MA585-HW5

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2022/3/11

1 & 2

```r
knitr::include_graphics("hw5-1.jpg")
```

1. Sample mean $\overline{X} = 0.157$ was computed from a sample of size 100 generated from a $MA(1)$ process with mean μ and $\theta = -0.6$, $\sigma^2 = 1$. Construct an approximate 95% CI for μ. Are the data compatible with the hypothesis that $\mu = 0$?

for MA(1): $X_t = \mu + e_t + \theta e_{t-1}$, we have $\bar{X} \sim N(\mu, \frac{\sigma^2 (\sum_{j=0}^{\infty} \psi_j)^2}{n})$

the CI for μ as $\bar{X} \pm 1.96 \frac{\sigma}{\sqrt{n}(1-\theta)} = \bar{X} \pm 1.96 \frac{1}{\sqrt{100}(1+0.6)} = 0.157 \pm 0.1225$

$\because$ $0 \notin$ CI for μ

$\therefore$ reject $H_0: \mu = 0$.

2. Suppose you have a sample of size 100 and obtained $\widehat{\rho}(1) = 0.432$ and $\widehat{\rho}(2) = 0.145$. Assuming that data was generated from a $MA(1)$ process, construct a 95% CI for $\rho(1)$ and $\rho(2)$. Based on these two confidence intervals, is the data consistent with a $MA(1)$ model with $\theta = 0.6$?

According to the Lecture, for MA(q): the CI for $\hat{\rho}(i)$ is $\pm 1.96 \frac{\sqrt{W_{ii}}}{\sqrt{n}}$

In practice, we often use $\pm \frac{1.96}{\sqrt{n}}$ to simplify

then. $\hat{\rho}(1)$: CI $= \pm 1.96 = (-0.196, 0.196)$

$\hat{\rho}(1) = 0.432 \notin$ CI $= (-0.196, 0.196)$ $\Rightarrow$ reject $H_0: \rho(1) = 0$

Similarly $\hat{\rho}(2) = 0.145 \in$ CI $= (-0.196, 0.196)$ $\Rightarrow$ can not reject $H_0: \rho(2) = 0$

So, ACF cuts-off after lag 1

for MA(1) with $\theta = 0.6$: $X_t = \mu + e_t + 0.6 e_{t-1}$

we have $\rho(1) = \frac{\theta}{1+\theta^2} = \frac{0.6}{1.36} \approx 0.44 \notin$ CI

So the data consistent with a MA(1) model with $\theta = 0.6$.

$n = 100$. $\hat{\rho}(1) = 0.432$, $\hat{\rho}(2) = 0.145$ MA(1) $W_{ii} =$

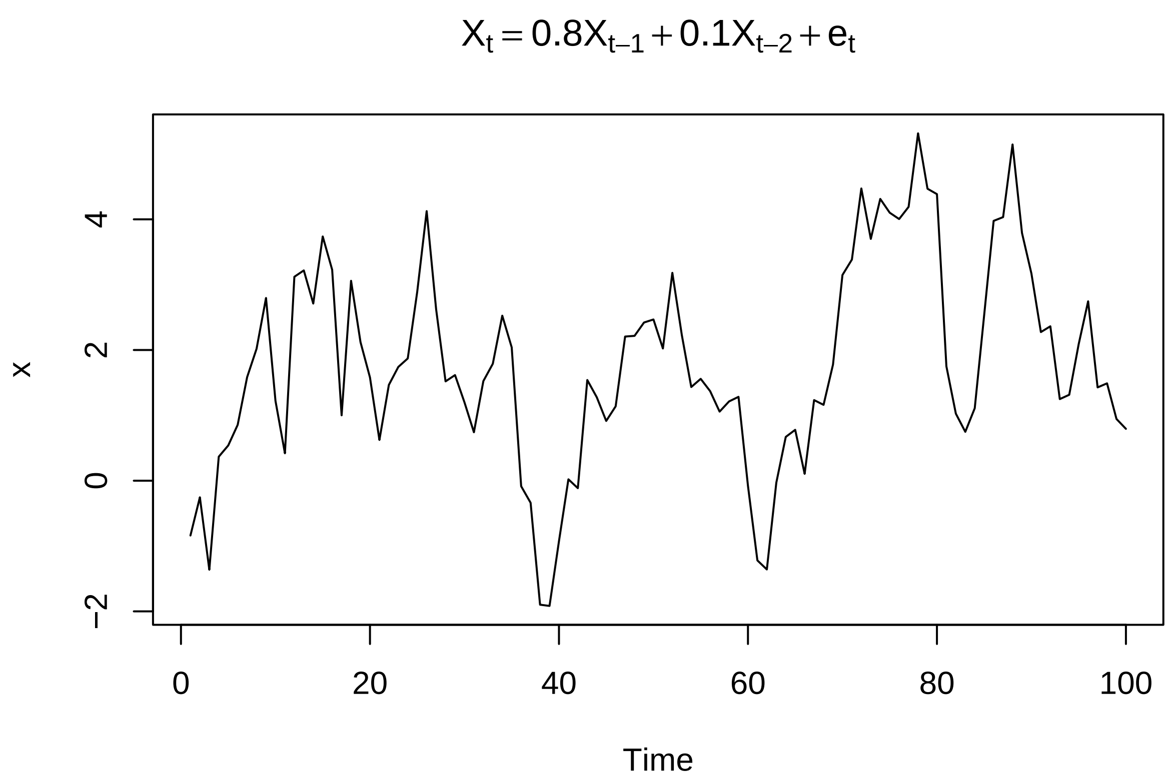
in practice, we also use $\pm \frac{1.96}{\sqrt{n}}$ (for this problem, this interval is enough)

3

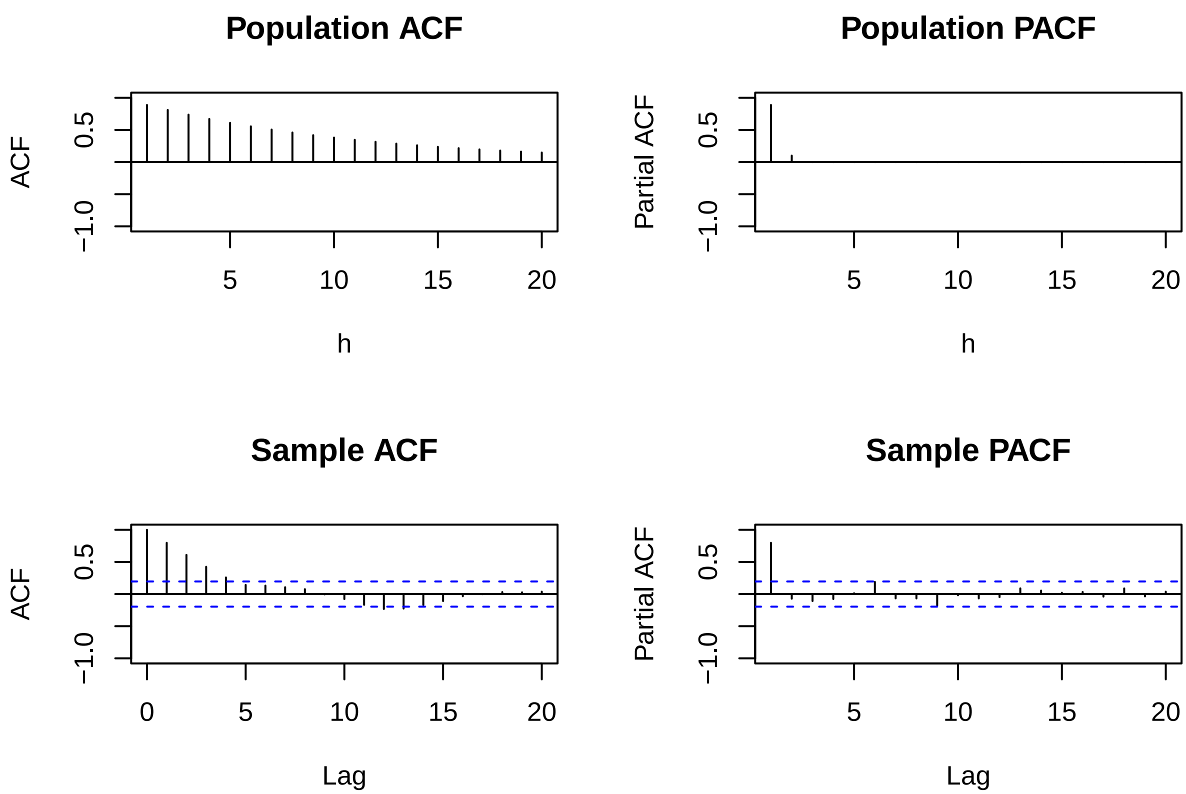
(i) AR(2) : $X_t + 0.8X_{t-1} + 0.1X_{t-2} = e_t$

For stationary, we need the characteristic roots lie outside the unit circle.

```
# polyroot(c(1,0.8,0.1)) # check
x=arima.sim(n=100, list(ar=c(0.8,0.1)))
plot.ts(x)
title(main=expression(X[t]==0.8*X[t-1]+0.1*X[t-2]+e[t]))
```




```
par(mfrow=c(2,2))
y = ARMAacf(ar=c(0.8,0.1),lag.max = 20); y = y[2:21]
plot(y, x = 1:20, type = "h", ylim = c(-1,1), xlab = "h",
ylab = "ACF", main = "Population ACF")
abline(h = 0)
y = ARMAacf(ar=c(0.8,0.1),lag.max = 20,pacf=T)
plot(y, x = 1:20, type = "h", ylim = c(-1,1), xlab = "h",
ylab = "Partial ACF", main = "Population PACF")
abline(h = 0)
acf(x,main="Sample ACF", ylim = c(-1,1))
pacf(x,main="Sample PACF", ylim = c(-1,1))
```



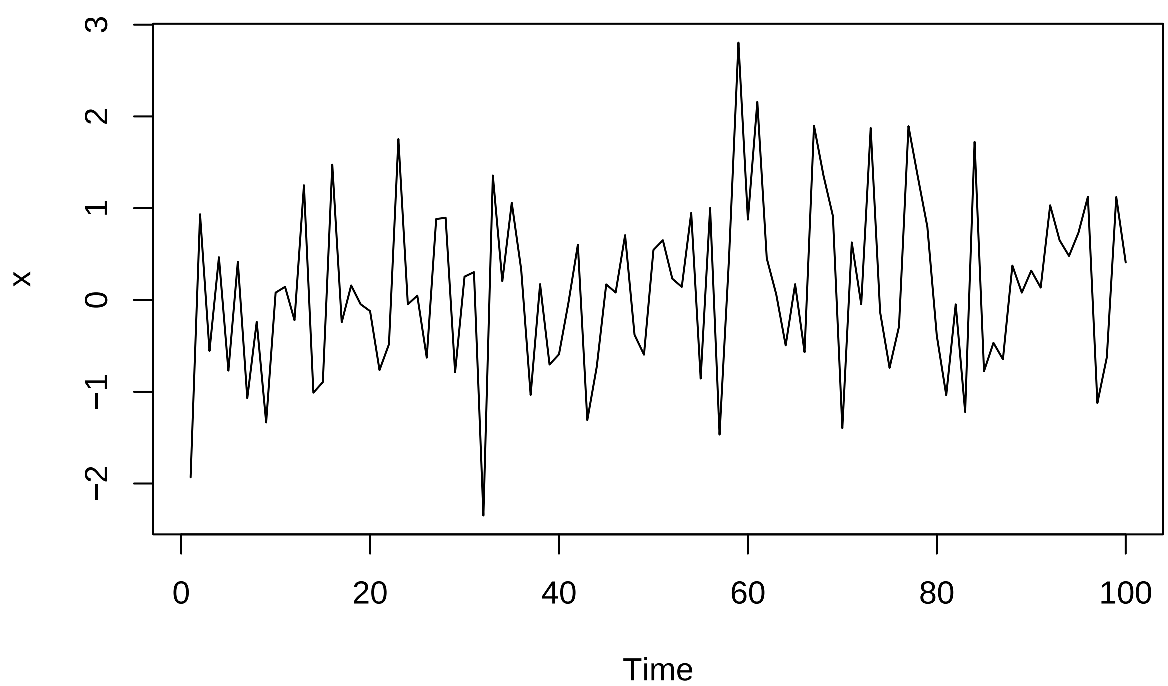
(ii) ARMA(1,1) : $X_t - 0.5X_{t-1} = e_t - 0.5e_{t-1}$

```
polyroot(c(1,-0.5))
```

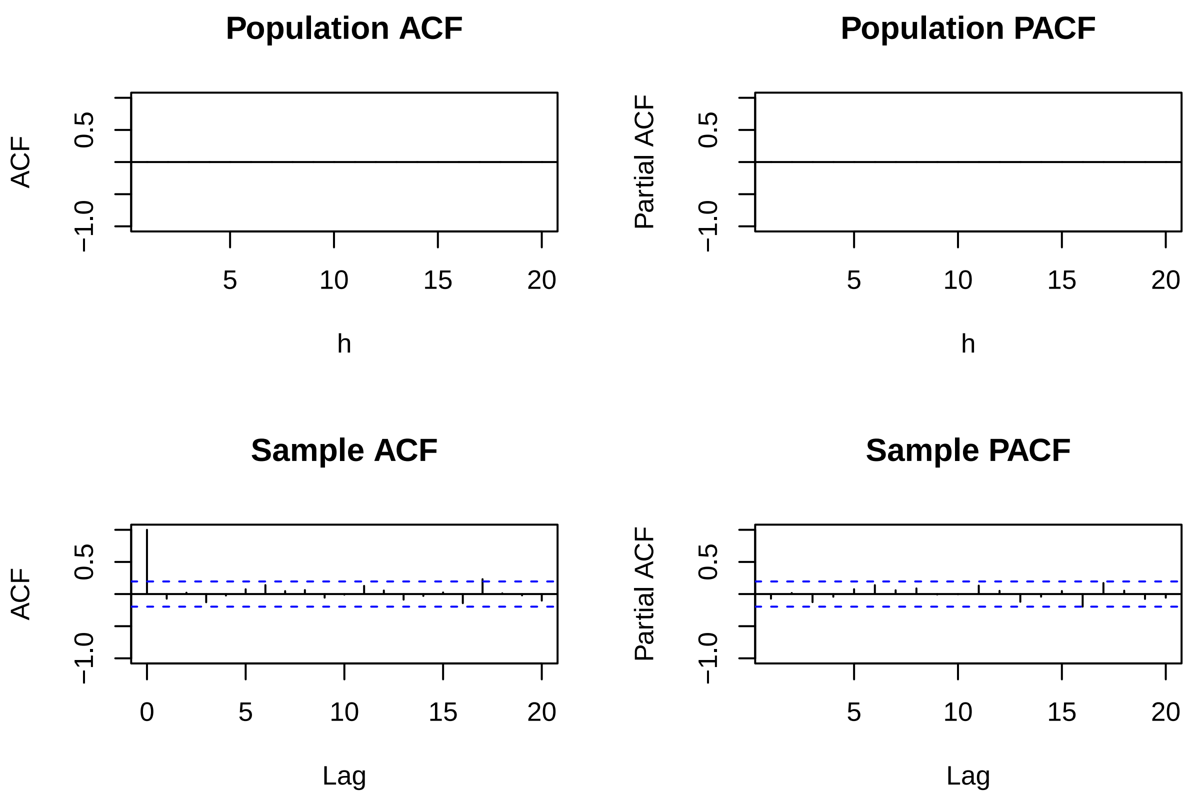
```
## [1] 2+0i
```

```
x=arima.sim(n=100, list(ar=c(0.5), ma=c(-0.5)))
plot.ts(x)
title(main=expression(X[t]-0.5*x[t-1]==e[t]-0.5*e[t-1]))
```

$X_t - 0.5x_{t-1} = e_t - 0.5e_{t-1}$



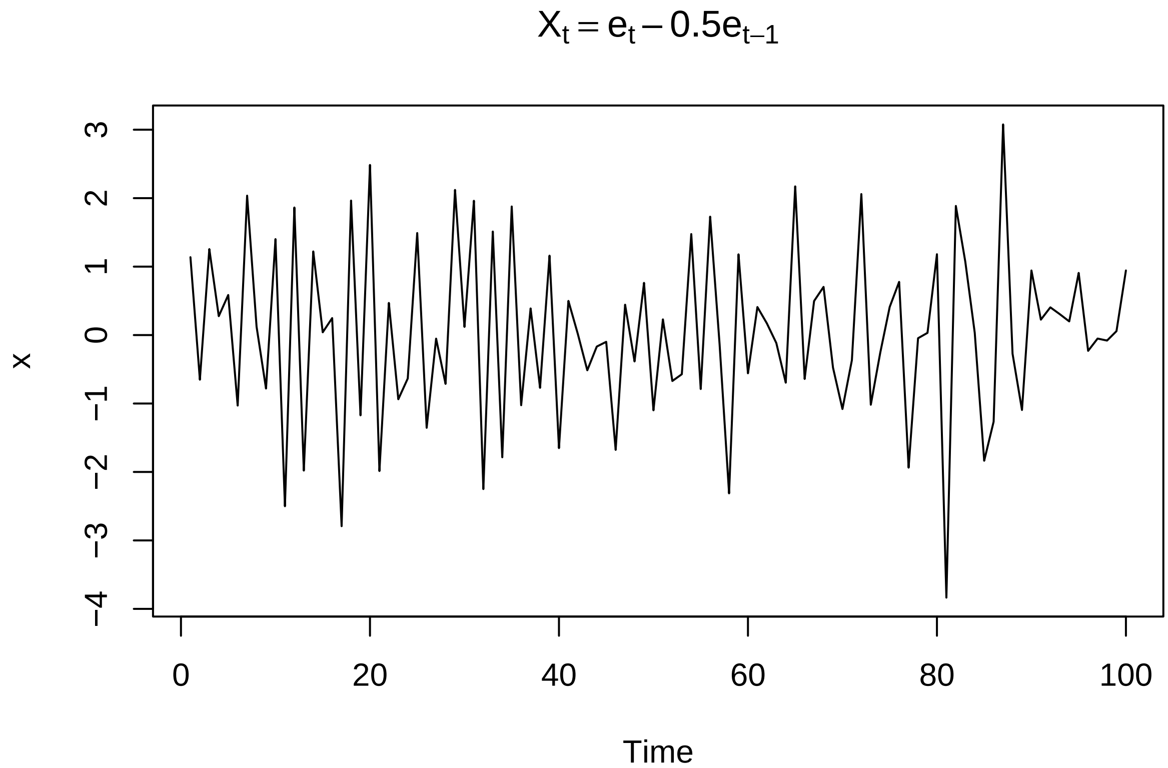

```
par(mfrow=c(2,2))
y = ARMAacf(ar=c(0.5), ma=c(-0.5),lag.max = 20); y = y[2:21]
plot(y, x = 1:20, type = "h", ylim = c(-1,1), xlab = "h",
ylab = "ACF", main = "Population ACF")
abline(h = 0)
y = ARMAacf(ar=c(0.5), ma=c(-0.5),lag.max = 20,pacf=T)
plot(y, x = 1:20, type = "h", ylim = c(-1,1), xlab = "h",
ylab = "Partial ACF", main = "Population PACF")
abline(h = 0)

acf(x,main="Sample ACF", ylim = c(-1,1))
pacf(x,main="Sample PACF", ylim = c(-1,1))
```



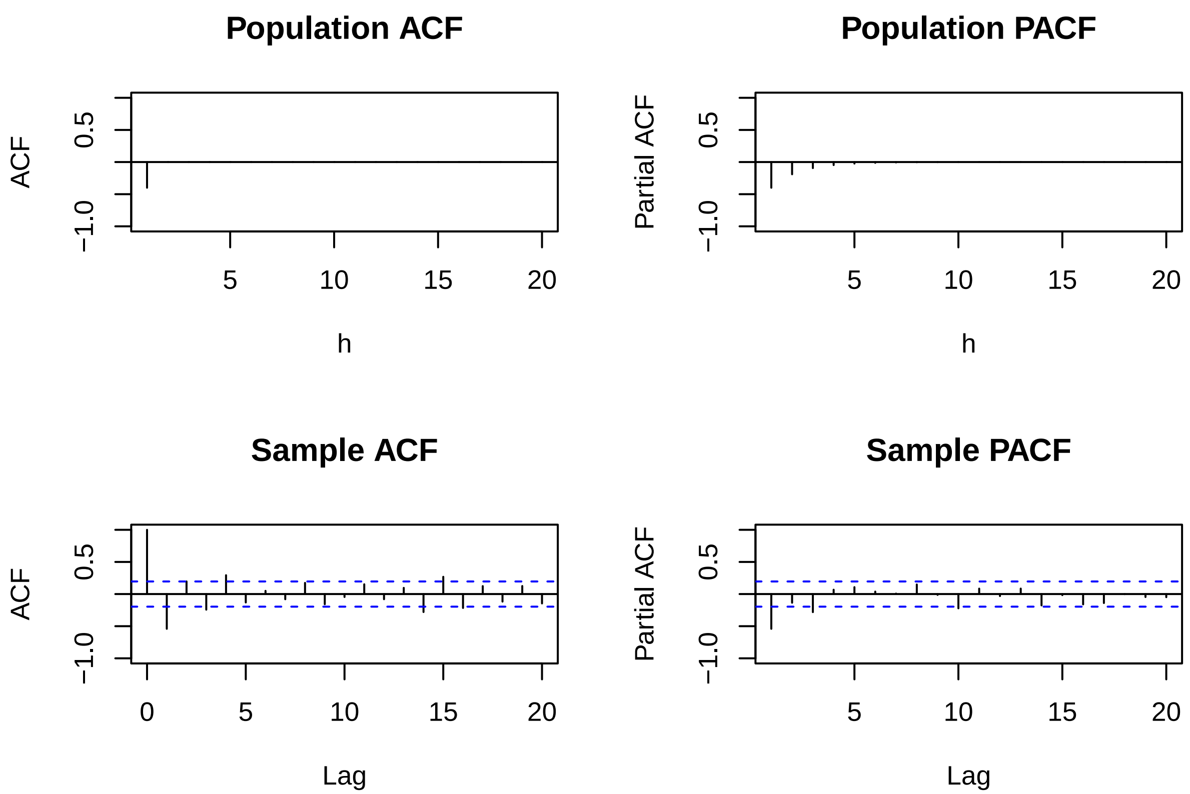
(iii) $\text{MA}(1): X_t = e_t - 0.5e_{t-1}$

For stationary, we need $|\theta| < 1$.

```
x=arima.sim(n=100, list(ma=c(-0.5)))
plot.ts(x)
title(main=expression(X[t]==e[t]-0.5*e[t-1]))
```

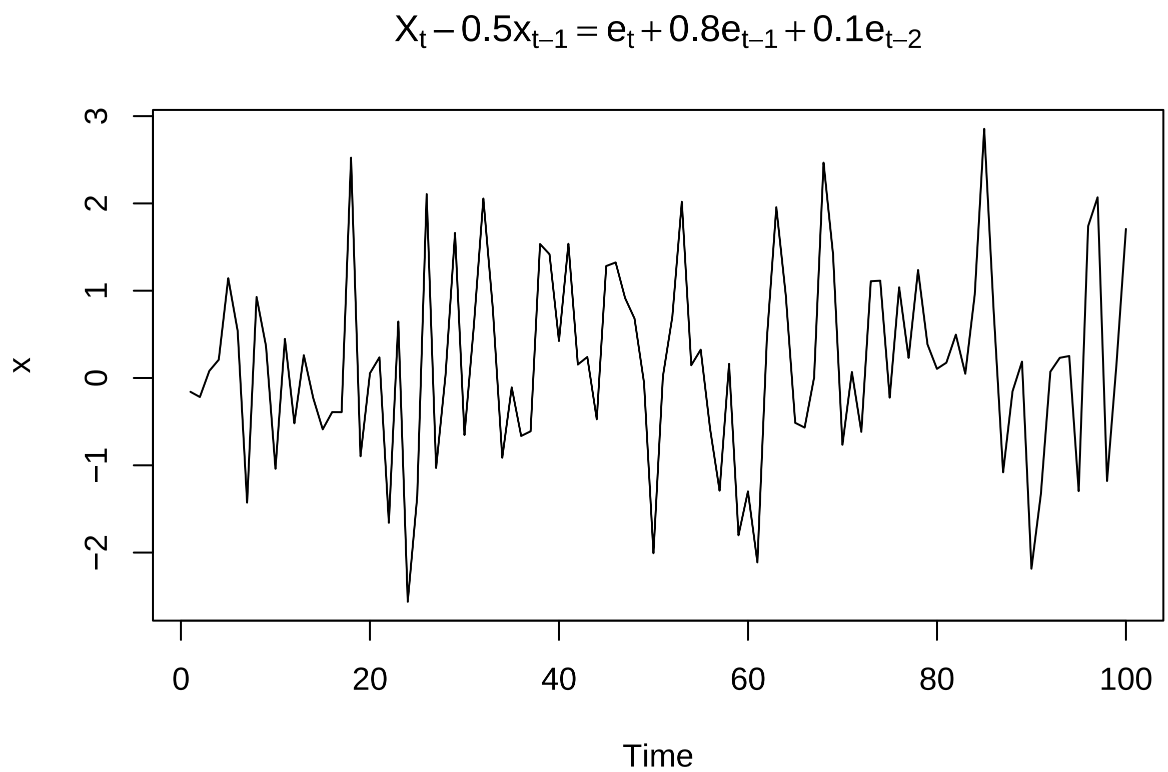



```
par(mfrow=c(2,2))
y = ARMAacf(ma=c(-0.5),lag.max = 20); y = y[2:21]
plot(y, x = 1:20, type = "h", ylim = c(-1,1), xlab = "h",
ylab = "ACF", main = "Population ACF")
abline(h = 0)
y = ARMAacf(ma=c(-0.5),lag.max = 20,pacf=T)
plot(y, x = 1:20, type = "h", ylim = c(-1,1), xlab = "h",
ylab = "Partial ACF", main = "Population PACF")
abline(h = 0)

acf(x,main="Sample ACF", ylim = c(-1,1))
pacf(x,main="Sample PACF", ylim = c(-1,1))
```

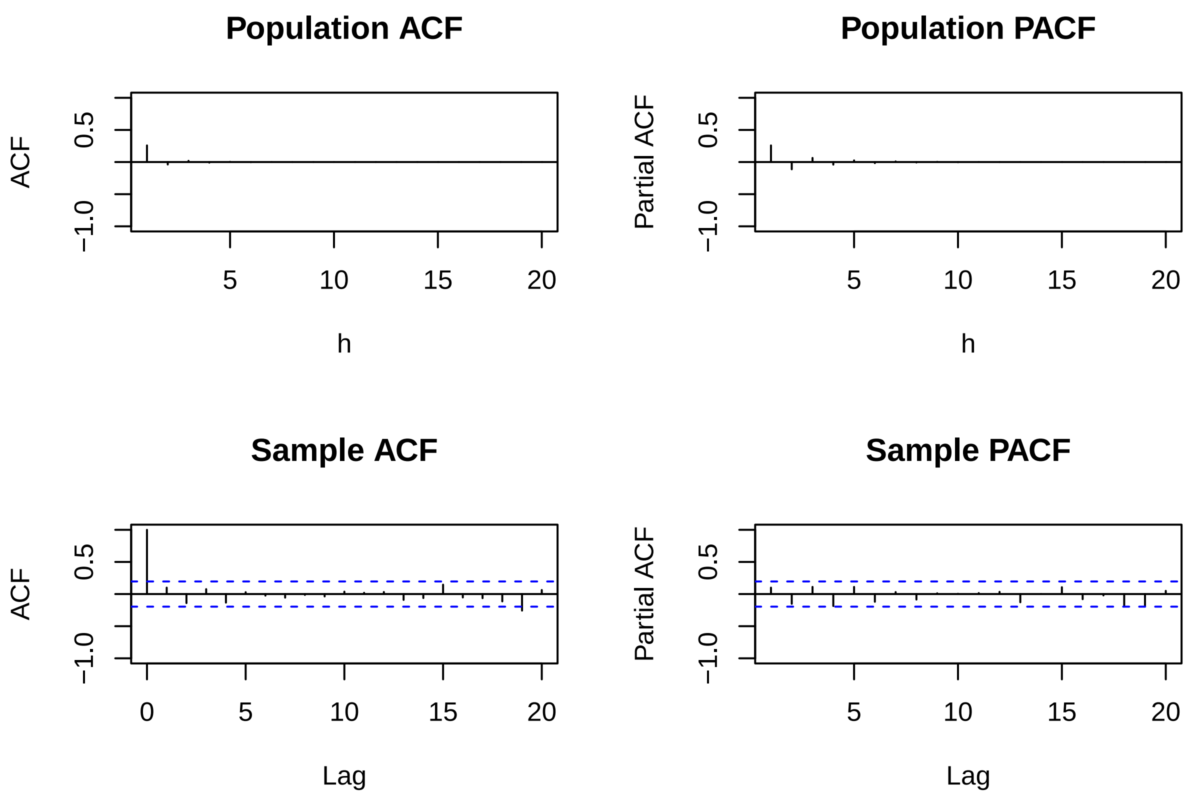


(iv) ARMA(1,2) : $X_t + 0.5X_{t-1} = e_t + 0.8e_{t-1} + 0.1e_{t-2}$

```
# polyroot(c(1,-0.5))
# polyroot(c(1,0.8,0.1))

x=arima.sim(n=100, list(ar=c(-0.5), ma=c(0.8,0.1)))
plot.ts(x)
title(main=expression(X[t]-0.5*x[t-1]==e[t]+0.8*e[t-1]+0.1*e[t-2]))
```




```
par(mfrow=c(2,2))
y = ARMAacf(ar=c(-0.5), ma=c(0.8,0.1),lag.max = 20); y = y[2:21]
plot(y, x = 1:20, type = "h", ylim = c(-1,1), xlab = "h",
ylab = "ACF", main = "Population ACF")
abline(h = 0)
y = ARMAacf(ar=c(-0.5), ma=c(0.8,0.1),lag.max = 20,pacf=T)
plot(y, x = 1:20, type = "h", ylim = c(-1,1), xlab = "h",
ylab = "Partial ACF", main = "Population PACF")
abline(h = 0)

acf(x,main="Sample ACF", ylim = c(-1,1))
pacf(x,main="Sample PACF", ylim = c(-1,1))
```

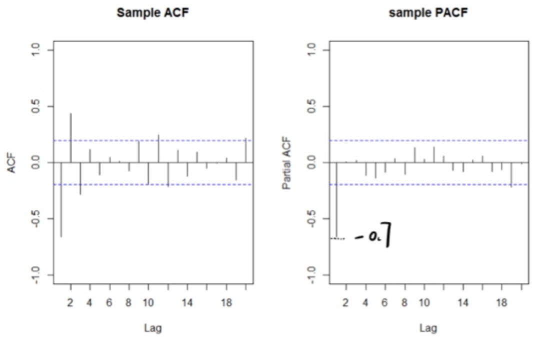


4

```
knitr::include_graphics("hw5-4.jpg")
```

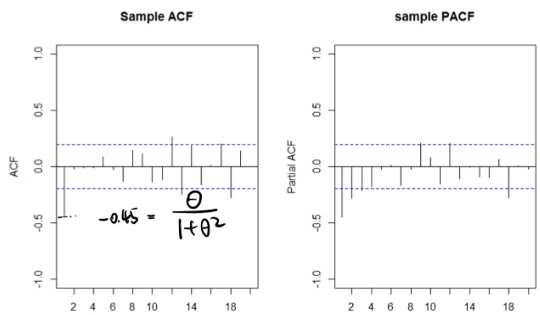
answer.

a.



Since ACF decay to 0, and PACF cuts off after lag 1, the process maybe AR(1). with $\phi = -0.7$, which is $X_t = -0.7X_{t-1} + e_t$

b.



$-0.45 = \frac{\theta}{1+\theta^2}$

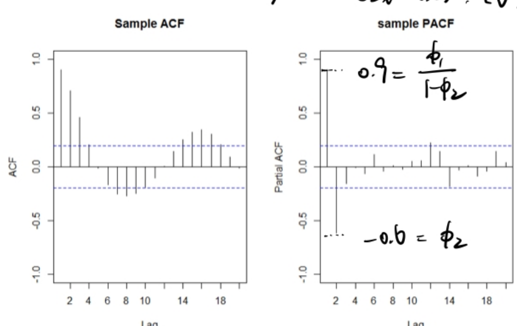
Since ACF is significant up to lag 1 and PACF decays to 0, the process maybe MA(1). with $\theta = -0.63$, which is $X_t = e_t - 0.63e_{t-1}$

$0.45\theta^2 + \theta + 0.45 = 0$

$\theta = \frac{-10 \pm \sqrt{19}}{9}$, $\theta_1 \approx 1.60$, will make the characteristic roots lie inside unit interval

$\theta_2 \approx -0.63$. (√)

c.



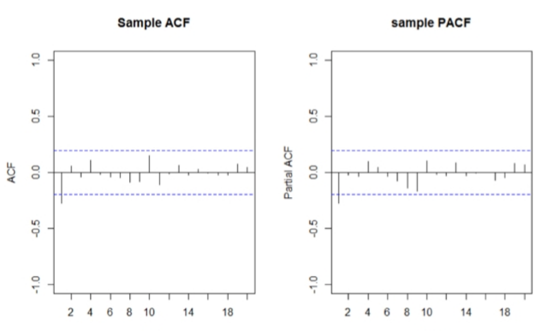
$0.9 = \frac{\phi_1}{1-\phi_2}$

$-0.6 = \phi_2$

Since ACF decays to 0 in sine wave, PACF cuts of after lag 2, the process maybe AR(2), with $\phi_1 = 1.44$, $\phi_2 = -0.6$, which is $X_t = 1.44X_{t-1} - 0.6X_{t-2} + e_t$

$\Rightarrow \phi_1 = 0.9(1-\phi_2) = 0.9 \times 1.6 = 1.44$

d.

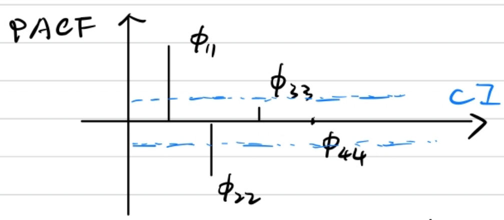


Since ACF and PACF both cuts off after lag 1, the process maybe AR(1), MA(1), or ARMA(1,1)

5 & 6

```r
knitr::include_graphics("hw5-5.jpg")
```

6. A stationary time series of length 121 produced sample partial autocorrelation of $\phi_{11} = 0.8$, $\phi_{22} = -0.6$, $\phi_{33} = 0.08$, and $\phi_{44} = 0.00$. Based on this information alone, what model would we tentatively specify for the series?



CI: $\pm \frac{1.96}{\sqrt{n}} = \pm \frac{1.96}{\sqrt{121}}$

$\approx (-0.18,\ 0.18)$

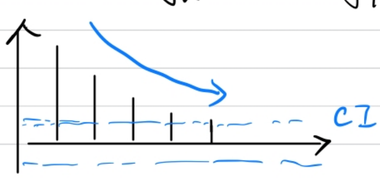
Based on the pattern of plot we assume the model is AR(2)

$\Rightarrow$ for AR(2). $\phi_{11} = \frac{\phi_1}{1-\phi_2} = 0.8$; $\phi_{22} = \phi_2 = -0.6$ $\longrightarrow$ $\phi_1 = 1.28$; $\phi_2 = -0.6$

So the model would be AR(2): $X_t = 1.28 X_{t-1} - 0.6 X_{t-2} + e_t$

5. For a series of length 169, we find that $\widehat{\rho}(1) = 0.41, \widehat{\rho}(2) = 0.32, \widehat{\rho}(3) = 0.26, \widehat{\rho}(4) = 0.21$, and $\widehat{\rho}(5) = 0.16$. What ARMA model fits this pattern of autocorrelations? Justify your answer.

CI: $\pm \frac{1.96}{\sqrt{n}} = \pm \frac{1.96}{\sqrt{169}} = (-0.15,\ 0.15)$



Based on the pattern of plot, we assume the model is AR or ARMA. model.

(1) If the model is AR.

$\rho(1) \approx 0.41$, $\rho(2) \approx 0.41^2 \approx 0.17$, $\rho(3) \approx 0.41^3 \approx 0.07$
$\rho(4) \approx 0.41^4 \approx 0.01$ (not similar as given)

or check the ratio $\frac{\rho(h+1)}{\rho(h)} = \phi$

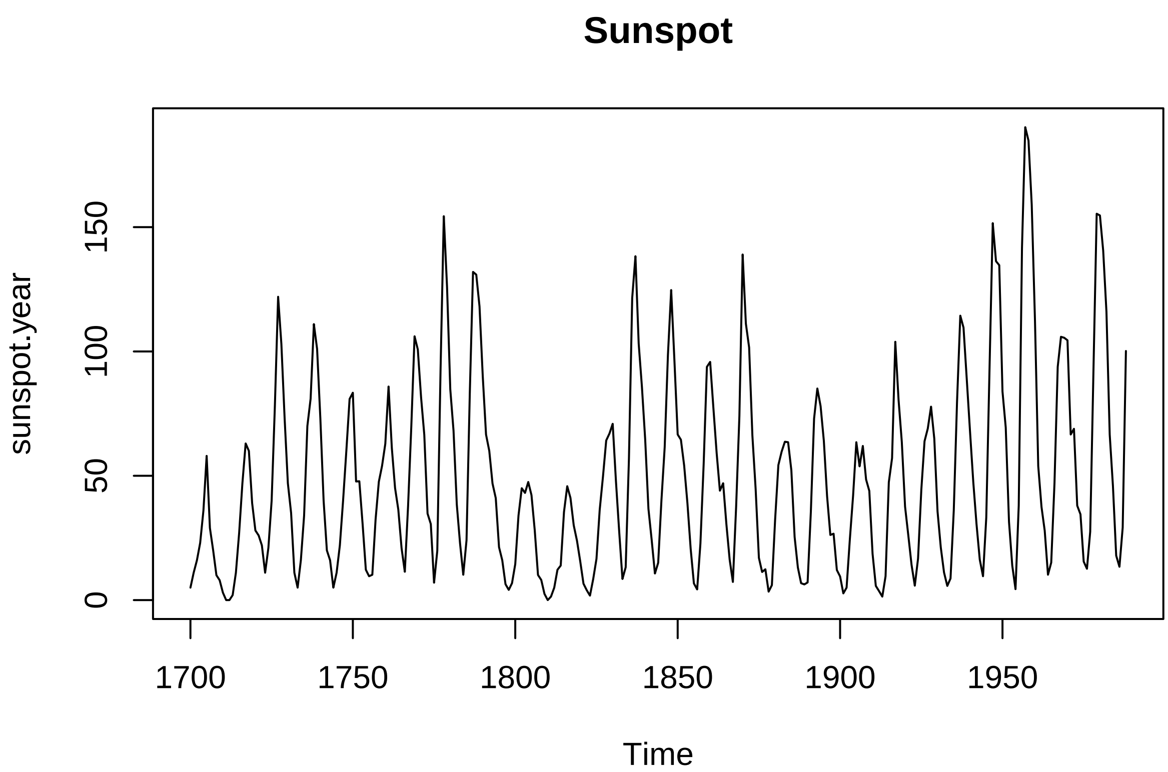
$\hat{\rho}(2)/\hat{\rho}(1) \approx 0.78$. $\hat{\rho}(3)/\hat{\rho}(2) = 0.81$. $\hat{\rho}(4)/\hat{\rho}(3) = 0.81$, $\hat{\rho}(5)/\hat{\rho}(4) = 0.76$
the ratio is not close to $\hat{\rho}(1) = 0.41$
So AR(1) may not be a candidate model

(2) If the model is ARMA
$\rho(2) = \phi\rho(1)$, $\rho(3) = \phi^2\rho(1)$, $\rho(4) = \phi^3\rho(1)$, ...
as we calculated before, ϕ is around 0.8
So ARMA (1,1) is a good candidate model.

7

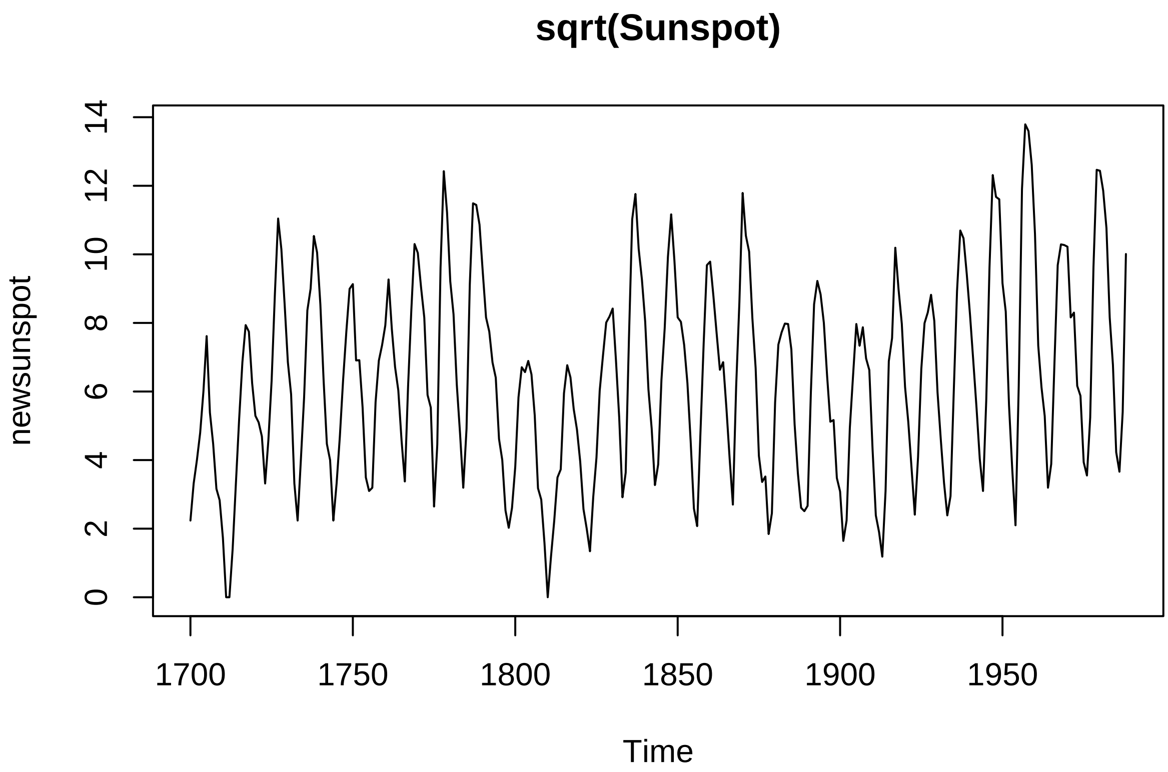
(i) According to the time series plot, we can see a clear seasonal trend.

```
plot.ts(sunspot.year, main = "Sunspot")
```



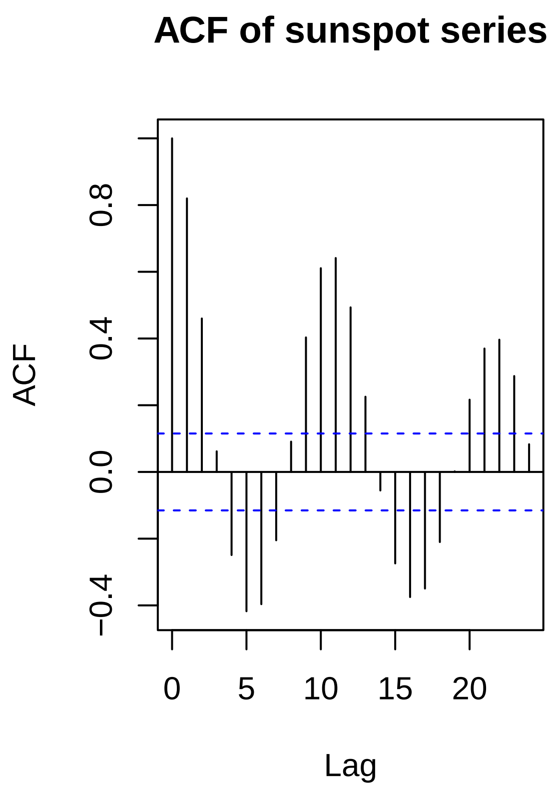
(ii) The square-root transformation is necessary, because it stabilize the variance of the change in time.

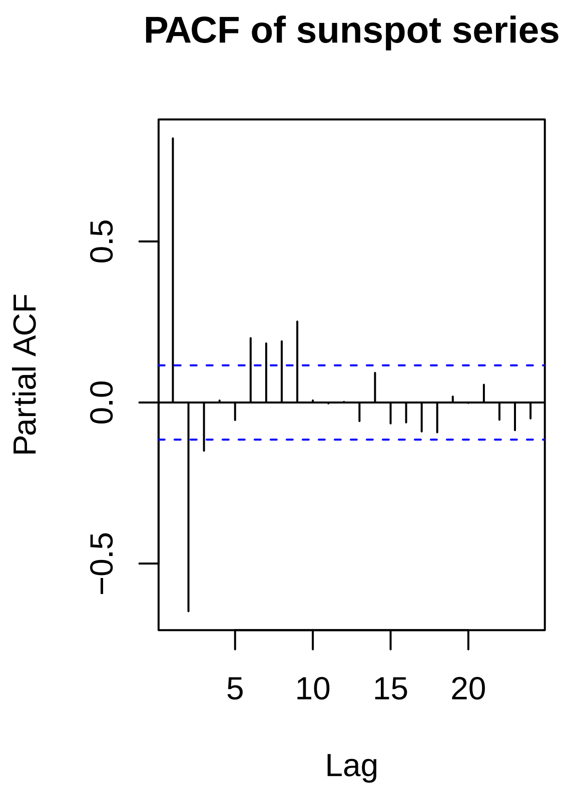
```
newsunspot = sqrt(sunspot.year)
plot.ts(newsunspot, main = "sqrt(Sunspot)")
```



(ii) Plot ACF and PACF of the transformed data. Based on these plots, propose a plausible model and justify your answer

```
par(mfrow=c(1,2))
acf(newsunspot, main = "ACF of sunspot series") # decay in sine wave9
pacf(newsunspot, main = "PACF of sunspot series") # cuts off after lag 2
```






```
# Assume the process is an AR(2) model,calculate phi_1 and phi_2 based on phi_11 and phi_22,
(phi2 <- -0.6) # phi_22 = -0.6
```

```
## [1] -0.6
```

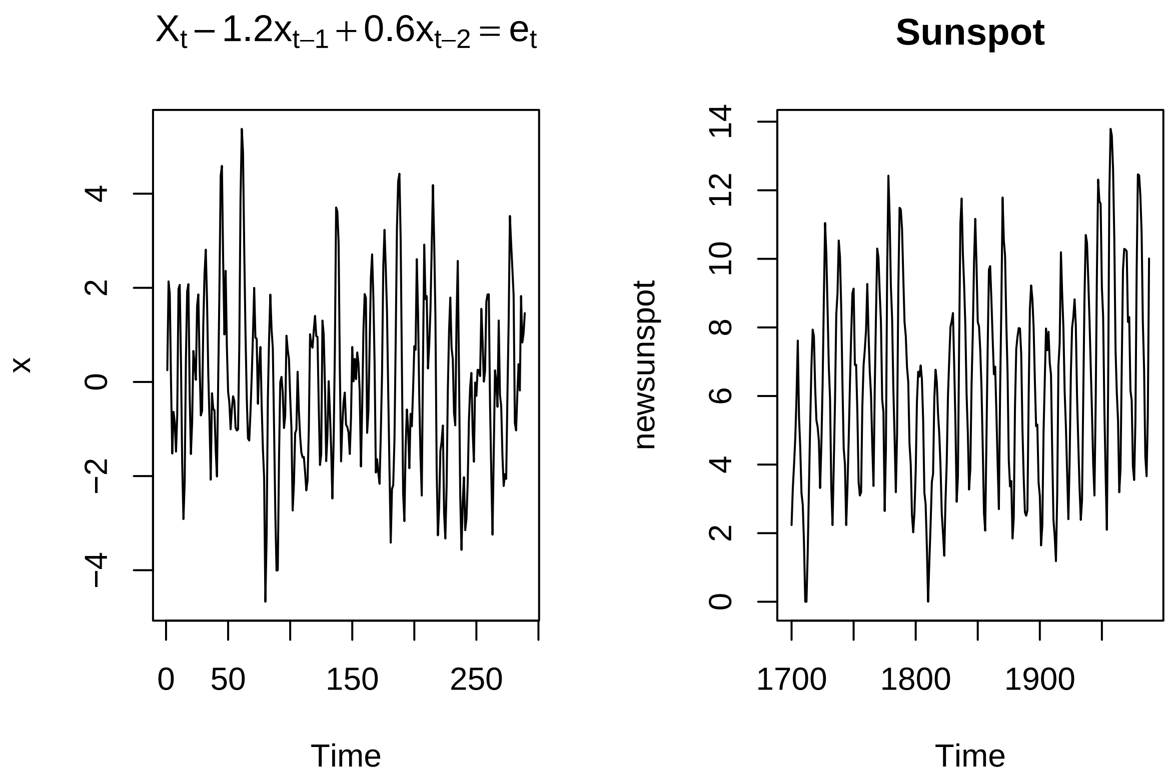
```
(phi1 <- 0.75*(1-phi2)) # phi_11 = 0.75
```

```
## [1] 1.2
```

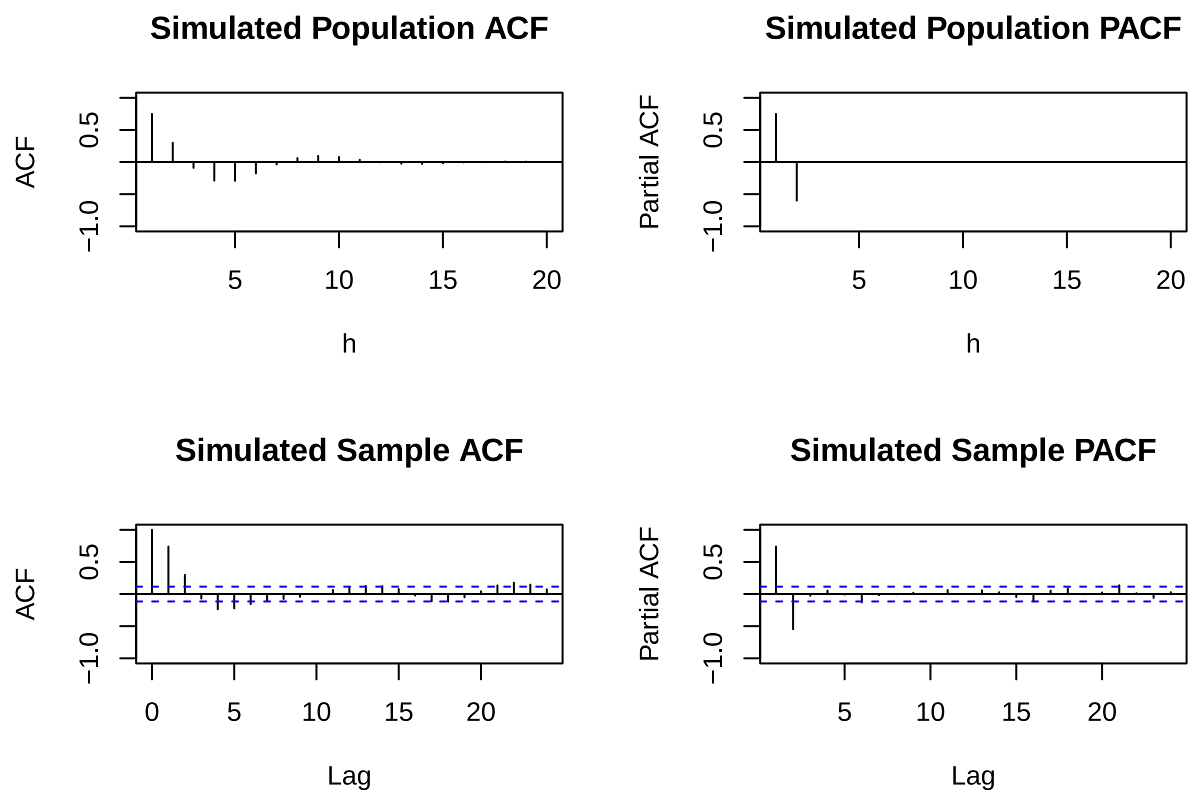
```
# check stationary: causal and invertible
polyroot(c(1,-1.2,0.6))
```

```
## [1] 1+0.816497i 1-0.816497i
```

```
# plot the ACF and PACF
x=arima.sim(n=length(newsunspot), list(ar=c(1.2,-0.6)))
plot.ts(x,main=expression(X[t]-1.2*x[t-1]+0.6*x[t-2]==e[t]))
plot.ts(newsunspot, main = "Sunspot")
```




```r
par(mfrow=c(2,2))
y = ARMAacf(ar=c(1.2,-0.6),lag.max = 20)
y = y[2:21]
plot(y, x = 1:20, type = "h", ylim = c(-1,1), xlab = "h",
ylab = "ACF", main = "Simulated Population ACF")
abline(h = 0)
y = ARMAacf(ar=c(1.2,-0.6),lag.max = 20,pacf=T)
plot(y, x = 1:20, type = "h", ylim = c(-1,1), xlab = "h",
ylab = "Partial ACF", main = "Simulated Population PACF")
abline(h = 0)
acf(x,main="Simulated Sample ACF", ylim = c(-1,1))
pacf(x,main="Simulated Sample PACF", ylim = c(-1,1))

# Check with EACF
TSA::eacf(newsunspot)
```




```
## AR/MA
##   0 1 2 3 4 5 6 7 8 9 10 11 12 13
## 0 x x o x x x x o x x x  x  x  o
## 1 x x o x x x x o x x x  x  x  o
## 2 x o o x x o x o o o o  x  o  o
## 3 x o o x o o x o o o o  x  o  o
## 4 x o o x o o o o o o o  x  o  o
## 5 x o x x x o o x o o o  x  o  o
## 6 x x x x x o o x o o o  x  o  o
## 7 x x o x x o o o o o o  o  o  o
```

At first, we assume the process is an AR(2) model based on the pattern of ACF and PACF plots, but the simulation of calculated parameters is not similar enough with the raw data, so we check the EACF table, and notice that it may also be an AR(2,1) model.