Theory of Computation

Mohammed Fahad

July 7, 2025

1 Why TOC?

- It helps us to understand the limits of what computer can do and how to model computation using mathematics

Q1: What is the motivation for studying theory behind computation? OR Needs of TOC?
A:

- Understanding the capability of a computer
- To find steps to solve a problem
- Increase efficiency while doing a task

Q2: List the problems that cannot be solved by a computer.
A:

1. Ethical problems. Eg: Self-driving car deciding to save the driver/passenger or the pedestrian
2. Generating truly original art of emotion

Automaton (pl.: Automata)

A simplified mathematical model of a machine (digital computer). It

- Accepts input
- Produces output
- Nay have some temporary storage
- can make descisions in transforming the input into the output

Q1: Why study computability and theory?
A:

- It helps to answer: "Can this be solved by a computer?"

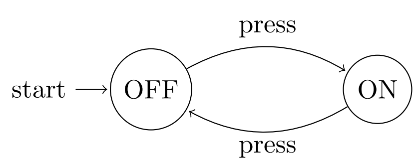
- Understand the principles behind algorithms and programs
- Explore the boundary between what is possible and impossible in computing

1.1 Need for mathematical modelling

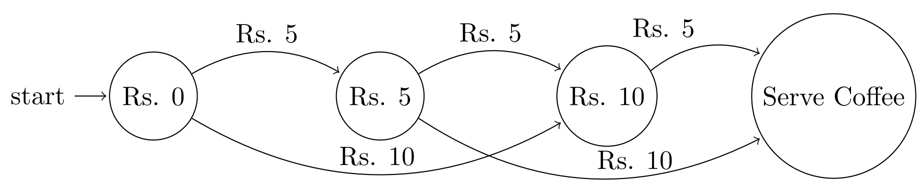
- Computers work on rules & logic
- We can represent computers using abstract models
- These models helps us study complex behaviour in a simplified way

2 Introduction to finite automata

1. ON/OFF Switch:



2. Coffee vending machine (Inputs: Rs. 5 & Rs. 10 — Rs. 15 for one coffee):



3 Formal Language

A formal language is an abstraction of the general characteristics of programming langauges.
A formal language consists of set of symbols and some rules of formation by which these symbols can be combined into entities called sentences.

4 Central concepts of automata theory

4.1 Alphabets

It a finite, non-empty set of symbols. Alphabets are represented by 'Σ'.

Binary alphabets can be represented as:

$$\Sigma = \{0, 1\}$$

Set of lowercase letters:

$$\Sigma = \{a, b, c, ..., z\}$$

4.2 Strings

A string is a finite seeunce of symbols chosen from some alphabets.
Eg: Let $\Sigma = \{0, 1\}$ be the alphabet.

Examples of strings in Σ:

$$0,\ 1,\ 00,\ 01,\ 10,\ 11,\ 000,\ 010,\ \ldots$$

4.2.1 Length of a string

The number of occurances of symbols in the string.

$$\begin{aligned} \text{Length one:} \quad & 0,\ 1 \\ \text{Length two:} \quad & 00,\ 01,\ 10,\ 11 \end{aligned}$$

The std. notation for length of a string w is $\|w\|$

4.2.2 Empty string (ε)

A string with zero occurances OR string with length '0'

4.2.3 Σ^*

Set of all strings over an alphabet

4.2.4 Σ^+

Set of all strings excluding empty string over an alphabet

$$\Sigma^* = \Sigma^0 \cup \Sigma^+$$

4.3 Powers of an alphabet

If Σ is an alphabet, Σ^k is the set of strings with length 'k', each of whose symbols is in 'Σ'.

$$\begin{aligned} &\Sigma^3 = \{000,\ 001,\ 010,\ 011,\ 100,\ 101,\ 110,\ 111\} \\ &\Sigma^0 = \{\varepsilon\} \end{aligned}$$

NOTE: $\Sigma \neq \Sigma^1$, Their definitions differentiates them.

5 Concatenation of strings

Let x and y be strings, then xy means combining both x and y.

$$\text{i.e., if } x = 01010 \text{ and } y = 110,$$
$$xy = 01010110$$

$$|x| = m \text{ and } |y| = n,$$
$$|xy| = m + n$$

For any string w, then the equation,

$$\varepsilon w = w\varepsilon = w$$

ie, ε is the identity of Concatenation

6 Languages

A set of strings all of which are chosen from Σ^*. If Σ is an alphabet, then $L \subset \Sigma^*$. Eg:

1. Set of all strings consisting of n 0s followed by n 1s, $n \geq 0$

$$L = \{\varepsilon, 01, 0011, 000111, ...\}$$

2. Set of all strings having equal number of 0s and 1s,

$$L = \{\varepsilon, 01, 0011, 0101, ...\}$$

NB: Σ^* is a language for any alphabet, Σ

$$L = \{\} \rightarrow \text{Empty language, } \varnothing$$
$$L = \{\varepsilon\} \rightarrow \text{Language containing the empty string}$$

There are 2 types of Languages:

1. **Infinte languages**. Eg: $\{0, 01, 001, 0001, 00001, ...\}$
2. **Finte langauges**. Eg: $\{\varepsilon, a, b, ab, ba\}$

7 Set-formers as a way to define language

$$\{w|w \text{ consists of equal number of 0s and 1st}\}$$

8 Problems

Problem is the question of deciding whether a given string is a member of some particular language.

9 Automata

An automaton is an abstract model of a digital computer.

9.1 Key components of automata

1. Input file
2. Control Ubit
3. storage
4. Output

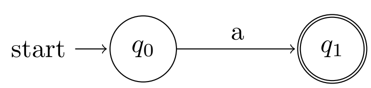
9.2 Types of automata

1. Deterministic (DFA): One move per configuration (Predictable)
2. Nondeterministic (NFA): Multiple possible moves

Then there is automata like:

- **Acceptor:** Says “yes” or “no” for an input.

  This simple automaton accepts the string ‘a‘:



- **Transducer:** Produces an output string based on input.