

Minimum Window Subsequence

Given strings S and T , find the minimum (contiguous) **substring** W of S , so that T is a **subsequence** of W .

If there is no such window in S that covers all characters in T , return the empty string `""`. If there are multiple such minimum-length windows, return the one with the left-most starting index.

Example 1:

Input:

$S = \text{"abcdebddde"}, T = \text{"bde"}$

Output: `"bcde"`

Explanation:

`"bcde"` is the answer because it occurs before `"bdde"` which has the same length.

`"deb"` is not a smaller window because the elements of T in the window must occur in order.

Note:

- All the strings in the input will only contain lowercase letters.
- The length of S will be in the range `[1, 20000]`.
- The length of T will be in the range `[1, 100]`.

Solution 1

For substring $S[0, i]$ and $T[0, j]$,
 $dp[i][j]$ is starting index k of the shortest postfix of $S[0, i]$, such that $T[0, j]$ is a subsequence of $S[k, i]$.
Here $T[0] = S[k]$, $T[j] = S[i]$. Otherwise, $dp[i][j] = -1$.

The goal is the substring with length of $\min(i - dp[i][n-1])$ for all $i < m$, where m is $S.size()$ and n is $T.size()$

Initial condition: $dp[i][0] = i$ if $S[i] = T[0]$, else -1

Equations: If $S[i] = T[j]$, $dp[i][j] = \max(dp[k][j-1])$ for all $k < i$; else $dp[i][j] = -1$;

$O(mn)$ space 82 ms

```
class Solution {
public:
    string minWindow(string S, string T) {
        int m = S.size(), n = T.size();
        vector<vector<int>> dp(n, vector<int>(m, -1));
        for (int i = 0; i < m; i++)
            if (S[i] == T[0]) dp[0][i] = i;
        for (int j = 1; j < n; j++) {
            int k = -1;
            for (int i = 0; i < m; i++) {
                if (k != -1 && S[i] == T[j]) dp[j][i] = k;
                if (dp[j-1][i] != -1) k = dp[j-1][i];
            }
        }
        int st = -1, len = INT_MAX;
        for (int i = 0; i < m; i++) {
            if (dp[n-1][i] != -1 && i - dp[n-1][i] + 1 < len) {
                st = dp[n-1][i];
                len = i - dp[n-1][i] + 1;
            }
        }
        return st == -1? "":S.substr(st, len);
    }
};
```

$O(m)$ space 53 ms

```

class Solution {
public:
    string minWindow(string S, string T) {
        int m = S.size(), n = T.size();
        vector<int> dp(m, -1);
        for (int i = 0; i < m; i++)
            if (S[i] == T[0]) dp[i] = i;
        for (int j = 1; j < n; j++) {
            int k = -1;
            vector<int> tmp(m, -1);
            for (int i = 0; i < m; i++) {
                if (k != -1 && S[i] == T[j]) tmp[i] = k;
                if (dp[i] != -1) k = dp[i];
            }
            swap(dp, tmp);
        }
        int st = -1, len = INT_MAX;
        for (int i = 0; i < m; i++) {
            if (dp[i] != -1 && i-dp[i]+1 < len) {
                st = dp[i];
                len = i-dp[i]+1;
            }
        }
        return st == -1? "":S.substr(st, len);
    }
};

```

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Solution 2

```
class Solution {
public:
    string minWindow(string s, string t) {
        int ns = s.size(), nt = t.size();
        int dp[ns+1][nt+1] = {};
        const int mxx = ns + 1;
        for (int i = 0; i <= ns; ++i) {
            for (int j = 1; j <= nt; ++j) {
                dp[i][j] = mxx;
                if (i) {
                    dp[i][j] = min(dp[i][j], 1 + dp[i-1][j]);
                    if (s[i-1] == t[j-1]) dp[i][j] = min(dp[i][j], 1 + dp[i-1][j-1]);
                }
            }
        }

        int ans = ns + 1, x = -1;
        for (int i = 0; i <= ns; ++i) if (dp[i][nt] < ans) {
            x = i;
            ans = dp[i][nt];
        }

        if (x < 0) return "";
        return s.substr(x-ans, ans);
    }
};
```

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Solution 3

```
class Solution {
    public String minWindow(String S, String T) {
        String output = "";
        int minLen = 20001;
        for (int i = 0; i <= S.length() - T.length(); i++) {
            while (i < S.length() && S.charAt(i) != T.charAt(0)) {
                i++;
            }
            int l = find(S.substring(i, Math.min(i + minLen, S.length())), T);
            if (l != -1 && l < minLen) {
                minLen = l;
                output = S.substring(i, i + l);
            }
        }
        return output;
    }

    private int find(String S, String T) {
        for (int i = 0, j = 0; i < S.length() && j < T.length(); i++) {
            if (S.charAt(i) == T.charAt(j)) {
                j++;
            }
            if (j == T.length()) {
                return i;
            }
        }
        return -1;
    }
}
```

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