Distributed Distortion-Aware Beamforming Designs for Cell-Free mMIMO Systems

Mengzhen Liu, Ming Li, Rang Liu, and Qian Liu

Convergence Analysis of Ring-topology-based Algorithm

To facilitate analysis, we start with constructing a comprehensive objective function at the b-th BS as:

$$\begin{aligned}&\mathcal{F}(\mathbf{w}_b, \mathbf{R}_b, \boldsymbol{\mu}_b, \boldsymbol{\zeta}_b; \overbrace{\widehat{\mathbf{Q}}_b, \widehat{\mathbf{p}}_b}^{\substack{\text{Known} \\ \text{local information}}}) \\ &\quad = -\sum_{k=1}^{K}\left(\log_2(1+\mu_{b,k}) - \mu_{b,k} - |\zeta_{b,k}|^2\sigma_k^2\right) - \delta + \sum_{l=1}^{B}\rho\|\mathbf{R}_l - \mathbf{w}_l\mathbf{w}_l^H\|_F^2,\end{aligned} \tag{1}$$

which is the combination of objective functions from subproblems for optimizing variables $\boldsymbol{\mu}_b, \boldsymbol{\zeta}_b$ in (15), $\mathbf{w}_b, \mathbf{R}_b$ in (40a). Then, the integrated optimization problem can be formulated as

$$\min_{\mathbf{w}_b, \mathbf{R}_b, \boldsymbol{\mu}_b, \boldsymbol{\zeta}_b} \quad \mathcal{F}(\mathbf{w}_b, \mathbf{R}_b, \boldsymbol{\mu}_b, \boldsymbol{\zeta}_b; \widehat{\mathbf{Q}}_b, \widehat{\mathbf{p}}_b) \tag{2a}$$

$$\text{s.t.} \quad \|\mathbf{w}_b\|_2^2 \le P_{\mathrm{t}}. \tag{2b}$$

In the following, we proceed to demonstrate that the value of the comprehensive objective function $\mathcal{F}(\mathbf{w}_b, \mathbf{R}_b, \boldsymbol{\mu}_b, \boldsymbol{\zeta}_b; \widehat{\mathbf{Q}}_b, \widehat{\mathbf{p}}_b)$ is progressively reduced with each iteration. According to the iteration process outlined in Algorithm 1, the following inequalities can be constructed sequentially as:

$$\begin{aligned}\mathcal{F}(\mathbf{w}_b^{(t)}, \mathbf{R}_b^{(t)}, \boldsymbol{\mu}_b^{(t)}, \boldsymbol{\zeta}_b^{(t)}; \widehat{\mathbf{Q}}_b^{(t)}, \widehat{\mathbf{p}}_b^{(t)}) &\overset{\text{(a)}}{\ge} \mathcal{F}(\mathbf{w}_b^{(t+1)}, \mathbf{R}_b^{(t)}, \boldsymbol{\mu}_b^{(t)}, \boldsymbol{\zeta}_b^{(t)}; \widehat{\mathbf{Q}}_b^{(t)}, \widehat{\mathbf{p}}_b^{(t)}) \\ &\overset{\text{(b)}}{\ge} \mathcal{F}(\mathbf{w}_b^{(t+1)}, \mathbf{R}_b^{(t+1)}, \boldsymbol{\mu}_b^{(t)}, \boldsymbol{\zeta}_b^{(t)}; \widehat{\mathbf{Q}}_b^{(t)}, \widehat{\mathbf{p}}_b^{(t)}) \\ &\overset{\text{(c)}}{\ge} \mathcal{F}(\mathbf{w}_b^{(t+1)}, \mathbf{R}_b^{(t+1)}, \boldsymbol{\mu}_b^{(t+1)}, \boldsymbol{\zeta}_b^{(t+1)}; \widehat{\mathbf{Q}}_b^{(t)}, \widehat{\mathbf{p}}_b^{(t)}),\end{aligned} \tag{3}$$

where (a) is obtained after solving problem (43), (b) is obtained after solving problem (48), and (c) can be derived after optimizing $\boldsymbol{\mu}_b$ and $\boldsymbol{\zeta}_b$ by (53) and (54), respectively. The non-increasing property is maintained at each step, since we can guarantee an optimal solution for each variable when solving the corresponding sub-problem. As a result, the objective function value decreases after the t-th iteration.

Using the ring topology, after the t-th iteration in the b-th BS, the variables are propagated to the $(b+1)$-th BS and thus the achieved objective value is retained as the starting objective value for the $(t+1)$-th iteration in the $(b+1)$-th BS (i.e., $(b+1):=(t+1) \bmod B+1$), which yields

$$\mathcal{F}(\mathbf{W}_b^{(t+1)}, \mathbf{R}_b^{(t+1)}, \boldsymbol{\mu}_b^{(t+1)}, \boldsymbol{\zeta}_b^{(t+1)}; \widehat{\mathbf{Q}}_b^{(t)}, \widehat{\mathbf{p}}_b^{(t)}) = \mathcal{F}(\mathbf{W}_{b+1}^{(t+1)}, \mathbf{R}_{b+1}^{(t+1)}, \boldsymbol{\mu}_{b+1}^{(t+1)}, \boldsymbol{\zeta}_{b+1}^{(t+1)}; \widehat{\mathbf{Q}}_{b+1}^{(t+1)}, \widehat{\mathbf{p}}_{b+1}^{(t+1)}). \tag{4}$$

Hence, we can conclude that the objective function is monotonically non-increasing after each iteration, i.e.

$$\mathcal{F}(\mathbf{W}_b^{(t)}, \mathbf{R}_b^{(t)}, \boldsymbol{\mu}_b^{(t)}, \boldsymbol{\zeta}_b^{(t)}; \widehat{\mathbf{Q}}_b^{(t)}, \widehat{\mathbf{p}}_b^{(t)}) \geq \mathcal{F}(\mathbf{W}_{b+1}^{(t+1)}, \mathbf{R}_{b+1}^{(t+1)}, \boldsymbol{\mu}_{b+1}^{(t+1)}, \boldsymbol{\zeta}_{b+1}^{(t+1)}; \widehat{\mathbf{Q}}_{b+1}^{(t+1)}, \widehat{\mathbf{p}}_{b+1}^{(t+1)}). \tag{5}$$

Moreover, the objective value is obviously lower bounded due to the power constraint, which ensures the convergence for the ring-topology-based fully-distributed beamforming design algorithm.

Convergence Analysis of Star-topology-based Algorithm

Combining the optimization of global variables $\{\mathbf{Q}_{\mathrm{C},b}\}_{b=1}^{B}$ in (60), local variables $\mathbf{W}_b, \mathbf{R}_b$ in (66), $\boldsymbol{\lambda}_b$ in (67) and $\boldsymbol{\mu}, \boldsymbol{\zeta}$ in (15), a unified objective function provided for both global and local optimization can be written as

$$\begin{aligned}
\mathcal{G}(\{\mathbf{Q}_{\mathrm{C},b}\}_{b=1}^{B}, \boldsymbol{\mu}, \boldsymbol{\zeta}; \overbrace{\{\mathbf{Q}_{\mathrm{L},b}, \mathbf{p}_{\mathrm{L},b}, \boldsymbol{\lambda}_b\}_{b=1}^{B}}^{\substack{\text{Known} \\ \text{local information}}}) &= \mathcal{L}_b(\mathbf{W}_b, \mathbf{R}_b, \boldsymbol{\lambda}_b; \overbrace{\mathbf{Q}_{\mathrm{C},b}, \widetilde{\mathbf{Q}}_{\mathrm{C},b}, \boldsymbol{\mu}, \boldsymbol{\zeta}}^{\substack{\text{Known} \\ \text{global information}}}) \\
&= -\sum_{k=1}^{K} \Big(\log_2(1+\mu_k) - \mu_k - |\zeta_k|^2 \sigma_k^2 \Big) - \delta \\
&\quad + \sum_{l=1}^{B} \rho \|\mathbf{R}_l - \mathbf{w}_l \mathbf{w}_l^H\|_F^2 + \frac{\varrho}{2} \sum_{l=1}^{B} \|\mathbf{q}_{\mathrm{C},l} - \mathbf{q}_{\mathrm{L},l} + \frac{\boldsymbol{\lambda}_l}{\varrho}\|_2^2.
\end{aligned} \tag{6}$$

Then, the global optimization problem is formulated as

$$\min_{\substack{\{\mathbf{Q}_{\mathrm{C},b}\}_{b=1}^{B} \\ \boldsymbol{\mu}, \boldsymbol{\zeta}}} \mathcal{G}(\{\mathbf{Q}_{\mathrm{C},b},\}_{b=1}^{B}, \boldsymbol{\mu}, \boldsymbol{\zeta}; \{\mathbf{Q}_{\mathrm{L},b}, \mathbf{p}_{\mathrm{L},b}, \boldsymbol{\lambda}_b\}_{b=1}^{B}). \tag{7a}$$

Similarly, the local optimization problem is formulated as

$$\min_{\mathbf{W}_b, \mathbf{R}_b, \boldsymbol{\lambda}_b} \quad \mathcal{L}_b(\mathbf{W}_b, \mathbf{R}_b, \boldsymbol{\lambda}_b; \mathbf{Q}_{\mathrm{C},b}, \widetilde{\mathbf{Q}}_{\mathrm{C},b}, \boldsymbol{\mu}, \boldsymbol{\zeta}) \tag{8a}$$

$$\text{s.t.} \quad \|\mathbf{w}_b\|_2^2 \leq P_{\mathrm{t}}. \tag{8b}$$

According to the global iteration process termed as information aggregation in Algorithm

2, we can acquire the subsequent inequations:

$$\begin{aligned}&\mathcal{G}(\{\mathbf{Q}_{\mathrm{C},b}^{(t)}\}_{b=1}^{B},\boldsymbol{\mu}^{(t)},\boldsymbol{\zeta}^{(t)};\{\mathbf{Q}_{\mathrm{L},b}^{(t)},\mathbf{p}_{\mathrm{L},b}^{(t)},\boldsymbol{\lambda}_b^{(t)}\}_{b=1}^{B})\\ &\overset{\text{(d)}}{\geq}\mathcal{G}(\{\mathbf{Q}_{\mathrm{C},b}^{(t+1)}\}_{b=1}^{B},\boldsymbol{\mu}^{(t)},\boldsymbol{\zeta}^{(t)};\{\mathbf{Q}_{\mathrm{L},b}^{(t)},\mathbf{p}_{\mathrm{L},b}^{(t)},\boldsymbol{\lambda}_b^{(t)}\}_{b=1}^{B})\\ &\overset{\text{(e)}}{\geq}\mathcal{G}(\{\mathbf{Q}_{\mathrm{C},b}^{(t+1)}\}_{b=1}^{B},\boldsymbol{\mu}^{(t+1)},\boldsymbol{\zeta}^{(t+1)};\{\mathbf{Q}_{\mathrm{L},b}^{(t)},\mathbf{p}_{\mathrm{L},b}^{(t)},\boldsymbol{\lambda}_b^{(t)}\}_{b=1}^{B})\end{aligned} \tag{9}$$

where (d) is obtained after solving problem (60), (e) can be derived after optimizing $\boldsymbol{\mu}$ and $\boldsymbol{\zeta}$ by (62) and (63), respectively. Similarly, we can guarantee an optimal solution for each variable in iterations. Therefore, the value of the global objective function shows a reduction after the information aggregation from the t-th iteration.

According to the local iteration process at the b-th BS in Algorithm 2, we can construct the subsequent inequations:

$$\begin{aligned}&\mathcal{L}_b(\mathbf{W}_b^{(t)},\mathbf{R}_b^{(t)},\boldsymbol{\lambda}_b^{(t)};\mathbf{Q}_{\mathrm{C},b}^{(t+1)},\widetilde{\mathbf{Q}}_{\mathrm{C},b}^{(t+1)},\boldsymbol{\mu}^{(t+1)},\boldsymbol{\zeta}^{(t+1)})\\ &\overset{\text{(f)}}{\geq}\mathcal{L}_b(\mathbf{W}_b^{(t+1)},\mathbf{R}_b^{(t+1)},\boldsymbol{\lambda}_b^{(t)};\mathbf{Q}_{\mathrm{C},b}^{(t+1)},\widetilde{\mathbf{Q}}_{\mathrm{C},b}^{(t+1)},\boldsymbol{\mu}^{(t+1)},\boldsymbol{\zeta}^{(t+1)})\\ &\overset{\text{(g)}}{\geq}\mathcal{L}_b(\mathbf{W}_b^{(t+1)},\mathbf{R}_b^{(t+1)},\boldsymbol{\lambda}_b^{(t+1)};\mathbf{Q}_{\mathrm{C},b}^{(t+1)},\widetilde{\mathbf{Q}}_{\mathrm{C},b}^{(t+1)},\boldsymbol{\mu}^{(t+1)},\boldsymbol{\zeta}^{(t+1)}),\end{aligned} \tag{10}$$

where (f) is obtained after solving problem (66), (g) can be derived after optimizing $\boldsymbol{\lambda}$ by (67). Therefore, the value of each local objective function is reduced after the local beamforming design is carried out in parallel.

Besides, it is obvious that the value of the global objective function after information aggregation equals the value of the local objective function before local optimization, i.e.,

$$\begin{aligned}&\mathcal{G}(\{\mathbf{Q}_{\mathrm{C},b}^{(t+1)}\}_{b=1}^{B},\boldsymbol{\mu}^{(t+1)},\boldsymbol{\zeta}^{(t+1)};\{\mathbf{Q}_{\mathrm{L},b}^{(t)},\mathbf{p}_{\mathrm{L},b}^{(t)},\boldsymbol{\lambda}_b^{(t)}\}_{b=1}^{B})\\ &=\mathcal{L}_b(\mathbf{W}_b^{(t)},\mathbf{R}_b^{(t)},\boldsymbol{\lambda}_b^{(t)};\mathbf{Q}_{\mathrm{C},b}^{(t+1)},\widetilde{\mathbf{Q}}_{\mathrm{C},b}^{(t+1)},\boldsymbol{\mu}^{(t+1)},\boldsymbol{\zeta}^{(t+1)}).\end{aligned} \tag{11}$$

It follows that the objective function exhibits a monotonically non-increasing trend after every iteration, i.e.,

$$\begin{aligned}&\mathcal{G}(\{\mathbf{Q}_{\mathrm{C},b}^{(t)}\}_{b=1}^{B},\boldsymbol{\mu}^{(t)},\boldsymbol{\zeta}^{(t)};\{\mathbf{Q}_{\mathrm{L},b}^{(t)},\mathbf{p}_{\mathrm{L},b}^{(t)},\boldsymbol{\lambda}_b^{(t)}\}_{b=1}^{B})\\ &\geq\mathcal{L}_b(\mathbf{W}_b^{(t+1)},\mathbf{R}_b^{(t+1)},\boldsymbol{\lambda}_b^{(t+1)};\mathbf{Q}_{\mathrm{C},b}^{(t+1)},\widetilde{\mathbf{Q}}_{\mathrm{C},b}^{(t+1)},\boldsymbol{\mu}^{(t+1)},\boldsymbol{\zeta}^{(t+1)}),\end{aligned} \tag{12}$$

which confirms the convergence of the star-topology-based partially-distributed beamforming design algorithm.