IISE Transactions

ISSN: (Print) (Online) Journal homepage: https://www.tandfonline.com/loi/uiie21

Flow Time in a Human-Robot Collaborative Assembly Process: Performance Evaluation, System Properties, and a Case Study*

Ya-jun Zhang, Ningjian Huang, Rober G. Radwin, Zheng Wang & Jingshan Li

Flow Time in a Human-Robot Collaborative Assembly Process: Performance Evaluation, System Properties, and a Case Study*

Ya-jun Zhang[a,b], Ningjian Huang[c], Rober G. Radwin[b], Zheng Wang[a] and Jingshan Li[b]

[a] School of Automation, Southeast University, Nanjing, Jiangsu, China

[b] Department of Industrial and Systems Eng., University of Wisconsin, Madison, WI, USA

[c] Manufacturing Systems Research Lab, General Motors R&D Center, Warren, MI, USA

*This work is supported in part by NSF Grants 1928245 and 2026478.

Corresponding author Jingshan Li jingshan.li@wisc.edu

Abstract

In this paper, an analytical method is introduced to evaluate the flow time of an assembly process with collaborative robots. In such a process, an operator and a collaborative robot can independently carry out preparation tasks first, then they work jointly to finish the assembly operations. To study the productivity performance of such systems, a stochastic process model is developed, where the joint work is modeled as an assembly merge process, and the task times of all operation steps are described by phase-type distributions. Closed-form solutions of system performance, such as flow time expectation and variability, as well as service rate, are derived analytically. The system properties of monotonicity, work allocation, and bottleneck identification are investigated. In addition, a case study is introduced to evaluate the performance of a front panel assembly process in automotive manufacturing.

Keywords: Assembly system, collaborative robots, flow time, service rate, phase-type distribution.

1 Introduction

Manufacturing industry is now facing tremendous challenges. In order to continuously improve productivity, quality, sustainability, and customer satisfaction, both industrial automation to maintain high efficiency and repeatability, and operation flexibility to deal with customization and variability are of critical importance, and should be balanced to ensure competitiveness. In responding to such challenges, in recent years, there has been an increasing trend to use collaborative robots in assembly operations. The collaborative robot market share has been growing significantly. It is projected that the global market will grow from USD 981 million in 2020 to USD 7,972 billion by 2026 (Collaborative Robots Market Report (2020)). Thus, the human-robot collaborative assembly process is gradually becoming an important element of smart manufacturing and factory automation, particularly beneficial to small- and medium-sized manufacturing enterprises (Marvel (2014)).

In a human-robot collaborative assembly process, the human operators and the robots share the same workspace and collaborate safely on a variety of tasks, such as pick-and-place, assembly, screwing, material handling, and inspection. Such robots are often referred to as collaborative robots or cobots. To address the assembly processes with cobots, numerous studies on human-robot collaborations have been conducted to discuss the issues and challenges, such as the concept, foundational framework, and various uses of cobots, as well as the safety, interfaces, and applications in industrial settings, and the enabling technologies for collaborations and cobot programming.

From an assembly system perspective, production planning and scheduling are of significant importance. Thus, conventional problems, such as line balancing, sequencing and task assignment, have attracted many research attentions under the collaborative robots environment. Many cobot scheduling studies use makespan as the performance measure to develop optimization models by considering cost, safety, strain index, logistic constraints and product characteristics. However, most of the works on collaborative robot systems focus on design and programming of cobots, or task allocation between robots and human operators, using detailed kinematics and aggregated task time, or assuming sequential process of tasks.

In spite of the available studies, modeling and analysis of human-robot collaborative assembly processes are still in an infant phase. Less research has been paid to evaluate the productivity or time performance of the cobot systems with both independent and collaborative features. Consider the following collaborative assembly scenario: The operators and the cobots first carry out preparation tasks independently, and then work jointly to finish the assembly process, as shown in Figure 1. Such collaborative assembly processes can be observed in automotive, appliance, battery, and equipment manufacturing systems. In addition, note that this type of collaborative assembly process is different with a traditional assembly system, since the preparation work (process 1 in Figure 1) and joint work (process 2 in Figure 1) need to use all resources (operator and robot) working on the same part, and are carried out by the same operator and cobot sequentially in the collaborative assembly process. In contrast, there are different operators and different robots (or machines) working on preparation and assembly processes on multiple workpieces simultaneously and independently in a traditional assembly system.

To study such a human-robot collaborative assembly process, the activities and the associated task times should be modeled and analyzed explicitly. Particularly, as the productivity performance, such as flow time or throughput, is critical for manufacturing operations to maintain competitiveness, developing an analytical model to study the flow time is necessary and important. Although there exist numerous studies of time performance in queueing networks addressing split and merge structures, many of them tackle production merge, where parts from different routes go through the merging node sequentially. However, in a collaborative assembly process (Figure 1), the merge process is an assembly merge, i.e., parts from different routes are assembled or integrated into one single part. Modeling and analysis of assembly merge are much more complicated. Most research on assembly systems only addresses the mean time performance under Markovian assumption of processing times. As manual operations and robot breakdown can introduce significant variability in task times, the flow time in each process may vary with non-Markovian behavior and affect system performance in a nonlinear manner. Therefore, analytical models to accommodate more detailed activities, general distributions and higher order variability measures are needed.

This paper is intended to contribute to this end by introducing a stochastic process model to analyze the flow time performance of such human-robot collaborative assembly processes. By assuming phase-type distribution of operation time, a complete distribution of the process flow time is derived, and system properties, such as monotonicity, work allocation, and bottlenecks, are investigated. The goal of this study is not to focus on the cognitive or collaborative part of cobots (which is represented through processing times), but on the resulting flow time of the whole process. In addition, a case study at an automotive panel assembly process is introduced to illustrate the applicability of the study.

The rest of the paper is organized as follows: Section 2 reviews the related literature. Section 3 introduces the model assumptions and formulates the problem. In Section 4, analytical methods with exponential and phase-type distribution task times are presented to evaluate flow time performance. The system properties are investigated in Section 5. A case study is described in Section 6. Finally, Section 7 summarizes the work. Due to space limitation, all proofs are provided in the Supplemental materials of the Transaction.

2 Literature Review

Collaborative assembly systems have received increasing research attention in recent years. Many surveys and numerous studies have been conducted on human robot collaborations. For instance, Chandrasekaran and Conrad (2015) survey the concept and various uses, and Villani et al. (2018) outline the safety, interfaces, and applications in industrial settings. Bauer et al. (2008) review the technologies to enable collaborations, such as machine learning, action planning, joint action, and intention estimation. Marvel (2014) predicts that many small- and medium-sized manufacturing enterprises have the greatest potential to benefit from using collaborative robots in various processes. In addition, Djuric et al. (2016) introduce a foundation and four tier framework, including system, work cell, machine, and worker levels, to facilitate design, development and integration of cobots, which can help enhance robotic and human abilities in the system, improve efficiency and adaptation to changes. Expecting that semi-automatic collaborative systems will be important for flexible and agile production automation, Bolmsjö et al. (2012) describe the methods, software, and hardware to support collaborations between robots and operators. Moreover, an overview of collaborative scenarios and cobot programming requirements for effective implementation is presented by El Zaatari et al. (2019), where detailed reviews on communication, optimization, and learning for cobot programming are conducted, and research gaps as well as future directions are identified. As safety is critically important in robot applications, Lasota et al. (2017) discuss the potential methods of ensuring safety during collaborations.

From a cobot assembly system perspective, planning and scheduling are of significant importance. Thus, job allocation and task assignment of cobots have attracted many research attentions. For example, Dalle Mura and Dini (2019) propose a genetic algorithm for assembly line balancing to minimize assembly line cost, the number of skilled workers, and the energy load variance among workers in the case of human-robot collaborative work. Weckenborg et al. (2019) also study line balancing with collaborative robots by developing a hybrid genetic algorithm in a mixed integer programming formulation, which can decide both assignment of collaborative robots to stations and distribution of workload to workers and robots to minimize cycle times. A hybrid optimization method is introduced by Maganha et al. (2019) to solve the sequencing and assignment problems in reconfigurable assembly line with collaborated human operators and mobile robots to define job schedules, assign tasks, and allocate robots to workstations. Tsarouchi et al. (2017) introduce a framework to execute collaborative tasks in hybrid assembly cells, in which a decision making method that allows allocation of sequential tasks assigned to a robot and a human operator in separate workspaces is proposed and integrated in a robot operating system. Moreover, Faccio et al. (2019) derive the conditions under which the collaborative assembly systems can perform better than traditional manual or automated lines, and also introduce a set of system variables and a mathematical model to estimate throughput and production cost by considering task allocation and interference during assembly.

Many cobot scheduling studies develop optimization models to minimize makespan. Chen et al. (2013) introduce a folk-joint task model to describe the sequential and parallel features and logic restrictions of human and robot collaborations. A logic mathematical method is used to quantify tradeoffs between assembly time cost and payment cost, and a genetic based revolutionary algorithm is developed to allocate subtasks in real-time to meet the required cost-effectiveness. Faccio et al. (2020) investigate the influence of product characteristics on interference between human operators and robots by introducing an algorithm to simulate assembly process and estimate makespan in different scenarios. Mokhtarzadeh et al. (2020) present a constraint programming based approach to allocate tasks to humans and robots to minimize makespan in print circuit boards industry. By seeking improvement in both makespan and strain index, Pearce et al. (2018) propose an optimization framework to generate solutions of task assignments and schedules for a human-robot team with priorities. In addition, a kinematic control strategy to enforce safety while maintaining the maximum level of productivity is introduced by Zanchettin et al. (2015), where an optimization-based real time algorithm controls the motion of the robots with safety as a hard constraint.

Although there exist numerous studies of time performance in queueing networks addressing split and merge structures, most of them consider production merge (see, for instance, Altiok and Perros (1986); Tsimashenka and Knottenbelt (2011); Fiorini and Lipsky (2015)). Similar issues exist in stochastic PERT networks as well (e.g., Shih (2005)). In a collaborative assembly system (Figure 1) where assembly merge is carried out, the problems become much more difficult. Even with extensive research in manufacturing systems (e.g., monographs by Viswanadham and Narahari (1992); Buzacott and Shanthikumar (1993); Papadopolous et al. (1993); Gershwin (1994); Li and Meerkov (2008) and reviews by Dallery and Gershwin (1992); Papadopoulos and Heavey (1996); Li et al. (2009); Papadopoulos et al. (2019)), most works address the mean time performance, and typically under Bernoulli or exponential assumption of processing times (such as representative papers by Gershwin (1991); Chiang et al. (2000a,b); Li and Meerkov (2001); Helber and Jusić (2004); Li (2005); Matta et al. (2005); Zhao and Li (2014); Jia et al. (2015); Ju et al. (2016)). Only limited work on general service time has been considered in assembly system studies. For example, Manitz (2008) introduces a decomposition approach to approximate the throughput of assembly system with finite buffers and general service times, where the two-station subsystems are analyzed by G/G/1/N queueing models to determine the virtual arrival and service rates as well as squared coefficients of variations. Manitz (2015) extends such an approach to multi-stage assembly/disassembly queueing networks to derive both throughput and variance of inter-departure times. Krishnamurthy et al. (2004) present an exact analysis of a fork/join station in a closed queuing network with inputs from servers having two-phase Coxian service distributions. Throughput and distributions of queue length and inter-departure times from the fork/join station are derived through queue length and departure processes. Additional studies on mean and variability measures of assembly systems with non-exponential processing times are also presented by De Boeck and Vandaele (2011); Manitz and Tempelmeier (2012); Tancrez (2020).

In spite of these efforts, there is still a need to model and analyze the flow time more explicitly by considering detailed activities, general distributions of task times, and higher order variability measures, which is the goal of this paper.

3 System Description

Consider a human-robot collaborative assembly station with one operator and one cobot illustrated in Figure 1. Such a station can be characterized by two independent preparation processes by the operator and cobot in parallel, and their joint collaborative process afterwards, as shown in Figure 2. The following assumptions define the collaborative assembly system under study.

(i) The human-robot collaborative assembly process includes one cobot and one human operator.

(ii) The operator and cobot first work independently to carry out the preparation tasks. Afterwards, they work jointly to finish the assembly task. The joint process is an assembly merge process that can only start after both individual preparatory processes are finished.

(iii) The processes of individual preparation work of the cobot and the operator are described by random processes *R* and *H*, respectively, while the joint work process is characterized by random process *C*.

(iv) Each random process, *R*, *H*, or *C*, is assumed to follow a phase-type distribution $f_i(t), i = h, r, c$, which can represent the multiple consecutive steps in a process. In other words, there exist m_i phases, $i = h, r, c$, in process *i*, and each phase follows an exponential distribution with parameter $\lambda_{i,j}$, $j = 1, \ldots, m_i$, corresponding the multiple steps to finish the process operation.

(v) The initial probability of each step in processes *R*, *H*, or *C*, is defined by $\alpha_{i,j}, i = h, r, c,\ j = 1, \ldots, m_i$, which can make the model applicable for more general cases. Typically $\alpha_{i,1} = 1$ (i.e., hypoexponential distribution case) and all $\alpha_{i,j} = 0,\ j = 2, \ldots, m_i, i = h, r, c$.

Remark 1. As shown in Li and Meerkov (2005), the process time in most manufacturing environment has coefficient of variation (CV) smaller than 1. In addition, a process can always be decomposed into a series of steps, which corresponds to multiple phases in a phase-type distribution. Thus, assumption (iv) introduces phase-type distributions to characterize processes *R*, *H* and *C*, which can describe the non-Markovian feature and have *CV* < 1. To fit or approximate such processes, the parameters of the phase-type distribution can be obtained by using the empirical data (see Lang and Arthur (1996) for details).

Remark 2. Although robot operation itself typically has few variability in processing time, the overall processing time may still be random due to unscheduled run-based (or operation-dependent) downtimes. Thus, for the robot preparation process, a phase-type distribution is still assumed.

Under assumptions (i)-(v), define the flow time to finish each process as t_k, $k = h, c, r$. Then the overall system flow time to finish all the processes (i.e., the system flow time) can be defined as

$$t = \max(t_h, t_r) + t_c. \qquad (1)$$

The expectation of flow time, *T*, needs to be evaluated. In addition, the variability, i.e., CV, and whether the assembly process can finish the work on time are important measures, where the last one is usually referred to as flow-time or assembly-time performance (FTP or ATP) or service rate *S*. In this study, *S* is defined as the probability to finish the whole process within a given time interval T_d. Then we have

$$T = E(t) \quad CV = \frac{\sqrt{E[(t-T)^2]}}{T}, \quad S(T_d) = P(t \le T_d). \quad (2)$$

These measures are functions of all system parameters, m_i, λ_{ij}, $\alpha_{i,j}, i = h, r, c,\ j = 1, \ldots, m_i$. Then the problem to be studied in this paper can be formulated as: *Under assumptions (i)-(v), develop a method to evaluate the performance of the human-robot collaborative assembly process, i.e., calculate T, CV, and $S(T_d)$ as functions of system parameters, and investigate system properties.*

4 Performance Evaluation

4.1 Analysis with Exponential Distribution Models

Exponential distribution is a special case that only has 1 phase in phase-type distribution, i.e., $m_i = 1,\ i = h, r, c$. The task time t_i, $i = h, r, c$, follows an exponential distribution with parameter λ_i. Under such an assumption, we first evaluate the service rate, i.e., the cumulative distribution function (CDF) of flow time.

Theorem 1. *Under assumptions (i)-(v) with 1 phase, i.e., $m_r = m_h = m_c = 1$, the system service rate for a given time period T_d, i.e., the CDF of flow time, can be calculated as*

$$S(T_d) = 1 - e^{-\lambda_c T_d} + \lambda_c(\gamma_{rh} - \gamma_r - \gamma_h), \quad (3)$$

where

$$\gamma_r = \begin{cases} \dfrac{e^{-\lambda_r T_d} - e^{-\lambda_c T_d}}{\lambda_c - \lambda_r}, & \text{if } \lambda_c \ne \lambda_r, \\ T_d e^{-\lambda_r T_d}, & \text{o/w}, \end{cases} \qquad \gamma_h = \begin{cases} \dfrac{e^{-\lambda_h T_d} - e^{-\lambda_c T_d}}{\lambda_c - \lambda_h}, & \text{if } \lambda_c \ne \lambda_h, \\ T_d e^{-\lambda_h T_d}, & \text{o/w}, \end{cases}$$
$$\gamma_{rh} = \begin{cases} \dfrac{e^{-(\lambda_r + \lambda_h) T_d} - e^{-\lambda_c T_d}}{\lambda_c - \lambda_r - \lambda_h}, & \text{if } \lambda_c \ne \lambda_r + \lambda_h, \\ T_d e^{-(\lambda_r + \lambda_h) T_d}, & \text{o/w}. \end{cases} \quad (4)$$

Using this result, the expected flow time can be evaluated.

Corollary 1. *Under assumptions (i)-(v) with 1 phase, i.e.,* $m_r = m_h = m_c = 1$*, the expected flow time of the system can be calculated as*

$$T = \frac{1}{\lambda_c} + \frac{1}{\lambda_r} + \frac{1}{\lambda_h} - \frac{1}{\lambda_r + \lambda_h}. \quad (5)$$

The first term in equation (5) represents the average task time of the joint process, while the last three terms characterize the maximum of task time of each preparation process, which can be understood as the preparation flow time needs to cover both individual processes and exclude the “overlapped part” (i.e., the term of $\lambda_r + \lambda_h$, which can be viewed as a combined rate).

Although the mean value is important, variability also impacts system performance significantly, particularly in a multi-process system, where one process’s variation can cause the whole system unbalanced. However, variance itself cannot directly measure the impact of variability since it depends on the mean. Therefore, we calculate the ratio of standard deviation and the mean, i.e., the CV, of system flow time under exponential distribution assumption.

Corollary 2. *Under assumptions (i)-(v) with 1 phase, i.e.,* $m_r = m_h = m_c = 1$*, the coefficient of variation of system flow time can be calculated as*

$$CV = \frac{\sqrt{\frac{1}{\lambda_h^2} + \frac{1}{\lambda_r^2} + \frac{1}{\lambda_c^2} - \frac{3}{(\lambda_r + \lambda_h)^2}}}{\frac{1}{\lambda_c} + \frac{1}{\lambda_r} + \frac{1}{\lambda_h} - \frac{1}{\lambda_r + \lambda_h}}. \quad (6)$$

4.2 Analysis with Phase-type Distribution Models

In practice, work times may not follow exponential distributions, and many processes need to be finished in multiple steps. Thus, a more general case, the phase-type distribution model, is studied to represent the variability of the process and serial nature of multiple steps. Specifically, consider a random process, *R*, *H*, or *C*, with m_i, $i = h, r, c$, steps following exponential distributions. Then each process can be described by a continuous time Markov chain with $m_i + 1$ states, where the first m_i states are transient, and the last one is an absorbing state. To characterize such a process, generator matrix $\boldsymbol{Q}_i$, $i = h, r, c$, is defined as

$$\boldsymbol{Q}_i = \begin{bmatrix} \boldsymbol{B}_i & \boldsymbol{b}_i \\ 0 & 0 \end{bmatrix}, \qquad (7)$$

where matrix $\boldsymbol{B}_i$ and vector $\boldsymbol{b}_i$ are defined as

$$\boldsymbol{B}_i = \begin{bmatrix} -\lambda_{i,1} & \lambda_{i,1} & 0 & \cdots & 0 \\ 0 & -\lambda_{i,2} & \lambda_{i,2} & \ddots & 0 \\ 0 & \ddots & \ddots & \ddots & \vdots \\ 0 & \ddots & \ddots & -\lambda_{i,m_{i-1}} & \lambda_{i,m_{i-1}} \\ 0 & \ddots & \ddots & \ddots & -\lambda_{i,m_i} \end{bmatrix}, \quad \boldsymbol{b}_i = -\boldsymbol{B}_i 1 = \begin{bmatrix} 0 \\ \vdots \\ \lambda_{i,m_i} \end{bmatrix}. \qquad (8)$$

Here 1 is a column vector whose entries are all ones, i.e., $1 = [1, \cdots, 1]^T$.

Define the initial probability vector for the m_i transient states as

$$\boldsymbol{\alpha}_i = \left[\alpha_{i,1}, \cdots, \alpha_{i,m_i} \right], \quad i = h, r, c,$$

where $\alpha_{i,j}$ represents the initial probability at state *j* of process *i*, $i = h, r, c,\ j = 1, \ldots, m_i$. Then the probability of the absorbing state will be $\alpha_{i,m_i+1} = 1 - \boldsymbol{\alpha}_i 1$.

Remark 3. When $m_i = 1$, the exponential distribution is obtained. Then subscript " 1" can be dropped from $\lambda_{i,1}$ so that $\boldsymbol{B}_i = -\lambda_i$ is used in Section 4.1.

The flow time to finish each process, t_i, $i = h, r, c$, i.e., the time to reach the absorbing state $m_i + 1$, is represented by a phase-type distribution. From (1), the total time *t* also follows a phase-type distribution. Then, based on the assumptions and definitions introduced in Section 3, the system performance measures, T, CV, and $S(T_d)$, can be evaluated.

Analogously to the exponential case, first, we derive the CDF of flow time, i.e., the service rate of the system, under the phase-type distribution assumption.

Theorem 2. *Under assumptions (i)-(v), the service rate for a given T_d can be calculated as*

$$S(T_d) = P(t \le T_d) = 1 - \boldsymbol{\alpha} e^{\boldsymbol{B} T_d} \mathbf{1}, \qquad (9)$$

where $\boldsymbol{B}$ is a square matrix with dimension m, and $\boldsymbol{\alpha}$ is a row vector, which are defined as follows:

$$m = m_h m_r + m_h + m_r + m_c,$$
$$\boldsymbol{\alpha} = \begin{bmatrix} \boldsymbol{\alpha}_r \otimes \boldsymbol{\alpha}_h & \boldsymbol{\alpha}_r \alpha_{h,m_i+1} & \boldsymbol{\alpha}_h \alpha_{r,m_i+1} & \rho \boldsymbol{\alpha}_c \end{bmatrix}, \qquad (10)$$
$$\boldsymbol{B} = \begin{bmatrix} \boldsymbol{B}_h \oplus \boldsymbol{B}_r & \boldsymbol{I}_h \otimes \boldsymbol{b}_r & \boldsymbol{b}_h \otimes \boldsymbol{I}_r & 0 \\ 0 & \boldsymbol{B}_h & 0 & \boldsymbol{b}_h \boldsymbol{\alpha}_c \\ 0 & 0 & \boldsymbol{B}_r & \boldsymbol{b}_r \boldsymbol{\alpha}_c \\ 0 & 0 & 0 & \boldsymbol{B}_c \end{bmatrix},$$

and $\boldsymbol{b}_r$ and $\boldsymbol{b}_h$ are column vectors

$$\rho = 1 - \begin{bmatrix} \boldsymbol{\alpha}_r \otimes \boldsymbol{\alpha}_h & \boldsymbol{\alpha}_r \alpha_{h,m_h+1} & \boldsymbol{\alpha}_h \alpha_{r,m_r+1} \end{bmatrix} \mathbf{1}, \quad \boldsymbol{b}_i = \begin{bmatrix} 0 \\ \vdots \\ \lambda_{i,m_i} \end{bmatrix}, \quad i = r, h. \qquad (11)$$

Remark 4. Note $\otimes$ represents the Kronecker product, whose formula is provided in the Supplemental materials.

Remark 5. For exponential model, $m_i = 1$, $\boldsymbol{B}_i = -\lambda_i$, and $\boldsymbol{\alpha}_i = 1$, we obtain

$$\boldsymbol{B}=\begin{bmatrix}-\lambda_h-\lambda_r & \lambda_r & \lambda_h & 0\\ 0 & -\lambda_h & 0 & \lambda_h\\ 0 & 0 & -\lambda_r & \lambda_r\\ 0 & 0 & 0 & -\lambda_c\end{bmatrix},\quad \boldsymbol{B}^{-1}=\begin{bmatrix}-\dfrac{1}{\lambda_h+\lambda_r} & -\dfrac{\lambda_r}{\lambda_h(\lambda_h+\lambda_r)} & -\dfrac{\lambda_h}{\lambda_r(\lambda_h+\lambda_r)} & -\dfrac{1}{\lambda_c}\\ 0 & -\dfrac{1}{\lambda_h} & 0 & -\dfrac{1}{\lambda_c}\\ 0 & 0 & -\dfrac{1}{\lambda_r} & -\dfrac{1}{\lambda_c}\\ 0 & 0 & 0 & -\dfrac{1}{\lambda_c}\end{bmatrix}.$$

It can be shown that the expressions derived in this subsection are equivalent to those obtained in Subsection 4.1. In addition, for the phase-type case, typically $\alpha_{i,1}=1$ when the operator and robot need to carry out all the subtasks in its process sequentially. However, in some cases, such as handling reworked parts, quality check, breakdowns, the operator and robot may skip some subtasks such that $\alpha_{i,1}<1$ where there exist subtasks $j\in\{2,\ldots,m_i\}$ with $\alpha_{i,j}>0$.

Using this result, for the mean and CV of flow time in the system, we obtain

Corollary 3. *Under assumptions (i)-(v), the expected flow time of the system can be calculated as*

$$T=E(t)=-\boldsymbol{\alpha}\boldsymbol{B}^{-1}\mathbf{1}. \quad (12)$$

Corollary 4. *Under assumptions (i)-(v), the CV of system flow time can be calculated as*

$$CV=\frac{\sqrt{2\boldsymbol{\alpha}\boldsymbol{B}^{-2}\mathbf{1}-\left(\boldsymbol{\alpha}\boldsymbol{B}^{-1}\mathbf{1}\right)^2}}{-\boldsymbol{\alpha}\boldsymbol{B}^{-1}\mathbf{1}}. \quad (13)$$

Remark 6. The above model can be extended to scenarios where more than one robots and/or more than one operators collaborate in a system, if all operators and robots are needed in the collaborative task. For example, two or three operators may collaborate to install the front panel in a vehicle in the assembly line. In this case, the proposed approach is still applicable. Assume there are k_r robots and k_h operators who will carry out their individual work first, then collaborate to finish the assembly. The system flow time can be characterized as $t = \max(t_{r,1} \ldots, t_{r,k_r}, t_{h,1}, \ldots, t_{h,k_h}) + t_c$. By applying the maximum function of two individual processes, we obtain a new process still following phase-type distribution. Then such a process is compared to the next individual process to obtain the maximums. Continue this procedure until the last individual process. Finally we can obtain the maximal work time of all independent processes. Then this time is summed up with the task time in the collaboration process, the overall flow time performance can be obtained. Moreover, if there is only one or no individual preparation process, the maximal function (or the whole term on preparation) can be ignored in flow time calculation. When the sequential collaborative works are carried out by an operator and a robot independently, by assigning different parameters for each operation, the proposed approach is still applicable.

Remark 7. Note that by adding constraints in a traditional assembly system, such as use of all resources (operator and robot), only one part in each preparation process and in the joint process, no part entry in preparation until finishing the joint work, dependent sequential processes, and the same single operator and single robot for both preparation and joint processes, etc., we may make an equivalence between the traditional assembly system and the human-robot collaborative assembly process described in this study. However, such constraints will make the analysis more difficult.

Remark 8. Although phase-type distribution can be used to model many random processes, there still exist some scenarios difficult to approximate, such as delayed exponential distribution (see Neuts (1994)), which can be observed on the factory floor (Inman (1999)). Extending the work to consider those distributions could be part of future work.

5 System Properties

5.1 Exponential Distribution Models

5.1.1 Monotonicity

Using the formulas introduced above, we investigate system properties. First, the exponential models are analyzed. By examining the expected flow time under exponential assumption, we observe monotonic properties with respect to its parameters. Such properties provide a foundation for work allocation and bottleneck analysis. Specifically, we expect that reducing work time of each step decreases the overall flow time.

Proposition 1. *Under assumptions (i)-(v) with 1 phase, i.e.,* $m_r = m_h = m_c = 1$*, the expected flow time of the system is monotonically decreasing with respect to* λ_r*,* λ_h*, and* λ_c*.*

Similar monotonic properties exist for the CV of flow time.

Proposition 2. *Under assumptions (i)-(v) with 1 phase, i.e.,* $m_r = m_h = m_c = 1$*, the CV of system flow time is monotonically decreasing with respect to* λ_r*,* λ_h*, and* λ_c*.*

5.1.2 Work allocation

If the total time to finish all the preparation tasks is a constant, what will be an optimal allocation of tasks to the operator and the cobot? To answer this question, we derive the following:

Proposition 3. *Under assumptions (i)-(v) with 1 phase, i.e.,* $m_r = m_h = m_c = 1$*, given constraint*

$$\frac{1}{\lambda_r} + \frac{1}{\lambda_h} = T_{prep} \qquad (14)$$

where T_{prep} *is the total work time of preparation tasks, then the system flow time is minimized if* $\lambda_r = \lambda_h$ *.*

This result indicates that the operator and the cobot should be allocated with the same amount of preparatory work (in terms of task time) so that they will have the same impact on the joint process.

5.1.3 Bottleneck analysis

When continuous improvement efforts are planned, a process that can lead to a larger decrease in total flow time becomes the bottleneck and should be mitigated. Thus, we define

Definition 1. *Under assumptions (i)-(v) with 1 phase, i.e.,* $m_r = m_h = m_c = 1$*, process i becomes the bottleneck if* $\forall i, k \in \{h, r, c\}$,

$$\left|\frac{\partial T}{\partial \lambda_i}\right| \geq \left|\frac{\partial T}{\partial \lambda_k}\right|, \quad k \neq i. \quad (15)$$

Under Definition 1, we first focus on the preparation processes.

Proposition 4. *Under assumptions (i)-(v) with 1 phase, i.e.,* $m_r = m_h = m_c = 1$*, then* $\forall i, k \in \{h, r\}$,

$$\left|\frac{\partial T}{\partial \lambda_i}\right| \geq \left|\frac{\partial T}{\partial \lambda_k}\right| \text{ if and only if } \lambda_i \leq \lambda_k. \quad (16)$$

Using this result, a bottleneck indicator can be obtained:

Bottleneck Indicator 1: In the collaborative assembly station with exponential process times, among the preparation processes, the one takes longer time is the bottleneck.

Next we investigate the bottleneck of all processes. Comparing changes in λ_c and λ_r or λ_h, we obtain:

Proposition 5. *Under assumptions (i)-(v) with 1 phase, i.e.,* $m_r = m_h = m_c = 1$*, then* $\forall k \in \{h, r\}$,

$$\left|\frac{\partial T}{\partial \lambda_k}\right| \geq \left|\frac{\partial T}{\partial \lambda_c}\right| \text{ if and only if } \frac{1}{\lambda_k^2} \geq \frac{1}{(\lambda_h + \lambda_r)^2} + \frac{1}{\lambda_c^2}. \quad (17)$$

This proposition implies that the time square of a bottleneck preparation process will be larger than the sum of time square of collaboration process and square of “combined” or “overlapped” time (see the explanation after Corollary 1 in Section 4.1). In other words, only when the preparation time is significantly longer than the collaboration time, then the longer preparation process will become the bottleneck.

To identify the system bottleneck, Indicator 1 can be used first to find the longer preparation process and then to compare with the collaboration process using Proposition 5 to determine the bottleneck. Thus, the following indicator is presented.

Bottleneck Indicator 2: In the collaborative assembly station with exponential process times, the longer preparation processes is bottleneck if its time is substantially longer than the collaboration time (i.e., using Proposition 5), otherwise the collaboration process is the bottleneck.

5.2 Phase-type Distribution Models

5.2.1 Monotonicity

When each process consists of more than one sub-processes, the phase-type distribution with multiple phases will be used. As we expected, the monotonic properties with respect to collaboration process task times still hold. In other words, reducing any collaboration sub-process task time (i.e., increasing task rate), the overall flow time will be decreased.

Proposition 6. *Under assumptions (i)-(v), the expected flow time of the system is monotonically decreasing with respect to* λ_{cj}, $j = 1, \ldots, m_c$.

The monotonicity with respect to task times in operator and robot preparation processes also holds. Analytical proofs for the case of smaller number of phases, such as $m_h, m_r \leq 4$, can be obtained. However, for processes with more phases, the expression of partial derivatives of complex matrices becomes difficult to obtain, which makes the derivation of closed form result all but impossible. Note that although an explicit expression of partial derivatives with respect to *T* may not be available for all cases, numerical calculation of the derivatives can always be carried out. Thus, based on extensive numerical experiments with various randomly generated parameters, we formulate the result as a numerical observation.

Numerical Observation 1. *Under assumptions (i)-(v), the expected flow time of the system is monotonically decreasing with respect to* λ_{ij}, $i = h, r$, *and* $j = 1, \ldots, m_i$.

An illustration of monotonic property with respect to $\lambda_{h,j}$ is provided in Figure 3, where

$$\boldsymbol{B}_h = \begin{bmatrix} -\lambda_{h,1} & \lambda_{h,1} & 0 & 0 & 0 \\ 0 & -0.2 & 0.2 & 0 & 0 \\ 0 & 0 & -\lambda_{h,3} & \lambda_{h,3} & 0 \\ 0 & 0 & 0 & -0.43 & 0.43 \\ 0 & 0 & 0 & 0 & -\lambda_{h,5} \end{bmatrix}, \quad \boldsymbol{B}_r = \begin{bmatrix} -0.1 & 0.1 & 0 & 0 \\ 0 & -0.25 & 0.25 & 0 \\ 0 & 0 & -0.32 & 0.32 \\ 0 & 0 & 0 & -0.66 \end{bmatrix},$$

$$\boldsymbol{B}_c = -1, \tag{18}$$

and the base values of $\lambda_{h,1}, \lambda_{h,3}$, and $\lambda_{h,5}$ are 0.1, 0.34, and 0.55, respectively. By varying one $\lambda_{h,i}$, *i* = 1, 3, 5, each time, we obtain the corresponding curve of flow time, showing as the red solid line, black dashed line, and green dotted line in Figure 3. As we can observe, all of them indicate decreasing patterns.

5.2.2 Work allocation

Concerning about task allocation, again when the preparation processes and tasks are symmetrically assigned, i.e., well balanced, then the overall flow time is minimized.

Proposition 7. *Under assumptions (i)-(v), given the constraint*

$$\sum_{j=1}^{m_r} \frac{1}{\lambda_{r,j}} + \sum_{j=1}^{m_h} \frac{1}{\lambda_{h,j}} = T_{prep}, \qquad (19)$$

where T_{prep} is the total work time of preparation tasks, then the system flow time is minimized if

$$m_r = m_h, \qquad \lambda_{r,j} = \lambda_{h,j}, \quad j = 1, \ldots, m_r. \qquad (20)$$

5.2.3 Bottleneck analysis

To identify a bottleneck in a phase-type distribution model, first, we consider the preparation process and the collaboration process individually.

Definition 2. *Under assumptions (i)-(v), step j becomes the bottleneck of process i if*

$$\left| \frac{\partial T}{\partial \lambda_{i,j}} \right| \geq \left| \frac{\partial T}{\partial \lambda_{i,k}} \right|, \quad k \neq j, \quad k, j \in \{1, \ldots, m_i\}, \quad \forall i \in \{h, r, c\}. \quad (21)$$

To identify a bottleneck, we first consider the collaboration process.

Proposition 8. *Under assumptions (i)-(v),* $\forall k \neq j$ *and* $k, j \in \{1, \ldots, m_c\}$*,*

$$\left| \frac{\partial T}{\partial \lambda_{c,j}} \right| \geq \left| \frac{\partial T}{\partial \lambda_{c,k}} \right|, \quad \text{if and only if} \quad \lambda_{c,j} \leq \lambda_{c,k}. \qquad (22)$$

Next, for the human and robot preparation processes, this property still holds, however, a rigorous proof is not available due to lack of closed form analytical expressions. Thus, we again formulate it as a numerical observation after extensive experiments.

Numerical Observation 2. *Under assumptions (i)-(v),* $\forall i \in \{r,h\}, k \neq j$ *, and* $k, j \in \{1,\ldots,m_i\}$ *,*

$$\left|\frac{\partial T}{\partial \lambda_{i,j}}\right| \geq \left|\frac{\partial T}{\partial \lambda_{i,k}}\right| \quad \text{if and only if} \quad \lambda_{i,j} \leq \lambda_{i,k}. \quad (23)$$

To present an example of this fact, consider the parameters shown in (18), and compare $\lambda_{h,1}$ and $\lambda_{h,3}$ under various values. As shown in Table 1, when $\lambda_{h,1} > \lambda_{h,3}$, we always obtain $\left|\frac{\partial T}{\partial \lambda_{h,1}}\right| > \left|\frac{\partial T}{\partial \lambda_{h,3}}\right|$.

The above results indicate that, in each processes, the step with the longest task time becomes the bottleneck in this process. Improving such a task time can lead to larger improvement in system flow time comparing with improving other task times. In addition, to identify the bottleneck, we can only compare the impact of improvement on the step with the longest task time in each process. Thus, we have:

Bottleneck Indicator 3: In the collaborative assembly station with phase-type process times, the step with the longest task time in each processes is the bottleneck of the corresponding process.

Finally, the bottleneck of the whole system is the step which can lead to the largest improvement of overall flow time. Thus, we define

Definition 3. *Under assumptions (i)-(v),* $\forall (k,l) \neq (i,j)$ *,*

$$\left|\frac{\partial T}{\partial \lambda_{i,j}}\right| \geq \left|\frac{\partial T}{\partial \lambda_{k,l}}\right|, \quad i,k, \in \{h,r,c\}, \quad j \in \{1,\ldots,m_i\}, \quad l \in \{1,\ldots,m_k\}. \quad (24)$$

Under Definition 3, using Proposition 8 and Numerical Observation 2, the system bottleneck must be among the three bottlenecks identified in each process. Then, using equation (24), the largest partial derivative can be selected from these three bottlenecks.

5.3 Comparisons

In addition, it is of interest to compare the models with exponential process time (Section 4.1) and phase-type process time (Section 4.2). Through extensive numerical experiments, we obtain

Numerical Observation 3. *Under assumptions (i)-(v), if*

$$\frac{1}{\lambda_i^{\exp}} = \sum_{j=1}^{m_i} \frac{1}{\lambda_{i,j}^{\mathrm{PH}}}, \quad i = h, r, c, \qquad (25)$$

then

$$T^{\exp} \geq T^{\mathrm{PH}}, \quad CV^{\exp} \geq CV^{\mathrm{PH}}, \quad (26)$$

where superscripts "exp" and "PH" indicate exponential and phase-type models, respectively.

An illustration of the numerical observation is presented in Figure 4 for both *T* and *CV*, where parameter $\lambda_h^{\exp} = 0.001$ in the exponential model, and the parameters in the phase-type model are from (18). To ensure the overall task time of human preparation process will be the same in both models, we set constraint

$$\sum_{i=1}^{5} \frac{1}{\lambda_{h,i}^{\mathrm{PH}}} = \frac{1}{\lambda_h^{\exp}}. \qquad (27)$$

When $\lambda_{h,4}^{\mathrm{PH}} = 0.02, \lambda_{h,5}^{\mathrm{PH}} = 0.07$, we vary $\lambda_{h,1}^{\mathrm{PH}}$ and $\lambda_{h,2}^{\mathrm{PH}}$. Then $\lambda_{h,3}^{\mathrm{PH}}$ will be determined due to constraint (27). As one can see from Figure 4, the resulting differences, $T^{\exp} - T^{\mathrm{PH}}$ in expected flow time and $CV^{\exp} - CV^{\mathrm{PH}}$ in CV, are all positive, which imply that the exponential model always provides higher values. In addition, when $\lambda_{h,1}^{PH}$ and $\lambda_{h,2}^{PH}$ are increased, shorter task time and flow time are expected in both phase-type and exponential cases due to constraint (34). Then the difference becomes smaller.

When more than 1 phases are included in each process (note that exponential model can be viewed as a phase-type model with only 1 phase), the process variability will be decreased, which will lead to smaller T and CV. This result indicates that when exponential model is used for performance evaluation, we may obtain the upper bounds (in terms of flow time and its variability) or lower bounds (in terms of throughput), which can be a good reference in designing the collaborative assembly processes.

6 Case Study

To illustrate the applicability of the model and method, a case study at a front panel assembly station in an automotive assembly plant is carried out.

6.1 System Description and Modeling

The following processes are conducted in the assembly station: The cobot moves automatically to the rack, picks up the panel, and transports the panel close to the car body on a moving line. While the cobot is transporting the panel, one operator prepares the necessary fitting in the car body. Then this operator holds the cobot to guide the panel into car body and set unto the right location, and the second operator, on the other side of the vehicle, secures the panel. Afterwards, the cobot is returning to the rack, while the first operator moves to the next vehicle to start preparation, and the second operator continues finishing the assembly tasks on the current vehicle. A graphical illustration of such an assembly station is presented in Figure 5, where the dashed lines represent the movement paths of the cobot and the operator.

To model such a collaborative assembly system, we characterize the activities into the following steps in each process, shown in Figure 6, where λ_{ij} represents the parameter of each step and i and j indicates the process ID and step number, respectively.

- Robot: The robot moves automatically to the rack; picks up the panel; and transports the panel close to the vehicle body on a moving assembly line.
- Human operators: When the robot moves to the rack and transports the panel, Operator 1 prepares necessary fitting in the current vehicle, and Operator 2 finishes assembly in the previous one.
- Collaboration: Operator 1 holds the robot to move the panel into the vehicle, and Operator 2 secures the panel and starts assembly. Then Operator 1 moves the robot out of the vehicle.

Note that although the second operator is working on the previous vehicle while the first operator and cobot start preparing for the current one, we can still count the work of the second operator as a “preparation” one since the operator can only start working on the current vehicle afterwards. Through observations of the operations, time stamps are recorded and the task times are evaluated and shown in Table 2. Note that due to confidentially reason, the data has been modified and is used for illustration only. In addition, exponential distribution is assumed for each subtask time. Thus, taking inverses of task times, λ_{ij}'s are obtained.

6.2 Performance Analysis

Using these parameters, we consider the individual processes of the operators and the cobot, i.e., H_1, H_2, and R, with task times t_{h_1}, t_{h_2}, and t_r, respectively. Introduce process H with $t_h = \max(t_{h_1}, t_{h_2})$. Then the system flow time can be defined as

$$t = \max(t_h, t_r) + t_c = \max(\max(t_{h_1}, t_{h_2}), t_r) + t_c,$$

where

$$m_{h_1} = 2, \quad m_{h_2} = 1, \quad m_r = 3, \quad m_c = 3, \quad \boldsymbol{\alpha}_{h_1} = [1, 0], \quad \alpha_{h_2} = 1, \quad \boldsymbol{\alpha}_r = \boldsymbol{\alpha}_c = [1, 0, 0],$$

$$\boldsymbol{B}_{h_1} = \begin{bmatrix} -\lambda_{h,11} & \lambda_{h,11} \\ 0 & -\lambda_{h,12} \end{bmatrix}, \quad \boldsymbol{B}_{h_2} = -\lambda_{h,21}, \quad \boldsymbol{B}_r = \begin{bmatrix} -\lambda_{r,1} & \lambda_{r,1} & 0 \\ 0 & -\lambda_{r,2} & \lambda_{r,2} \\ 0 & 0 & -\lambda_{r,3} \end{bmatrix},$$

$$\boldsymbol{B}_c = \begin{bmatrix} -\lambda_{c,1} & \lambda_{c,1} & 0 \\ 0 & -\lambda_{c,2} & \lambda_{c,2} \\ 0 & 0 & -\lambda_{c,3} \end{bmatrix}.$$

To evaluate the system performance, first, we obtain parameters m_h, $\boldsymbol{\alpha}_h$, $\boldsymbol{B}_h$ for process H using $H = \max(H_1, H_2)$. From the proof of Theorem 2 in the Supplemental materials, we have

$$m_h = m_{h_1} m_{h_2} + m_{h_1} + m_{h_2} = 5, \quad \boldsymbol{\alpha}_h = [1, 0, 0, 0, 0],$$

$$\boldsymbol{B}_h = \begin{bmatrix} \lambda_{h,11}\lambda_{h,21} & -\lambda_{h,11}\lambda_{h,21} & \lambda_{h,21} & 0 & 0 \\ 0 & \lambda_{h,12}\lambda_{h,21} & 0 & \lambda_{h,21} & \lambda_{h,12} \\ 0 & 0 & -\lambda_{h,11} & \lambda_{h,11} & 0 \\ 0 & 0 & 0 & -\lambda_{h,12} & 0 \\ 0 & 0 & 0 & 0 & -\lambda_{h,21} \end{bmatrix}.$$

Solving $\boldsymbol{B}$ and $\boldsymbol{\alpha}$, and considering processes of operators (H), robot (R), and collaboration (C), from Corollaries 3 and 4, the mean and CV of flow time for the panel assembly process can be calculated.

$$T^{\mathrm{PH}} = 65.8869, \qquad CV^{\mathrm{PH}} = 0.4452.$$

Similarly, for a given T_d, service rate $S^{\mathrm{PH}}(T_d)$ can be derived as shown in (28).

$$
\begin{aligned}
S^{\mathrm{PH}}(T_d) = {} & 1 + 0.0206e^{-\frac{2T_d}{5}} + 346.3643e^{-\frac{21T_d}{100}} - 123.6768e^{-\frac{18T_d}{125}} - 0.0070e^{-\frac{68T_d}{125}} \\
& - 15.6311e^{-\frac{13T_d}{200}} + 6760.5217e^{-\frac{29T_d}{250}} - 2530.9393e^{-\frac{33T_d}{250}} + 1244.44386e^{-\frac{37T_d}{250}} \\
& - 537.2332e^{-\frac{41T_d}{250}} - 0.0240e^{-\frac{93T_d}{200}} + 111.6333e^{-\frac{49T_d}{500}} - 5359e^{-\frac{61T_d}{500}} \\
& - 251.1430e^{-\frac{97T_d}{500}} + 382.0859e^{-\frac{111T_d}{500}} + 0.0176e^{-\frac{249T_d}{500}} - 2.2071e^{-\frac{33T_d}{1000}} \\
& - 122.0238e^{-\frac{83T_d}{1000}} + 350.0769e^{-\frac{99T_d}{1000}} + 347.6321e^{-\frac{111T_d}{1000}} - 436.1801e^{-\frac{181T_d}{1000}} \\
& + 390.8651e^{-\frac{197T_d}{1000}} - 661.2954e^{-\frac{227T_d}{1000}} + 104.6863e^{-\frac{243T_d}{1000}} - 0.0139e^{-\frac{433T_d}{1000}} \\
& + 0.0092e^{-\frac{511T_d}{1000}} - 70.2142te^{-\frac{61T_d}{500}} + 1.
\end{aligned} \tag{28}
$$

To compare with the exponential distribution model, we also obtain the performance measure under 1 phase task time for each process,

$$T^{\exp} = 69.5674, \qquad CV^{\exp} = 0.5414.$$

For a given T_d, service rate $S^{\exp}(T_d)$ is shown in (29).

$$
\begin{aligned}
S^{\exp}(T_d) = {} & 1 + 0.7166e^{-\frac{229T_d}{2000}} - 6.8286e^{-\frac{51T_d}{1250}} - 6.4595e^{-\frac{101T_d}{2500}} - 1.4311e^{-\frac{203T_d}{2500}} \\
& + 19.9621e^{-\frac{239T_d}{5000}} - 3.2966e^{-\frac{333T_d}{10000}} - 1.8456e^{-\frac{737T_d}{10000}} - 1.8175e^{-\frac{741T_d}{10000}}.
\end{aligned} \tag{29}
$$

Both results are verified through simulation experiments. As we can see, the mean flow time and CV in the exponential model are both higher than those in the phase-type model. This is also visible from the flow time distributions. From the derived service rates (see equations (28) and (29)), we can derive the complete distributions of system flow time, shown in Figures 7(a) and (b) for cumulative distribution function (S) and probability density function (s), respectively. The flow times of phase-type and exponential distribution models are depicted by red dashed line and blue solid line, respectively, in both figures. These figures indicate that the exponential model has a higher variability, which is due to higher CV in each process. Particularly, the service rate becomes lower in exponential model when T_d becomes longer. Such results also verify Numerical Observation 3.

6.3 Continuous Improvement

To improve the collaborative assembly process performance by reducing system flow time, bottleneck analysis is carried out to seek the largest potential improvement.

Using Bottleneck Indicator 3, we identify steps R_2 and H_{12} as the bottlenecks in robot and Operator 1 processes, respectively. As C_1 or C_2 have the same task time, they both are bottlenecks in the collaboration process. Operator 2 only has one process, thus H_{21} is the only choice. In addition, we observe that Operator 2 (H_{21}) has a much longer process than others. Thus, it becomes the bottleneck of the whole system. Speeding up to reduce Operator 2's task time or allocating part of Operator 2's task to Operator 1 or the collaboration process, the overall flow time can be decreased. The feasibility of potential time reduction or task redistribution in practice is under investigation.

7 Conclusions

Collaborative robots are becoming prevalent in many manufacturing industries. Effective modeling and analysis of assembly systems with collaborative robots are necessary and important. In this paper, a system-theoretic method is proposed to evaluate flow time of human-robot collaborative assembly processes. Analytical formulas for mean flow time and its variation, and service rate are derived under both exponential and phase-type distributions of process times. System properties are investigated and bottleneck analyses are introduced. A case study at an automotive front panel assembly station is presented to illustrate the applicability of the method. The development of such methods provide quantitative tools for production engineers and managers to design and evaluate tasks in human-robot collaborative processes.

In future work, the following directions can be considered:

- developing methods for multi-stage collaborative assembly processes, where each stage can be either robotic, manual, or collaboration operations. A possible approach is to derive the performance for a two-stage system first, and then extend to multiple stages;
- including not only flow time but also ergonomic and safety measurements in the model, e.g., to also evaluate stress and fatigue indices in each process. The challenge lies in how to integrate the time and ergonomic measurements into one performance index;
- considering cognitive issues in human cobot collaboration activities, which can affect both time and ergonomic performances;
- validating the assumptions of the model through extensive data collection and analysis;
- extending to other random process models, such as delayed exponential distribution, and particularly, including possible robot breakdowns (both run-based and time-based);
- investigating the possibility of real-time reallocation of tasks to minimize waiting times, such as starting joint work earlier when human preparation finishes first;
- introducing multi-objective optimization models for system design to assign tasks to human operators and cobots to minimize flow time and reduce fatigue and injuries, and
- finally, applying models on the factory floor to improve both productivity and ergonomic performances.

References

Altiok, T. and H. G. Perros (1986). Open networks of queues with blocking: Split and merge configurations. *IIE Transactions 18* (3), 251–261.

Bauer, A., D. Wollherr, and M. Buss (2008). Human–robot collaboration: a survey. *International Journal of Humanoid Robotics 5* (01), 47–66.

Bolmsjö, G., F. Danielsson, and B. Svensson (2012). Collaborative robots to support flexible operation in a manufacturing system. In *22nd International Conference on Flexible Automation and Intelligent Manufacturing*, pp. 531–538. Tampere University.

Buzacott, J. A. and J. G. Shanthikumar (1993). *Stochastic models of manufacturing systems*. Pearson College Division.

Chandrasekaran, B. and J. M. Conrad (2015). Human-robot collaboration: A survey. In *SoutheastCon 2015*, pp. 1–8. IEEE.

Chen, F., K. Sekiyama, F. Cannella, and T. Fukuda (2013). Optimal subtask allocation for human and robot collaboration within hybrid assembly system. *IEEE Transactions on Automation Science and Engineering 11* (4), 1065–1075.

Chiang, S.-Y., C.-T. Kuo, J.-T. Lim, and S. Meerkov (2000a). Improvability of assembly systems i: Problem formulation and performance evaluation. *Mathematical Problems in Engineering 6* (4), 321–357.

Chiang, S.-Y., C.-T. Kuo, J.-T. Lim, and S. Meerkov (2000b). Improvability of assembly systems ii: Improvability indicators and case study. *Mathematical Problems in Engineering 6* (4), 359–393.

Collaborative Robots Market Report (2020). https://www.marketsandmarkets.com/Market-Reports/collaborative-robot-market-194541294.html. Accessed: April 30, 2020.

Dalle Mura, M. and G. Dini (2019). Designing assembly lines with humans and collaborative robots: A genetic approach. *CIRP Annals 68* (1), 1–4.

Dallery, Y. and S. B. Gershwin (1992). Manufacturing flow line systems: a review of models and analytical results. *Queueing Systems 12* (1), 3–94.

De Boeck, L. and N. Vandaele (2011). Analytical analysis of a generic assembly system. *International Journal of Production Economics 131* (1), 107–114.

Djuric, A. M., R. Urbanic, and J. Rickli (2016). A framework for collaborative robot (cobot) integration in advanced manufacturing systems. *SAE International Journal of Materials and Manufacturing 9* (2), 457–464.

El Zaatari, S., M. Marei, W. Li, and Z. Usman (2019). Cobot programming for collaborative industrial tasks: An overview. *Robotics and Autonomous Systems 116*, 162–180.

Faccio, M., M. Bottin, and G. Rosati (2019). Collaborative and traditional robotic assembly: a comparison model. *The International Journal of Advanced Manufacturing Technology 102* (5), 1355–1372.

Faccio, M., R. Minto, G. Rosati, and M. Bottin (2020). The influence of the product characteristics on human-robot collaboration: a model for the performance of collaborative robotic assembly. *The International Journal of Advanced Manufacturing Technology 106* (5), 2317–2331.

Fiorini, P. M. and L. Lipsky (2015). Exact analysis of some split-merge queues. *ACM SIGMETRICS Performance Evaluation Review 43* (2), 51–53.

Gershwin, S. B. (1991). Assembly/disassembly systems: An efficient decomposition algorithm for tree-structured networks. *IIE Transactions 23* (4), 302–314.

Gershwin, S. B. (1994). *Manufacturing systems engineering*. Prentice Hall.

Helber, S. and H. Jusić (2004). A new decomposition approach for non-cyclic continuous material flow lines with a merging flow of material. *Annals of Operations Research 125* (1), 117–139.

Inman, R. R. (1999). Empirical evaluation of exponential and independence assumptions in queueing models of manufacturing systems. *Production and Operations Management 8* (4), 409–432.

Jia, Z., L. Zhang, J. Arinez, and G. Xiao (2015). Performance analysis of assembly systems with bernoulli machines and finite buffers during transients. *IEEE Transactions on Automation Science and Engineering 13* (2), 1018–1032.

Ju, F., J. Li, and W. Deng (2016). Selective assembly system with unreliable bernoulli machines and finite buffers. *IEEE Transactions on Automation Science and Engineering 14* (1), 171–184.

Krishnamurthy, A., R. Suri, and M. Vernon (2004). Analysis of a fork/join synchronization station with inputs from coxian servers in a closed queuing network. *Annals of Operations Research 125* (1), 69–94.

Lang, A. and J. L. Arthur (1996). Parameter approximation for phase-type distributions. *Lecture Notes in Pure and Applied Mathematics,* 151–206.

Lasota, P. A., T. Fong, J. A. Shah, et al. (2017). *A survey of methods for safe human-robot interaction*. Now Publishers.

Li, J. (2005). Overlapping decomposition: A system-theoretic method for modeling and analysis of complex manufacturing systems. *IEEE Transactions on Automation Science and Engineering 2* (1), 40–53.

Li, J., D. E. Blumenfeld, N. Huang, and J. M. Alden (2009). Throughput analysis of production systems: recent advances and future topics. *International Journal of Production Research 47* (14), 3823–3851.

Li, J. and S. M. Meerkov (2001). Customer demand satisfaction in production systems: A due-time performance approach. *IEEE Transactions on Robotics and Automation 17* (4), 472–482.

Li, J. and S. M. Meerkov (2005). On the coefficients of variation of uptime and downtime in manufacturing equipment. *Mathematical Problems in Engineering 2005* (1), 1–6.

Li, J. and S. M. Meerkov (2008). *Production systems engineering*. Springer Science & Business Media.

Maganha, I., C. Silva, N. Klement, A. B. dit Eynaud, L. Durville, and S. Moniz (2019). Hybrid optimisation approach for sequencing and assignment decision-making in reconfigurable assembly lines. *IFAC-PapersOnLine 52* (13), 1367–1372.

Manitz, M. (2008). Queueing-model based analysis of assembly lines with finite buffers and general service times. *Computers & Operations Research 35* (8), 2520–2536.

Manitz, M. (2015). Analysis of assembly/disassembly queueing networks with blocking after service and general service times. *Annals of Operations Research 226* (1), 417–441.

Manitz, M. and H. Tempelmeier (2012). The variance of inter-departure times of the output of an assembly line with finite buffers, converging flow of material, and general service times. *OR Spectrum 34* (1), 273–291.

Marvel, J. A. (2014). Collaborative robots: A gateway into factory automation. *National Institute of Standards and Technology.*

Matta, A., Y. Dallery, and M. Di Mascolo (2005). Analysis of assembly systems controlled with kanbans. *European Journal of Operational Research 166* (2), 310–336.

Mokhtarzadeh, M., R. Tavakkoli-Moghaddam, B. Vahedi-Nouri, and A. Farsi (2020). Scheduling of human-robot collaboration in assembly of printed circuit boards: a constraint programming approach. *International Journal of Computer Integrated Manufacturing 33* (5), 460–473.

Neuts, M. F. (1994). *Matrix-geometric solutions in stochastic models: an algorithmic approach*. Courier Corporation.

Papadopolous, H., C. Heavey, and J. Browne (1993). *Queueing theory in manufacturing systems analysis and design*. Springer Science & Business Media.

Papadopoulos, C. T., J. Li, and M. E. O'Kelly (2019). A classification and review of timed markov models of manufacturing systems. *Computers & Industrial Engineering 128*, 219–244.

Papadopoulos, H. and C. Heavey (1996). Queueing theory in manufacturing systems analysis and design: A classification of models for production and transfer lines. *European Journal of Operational Research 92* (1), 1–27.

Pearce, M., B. Mutlu, J. Shah, and R. Radwin (2018). Optimizing makespan and ergonomics in integrating collaborative robots into manufacturing processes. *IEEE Transactions on Automation Science and Engineering 15* (4), 1772–1784.

Shih, N. (2005). Estimating completion-time distribution in stochastic activity networks. *Journal of the Operational Research Society 56* (6), 744–749.

Tancrez, J.-S. (2020). A decomposition method for assembly/disassembly systems with blocking and general distributions. *Flexible Services and Manufacturing Journal 32* (2), 272–296.

Tsarouchi, P., A.-S. Matthaiakis, S. Makris, and G. Chryssolouris (2017). On a human-robot collaboration in an assembly cell. *International Journal of Computer Integrated Manufacturing 30* (6), 580–589.

Tsimashenka, I. and W. J. Knottenbelt (2011). Reduction of variability in split-merge systems. In *ICCSW*, pp. 101–107. Citeseer.

Villani, V., F. Pini, F. Leali, and C. Secchi (2018). Survey on human–robot collaboration in industrial settings: Safety, intuitive interfaces and applications. *Mechatronics 55*, 248–266.

Viswanadham, N. and Y. Narahari (1992). *Performance Modeling of Automated Manufacturing Systems-*. Prentice Hall.

Weckenborg, C., K. Kieckhäfer, C. Müller, M. Grunewald, and T. S. Spengler (2019). Balancing of assembly lines with collaborative robots. *Business Research,* 1–40.

Zanchettin, A. M., N. M. Ceriani, P. Rocco, H. Ding, and B. Matthias (2015). Safety in human-robot collaborative manufacturing environments: Metrics and control. *IEEE Transactions on Automation Science and Engineering 13* (2), 882–893.

Zhao, C. and J. Li (2014). Analysis and improvement of multi-product assembly systems: An application study at a furniture manufacturing plant. *International Journal of Production Research 52* (21), 6399–6413.

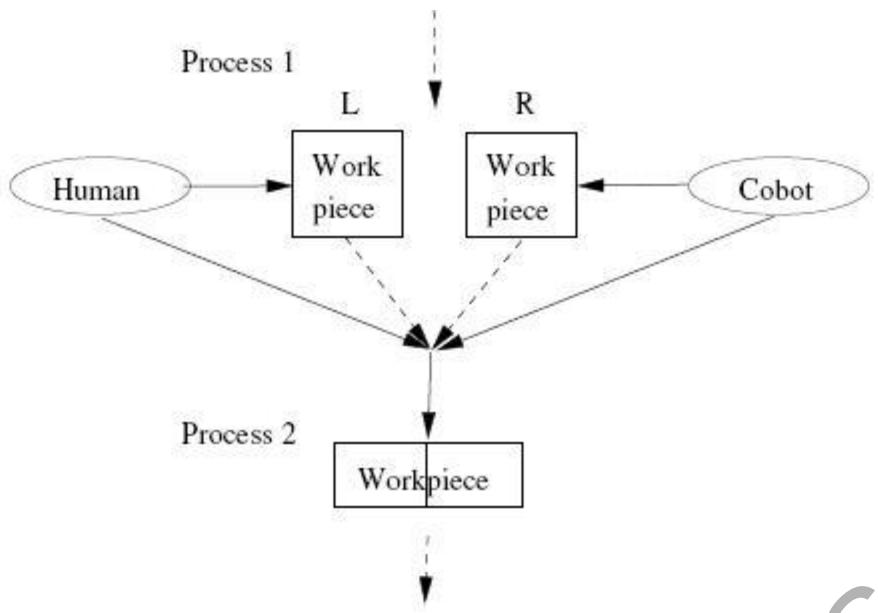


Fig. 1 A collaborative scenario in an assembly system

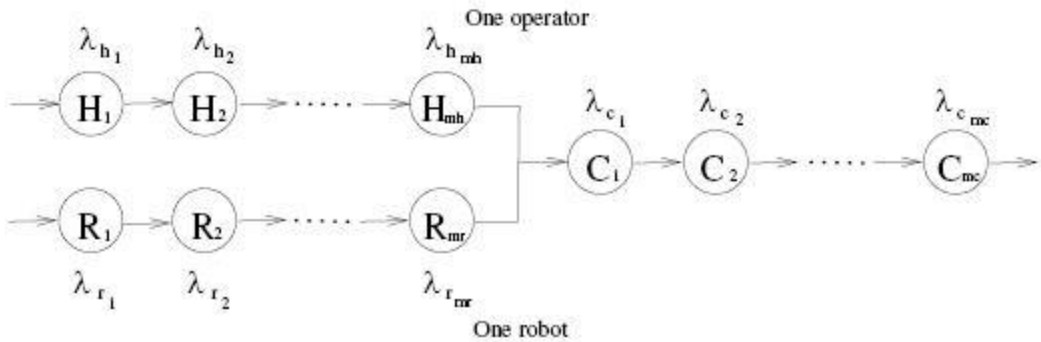


Fig. 2 A collaborative assembly system

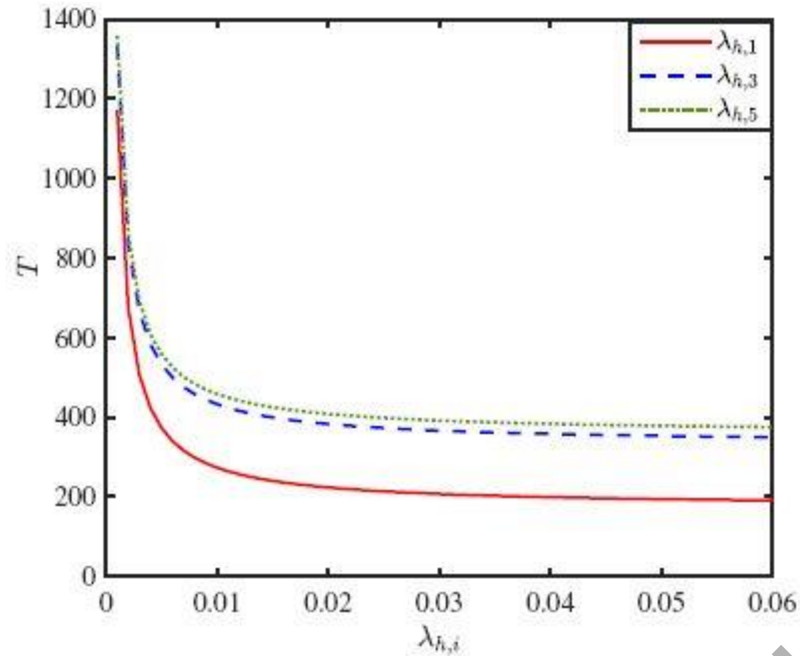
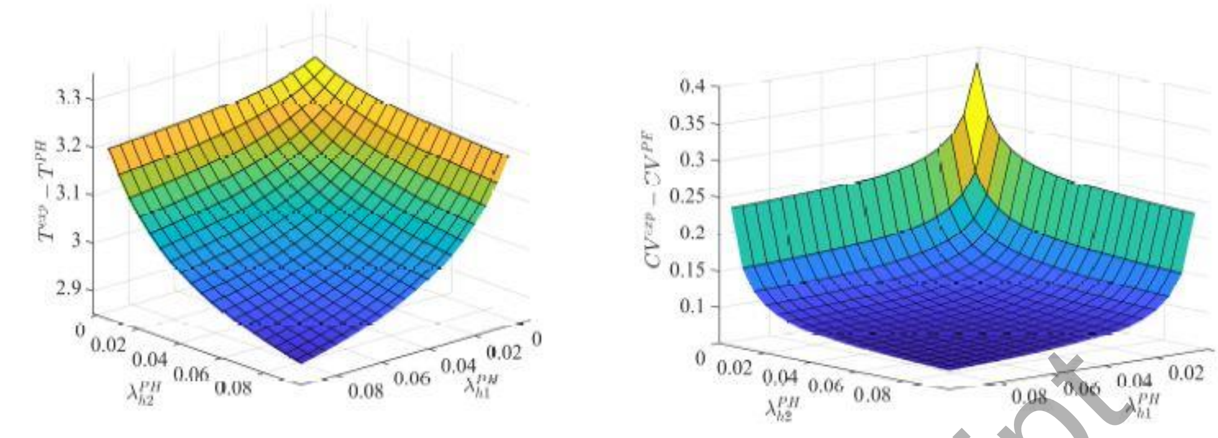


Fig. 3 Monotonicity



(a) Comparison of expected flow time (b) Comparison of CV

Fig. 4 Comparisons between exponential and phase-type models

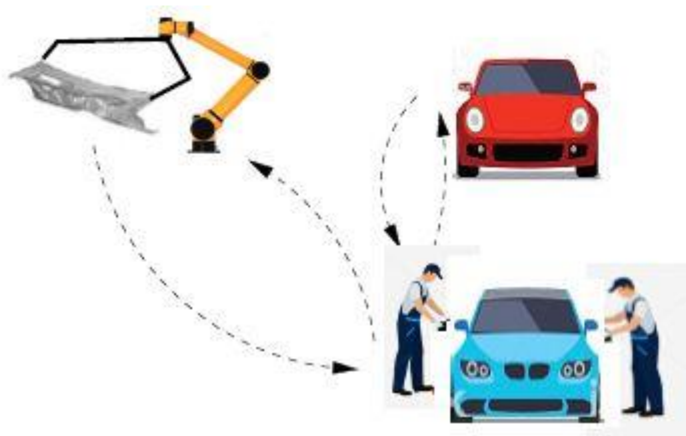


Fig. 5 Panel assembly system model

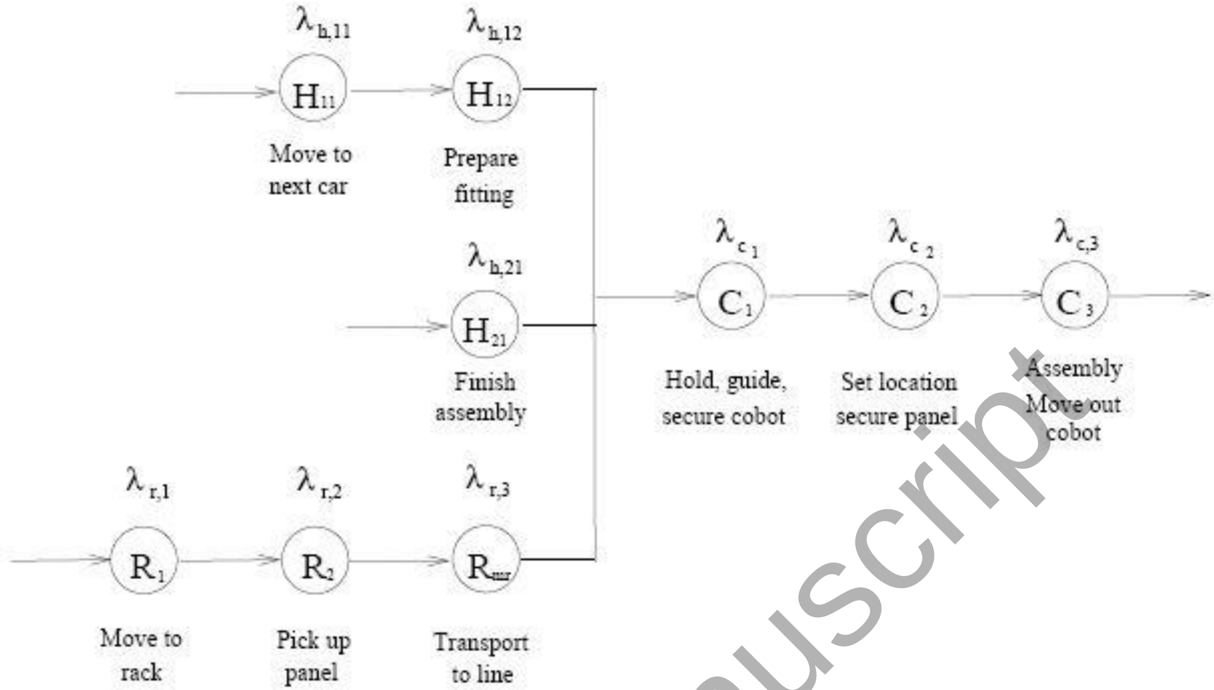


Fig. 6 Panel assembly system model

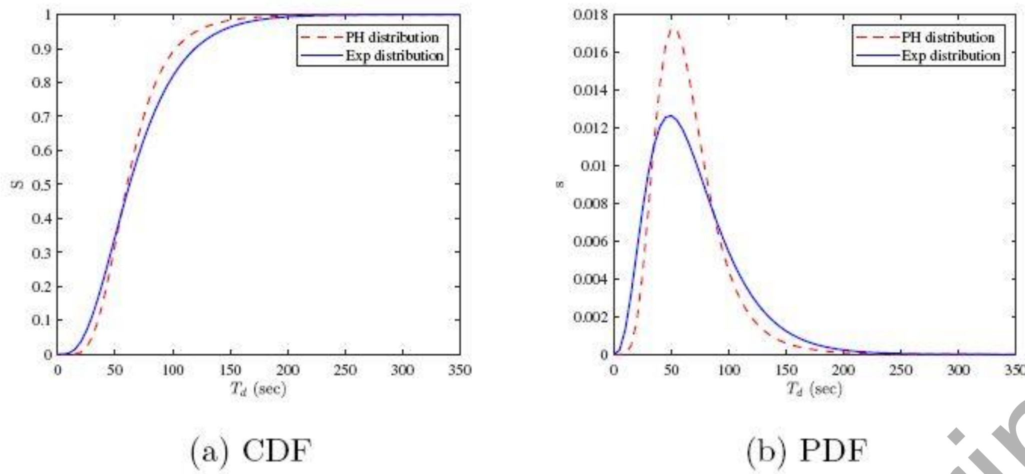


Fig. 7 CDF and PDF of system flow time

Table 1 Examples of Numerical Observation 2

$\lvert\frac{\partial T}{\partial \lambda_{h,1}}\rvert$		$\lambda_{h,1}$				
$-\lvert\frac{\partial T}{\partial \lambda_{h,3}}\rvert$		0.05	0.10	0.15	0.20	0.25
	0.05	0	-283.13	-329.87	-343.99	-349.46
	0.10	283.13	0	-47.29	-62.00	-68.00
$\lambda_{h,3}$	0.15	329.87	47.29	0	-14.85	-21.01
	0.20	343.99	62.00	14.86	0	-6.20
	0.25	349.46	68.00	21.01	6.20	0

Table 2 Task description and work time (sec)

	R_1:	**Move to rack automatically**	**10.1**	$\lambda_{r,1}: 0.099$
Robot	R_2:	Pick up panel	12.1	$\lambda_{r,2}: 0.0826$
	R_3:	Transport panel to the line	2.5	$\lambda_{r,3}: 0.4$
	H_{11} (Op. 1):	Walk to the next car	9	$\lambda_{h,11}: 0.1111$
Human	H_{12} (Op. 1):	Prepare fitting	15.5	$\lambda_{h,12}: 0.0645$
	H_{21} (Op. 2):	Finish assembly of the previous car	30	$\lambda_{h,21}: 0.0333$
	C_1:	Hold, guide, and secure cobot	8.2	$\lambda_{c,1}: 0.1220$
Collaboration	C_2:	Set location and secure panel	8.2	$\lambda_{c,2}: 0.1220$
	C_3:	Assembly and move out cobot	4.5	$\lambda_{c,3}: 0.2222$