_1 and X_2 have the same value and it doesn't consider X_3, Hence the function $f: X \to Y$ cannot be used therefore if we need to determine the complexity we need to use what we learned about PAC to learn a function with error ε and confidence $1-\delta$ we use the $m(\varepsilon,\delta)=O\left(\frac{d+\log(1/\delta)}{\varepsilon}\right)$ formula and now we substitute $d=2$ because the VC-dimension is 2 since we are using only X_1 and X_2 and ignoring X_3. in conclusion the complexity is : $m(\varepsilon,\delta)=O\left(\frac{2+\log(1/\delta)}{\varepsilon}\right)$

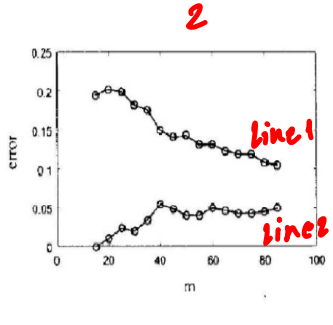
Section 3:

Dana was given a hard classification problem and she decided to use SVM with polynomial kernel with $d=2, 10, 20$. For each degree she tried 15 to 85 training samples, with jumps of 5 (15, 20, ...). The following graphs describe the train and test error for each d separately. However, she forgot which graph belongs to which d, and for each graph, what line is the train and the test. Your task is to match each graph to the correct d and mark which lines are the test and the train.

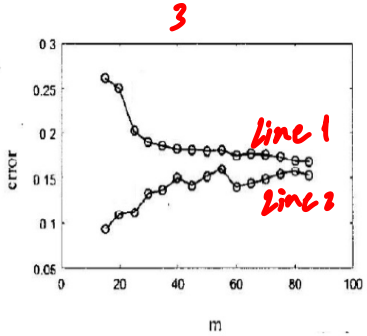
1



2



3



d	graph	train	test
2	1	line 1	line 2
10	3	line 2	line 1
20	2	line 2	line 1