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Memetic niching-based evolutionary algorithms for solving nonlinear equation system

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ABSTRACT

In numerical computation, finding multiple roots of nonlinear equation systems (NESs) in a single run is a fundamental and difficult problem. Recently, evolutionary algorithms (EAs) have been applied to solve NESs. However, due to the diversity preservation mechanism that EAs use, the accuracy of the roots may be reduced. To remedy this drawback, we propose a generic framework of memetic niching-based EA, referred to as MENI-EA. The main features of the framework are: i) the numerical method for a NES is integrated into an EA to obtain highly accurate roots; ii) the niching technique is employed to improve the diversity of the population; iii) different roots of the NESs are located simultaneously in a singe run; and iv) different numerical methods and different niching techniques can be used in the framework. To evaluate the performance of our approach, thirty NESs were chosen from the literature as the test suite. Experimental results show that the proposed approach is capable of yielding promising performance for different NESs in both the root ratio and success rate.

1. Introduction

Nonlinear equation systems (NESs) exist in many real-world applications (Facchinei & Kanzow, 2007; Kastner, 2008; Wales & Scheraga, 1999). For example, the mission design for Geosynchronous bistatic synthetic aperture radar (GEO-BiSAR) to obtain the desired imaging performance can be converted into a NES. The results of the mission design can be directly used to guide the pilot to fly for the required imaging performance (Sun et al., 2015). Generally, most NESs contain multiple roots. Fig. 1 describes a basic NES problem that has six roots (Liao, Gong, Yan, Wang, & Hu, 2018). It can be seen that the roots are randomly distributed in the search area. Multiple roots represent diverse optimal alternatives that can be provided to decision-makers to make appropriate decisions (Li, Epitropakis, Deb, & Engelbrecht, 2017). The goal of solving a NES is to locate multiple roots in a single run. However, it is challenging work in numerical computation.

Several numerical methods, such as the Newton-type method (Darvishi & Barati, 2007) and the iterative and recursive methods (Knoll & Keyes, 2004), have been presented to deal with NES problems. Although these methods bring significant performance in solving NESs, they have some drawbacks. For example, most them are single-point based algorithms, which cannot locate multiple roots in a single run. In addition, the choice of initial guess influences the performance of numerical methods. If an initial guess is far away from the optimal solution, it is hard to find the exact roots with the numerical method. However, the selection of the proper initial guess is trivial work.

Evolutionary algorithms (EAs) are a class of methods to handle a variety of optimization problems (Aslantas & Kurban, 2010; Civicioglu & Besdok, 2019; Yildizdan & Ömersss Kaan Baykan, 2020). It is natural to employ EAs to locate the roots of NESs (Abdollahi, Isazadeh, & Abdollahi, 2013; Zhang, Wan, & Fan, 2017a). EAs are insensitive to the shapes of objective functions such as non-convexity and discontinuity. Thus, using EAs to solve NESs has received widespread attention during the last few decades. However, most EAs are the single-point method, in which locating multiple roots in a single run is a challenge.

Several niching techniques have been introduced to EAs to locate multiple optima (Qu, Suganthan, & Liang, 2012). Niching techniques can divide the whole population into diverse subpopulations and thereby maintain population diversity. Classical niching methods include: crowding (Thomsen, 2004), speciation (Li, 2005) and fitness share (Goldberg & Richardson, 1987). However, it is possible to obtain a lower precision root by directly using niching-based EAs to solve NESs.

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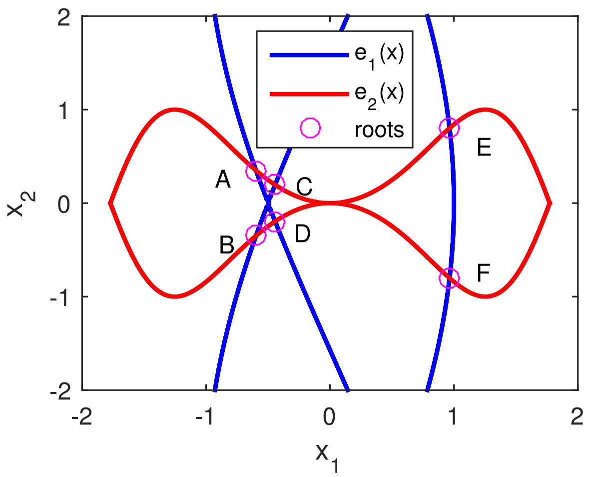

Fig. 1. An example of a NES problem with multiple roots, where the circles represent the roots of the problem.

In this paper, we propose a general framework of memetic niching-based EAs, referred to as MENI-EA, to solve NESs. In MENI-EA, a memetic algorithm (MA) is selected as the baseline algorithm. The reason is that MAs have been successfully applied to a wide range of application domains (Chen, Ong, Lim, & Tan, 2011; Demirel & Deveci, 2017; Deveci & Demirel, 2018; Ghosh, Begum, Sarkar, Chakraborty, & Maulik, 2019; Gonzalez-Sanchez, Vega-Rodríguez, & Santander-Jiménez, 2019). Generally, the numerical method for NESs endows a MA with favorable exploitation ability and the ability to locates high-quality roots during the run. However, previous studies mainly focus on searching for one root (Luo, Tang, & Zhou, 2008). In contrast, MENI-EA adopts a niching-based EA that enables parallel searches for multiple roots. Moreover, MENI-EA is a generic framework, in which different niching-based EAs and different numerical methods can be easily integrated. To evaluate the performance of the method, extensive experiments were carried out on 30 NESs with diverse characteristics, including being influenced by different numerical methods, the effect of different niching-based EAs, comparison with other state-of-the-art methods, comparison with different parameters, and discussions on different local search methods.

The primary contributions of this paper are as follows:

- The core contribution is that numerical methods for NESs and niching-based EAs are combined to solve NESs. To the best of our knowledge, this is the first attempt to combine the numerical methods with niching-based EAs to locate multiple roots of NESs.
- A general framework, MENI-EA, is proposed, where different numerical methods and different niching-based EAs can be easily used.

The rest of the paper is organized as follows. Section 2 briefly reviews the related work for solving NESs. In Section 3, the proposed MENI-EA is described in detail. In Section 4, extensive experiments and comprehensive analysis are conducted to evaluate the performance of our approach. In Section 5, the effects of parameter settings are discussed. Finally, the conclusion is given in Section 6.

2. Related work

2.1. Problem statement

Generally, a NES can be written as:

$$\mathbf{e}(\mathbf{x}) = \begin{cases} e_1(x_1, x_2, \dots, x_m) & = & 0 \\ e_2(x_1, x_2, \dots, x_m) & = & 0 \\ & \vdots & \\ e_n(x_1, x_2, \dots, x_m) & = & 0 \end{cases} \tag{1}$$

where n is number of equations; $\mathbf{x} = (x_1, x_2, \dots, x_m)$ is an m-dimensional decision vector; $\mathbf{x} \in \mathbb{S}$, and $\mathbb{S} \subseteq \mathbb{R}^m$ represents the search space. Generally,

$\mathbb{S} = [\underline{x}_j, \overline{x}_j]^m$,

where $j = 1, \cdots, m$, $\underline{x}_j$ and $\overline{x}_j$ are the lower and upper bounds of x_j, respectively.

Before applying the optimization algorithm to solve a NES, it is usually transformed into a minimization problem as follows:

$$\text{minimize} \quad f(\mathbf{x}) = \sum_{i=1}^{n} e_i^2(\mathbf{x}). \tag{2}$$

Subsequently, solving NES is equivalent to locating the global minimizers of the transformed optimization problem in (2).

2.2. Classic niching methods

In this subsection, two classical niching methods are briefly introduced.

2.2.1. Crowding

One classic niching method is crowding, which has been applied to multimodal problems (Thomsen, 2004). This approach allows competition between similar individuals in a population for limited resources. In general, the similarity is measured by the distance between individuals. The algorithm compares the offspring with the nearest individual in the current population. If the offspring is a superior individual, the nearest individual will be replaced. Although crowding is simplicity, replacement error is the main disadvantage.

2.2.2. Speciation

Speciation is a commonly used concept to deal with multimodal optimization problems (Li, 2005). This approach relies primarily on a distance parameter *rs*. Species seed and the individuals that fall within *rs* from the species seed form a species. Species seed represents the center of the species. In this way, the whole population is divided into different sub-populations. Although speciation can preserve population diversity, setting a reasonable *rs* remains a thorny issue.

2.3. Evolutionary algorithms for NESs

Because they are so effective at solving optimization problems, it is natural to use EAs to solve NESs. Wang used differential evolution to locate the root of NESs (Wang, 2010). Raja et al. designed a new approach which used an evolutionary computational technique based on variants of a genetic algorithm to solve NESs. Numerical experiments have been performed to validate the convergence, accuracy and robustness (Raja, Sabir, Mehmood, Al-Aidarous, & Khan, 2015). In the same way, Abdollahi et al. used imperialist competitive algorithm to solve NESs and presented some famous problems to demonstrate the efficiency of this algorithm (Abdollahi et al., 2013). Zhang et al. employed the niche strategy to improve the capability of cuckoo search algorithm for solving NESs (Zhang et al., 2017a). Chao et al. proposed a maximum entropy harmony search algorithm for the root (Chao-Yan, Lai, & Zhou, 2011). Zhou et al. designed a hybrid approach based on a combination of differential evolution and an invasive weed optimization algorithm to locate the root (Zhou, Luo, & Chen, 2013). Wu et al. introduced the Metropolis rule into the social emotional optimization algorithm in order to escape local optima (Wu, Cui, & Liu, 2011). Wang and Kang proposed a fast and elitist parallel evolutionary algorithm to determine the root of NESs, where multi-parent crossover and elite-preserve strategy were exploited to share the information of population and improve convergence of the algorithm (Wu & Kang, 2003). Mo et al. developed a novel and efficient approach by combining a conjugate direction method with particle swarm optimization to handle high-dimensional optimization problems and successfully solved nonlinear benchmark models (Mo, Liu, & Wang, 2009). However, most of these methods can only find one root. Liao, Gong, Wang, Yan, and Hu (2019) presented a decomposition based differential evolution (DE) with reinitialization for NESs.

2.4. Other approaches for NESs

In recent years, several methods have been developed to locate multiple roots of NESs. These methods can be divided into three categories:

- Repulsion-based methods: The repulsion technique can generate repulsive regions near the found roots and be an effective method for locating multiple roots. Numerous methods based on repulsion techniques have been proposed to solve NESs (Hirsch, Pardalos, & Resende, 2009; Pourjafari & Mojallali, 2012; Ramadas, Fernandes, & Rocha, 2014).
- Clustering-based methods: The clustering technique groups similar individuals into different sub-populations, thereby obtaining population diversity. Several clustering-based methods have been developed to solve NESs. For example, Pourjafari and Mojallali introduced a new method, which used invasive weed optimization and clustering to locate all roots of NESs (Pourjafari & Mojallali, 2012). Arek et al. developed the clustering-based Minfinder to solve NESs (Arek, George, Ron, Charles, & Katherine, 2010). Sacco and Henderson presented a hybrid metaheuristic with fuzzy clustering means to solve the problems (Sacco & Henderson, 2011).
- Multi-objective optimization-based methods: The main task of a multi-objective optimization problem is to find a set of optimal roots, which is similar to locating numerous roots of NESs. Thus, several approaches have been presented to deal with NESs by using multi-objective optimization-based methods. For example, Grosan et al. transformed a NES into a multi-objective optimization problem for locating multiple roots (Grosan & Abraham, 2008). Subsequently, Song et al. developed a bi-objective transformation technique (MONES) to deal with NESs (Song, Wang, Li, & Cai, 2015). Gong et al. designed the weighted bi-objective transformation technique (A-WeB) for NESs (Gong, Wang, Cai, & Yang, 2017).

3. Our approach

In this section, the motivations of our approach are illustrated firstly. Section 3.2 discusses four issues in memetic algorithms. Then, five numerical methods are introduced in Section 3.3. Finally, in Section 3.4, the framework (MENI-EA) is given in detail.

3.1. Motivation

Maintaining population diversity is beneficial for finding multiple roots since the optimization algorithm can seek different promising regions in the search space. The niching technique which has been incorporated into EAs to find global and local optima of multi-modal problems, is the common diversity-preserving mechanism. Li et al. (2017). However, the niching technique may obtain low-quality roots when directly using them to deal with NESs. In contrast, numerical methods can obtain high-quality roots. However, they are single point-based methods and need to give the initial guess in advance. Previous studies have successfully applied MAs to a wide range of fields, but it is still rare to use the combination of niching-based EAs and numerical methods to locate multiple roots of NES.

Motivated by these considerations, we propose the memetic niching-based EA (MENI-EA) for solving NESs. In MENI-EA, the niching technique can maintain multiple groups within the whole population. As the algorithm runs, individuals gradually converge toward different subregions which may contain the optimal solutions. If there are individuals that meet the corresponding condition, they can be regarded as initial guesses. The numerical method is then exploited to refine them and obtain the exact roots. Consequently, different methods when combined together in a synergistic manner can greatly enhance the performance of solving NESs.

3.2. Issues in memetic algorithm

Tang et al. put forward to four issues that must be addressed when using local search methods in memetic algorithms for optimization problems (Tang, Meng, & Ong, 2006). In the same way, these four issues are also critical to locate multiple NESs roots: i) How often should local learning be applied, *i.e.*, local search frequency? ii) On which solutions should the local learning be applied? iii) How long should the local learning be run, *i.e.*, local search intensity? iv) Which local learning procedure or local search should be used? Next, we briefly discuss how to deal with these issues:

- First, the frequency of using local search should be reasonable. If the local search is applied broadly at the beginning stage, it will cause the waste of computational resources. In addition, several individuals may be far away from the roots. If the local search method is exploited directly to refine the roots, it may result in trapping in local optima.
- Secondly, individuals that meet certain conditions, such as the fitness value being less than ϵ, can be selected as initial guesses for local optimization. ϵ is a fixed small positive number. If the fitness value is greater than ϵ, the individual is considered unpromising and cannot be selected as initial guess.
- Thirdly, if the fitness value is less than ξ, the individual is considered as a candidate root, and the local search procedure will terminate. ξ is a fixed small positive number. In addition, if the maximal number of function evaluations is reached, then the procedure ends.
- Finally, since we mainly focus on solving NESs, in this work, numerical methods for NESs instead of local search are chosen to refine the initial guesses. In the following subsection, these numerical methods are briefly described.

3.3. Numerical methods

In this work, five numerical methods are used in the proposed framework. These are as follows:

1) Trust-Region with DogLeg (TRD). TRD is derives from the Powell dogleg method (Powell, 1968). The critical feature of this method is to use the Powell dogleg procedure for minimizing the trust-region subproblem. It is an efficient optimization method.

2) Trust-Region-Reflective (TRR). TRR is a subspace trust-region method based on the interior-reflective Newton method

(Coleman & Li, 1996). In TRR, the optimum values can be obtained by means of several termination tolerances and the iterations number in the optimization process. The termination tolerances may be some fixed values and the iterations number can also vary. TRR is a simple and powerful method, which is employed to solve nonlinear minimization problems.

3) Levenberg-Marquardt (LM). LM was first created by Levenberg, and developed by Marquardt (Ranganathan, 2013). It is interpolates between the Gauss-Newton method (GNM) and the gradient decent method (GDM). With regard to robustness, LM performs better than the GNM, which illustrates that such an approach can locate a root even if the start point is far away from the minimum. Additionally, LM can deal with the ill-condition problem and is expert in handling nonlinear problems.

4) Homotopy Continuation (HC). HC is part of the family of continuation methods (Alexander & Yorke, 1978). It finds a solution of the problem by first transforming a hard problem into a much simpler one, and then the simpler problem gradually deforms into the original problem. HC is an effective method for finding the minimum optimization function.

5) Newton Method. Newton's method (Sorensen, 1982) is a classical numerical method and is considered as a powerful method for solving the equation $f(\mathbf{x}) = 0$. One of the advantages is that it is not too complicated in form. Thus, Newton's method can be used to deal with a variety of problems.

There are two reasons for choosing these methods: i) these methods are classical and representative numerical methods; ii) they are also expert in dealing with nonlinear problems. Hence, we choose these numerical methods to improve the performance of the proposed algorithm.

3.4. The proposed framework: MENI-EA

In this work, MENI-EA is proposed to locate multiple roots of NESs in a single run. The reasons are three-fold: i) the niche technique can maintain population diversity and provide the initial guesses for the numerical method; ii) numerical methods can improve the accuracy of initial guesses; iii) EAs have been used to solve NESs and have shown their promising performance.

The framework of our presented MENI-EA is illustrated in Algorithm 1, where *NP* is the population size; *maxFEs* is the maximal number of function evaluations;*NFEs* represents the number of function evaluations. *NFEs_LS* is the number of fitness evaluations consumed by the numerical method.

Algorithm 1: The framework of MENI-EA.

Input: Control parameters: *NP*, *maxFEs*
Output: The solutions in the final population
1 Set $NFEs = 0$;
2 Randomly generate an initial population $\mathcal{P}$;
3 Calculate $f(\mathbf{x}_i)$ $(i = 1, 2, \ldots, NP)$ of each individual $\mathbf{x}_i$ in $\mathcal{P}$;
4 $NFEs = NFEs + NP$;
5 **while** $NFEs < maxFEs$ **do**
6   **for** $j = 1$ *to* NP **do**
7   Generate the trial vector $\mathbf{u}_j$ via niching-based EA;
8   Using Equation (2) to calculate $f(\mathbf{u}_j)$;
9         $NFEs = NFEs + 1$;
10   **if** $f(\mathbf{u}_j) < \epsilon$ **then**
11   Calculate the exact roots $\mathbf{e}_j$ via numerical methods;
12             $NFEs = NFEs + NFEs\_LS$;
13   **if** $f(\mathbf{e}_j) < f(\mathbf{u}_j)$ **then**
14                 $\mathbf{u}_j = \mathbf{e}_j$

MENI-EA works as follows:

In line 2, a random population containing *NP* individuals is generated. Subsequently, the fitness of each individual $\mathbf{x}_i$ is calculated by using Eq. (2) in line 3. In line 4, *NFEs* is updated. In line 7, the niching-based EA generates the offspring in different subregions. In lines 8–9, the main work is to calculate the fitness of offspring $\mathbf{u}_j$ with Eq. (2) and update *NFEs*. In line 10, if $f(\mathbf{u}_j)$ is less than ϵ, $\mathbf{u}_j$ is considered as an initial guess and the numerical method is triggered. In this paper, $\epsilon = 0.5$. Thus, line 11 employs the numerical method to improve the quality of $\mathbf{u}_j$. In line 12, *NFEs* is updated. It is noteworthy that *NFEs_LS* is not a fixed value; different numerical methods will return different *NFEs_LS*. In lines 13–14, if $\mathbf{e}_j$ is better than $\mathbf{u}_j$, $\mathbf{u}_j$ is set to $\mathbf{e}_j$.

Note that, MENI-EA is a general framework, where different niching-based EAs can be used in line 7; moreover, in line 11, different numerical methods are also employed to obtain high-quality roots.

To clearly describe the structure of the algorithm, Fig. 2 shows the flow chart of MENI-EA. The niching-based EA is used to generate the trial vector $\mathbf{u}_j$ to enhance the populations diversity, whereas the numerical method can refine the fitness value of $\mathbf{u}_j$ that approaches the optima. Thus, MENI-EA can obtain multiple high-quality roots through the combination of these two methods.

4. Experimental results and analysis

As mentioned previously, MENI-EA is a generic framework, in which numerical methods and different niching-based EAs can be used. In this section, we carry out experiments to evaluate the effect of the different numerical methods and niching-based EAs. Moreover, MENI-EA is compared with several state-of-the-art methods for NESs.

4.1. Test problems

To validate the effectiveness of different algorithms, we selected 30 NESs from the literature as the test suite. They have different features, such as multiple roots and different dimensional decision vectors, which may challenge the MENI-EA's ability to locate multiple roots in a single run. Additionally, some test problems come from real-world fields. For example, F03 is the interval arithmetic problem (Grosan & Abraham, 2008) and F07 is a kind of robot kinematics problem (Wang, Luo, Wu, & Han, 2011). Table 2 introduces the characteristics of the test problems used in this work. It is worth noting that max *FEs* are different for different problems due to their different difficulties. The detailed information of the NESs is reported in Appendix A.

4.2. Performance criteria

In this problem, after the conversion, two performance criteria employed to evaluate the performance of different algorithms (Li, Engelbrecht, & Epitropakis, 2013).

4.2.1. Root ratio (RR)

The root ratio calculates average ratio of solutions located over multiple runs within *maxFEs*.

$$RR = \frac{\sum_{i=1}^{N_r} NoR_i}{NoR \cdot N_r}, \tag{3}$$

where N_r is the number of runs; NoR_i is the number of roots located in the *i*th run, and *NoR* is the number of the known roots of a NES. If $f(\mathbf{x}) < \xi$, the solution $\mathbf{x}$ is considered as a root. In this work, $\xi = 1e-05$. Each algorithm is performed over $Nr = 30$ independent runs for each NES to cause meaningful comparison.

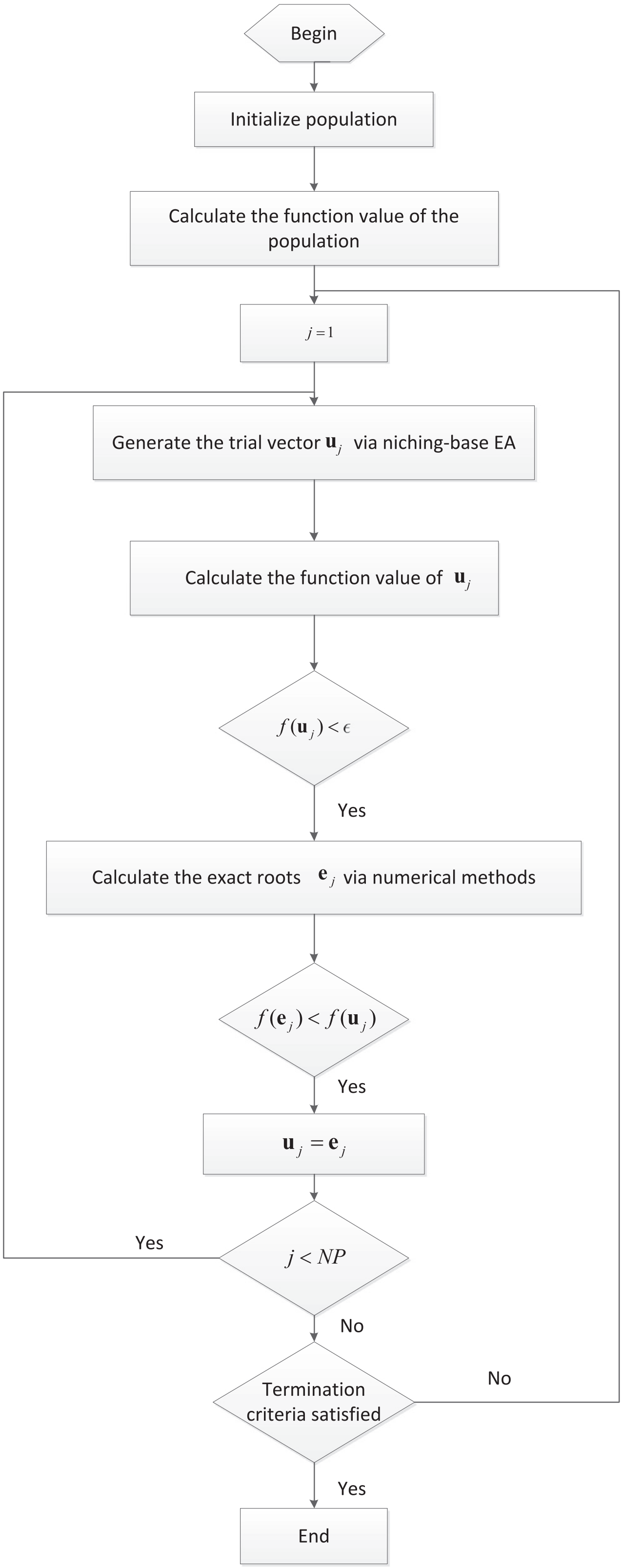

Fig. 2. The flowchart of MENI-EA.

4.2.2. Success rate (SR)

The success rate evaluates the ratio of successful runs[1] out of all runs:

$$SR = \frac{N_{sr}}{N_r}. \tag{4}$$

where N_{sr} is the number of successful runs.

We executed the multiple problem Wilcoxon's test and Friedman's test via keel software to test the statistical differences among different methods according to *RR* and *SR* (Alcalá-Fdez et al., 2009). In the multiple-problem Wilcoxon's test, $p < 0.05$ denotes that two compared methods is a significant difference.

4.3. Influence of different numerical methods

In this subsection, the influence of different numerical methods in MENI-EA is evaluated. NCDE (Qu et al., 2012) is selected as a basic niching-based EA. The reasons are: 1) NCDE obtains promising results for multi-modal optimization problems; and 2) it is very simple and can be implemented easily. Then, NCDE is integrated into MENI-EA, the resultant method is referred to as MENI-NCDE.

Four numerical methods introduced in Section 3.3 are integrated into MENI-NCDE, called MENI-NCDE-* collectively. The five MENI-NCDE-* variants are respectively referred to as MENI-NCDE-TRD with Trust-Region with DogLeg, MENI-NCDE-TRR with Trust-Region-Reflective, MENI-NCDE-LM with Levenberg-Marquardt, MENI-NCDE-HC with Homotopy Continuation method, and MENI-NCDE-NEWTON with Newton's method.

The detailed results are repectively reported in Tables B.1 and B.2 in Appendix B. From Tables B.1 and B.2, the variants of MENI-NCDE-*, such as MENI-NCDE-TRD, MENI-NCDE-TRR, MENI-NCDE-LM, MENI-NCDE-HC, and MENI-NCDE-NEWTON obtained better results than numerical methods in terms of *RR* and *SR*. On the whole, MENI-NCDE-LM received the highest average results in both *RR* and *SR*. In addition, the results yielded by the multiple-problem Wlilcoxon test are shown in Table 3. The results demonstrate that the performance of MENI-NCDE-* was better than different numerical methods since MENI-NCDE-* obtained the higher R^+ values than R^- values in all cases. MENI-NCDE-LM especially demonstrated obviously better results since the *p*-value is less than 0.05 in terms of *RR* and *SR*.

Algorithm 2: The framework of numerical method.

Input: Control parameters: *NP*, *maxFEs*
Output: The solutions in the final population
Randomly generate an initial population $\mathcal{P}$;
Calculate $f(\mathbf{x}_i)$ of each individual $\mathbf{x}_i$ in $\mathcal{P}$;
Set $NFEs = NFEs + NP$;
while $NFEs < maxFEs$ **do**
  for $i = 1$ *to* NP **do**
    Calculate the exact roots $\mathbf{e}_i$ via numerical methods;
    $NFES = NFEs + NFEs\_LS$;
    if $f(\mathbf{e}_i) < f(\mathbf{x}_i)$ **then**
      $\mathbf{x}_i = \mathbf{e}_i$
  Selection between the parent population and the offspring population to form new parent population $\mathcal{P}$;

In Algorithm 2, at the beginning stage, all the individuals are taken as the initial guesses, and the numerical method is adopted to refine them. However, direct use of the numerical method may

[1] A successful run is defined as a run where all known roots of a NES are found.

Table 1
Parameter settings for different methods. Note that all other parameters used in different methods are kept the same as used in their original literature.

Method	Parameter settings
MENI-EA	$NP = 100, F = 0.9, CR = 0.1, \epsilon = 0.5$
NCDE	$NP = 100, F = 0.9, CR = 0.1$
NSDE	$NP = 100, F = 0.9, CR = 0.1$
LIPS	$NP = 100, w = 0.729843788$
R3PSO	$NP = 100, w = 0.729843788, c1 = c2 = 2.05$
A-WeB	$NP = 100, H_m = \mu$
MONES	$NP = 100, H_m = \mu$
I-HS	$NP = 10$, HMCR $= 0.95$, $\text{PAR}_{\text{min}} = 0.35$, $\text{PAR}_{\text{max}} = 0.99$, $\text{BW}_{\text{min}} = 10^{-6}$, $\text{BW}_{\text{max}} = 5$
RADE	$NP = 100, H_m = 200$
DR-JADE	$NP = 10, t_{\text{max}} = 30$
Trust-Region with DogLeg	$\lambda = 0.05, \eta_1 = 0.05, \eta_2 = 0.9, \gamma_1 = 0.25, \gamma_2 = 2.5$
Trust-Region-Reflective	$\lambda = 0.05, \eta_1 = 0.05, \eta_2 = 0.9, \gamma_1 = 0.25, \gamma_2 = 2.5$
Levenberg-Marquardt	$\lambda = 0.05, \eta_1 = 0.05, \eta_2 = 0.9, \gamma_1 = 0.25, \gamma_2 = 2.5$
Homotopy Continuation	$\lambda = 0.05$
Nelder-Meads	$\rho = 1, \chi = 2, \psi = 0.5, \sigma = 0.5$
pattern search	$\lambda = 0.5, \delta = 2$
simplex crossover	$n_p = 3, \varepsilon = 1$

Table 2
Brief descriptions of the test problems used in this work, where "*D*" is the number of decision variables; "$\mathbb{S}$" is the decision space; "*LE*" and "*NE*" are respectively the number of linear and nonlinear equations; "*NoR*" is the number of roots; and "*maxFEs*" is the maximal number of function evaluations.

Prob.	*D*	$\mathbb{S}$	*LE*	*NE*	*NoR*	*maxFEs*
F01	2	$[-1, 1]^m$	1	1	11	50,000
F02	2	$[-10, 10]^m$	0	2	15	50,000
F03	10	$[-2, 2]^m$	0	10	1	50,000
F04	4	$[0, 5]^m$	0	4	1	50,000
F05	3	$[-20, 20]^m$	0	3	12	50,000
F06	2	$[0, 2\pi]^m$	0	2	13	50,000
F07	8	$[-1, 1]^m$	0	8	16	100,000
F08	3	$[-20, 20]^m$	0	3	7	50,000
F09	2	$[0, 1], [-10, 0]$	0	2	2	50,000
F10	2	$[-30, 30]^m$	0	2	4	50,000
F11	2	$[-1, 1], [-10, 10]$	0	2	4	50,000
F12	20	$[-1, 1]^m$	0	2	2	100,000
F13	5	$[-2, 2]^m$	0	2	2	50,000
F14	2	$[0, 2], [-10, 10], [-1, 1]$	0	2	2	50,000
F15	20	$[-2, 2]^m$	0	2	2	100,000
F16	2	$[0, 4], [-3, 4]$	0	2	2	50,000
F17	3	$[0, 5]^m$	0	3	2	50,000
F18	3	$[-5, 5]^m$	0	3	2	50,000
F19	3	$[-5, 5]^m$	0	3	2	50,000
F20	3	$[-10, 10]^m$	0	3	4	50,000
F21	2	$[-2, 2]^m$	0	2	10	50,000
F22	2	$[-2, 2]^m$	0	2	13	50,000
F23	2	$[-2, 2]^m$	0	2	4	50,000
F24	3	[0,10],[0,10],[0,1]	0	3	8	50,000
F25	2	$[-20, 20]^m$	0	2	16	50,000
F26	2	$[-5, 5]^m$	0	2	6	50,000
F27	2	$[-20, 20]^m$	0	2	18	50,000
F28	2	$[-15, 15]^m$	0	2	18	50,000
F29	2	$[-5, 5]^m$	0	2	4	50,000
F30	2	$[-2, 2]^m$	0	2	6	50,000

obtain low-quality roots because some initial guesses are further away from the roots. Since the numerical method is sensitive to the initial guess, the poor initial guess may lead to low-quality roots. Additionally, it also results in the waste of computational resources. Conversely, in Algorithm 1, the numerical method does not work at the beginning stage while NCDE guides the individuals to search for different promising regions. Once the fitness value of an individual is less than ϵ, it demonstrates that such an individual lies in the vicinity of the optimal solution. Thus, the numerical method can make good use of it as an initial guess to yield high-quality roots. Due to the features of NCDE, individuals gradually converge to the vicinity of different optimal solutions with an increase in the iteration number. Therefore, the numerical method can use different initial guesses from different regions to obtain multiple roots of NESs.

4.4. Effect of different niching-based EAs

In this subsection, other niching-based EAs, *i.e.*, NCDE, NSDE (Qu et al., 2012), LIPS (Qu, Suganthan, & Das, 2013), R3PSO (Li, 2010) and Fast_NCDE (Zhang, Gong, Zhang, Gu, & Zhang, 2017b), are integrated into MENI-EA to study the effectiveness of our approach.[2] Additionally, since MENI-NCDE-LM obtained the best results in Section 4.3, LM is considered as a representative numerical method and is selected to combine with other niching-based EAs. Thus, the resultant methods are called MENI-NCDE-LM, MENI-NSDE-LM, MENI-LIPS-LM, MENI-R3PSO-LM, and MENI-Fast_NCDE-LM. Parameter settings of different niching-based EAs are shown in Table 1.

The detailed *RR* and *SR* results are provided in Tables B.3 and B.4 in Appendix B. According to the average results of both *RR* and *SR*, different MENI-EAs receive better results than original niching-based EAs. For example, MENI-NCDE-LM obtains the average *RR* value, 0.9786 and the average *SR* value, 0.9089, which are much higher than the results obtained from NCDE. Other niching-based EAs in the framework of MENI-EA, such as MENI-NSDE-LM, MENI-LIPS-LM, MENI-R3PSO-LM and MENI-Fast_NCDE-LM, also receive better results than their own original algorithms in terms of *RR* and *SR*.

The summarized results obtained by the Wilcoxon test are demonstrated in Table 4. Compared with their initial niching-based

Table 3
Results obtained by the Wilcoxon test in terms of *RR* and *SR* between the different MENI-NCDE-* and original numerical methods.

MENI-NCDE-* VS *	*RR*			*SR*		
	R^+	R^-	*p*-value	R^+	R^-	*p*-value
*: TRD	295.0	170.0	*1.73E−01*	315.5	149.5	*8.42E−02*
*: TRR	283.0	182.0	*2.94E−01*	299.0	166.0	*1.45E−01*
*: LM	354.5	110.5	**1.17E−02**	340.5	124.5	**1.70E−02**
*: HC	334.5	130.5	**2.66E−02**	320.5	144.5	*5.50E−02*
*: NEWTON	288.5	176.5	*2.45E−01*	287.5	177.5	*2.48E−01*

EAs, the MENI-EA variants also obtain significantly better results in all cases according to the Wilcoxon test at $\alpha = 0.05$ shown in Table 4. Therefore, the proposed memetic niching-based EAs are also capable of improving the performance of niching-based EAs.

Niching-based EAs can guide individuals to search for different promising regions. However, they are easily trapped in local optima during the run and may obtain low-quality roots. Due to the superior performance of numerical method for solving NESs, it is natural that the numerical method is integrated into MENI-EA for improving the quality of the roots. Experiment results also verify that MENI-EA can be beneficial to the enhancement of original niching-based EAs.

Note that the proposed approach obtains satisfactory results in most of the test problems. However, MENI-EAs have shown worse results in some test problems, such as F15, F27 and F28. The reason lies in two aspects: 1) high dimensional problems (F15) increase the complexity of computation, which may result in decreasing the performance of niching-based EAs; 2) the test problems (F27 and F28) have the characteristics of a large decision space and multiple roots. Locating multiple roots of these problems in a single run

[2] For the five EAs, NCDE, NSDE and Fast_NCDE are their DE variants, and LIPS and R3PSO are the enhanced particle swarm optimization (PSO) methods. All of the five methods obtained appealing results in the literature.

Table 4
Results obtained by the Wilcoxon test in terms of *RR* and *SR* between the MENI-EA variants and other niching-based EAs.

*-LM VS *	*RR*			*SR*		
	R^+	R^-	*p*-value	R^+	R^-	*p*-value
*: MENI-NCDE	448.5	16.5	**3.01E−06**	448.5	16.5	**0.00E+00**
*: MENI-NSDE	429.5	35.5	**1.80E−05**	447.0	18.0	**5.00E−06**
*: MENI-LIPS	412.0	53.0	**9.91E−05**	414.5	50.5	**1.55E−04**
*: MENI-R3PSO	448.5	16.5	**3.01E−06**	432.0	33.0	**1.20E−05**
*: MENI-Fast_NCDE	450.0	15.0	**2.01E−06**	454.5	10.5	**3.00E−06**

Table 5
Average rankings of MENI-NCDE-LM, A-WeB, MONES, I-HS, RADE and DR-JADE obtained by the Friedman test for both *RR* and *SR*.

Algorithm	Ranking (*RR*)	Ranking (*SR*)
MENI-NCDE-LM	**1.8667**	**1.8667**
A-WeB	3.6333	3.6167
MONES	4.3333	4.1000
I-HS	4.5167	4.4833
RADE	2.7167	2.8500
DR-JADE	3.9333	4.0833

Table 6
Results obtained by the Wilcoxon test for algorithm MENI-NCDE-LM in terms of *RR* and *SR* compared with A-WeB, MONES, I-HS, RADE and DR-JADE.

VS	*RR*			*SR*		
	R^+	R^-	*p*-value	R^+	R^-	*p*-value
A-WeB	422.0	43.0	**9.31E−05**	417.0	48.0	**5.30E−05**
MONES	432.0	33.0	**1.41E−05**	437.5	27.5	**8.00E−06**
I-HS	454.5	10.5	**4.00E−06**	439.5	25.5	**0.00E+00**
RADE	401.5	63.5	**4.73E−04**	381.0	84.0	**3.58E−04**
DR-JADE	447.0	18.0	**9.01E−06**	447.0	18.0	**6.01E−06**

may require extremely high computational cost. Inspired by these, we will direct our attention to designing a more powerful algorithm to enhance its search ability in the future.

4.5. Comparison with other methods for NESs

In this subsection, MENI-EA is compared with other state-of-the-art methods for NESs. Five methods are compared with MENI-NCDE-LM: A-WeB (Gong et al., 2017), MONES (Song et al., 2015), I-HS (Ramadas et al., 2014), RADE (Gong, Wang, Cai, & Wang, 2018), and DR-JADE (Liao et al., 2018). Among these methods, A-WeB and MONES are the multi-objective-optimization-based methods. I-HS is a repulsion-based harmony search method. RADE is a new approach that combines the repulsion technique, neighborhood based mutation and adaptive parameter control. It achieved significant results compared with other methods for NESs. DR-JADE adopted dynamic repulsive radius to locate the roots of NESs and has shown appealing results (Liao et al., 2018). The parameter settings of A-WeB, MONES, I-HS, RADE, and DR-JADE are shown in Table 1.

In Tables B.5 and B.6 of the Appendix B, the average value of both *RR* and *SR* yielded by the different approaches for solving NESs are reported. In addition, the ranking results based on the Friedman test are shown in Table 5 and the statistical results obtained by the Wilcoxon test are reported in Table 6. From Tables B.5 and B.6, the results show that MENI-NCDE-LM gains the best average values in both *RR* and *SR*. MENI-NCDE-LM also obtains the best ranking compared with the other five methods by the Friedman test as shown in Table 5. Meanwhile, in Table 6, MENI-NCDE-LM significantly outperforms the other five methods in terms of both *RR* and *SR* since all $p-$values are less than 0.05. Therefore, we can conclude that our proposed approach can be an effective alternative to simultaneously locating multiple roots of NESs in a single run.

5. Discussion

In the experimental analysis, the effectiveness of the MENI-EA has been verified. In this section, the effect of different parameters on MENI-EA is discussed, including the accuracy level ξ and the size of ϵ. Moreover, other local search methods used in MENI-EA are studied.

5.1. Comparison with different accuracy

In the previous experiment, $\xi = 1e-05$. If the fitness value is less than ξ, the individual is considered as a candidate solution. In this subsection, the influence of different accuracy levels on the performance of MENI-EA is studied. Four different solution accuracy levels: $1e-06$, $1e-07$, $1e-08$, $1e-09$, are used in the experiment. In addition, RADE and DR-JADE are chosen to compare with MENI-NCDE-LM at different accuracy levels.

For MENI-NCDE-LM, the resultant methods are respectively called as MENI-NCDE-LM-6, MENI-NCDE-LM-7, MENI-NCDE-LM-8, MENI-NCDE-LM-9 in different accuracy levels. Similarly, the resultant methods of RADE are known as RADE-6, RADE-7, RADE-8, and RADE-9. DR-JADE at different accuracy levels are named as DR-JADE-6, DR-JADE-7, DR-JADE-8, and DR-JADE-9.

The detailed *RR* and *SR* results are reported in Tables B.7 and B.8 in Appendix B, respectively. Additionally, the summarized results based on the Wilcoxon test are shown in Table 7. From Tables B.7 and B.8, we can see that the average values of *RR* and *SR* of MENI-NCDE-LM, RADE and DR-JADE gradually decrease as the accuracy increases. However, the average *RR* and *SR* of MENI-NCDE-LM are higher than RADE and DR-JADE, which demonstrates that the performance of MENI-NCDE-LM for solving NESs is better than the other two methods at different accuracy levels. Moreover, in addition to the accuracy level of $1e-09$, MENI-NCDE-LM receives better results than RADE and DR-JADE in all cases according to the Wilcoxon test at $\alpha = 0.05$ as shown in Table 7. Thus, we can obtain two conclusions:

- The performance of different approaches for solving NESs will decrease with the increase of accuracy level ξ.
- Compared with RADE and DR-JADE, MENI-NCDE-LM obtains better results at different accuracy levels, which signifies that MENI-NCDE-LM is superior to RADE and DR-JADE.

5.2. Influence of different ϵ

Parameter ϵ indicates whether the individual is close to or far from the optimal solution. A large ϵ indicates that the individual is far away from the optimal solution, and vice versa. In this subsection, we will discuss the influence of different ϵ.

Table 7
Results obtained by the Wilcoxon test in terms of *RR* and *SR* at different accuracy levels.

MENI-NCDE-LM-6 VS *	*RR*			*SR*		
	R^+	R^-	*p*-value	R^+	R^-	*p*-value
*: RADE-6	367.5	97.5	**3.39E−03**	366.5	98.5	**3.19E−03**
*: DR-JADE-6	433.0	32.0	**3.10E−05**	433.0	32.0	**1.71E−05**
MENI-NCDE-LM-7 VS *	R^+	R^-	*p*-value	R^+	R^-	*p*-value
*: RADE-7	342.0	123.0	**2.33E−02**	358.0	107.0	**8.65E−03**
*: DR-JADE-7	374.0	91.0	**3.40E−03**	400.0	65.0	**1.74E−04**
MENI-NCDE-LM-8 VS *	R^+	R^-	*p*-value	R^+	R^-	*p*-value
*: RADE-8	358.0	107.0	**8.65E−03**	352.5	112.5	**3.58E−03**
*: DR-JADE-8	400.0	65.0	**1.74E−04**	380.5	84.5	**1.06E−03**
MENI-NCDE-LM-9 VS *	R^+	R^-	*p*-value	R^+	R^-	*p*-value
*: RADE-9	341.5	150.5	*7.05E−02*	298.0	167.0	*1.12E−01*
*: DR-JADE-9	310.0	155.0	*1.07E−01*	314.0	151.0	*8.50E−02*

Table 8
Average rankings of MENI-EA with different ϵ obtained by the Friedman test for both *RR* and *SR*.

Parameter Setting	Ranking (*RR*)	Ranking (*SR*)
$\epsilon = 0.1$	5.1833	5.0333
$\epsilon = 0.2$	4.8333	4.7000
$\epsilon = 0.3$	4.6000	4.6167
$\epsilon = 0.4$	4.6000	4.6333
$\epsilon = 0.5$	**3.9500**	**4.0833**
$\epsilon = 0.6$	4.0833	4.1167
$\epsilon = 0.7$	4.2500	4.2167
$\epsilon = 0.8$	4.5000	4.6000

Table 9
Average rankings of MENI-NCDE-LM, MENI-NCDE-NM, MENI-NCDE-PS, and MENI-NCDE-SPX obtained by the Friedman test for both *RR* and *SR*.

Algorithm	Ranking (*RR*)	Ranking (*SR*)
MENI-NCDE-LM	**1.6167**	**1.7000**
MENI-NCDE-NM	2.1667	2.1000
MENI-NCDE-PS	2.8000	2.8333
MENI-NCDE-SPX	3.8667	3.9667

Table 10
Results obtained by the Wilcoxon test for MENI-NCDE-LM in terms of *RR* and *SR* compared with MENI-NCDE-NM, MENI-NCDE-PS, and MENI-NCDE-SPX.

VS	*RR*			*SR*		
	R^+	R^-	*p*-value	R^+	R^-	*p*-value
MENI-NCDE-NM	378.0	87.0	**2.67E−03**	352.5	112.5	**1.09E−02**
MENI-NCDE-PS	413.0	52.0	**1.90E−04**	398.0	67.0	**3.07E−04**
MENI-NCDE-SPX	448.5	16.5	**3.00E−06**	460.0	5.0	**0.00E+00**

Table 11
Results obtained by the Wilcoxon test in terms of *RR* and *SR* between the MENI-EA variants and other niching-based DE.

MENI-SHNCDE-LM VS *	*RR*			*SR*		
	R^+	R^-	*p*-value	R^+	R^-	*p*-value
*: SHNCDE	449.0	16.0	**6.99E−05**	437.0	28.0	**0.00E+00**
*: NCDE	458.5	6.5	**3.01E−05**	445.5	19.5	**0.00E+00**
MENI-SHNSDE-LM VS *	R^+	R^-	*p*-value	R^+	R^-	*p*-value
*: SHNSDE	428.0	37.0	**5.10E−05**	416.0	49.0	**4.16E−05**
*: NSDE	459.0	6.0	**1.01E−06**	438.5	26.5	**4.63E−06**

Previously, the value $\epsilon = 0.5$ is used in MENI-NCDE-LM. In this subsection, ϵ is a set of different values, such as $\epsilon = \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8\}$ used to study the effect of the algorithm's performance. The detailed results of *RR* and *SR* are shown in Tables B.9 and B.10 in Appendix B. The average rankings obtained by the Friedman test are outlined in Table 8.

From Tables B.9 and B.10, $\epsilon = 0.6$ obtains the highest average *RR* and highest *SR*, whereas $\epsilon = 0.5$ achieves the best ranking in Table 8. The results yielded by Friedman test reveal that the appropriate value of ϵ, $\epsilon \in [0.5, 0.7]$ achieved better results. The reason is that if ϵ is too large, the initial guess is far away from the optimal solution. If the numerical method uses it directly as the initial guess, it may not find the exact root. Conversely, if ϵ is too small, many computing resources are wasted on niching-based EAs, which makes it impossible to make full use of numerical methods to solve NESs. Therefore, the performance of MENI-EA declined.

5.3. Discussion on different local search methods

Previously, numerical methods for NESs combined with niching-based EAs have obtained promising results. In this subsection, the influence of local search methods in MENI-EA is investigated. For this purpose, three different local search methods, such as Nelder-Meads (NM), pattern search (PS) and simplex crossover (SPX) operation, are integrated into MENI-EA for solving NESs (Audet & Jr, 2006; Lagarias, Reeds, Wright, & Wright, 1998; Noman & Iba, 2008). Similarly, NCDE, as the basic niching-based EA, guides the individuals to search toward different promising regions. The resultant methods of three local search methods in MENI-EA are respectively called MENI-NCDE-NM, MENI-NCDE-PS, and MENI-NCDE-SPX. The parameter settings of local search methods are shown in Table 1.

The detailed results in terms of *RR* and *SR* are shown in Tables B.11 and B.12 in Appendix B. In addition, the ranking results based on the Friedman test are shown in Table 9 and the statistical results obtained by the Wilcoxon test are reported in Table 10. From Tables B.11 and B.12, MENI-NCDE-LM gets the best average values of *RR* and *SR*. It also receives the best ranking as shown in Table 9. Moreover, in Table 10, MENI-NCDE-LM obtains significantly better results than these three variants since the $p < 0.05$ in all cases.

Comparing MENI-NCDE-LM with three variants of MENI-EA, MENI-NCDE-LM achieved better performance. The reason may be that the numerical method is specially developed for NESs, which is the most effective method for finding high-quality roots. Although local search can improve the performance of the niching-based EA, it is not as good as the numerical method. Especially, from Tables B.11 and B.12 of Appendix B, MENI-NCDE-LM can obtain all the roots of F20, while MENI-NCDE-NM, MENI-NCDE-PS and MENI-NCDE-SPX lose some roots as the search proceeds. Thus, the numerical method plays an important role in MENI-EA, which can influence the performance of MENI-EA.

5.4. On the effectiveness of other niching methods

In recent years, other state-of-the-art niching approaches have been proposed to deal with multi-modal optimization of constrained functions (Poole & Allen, 2019). To further verify the effectiveness of our approach, in this section, two advanced niching methods (Poole & Allen, 2019), named SHNCDE and SHNSDE, are used in the proposed framework. In SHNCDE, the combination of the NCDE algorithm and SHADE (Tanabe & Fukunaga, 2013) mutation strategy is used to solve constrained multi-modal problems, and so is SHNSDE. In this section, SHNCDE and SHNSDE are combined with LM to solve NESs. The resultant methods are called MENI-SHNCDE-LM and MENI-SHNCDE-LM.

The detailed *RR* and *SR* results are given in Tables B.13and in Appendix B. Moreover, the statistical results obtained by the Wilcoxon test are given in Table 11. From the results, it can be observed that:

- Compared with SHNCDE and NCDE, it is clear that MENI-SHNCDE-LM had a significant improvement, whether it was *RR* or *SR*. In addition, MENI-SHNSDE-LM was also greatly improved compared with SHNSDE and NSDE. Hence, LM combined with SHNCDE and SHNSDE can improve the performance for solving NESs, and further verifies the effectiveness of the proposed framework.
- SHNCDE and SHNSDE also obtained the better *RR* and *SR* values than NCDE and NSDE, respectively. Thus, it is clear that NCDE and NSDE combined with the SHADE mutation strategy also improve the ability to solve NESs. This motivates us to adopt different mutation strategies in niching-based EAs to locate multiple roots in the future.

Meanwhile, as shown in Table 11, MENI-SHNCDE-LM and MENI-SHNSDE-LM significantly outperformed their niching-based EAs in terms of both *RR* and *SR* since all *p*-values are less than 0.05. Therefore, we can conclude that the combination of the numerical method and niching-based EAs can improve the ability to locate multiple roots of NESs.

6. Conclusion

Locating multiple roots of NESs is a challenging task in numerical optimization. Numerous previous works have focused on using EAs and memetic algorithms to determine a root of NES. However, little work to date has been presented on locating multiple roots by using the memetic algorithm. This paper has presented a memetic niching-based EA, named MENI-EA, in which the niching-based EAs and the numerical methods are combined to solve NESs effectively. To verify the effectiveness of our approach, different numerical methods and different niching-based EAs were integrated into MENI-EA. Additionally, 30 NESs with different characteristics were selected as the test suite. Experimental results showed that the proposed method was effective and could enhance the performance of numerical methods for solving NESs. Moreover, different niching-based EAs, *i.e.*, MENI-NSDE-LM, MENI-LIPS-LM, MENI-R3PSO-LM, and MENI-Fast_NCDE-LM, were used in MENI-EA to locate numerous roots of NESs in one run. Furthermore, compared with other state-of-the-art methods, MENI-EA provided significantly competitive performance. Therefore, MENI-EA can be an effective alternative to solve NESs.

Niching-based EAs in MENI-EA play a vital role in searching for different promising regions. In the future, we will design other powerful niching-based methods for preserving population diversity. Additionally, we will apply MENI-EA to solve complex real-world NES problems, such as parameter estimation for nonlinear signal processing systems, simulation of nonlinear resistive circuits, and hypersurface exploration of the potential energy landscape.

The source code used in this paper can be obtained from the authors upon request.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Credit authorship contribution statement

Zuowen Liao: Data curation, Writing - original draft. **Wenyin Gong:** Conceptualization, Methodology, Software, Writing - original draft, Writing - review & editing. **Ling Wang:** Writing - review & editing.

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Appendix A. Test problems

F01

$$\begin{cases} x_1 - \sin(5\pi x_2) = 0 \\ x_1 - x_2 = 0 \end{cases} \tag{A.1}$$

where $x_i \in [-1, 1]$, $i = 1, \cdots, D$, and $D = 2$. It has 11 roots as shown in Table A.12 Song et al. (2015).

F02

$$\begin{cases} x_1 - \cos(4\pi x_2) = 0 \\ x_1^2 + x_2^2 - 1 = 0 \end{cases} \tag{A.2}$$

where $x_i \in [-10, 10]$, $i = 1, \cdots, D$, and $D = 2$. It has 15 roots as shown in Table A.13 Song et al. (2015).

Table A12
The roots of F01.

x_1	x_2
−0.924840	−0.924840
−0.866760	−0.866760
−0.562010	−0.562010
−0.428168	−0.428168
−0.187960	−0.187960
0.000000	0.000000
0.187960	0.187960
0.428168	0.428168
0.562010	0.562010
0.866760	0.866760
0.924840	0.924840

Table A13
The roots of F02.

x_1	x_2
0.416408	−0.909178
−0.561364	−0.827569
−0.724322	−0.689462
0.837812	−0.545959
0.886984	−0.461799
−0.962322	−0.271914
−0.972855	−0.231415
1.000000	0.000000
−0.972855	0.231416
−0.962322	0.271914
0.886984	0.461799
0.837812	0.545959
−0.724322	0.689462
−0.561364	0.827569
0.416408	0.909178

F03

$$\begin{cases} x_1 - 0.25428722 - 0.18324757x_4x_3x_9 = 0 \\ x_2 - 0.37842197 - 0.16275449x_1x_{10}x_6 = 0 \\ x_3 - 0.27162577 - 0.16955071x_1x_2x_{10} = 0 \\ x_4 - 0.19807914 - 0.15585316x_7x_1x_6 = 0 \\ x_5 - 0.44166728 - 0.19950920x_7x_6x_3 = 0 \\ x_6 - 0.14654113 - 0.18922793x_8x_5x_{10} = 0 \\ x_7 - 0.42937161 - 0.21180486x_2x_5x_8 = 0 \\ x_8 - 0.07056438 - 0.17081208x_1x_7x_6 = 0 \\ x_9 - 0.34504906 - 0.19612740x_{10}x_6x_8 = 0 \\ x_{10} - 0.42651102 - 0.21466544x_4x_8x_1 = 0 \end{cases} \tag{A.3}$$

where $x_i \in [-2, 2]$, $i = 1, \cdots, D$, and $D = 10$. It has one root: (0.257833, 0.381097, 0.278745, 0.200669, 0.445251, 0.149184, 0.432010, 0.073403, 0.345967, 0.427326) Grosan and Abraham (2008).

F04

$$\begin{cases} 3.0 - x_1x_3^2 = 0 \\ x_3 \sin(\pi/x_2) - x_3 - x_4 = 0 \\ -x_2x_3 \exp(1.0 - x_1x_3) + 0.2707 = 0 \\ 2x_1^2x_3 - x_2^4x_3 - x_2 = 0 \end{cases} \tag{A.4}$$

where $x_i \in [0, 5]$, $i = 1, \cdots, D$, and $D = 4$. It has one root: (3, 2, 1, 0) Pourjafari and Mojallali (2012).

F05

$$\begin{cases} 4x_1^3 + 4x_1x_2 + 2x_2^2 - 42x_1 - 14 = 0 \\ 4x_2^3 + 2x_1^2 + 4x_1x_2 - 26x_2 - 22 = 0 \end{cases} \tag{A.5}$$

where $x_i \in [-20, 20]$, $i = 1, \cdots, D$, and $D = 2$. It has 9 roots as shown in Table A.14 Sacco and Henderson (2011); Wang et al. (2011).

F06

$$\begin{cases} -\sin(x_1)\cos(x_2) - 2\cos(x_1)\sin(x_2) = 0 \\ -\cos(x_1)\sin(x_2) - 2\sin(x_1)\cos(x_2) = 0 \end{cases} \tag{A.6}$$

where $x_i \in [0, 2\pi]$, $i = 1, \cdots, D$, and $D = 2$. It has 13 roots as shown in Table A.15 Hirsch et al. (2009); Sacco and Henderson (2011).

Table A14
The roots of F05.

x_1	x_2
−0.127961	−1.953715
−0.270845	−0.923039
0.086678	2.884255
3.385154	0.073852
3.584428	−1.848127
3.000000	2.000000
−3.779310	−3.283186
−3.073026	−0.081353
−2.805118	3.131313

Table A15
The roots of F06.

x_1	x_2
0.000000	0.000000
3.141593	0.000000
1.570796	1.570796
6.283185	0.000000
0.000000	3.141593
4.712389	1.570796
3.141593	3.141593
1.570796	4.712389
6.283185	3.141593
0.000000	6.283185
4.712389	4.712389
3.141593	6.283185
6.283185	6.283185

F07

$$\begin{cases} x_1^2 + x_2^2 - 1.0 = 0 \\ x_3^2 + x_4^2 - 1.0 = 0 \\ x_5^2 + x_6^2 - 1.0 = 0 \\ x_7^2 + x_8^2 - 1.0 = 0 \\ 4.731 \cdot 10^{-3}x_1x_3 - 0.3578x_2x_3 - 0.1238x_1 + x_7 \\ -1.637 \cdot 10^{-3}x_2 - 0.9338x_4 - 0.3571 = 0 \\ 0.2238x_1x_3 + 0.7623x_2x_3 + 0.2638x_1 - x_7 \\ -0.07745x_2 - 0.6734x_4 - 0.6022 = 0 \\ x_6x_8 + 0.3578x_1 + 4.731 \cdot 10^{-3}x_2 = 0 \\ -0.7623x_1 + 0.2238x_2 + 0.3461 = 0 \end{cases} \tag{A.7}$$

where $x_i \in [-1, 1]$, $i = 1, \cdots, D$, and $D = 8$. It has 16 roots as shown in Table A.16.

F08

$$x_i - \cos\left(2x_i - \sum_{j=1}^{D} x_j\right) = 0, \quad i = 1, \cdots, D \tag{A.8}$$

where $x_i \in [-20, 20]$, $i = 1, \cdots, D$, and $D = 3$. It has 7 roots as shown in Table A.17 Sharma and Arora (2013).

F09

$$\begin{cases} x_1^2 - x_2 - 2 = 0 \\ x_1 + \sin\left(\frac{\pi}{2}x_2\right) = 0 \end{cases} \tag{A.9}$$

where $x_1 \in [0, 1]$ and $x_2 \in [-10, 0]$. It has two roots: (0, −2) and (0.707660, −1.5) Pourjafari and Mojallali (2012).

F10

$$\begin{cases} x_1^2 + x_2^2 + x_1 + x_2 - 8 = 0 \\ x_1|x_2| + x_1 + |x_2| - 5 = 0 \end{cases} \tag{A.10}$$

Table A16
The roots of F07.

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
0.1644	−0.9864	−0.9471	−0.3210	−0.9982	−0.0594	0.4110	0.9116
0.1644	−0.9864	−0.9471	−0.3210	−0.9982	0.0594	0.4110	−0.9116
0.1644	−0.9864	−0.9471	−0.3210	0.9982	−0.0594	0.4110	0.9116
0.1644	−0.9864	−0.9471	−0.3210	0.9982	0.0594	0.4110	−0.9116
0.1644	−0.9864	0.7185	−0.6956	−0.9980	−0.0638	−0.5278	0.8494
0.1644	−0.9864	0.7185	−0.6956	−0.9980	0.0638	−0.5278	−0.8494
0.1644	−0.9864	0.7185	−0.6956	0.9980	−0.0638	−0.5278	0.8494
0.1644	−0.9864	0.7185	−0.6956	0.9980	0.0638	−0.5278	−0.8494
0.6716	0.7410	−0.6516	−0.7586	−0.9625	−0.2711	−0.4376	0.8992
0.6716	0.7410	−0.6516	−0.7586	−0.9625	0.2711	−0.4376	−0.8992
0.6716	0.7410	−0.6516	−0.7586	0.9625	−0.2711	−0.4376	0.8992
0.6716	0.7410	−0.6516	−0.7586	0.9625	0.2711	−0.4376	−0.8992
0.6716	0.7410	0.9519	−0.3064	−0.9638	−0.2666	0.4046	0.9145
0.6716	0.7410	0.9519	−0.3064	−0.9638	0.2666	0.4046	−0.9145
0.6716	0.7410	0.9519	−0.3064	0.9638	0.2666	0.4046	−0.9145
0.6716	0.7410	0.9519	−0.3064	0.9638	−0.2666	0.4046	0.9145

Table A17
The roots of F08.

x_1	x_2	x_3
0.810561	0.810561	−0.625687
0.810561	−0.625687	0.810561
−0.625687	0.810561	0.810561
0.543850	0.995778	0.543850
0.543850	0.543850	0.995778
0.995778	0.543850	0.543850
0.739086	0.739086	0.739086

Table A18
The roots of F14.

x_1	x_2	x_3
0.825297	−0.859034	−0.151946
1.299490	0.525835	−0.642769
1.533662	−1.648068	0.499604
1.981360	−2.172180	0.775731
1.983283	0.983378	−0.016762

where $x_1 \in [-30, 30]$ and $x_2 \in [-30, 30]$. It has 4 roots: (0.404634, −3.271577), (2.403604, −0.762837), (1, 2), and (2, 1). This problem is modified from Oliveira Jr and Petraglia (2013).

F11

$$\begin{cases} x_1^2 - |x_2| + 1 + \frac{1}{9}|x_1 - 1| = 0 \\ x_2^2 + 5x_1^2 - 7 + \frac{1}{9}|x_2| = 0 \end{cases} \tag{A.11}$$

where $x_1 \in [-1, 1]$ and $x_2 \in [-10, 10]$. It has 4 roots: (−0.814326, −1.864719), (0.861828, −1.758100), (−0.814326, 1.864719), and (0.861828, 1.758100). This problem is modified from Oliveira Jr and Petraglia (2013).

F12

$$\begin{cases} \sum_{i=1}^{D} x_i^2 - 1 = 0 \\ |x_1 - x_2| + \sum_{i=3}^{D} x_i^2 = 0 \end{cases} \tag{A.12}$$

where $x_i \in [-1, 1]$, $i = 1, \cdots, D$, and $D = 20$. It has two roots: (−0.707107, −0.707107, 0, ⋯, 0) and (0.707107, 0.707107, 0, ⋯, 0) Song et al. (2015).

F13

$$\begin{cases} 2x_1 + x_2 + x_3 + x_4 + x_5 - 6.0 = 0 \\ x_1 + 2x_2 + x_3 + x_4 + x_5 - 6.0 = 0 \\ x_1 + x_2 + 2x_3 + x_4 + x_5 - 6.0 = 0 \\ x_1 + x_2 + x_3 + 2x_4 + x_5 - 6.0 = 0 \\ x_1 x_2 x_3 x_4 x_5 - 1.0 = 0 \end{cases} \tag{A.13}$$

where $x_i \in [-2, 2]$, $i = 1, \cdots, D$, and $D = 5$. It has two roots: (1, 1, 1, 1, 1), (0.916355, 0.916355, 0.916355, 0.916355, 1.418227). Morgan and Shapiro (1987), Turgut, Turgut, and Coban (2014).

F14

$$\begin{cases} x_1^2 - x_1 - x_2^2 - x_2 + x_3^2 = 0 \\ \sin(x_2 - \exp(x_1)) = 0 \\ x_3 - \log|x_2| = 0 \end{cases} \tag{A.14}$$

where $x_1 \in [0, 2]$, $x_2 \in [-10, 10]$, and $x_3 \in [-1, 1]$. It has 5 roots shown in Table A.18. This problem is modified from Grau-Sánchez, Grau, and Noguera (2011).

F15

$$\begin{cases} x_i + \sum_{j=1}^{D} x_j - (D+1) = 0 \quad i = 1, \cdots, D-1 \\ \left[\prod_{j=1}^{D} x_j \right] - 1 = 0 \end{cases} \tag{A.15}$$

where $x_i \in [-2, 2]$, $i = 1, \cdots, D$, and $D = 20$.

It has two roots: (1, ⋯, 1) and (0.994922, ⋯, 0.994922, 1.101551).

F16

$$\begin{cases} x_1 - x_2^2 + 3\log(x_1) = 0 \\ 1 - 5x_1 + 2x_2^2 - x_1 x_2 = 0 \end{cases} \tag{A.16}$$

where $x_1 \in [0, 4]$, $x_2 \in [-3, 4]$. It has two roots: (3.75683, 2.779849) and (1.37347, −1.52496) Frontini and Sormani (2004)

F17

$$\begin{cases} \cos(x_2) - \sin(x_1) = 0 \\ x_3^{x_1} - \frac{1}{x_2} = 0 \\ \exp(x_1) - x_3^2 = 0 \end{cases} \tag{A.17}$$

where $x_1 \in [0, 5]$, $x_2 \in [0, 5]$, $x_3 \in [0, 5]$. It has two roots: (0.9096, 0.6612, 1.5758) and (1.7770, 0.2062, 2.4315) Gordji, Ebadian, Ghaemi, and Shokri (2009)

F18

$$\begin{cases} (x_1-1)^4\exp(x_2)=0 \\ (x_2-2)^5(x_1x_2-1)=0 \\ (x_3+4)^6=0 \end{cases} \tag{A.18}$$

where $x_1 \in [-5, 5]$, $x_2 \in [-5, 5]$,$x_3 \in [-5, 5]$. It has two roots: (0.99742 1.00259 −4.02826) and (1, 2, 4) Hueso, Martínez, and Torregrosa (2009)

F19

$$\begin{cases} \exp(x_1^2)-8x_1=0 \\ x_1+x_2-1=0 \\ (x_3-1)^3=0 \end{cases} \tag{A.19}$$

where $x_1 \in [-5, 5]$, $x_2 \in [-5, 5]$, $x_3 \in [-5, 5]$. It has two roots: (0.1756, 0.8244, 1) and (0.7042, 0.2958, 0.9999) Hueso et al. (2009)

F20

$$\begin{cases} x_1^3-x_1x_2x_3=0 \\ x_2^2-x_1x_3=0 \\ 10x_1x_2x_3-x_1-0.1=0 \end{cases} \tag{A.20}$$

where $x_1 \in [-2, 2]$, $x_2 \in [-2, 2]$,$x_3 \in [-10, 10]$. It has three roots: (−0.1153,−0.1153,−0.1153),(0.3577, 0.3577, 0.3577) and (−0.0998, 0, 0) Waziri, Leong, Hassan, and Monsi (2010)

F21

$$\begin{cases} \sin(x_1^3)-3x_1x_2^2-1=0 \\ \cos(3x_1^2x_2)-|x_2^3|+1=0 \end{cases} \tag{A.21}$$

where $x_1 \in [-2, 2]$, $x_2 \in [-2, 2]$. It has 10 roots shown in Table A.19.

F22

$$\begin{cases} 4x_1^3-3x_1-\cos(x_2)=0 \\ \sin(x_1^2)-|x_2|=0 \end{cases} \tag{A.22}$$

where $x_1 \in [-2, 2]$, $x_2 \in [-2, 2]$. It has 6 roots shown in Table A.20.

Table A19
The roots of F21.

x_1	x_2
−1.810885	−0.349092
−1.810885	0.349092
−1.502221	−0.409077
−1.502221	0.409077
−1.791302	0.301926
−1.791302	−0.301926
−0.947268	0.78502
−0.947268	−0.78502
−0.213057	1.256845
−0.213057	−1.256845

Table A20
The roots of F22.

x_1	x_2
−0.597167	−0.349098
−0.597167	0.349098
−0.442758	−0.194781
−0.442758	0.194781
0.964499	−0.801774
0.964499	0.801774

Table A21
The roots of F23.

x_1	x_2
0.8450	−0.6201
−0.4641	0.9397
−1.0352	−0.1637
0.9793	0.3735
−0.0018	−1.04814
−1.0171	0.2528

Table A22
The roots of F24.

x_1	x_2	x_3
0	0	0
0.488122	0.959435	0.149452
0.540304	0.953754	0.169399
0.959447	0.149373	0.487917
0.14944	0.488092	0.95944
0.953781	0.169343	0.540157
0.169254	0.539937	0.953788
0.739584	0.739584	0.739574

F23

$$\begin{cases} \exp(x_1^2+x_2^2)-3=0 \\ |x_2|+x_1+x_2-2\sin(3|x_2|+x_1)=0 \end{cases} \tag{A.23}$$

where $x_1 \in [-2, 2]$, $x_2 \in [-2, 2]$. It has 6 roots shown in Table A.21.

F24

$$\begin{cases} -3.84x_1^2+3.84x_1-x_2=0 \\ -3.84x_2^2+3.84x_2-x_3=0 \\ -3.84x_3^2+3.84x_3-x_1=0 \end{cases} \tag{A.24}$$

where $x_1 \in [0, 10]$, $x_2 \in [0, 10]$ and $x_3 \in [0, 1]$. It has 8 roots shown in Table A.22.

F25

$$\begin{cases} x_1^4+x_2^4-x_1x_2^3-6=0 \\ |1-x_1^2x_2^2|-0.6787=0 \end{cases} \tag{A.25}$$

where $x_1 \in [-20, 20]$, $x_2 \in [-20, 20]$. It has 16 roots shown in Table A.23.

F26

$$\begin{cases} 0.5x_1^2+0.5x_2^2+x_1+x_2-8=0 \\ |x_1|^{x_2}+x_1+|x_2|^{x_1}-5=0 \end{cases} \tag{A.26}$$

where $x_1 \in [-5, 5]$, $x_2 \in [-5, 5]$. It has 6 roots shown in Table A.24.

Table A23
The roots of F25.

x_1	x_2
0.38408	−1.47585
0.73305	1.76745
−0.7330	−1.76745
−0.38401	1.47588
−1.59171	−0.81397
1.591749	0.81397
−1.56878	−0.36131
0.999402	−1.29642
−1.55902	0.36358
0.34203	1.65724
−1.43952	0.900053
1.55902	−0.36358
1.56878	0.36131
−0.99944	1.296421
−0.34203	−1.65724
1.439526	−0.90005

Table A24
The roots of F26.

x_1	x_2
−1.82535	3.16158
1.04266	2.71853
3.24058	−1.13217
−4.43124	1.49535
2.93865	0.57701
−4.89909	0.67243

Table A25
The roots of F27.

x_1	x_2
1.82668	3.41518
−2.97476	2.48055
2.97436	−2.48055
0.46246	3.84527
2.12002	3.24121
−3.27714	−2.06406
−1.2827	3.65437
−1.82661	−3.41518
1.08376	−3.71827
1.28276	−3.65437
−1.08379	3.71827
−0.46246	−3.84527
−2.12002	−3.24121
2.55955	−2.90665
−0.326176	−3.85922
0.326176	3.85922
−2.55955	2.90665
3.27714	2.06406

F27

$$\begin{cases} 4\sin(4x_1) - x_2 = 0 \\ x_1^2 + x_2^2 - 15 = 0 \end{cases} \tag{A.27}$$

where $x_1 \in [-20, 20]$, $x_2 \in [-20, 20]$. It has 18 roots shown in Table A.25.

F28

$$\begin{cases} \cos(2x_1) - \cos(2x_2) - 0.4 = 0 \\ 2(x_2 - x_1) + \sin(2x_2) - \sin(2x_1) - 1.2 = 0 \end{cases} \tag{A.28}$$

where $x_1 \in [-15, 15]$, $x_2 \in [-15, 15]$. It has 18 roots shown in Table A.26.

Table A26
The roots of F28.

x_1	x_2
−12.409851	−12.072994
−11.886135	−10.306380
−9.268258	−8.931402
−8.744542	−7.164787
−6.126665	−5.789809
−5.602950	−4.023195
−2.985073	−2.648216
−2.461357	−0.881602
0.156520	0.493376
0.680236	2.259991
3.298113	3.634969
3.821828	5.401583
6.439705	6.776562
6.963421	8.543176
9.581298	9.918154
10.105014	11.684769
12.722891	13.059747
13.246606	14.826361

Table A27
The roots of F29.

x_1	x_2
2.33387	2.30916
−1.99146	−3.73926
4.00916	−1.40771
4.69774	−0.77751

Table A28
The roots of F30.

x_1	x_2
0.35969	−0.93306
−0.99874	−0.05012
−0.20230	0.979322
−0.35969	−0.93306
0.20230	0.97932
0.99874	−0.05016

F29

$$\begin{cases} x_1 + 0.5x_2^2 - 5 = 0 \\ x_1 + 5\sin(\pi x_2/2) = 0 \end{cases} \tag{A.29}$$

where $x_1 \in [-5, 5]$, $x_2 \in [-5, 5]$. It has 4 roots shown in Table A.27.

F30

$$\begin{cases} x_1^2 + x_2^2 - 1 = 0 \\ 20x_1^2 x_2 + 2x_2^5 + 1 = 0 \end{cases} \tag{A.30}$$

where $x_1 \in [-2, 2]$, $x_2 \in [-2, 2]$. It has 6 roots shown in Table A.28.

Appendix B. Detailed results

Table B.1
Influence of different numerical methods in MENI-EA in terms of the peak ratio.

Prob	MENI-NCDE-TRD	TRD	MENI-NCDE-TRR	TRR	MENI-NCDE-LM	LM	MENI-NCDE-HC	HC	MENI-NCDE-NEWTON	NEWTON
F1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F2	**0.9889**	0.8489	**0.9933**	0.7222	**0.9889**	0.6956	**0.9978**	0.6422	0.9714	**1.0000**
F3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F4	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	**1.0000**	0.9333	1.0000	1.0000
F5	**0.9630**	0.9037	**0.9593**	0.8889	**0.9778**	0.8741	**0.0037**	0.0000	1.0000	1.0000
F6	**0.9974**	0.9872	**0.9974**	0.9718	**1.0000**	0.9897	0.9974	**1.0000**	0.9000	**1.0000**
F7	**1.0000**	0.9958	**1.0000**	0.9979	**1.0000**	0.9979	1.0000	1.0000	1.0000	1.0000
F8	**0.9810**	0.9381	0.9905	**1.0000**	**1.0000**	0.9952	**0.9857**	0.5000	0.9714	**1.0000**
F9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000	0.0000	0.9000	**1.0000**
F13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F14	1.0000	1.0000	**1.0000**	0.9733	**1.0000**	0.9933	1.0000	1.0000	**1.0000**	0.9867
F15	0.0000	**0.5000**	0.0000	**0.5000**	0.5000	0.5000	0.0667	**0.5000**	0.5000	0.5000
F16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	**1.0000**	0.9667	1.0000	1.0000
F19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F20	0.9778	**1.0000**	0.9778	**1.0000**	1.0000	1.0000	**0.7000**	0.6444	1.0000	1.0000
F21	**0.9933**	0.9667	**1.0000**	0.9767	**1.0000**	0.9733	**0.9933**	0.9467	**0.9867**	0.9600
F22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	**0.0056**	0.0000	1.0000	1.0000
F23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.8333	0.8333
F24	**0.9833**	0.8750	**0.9958**	0.8750	**0.9958**	0.8750	**0.9792**	0.8542	**0.9833**	0.8750
F25	**0.9667**	0.6125	**0.9583**	0.5646	**0.9875**	0.5688	**0.9542**	0.5833	**0.9667**	0.6042
F26	0.9833	**1.0000**	0.9722	**1.0000**	0.9722	**1.0000**	0.9444	**0.9889**	0.9333	**1.0000**
F27	**0.9259**	0.7815	**0.9444**	0.7167	**0.9537**	0.7019	**0.0333**	0.0222	**0.9519**	0.7370
F28	0.9413	**0.9440**	0.9493	**0.9773**	**0.9827**	0.9800	**0.9400**	0.5600	0.9680	**0.9760**
F29	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Avg.	**0.9567**	0.9451	**0.9579**	0.9388	**0.9786**	0.9382	**0.8200**	0.7714	**0.9655**	0.9348

Table B.2
Influence of different numerical methods in MENI-EA in terms of the success rate.

Prob	MENI-NCDE-TRD	TRD	MENI-NCDE-TRR	TRR	MENI-NCDE-LM	LM	MENI-NCDE-HC	HC	MENI-NCDE-NEWTON	NEWTON
F1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F2	**0.8333**	0.1333	**0.9000**	0.0333	**0.8333**	0.0333	**0.9667**	0.0000	**1.0000**	0.0667
F3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F4	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	**1.0000**	0.9333	1.0000	1.0000
F5	**0.6667**	0.4000	**0.7000**	0.2333	**0.8000**	0.2333	0.0000	0.0000	**0.7333**	0.1333
F6	**0.9667**	0.8333	**0.9667**	0.6667	**1.0000**	0.8667	0.9667	**1.0000**	**1.0000**	0.7333
F7	**1.0000**	0.9333	**1.0000**	0.9667	**1.0000**	0.9667	1.0000	1.0000	1.0000	1.0000
F8	**0.8667**	0.6333	0.9333	**1.0000**	**1.0000**	0.9667	**0.9000**	0.0000	0.8000	**1.0000**
F9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000	0.0000	0.8000	**1.0000**
F13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F14	1.0000	1.0000	**1.0000**	0.8667	**1.0000**	0.9667	1.0000	1.0000	**1.0000**	0.9333
F15	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
F16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	**1.0000**	0.9333	1.0000	1.0000
F19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F20	0.9333	**1.0000**	0.9333	**1.0000**	1.0000	1.0000	**0.1000**	0.0000	1.0000	1.0000
F21	**0.9333**	0.8333	**1.0000**	0.8667	**1.0000**	0.8667	**0.9667**	0.7333	**0.9333**	0.8000
F22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000	0.0000	1.0000	1.0000
F23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000	0.0000
F24	**0.8667**	0.0000	**0.9667**	0.0000	**0.9667**	0.0000	**0.8333**	0.0000	**0.8667**	0.0000
F25	**0.5333**	0.0000	**0.4333**	0.0000	**0.8000**	0.0000	**0.4333**	0.0000	**0.6667**	0.0000
F26	0.9000	**1.0000**	0.8333	**1.0000**	0.8333	**1.0000**	0.6667	**0.9333**	0.6000	1.0000
F27	**0.2000**	0.0000	**0.4000**	0.0000	**0.4000**	0.0000	0.0000	0.0000	**0.3333**	0.0000
F28	0.0333	**0.1333**	0.2000	**0.5333**	0.6333	0.6333	**0.0667**	0.0000	0.3333	**0.5333**
F29	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Avg.	**0.8578**	0.7633	**0.8756**	0.7722	**0.9089**	0.7844	**0.7300**	0.6178	0.8356	0.7400

Table B.3
Effect of different niching-based EAs in MENI-EA in terms of the peak ratio.

Prob	MENI-NCDE-LM	NCDE	MENI-NSDE-LM	NSDE	MENI-LIPS-LM	LIPS	MENI-R3PSO-LM	R3PSO	MENI-Fast_NCDE-LM	Fast_NCDE
F1	**1.0000**	0.8182	**0.9970**	0.8697	**1.0000**	0.9030	**1.0000**	0.6909	**0.9424**	0.1727
F2	**0.9889**	0.4022	**1.0000**	0.7911	**1.0000**	0.9778	**0.9867**	0.5178	**0.7867**	0.0400
F3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000	0.0000	**1.0000**	0.0000
F4	**1.0000**	0.0000	**1.0000**	0.1000	**1.0000**	0.0000	**0.3667**	0.0000	**1.0000**	0.0000
F5	**0.9778**	0.5037	**0.9444**	0.7630	**0.9815**	0.9407	**0.6148**	0.1889	**0.1370**	0.0000
F6	**1.0000**	0.9128	**0.9923**	0.8974	**0.8897**	0.3949	**0.8744**	0.3231	**0.9308**	0.4077
F7	**1.0000**	0.0313	**0.9854**	0.2500	**0.8771**	0.0021	**0.0333**	0.0000	**0.5021**	0.0000
F8	**1.0000**	0.2667	**1.0000**	0.8619	**1.0000**	0.9762	**0.7952**	0.0048	**0.7429**	0.0048
F9	**1.0000**	0.6667	**1.0000**	0.8500	1.0000	1.0000	**1.0000**	0.9000	**1.0000**	0.6833
F10	**1.0000**	0.5083	**1.0000**	0.8500	1.0000	1.0000	**1.0000**	0.4917	**0.9250**	0.0000
F11	**1.0000**	0.0083	**1.0000**	0.5000	1.0000	1.0000	**1.0000**	0.9083	**1.0000**	0.0167
F12	**1.0000**	0.9500	0.7333	**1.0000**	0.7333	**1.0000**	0.0000	0.0000	**0.5833**	0.0000
F13	**1.0000**	0.0000	**1.0000**	0.0000	**1.0000**	0.6833	**0.5667**	0.0000	**0.8167**	0.0000
F14	**1.0000**	0.3933	**1.0000**	0.8267	**0.9533**	0.6000	**0.8933**	0.0467	**0.9533**	0.0133
F15	**0.5000**	0.0000	0.0000	0.0000	**0.5000**	0.0000	0.0000	0.0000	0.0000	0.0000
F16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	**1.0000**	0.9167	**1.0000**	0.2833
F17	**1.0000**	0.0167	**1.0000**	0.3167	**0.8333**	0.1833	**0.9667**	0.0500	**0.7500**	0.0000
F18	**1.0000**	0.7500	**1.0000**	0.8167	**1.0000**	0.5167	**1.0000**	0.5500	**1.0000**	0.9500
F19	**1.0000**	0.8333	**1.0000**	0.8167	1.0000	1.0000	**1.0000**	0.1167	**0.9500**	0.1000
F20	**1.0000**	0.7556	**0.9778**	0.8444	**1.0000**	0.9333	**0.9889**	0.5111	**0.8889**	0.4778
F21	**1.0000**	0.4033	**1.0000**	0.5933	**0.9400**	0.7067	**0.9933**	0.5300	**0.7467**	0.0233
F22	**1.0000**	0.9556	**1.0000**	0.9833	1.0000	1.0000	**1.0000**	0.8000	**0.9778**	0.3944
F23	**1.0000**	0.9500	**1.0000**	0.9500	1.0000	1.0000	**1.0000**	0.7889	**0.9667**	0.0500
F24	**0.9958**	0.4875	**1.0000**	0.8458	**0.8833**	0.3333	**0.8667**	0.0000	**0.9375**	0.1167
F25	**0.9875**	0.2854	**0.9667**	0.3792	**0.9833**	0.8479	**0.9500**	0.2021	**0.5792**	0.0063
F26	**0.9722**	0.7389	**0.9333**	0.8167	**0.9167**	0.8333	**0.8444**	0.4667	**0.7667**	0.0167
F27	**0.9537**	0.2667	**0.9333**	0.4389	**0.9167**	0.6833	**0.8037**	0.1574	**0.3259**	0.0000
F28	**0.9827**	0.2867	**0.9600**	0.3707	**0.9680**	0.8280	**0.8920**	0.1680	**0.9600**	0.3707
F29	**1.0000**	0.7833	**1.0000**	0.9417	**1.0000**	0.9667	**1.0000**	0.7583	**0.9667**	0.0500
F30	**1.0000**	0.3278	**1.0000**	0.6000	1.0000	1.0000	**1.0000**	0.4833	**1.0000**	0.0222
Avg.	**0.9786**	0.5101	**0.9475**	0.6758	**0.9459**	0.7437	**0.7812**	0.3524	**0.8045**	0.1400

Table B.4
Effect of different niching-based EAs in MENI-EA in terms of the success rate.

Prob	MENI-NCDE-LM	NCDE	MENI-NSDE-LM	NSDE	MENI-LIPS-LM	LIPS	MENI-R3PSO-LM	R3PSO	MENI-Fast_NCDE-LM	Fast_NCDE
F1	**1.0000**	0.1333	**0.9667**	0.2333	**1.0000**	0.2333	**1.0000**	0.0000	**0.6333**	0.0000
F2	**0.8333**	0.0000	**1.0000**	0.0333	**1.0000**	0.7333	**0.8000**	0.0000	**0.0667**	0.0000
F3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000	0.0000	**1.0000**	0.0000
F4	**1.0000**	0.0000	**1.0000**	0.1000	**1.0000**	0.0000	**0.3667**	0.0000	**1.0000**	0.0000
F5	**0.8000**	0.0000	**0.6000**	0.1000	**0.8333**	0.5667	0.0000	0.0000	0.0000	0.0000
F6	**1.0000**	0.2667	**0.9000**	0.1333	**0.1000**	0.0000	**0.0333**	0.0000	**0.3667**	0.0000
F7	**1.0000**	0.0000	**0.8000**	0.0000	**0.1000**	0.0000	0.0000	0.0000	0.0000	0.0000
F8	**1.0000**	0.0000	**1.0000**	0.4333	**1.0000**	0.8333	**0.2000**	0.0000	**0.1667**	0.0000
F9	**1.0000**	0.3333	**1.0000**	0.7000	1.0000	1.0000	**1.0000**	0.8000	**1.0000**	0.3667
F10	**1.0000**	0.0000	**1.0000**	0.4333	1.0000	1.0000	**1.0000**	0.1000	**0.7333**	0.0000
F11	**1.0000**	0.0000	**1.0000**	0.0333	1.0000	1.0000	**1.0000**	0.7000	**1.0000**	0.0000
F12	**1.0000**	0.9000	0.5333	**1.0000**	0.4667	**1.0000**	0.0000	0.0000	**0.2667**	0.0000
F13	**1.0000**	0.0000	**1.0000**	0.0000	**1.0000**	0.5333	**0.3000**	0.0000	**0.6333**	0.0000
F14	**1.0000**	0.0000	**1.0000**	0.4333	**0.7667**	0.0000	**0.5000**	0.0000	**0.7667**	0.0000
F15	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
F16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	**1.0000**	0.8333	**1.0000**	0.0333
F17	**1.0000**	0.0000	**1.0000**	0.0000	**0.6667**	0.0000	**0.9333**	0.0000	**0.5000**	0.0000
F18	**1.0000**	0.5000	**1.0000**	0.6333	**1.0000**	0.0333	**1.0000**	0.1333	**1.0000**	0.9000
F19	**1.0000**	0.7000	**1.0000**	0.6333	1.0000	1.0000	**1.0000**	0.0000	**0.9000**	0.0000
F20	**1.0000**	0.3000	**0.9333**	0.5333	**1.0000**	0.8000	**0.9667**	0.0333	**0.6667**	0.0000
F21	**1.0000**	0.0000	**1.0000**	0.0000	**0.7000**	0.0333	**0.9667**	0.0000	**0.1000**	0.0000
F22	**1.0000**	0.7333	**1.0000**	0.9000	1.0000	1.0000	**1.0000**	0.2333	**0.9000**	0.0333
F23	**1.0000**	0.7000	**1.0000**	0.7000	1.0000	1.0000	**1.0000**	0.2333	**0.8000**	0.0000
F24	**0.9667**	0.0000	**1.0000**	0.1333	**0.0667**	0.0000	0.0000	0.0000	**0.6667**	0.0000
F25	**0.8000**	0.0000	**0.6667**	0.0000	**0.7333**	0.0333	**0.3667**	0.0000	0.0000	0.0000
F26	**0.8333**	0.0333	**0.6000**	0.0667	**0.5000**	0.0000	**0.2667**	0.0000	**0.1000**	0.0000
F27	**0.4000**	0.0000	**0.3000**	0.0000	**0.2000**	0.0000	0.0000	0.0000	0.0000	0.0000
F28	**0.6333**	0.0000	**0.3333**	0.0000	**0.3667**	0.0000	0.0000	0.0000	**0.3333**	0.0000
F29	**1.0000**	0.1667	**1.0000**	0.8000	**1.0000**	0.8667	**1.0000**	0.2333	**0.8667**	0.0000
F30	**1.0000**	0.0000	**1.0000**	0.0333	1.0000	1.0000	**1.0000**	0.0000	**1.0000**	0.0000
Avg.	**0.9089**	0.2256	**0.8544**	0.3355	**0.7500**	0.4889	**0.5567**	0.1100	**0.5489**	0.0444

Table B.5
Comparison between MENI-NCDE-LM and other state-of-the-art methods with respect to peak ratio.

Prob.	MENI-NCDE-LM	A-WeB	MONES	I-HS	RADE	DR-JADE
F01	**1.0000**	**1.0000**	0.9697	0.8758	**1.0000**	0.8727
F02	**0.9889**	0.9500	0.9489	0.8533	0.9844	0.8267
F03	**1.0000**	**1.0000**	**1.0000**	0.0333	**1.0000**	**1.0000**
F04	**1.0000**	0.4200	0.0000	0.0333	0.9333	**1.0000**
F05	**0.9778**	0.9733	0.0407	0.8000	0.9926	0.6519
F06	**1.0000**	**1.0000**	**1.0000**	0.3462	0.9974	0.7333
F07	**1.0000**	0.6688	0.0500	0.7313	0.9500	0.8396
F08	**1.0000**	0.9514	0.5810	0.7476	0.8381	0.7571
F09	**1.0000**	**1.0000**	0.9167	**1.0000**	**1.0000**	**1.0000**
F10	**1.0000**	**1.0000**	0.8417	**1.0000**	0.4917	0.9750
F11	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F12	**1.0000**	0.6200	**1.0000**	0.0000	**1.0000**	0.3167
F13	**1.0000**	0.8900	0.0667	0.0000	**1.0000**	**1.0000**
F14	**1.0000**	0.9320	0.1600	0.0467	**1.0000**	0.7800
F15	0.5000	**0.6200**	0.0000	0.0000	0.0000	0.0000
F16	**1.0000**	**1.0000**	**1.0000**	0.9167	**1.0000**	0.9833
F17	**1.0000**	0.3667	0.0667	0.0500	0.6667	0.5500
F18	**1.0000**	0.0167	**1.0000**	0.5000	0.8333	0.5000
F19	**1.0000**	0.9667	**1.0000**	**1.0000**	0.5000	**1.0000**
F20	**1.0000**	0.0000	0.9444	0.3222	0.9222	0.7111
F21	**1.0000**	0.0000	0.8267	0.9300	0.7633	0.8767
F22	**1.0000**	0.9833	**1.0000**	0.9333	**1.0000**	0.7611
F23	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F24	**0.9958**	0.7750	0.6000	0.1708	0.8792	0.9500
F25	**0.9875**	0.8000	0.4479	0.8604	0.8167	0.7229
F26	**0.9722**	0.8667	0.8611	0.8556	0.9833	0.7944
F27	**0.9537**	0.8981	0.2667	0.7185	0.9333	0.7296
F28	**0.9827**	0.5400	0.1693	0.0000	0.7360	0.5227
F29	**1.0000**	**1.0000**	0.8083	**1.0000**	**1.0000**	**1.0000**
F30	**1.0000**	0.9000	0.8944	0.9944	**1.0000**	0.9778
Avg.	**0.9786**	0.7713	0.6487	0.5906	0.8741	0.7944

Table B.6
Comparison between MENI-NCDE-LM and other state-of-the-art methods with respect to success rate.

Prob.	MENI-NCDE-LM	A-WeB	MONES	I-HS	RADE	DR-JADE
F01	**1.0000**	**1.0000**	0.8000	0.1333	**1.0000**	0.2667
F02	**0.8333**	0.5800	0.6333	0.0333	0.8000	0.0000
F03	**1.0000**	**1.0000**	**1.0000**	0.0333	**1.0000**	**1.0000**
F04	**1.0000**	0.4200	0.0000	0.0333	0.9333	**1.0000**
F05	**0.8000**	0.7600	0.0000	0.1000	0.9333	0.0000
F06	**1.0000**	**1.0000**	**1.0000**	0.0000	0.9667	0.0000
F07	**1.0000**	0.0000	0.0000	0.0000	0.4333	0.0667
F08	**1.0000**	0.7000	0.0000	0.0667	0.4333	0.1667
F09	**1.0000**	**1.0000**	0.8333	**1.0000**	**1.0000**	**1.0000**
F10	**1.0000**	**1.0000**	0.4667	**1.0000**	0.0000	0.9000
F11	**1.0000**	**1.0000**	**1.0000**	1.0000	**1.0000**	**1.0000**
F12	**1.0000**	0.3600	**1.0000**	0.0000	**1.0000**	0.0667
F13	**1.0000**	0.6800	0.0000	0.0000	**1.0000**	**1.0000**
F14	**1.0000**	0.6600	0.0000	0.0000	**1.0000**	0.3000
F15	0.0000	**0.2400**	0.0000	0.0000	0.0000	0.0000
F16	**1.0000**	**1.0000**	**1.0000**	0.8333	**1.0000**	0.9667
F17	**1.0000**	0.0333	0.0000	0.0000	0.3333	0.1000
F18	**1.0000**	0.0000	**1.0000**	0.0000	0.6667	0.0000
F19	**1.0000**	0.9300	**1.0000**	**1.0000**	0.0000	**1.0000**
F20	**1.0000**	0.0000	0.8333	0.0333	0.7667	0.2333
F21	**1.0000**	0.0000	0.2667	0.5000	0.2500	0.2000
F22	**1.0000**	0.9000	**1.0000**	0.6333	**1.0000**	0.1000
F23	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F24	**0.9667**	0.2000	0.0333	0.0000	0.0333	0.7000
F25	**0.8000**	0.1000	0.0000	0.0667	0.0333	0.0000
F26	**0.8333**	0.4000	0.3667	0.3000	0.9333	0.0333
F27	**0.4000**	0.0000	0.0000	0.0000	0.3000	0.0000
F28	**0.6333**	0.0000	0.0000	0.0000	0.0000	0.0000
F29	**1.0000**	**1.0000**	0.4000	**1.0000**	**1.0000**	**1.0000**
F30	**1.0000**	0.5000	0.6667	0.9667	**1.0000**	0.8667
Avg.	**0.9089**	0.5488	0.4767	0.3244	0.6606	0.4322

Table B.7
Influence of different accuracy levels in MENI-NCDE-LM, RADE and DR-JADE with respect to the peak ratio.

Prob	MENI-EA-6	RADE-6	DR-JADE-6	MENI-EA-7	RADE-7	DR-JADE-7	MENI-EA-8	RADE-8	DR-JADE-8	MENI-EA-9	RADE-9	DR-JADE-9
F1	**1.0000**	**1.0000**	0.8606	**1.0000**	**1.0000**	0.8394	**1.0000**	**1.0000**	0.8424	0.6364	**0.9939**	0.8394
F2	**0.9956**	0.9288	0.8222	**0.9911**	0.8622	0.8133	**0.9978**	0.7556	0.8156	**0.9889**	0.5556	0.7978
F3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.8000	**1.0000**	**1.0000**	0.0000
F4	**1.0000**	0.4000	**1.0000**	**1.0000**	0.0667	**1.0000**	**1.0000**	0.0000	**1.0000**	**1.0000**	0.0000	**1.0000**
F5	0.9481	**0.9778**	0.6519	**0.9741**	0.9593	0.6444	**0.9519**	0.9333	0.6407	0.7667	**0.9037**	0.6370
F6	**1.0000**	0.9974	0.7487	**1.0000**	0.9538	0.7795	**0.9949**	0.8923	0.7641	**0.9949**	0.8128	0.7641
F7	**1.0000**	0.9500	0.8396	0.1438	**0.8542**	0.8125	0.0104	0.7333	**0.8417**	0.0021	**0.6104**	0.8229
F8	**0.9905**	0.7143	0.7524	**0.9952**	0.5524	0.7667	**0.9905**	0.4857	0.7571	**0.9857**	0.4476	0.8048
F9	**1.0000**	**1.0000**	0.9833	0.5000	**1.0000**	**1.0000**	0.5000	**1.0000**	**1.0000**	0.5000	**1.0000**	**1.0000**
F10	**1.0000**	0.4833	0.9833	**1.0000**	0.4583	0.9917	**1.0000**	0.4250	0.9917	**1.0000**	0.3333	0.9917
F11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F12	0.2000	**1.0000**	0.2833	0.0000	**0.9667**	0.3167	0.0000	**1.0000**	0.3333	0.0000	**0.9500**	0.0333
F13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	**1.0000**	0.9500	**1.0000**	**1.0000**	0.3833	**1.0000**
F14	**1.0000**	0.9933	0.7400	**1.0000**	0.9667	0.7267	0.8000	**0.8867**	0.7600	0.8000	**0.8267**	0.7533
F15	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
F16	1.0000	1.0000	1.0000	**1.0000**	**1.0000**	0.9833	**1.0000**	1.0000	1.0000	0.5000	**1.0000**	0.9833
F17	**1.0000**	0.5333	0.5000	**1.0000**	0.5167	0.5167	**1.0000**	0.4833	0.5000	**1.0000**	0.4333	0.5500
F18	**1.0000**	0.7500	0.5000	**1.0000**	0.7333	0.5000	**1.0000**	0.9333	0.5000	**1.0000**	0.9333	0.5167
F19	**1.0000**	0.5000	**1.0000**	**1.0000**	0.5000	**1.0000**	**1.0000**	0.5000	**1.0000**	**1.0000**	0.5000	**1.0000**
F20	**1.0000**	0.9000	0.7222	**0.9889**	0.3889	0.4000	**0.6667**	0.1667	0.3778	**0.6667**	0.0889	0.3889
F21	**1.0000**	0.7400	0.8400	**0.9867**	0.7333	0.8567	**0.9900**	0.5600	0.8533	0.7833	0.4800	**0.8700**
F22	**1.0000**	**1.0000**	0.7722	**1.0000**	**1.0000**	0.7444	**1.0000**	**1.0000**	0.7611	**1.0000**	**1.0000**	0.8000
F23	**1.0000**	**1.0000**	0.9889	**1.0000**	**1.0000**	0.9778	**1.0000**	**1.0000**	0.9722	**1.0000**	**1.0000**	0.9722
F24	**0.9833**	0.8750	0.9708	**0.9833**	0.8750	0.9792	**0.9917**	0.8583	0.9667	0.6250	0.5417	**0.9792**
F25	**0.9625**	0.7375	0.7208	**0.9708**	0.6667	0.6938	**0.9667**	0.5791	0.7146	**0.9542**	0.5125	0.6854
F26	0.9778	**0.9833**	0.7944	0.9278	**0.9611**	0.8000	**0.9667**	0.9222	0.7722	**0.9500**	0.8833	0.7889
F27	**0.9556**	0.8518	0.7185	**0.9370**	0.7315	0.6981	**0.9315**	0.6092	0.7056	**0.9296**	0.5000	0.6556
F28	**0.9547**	0.6427	0.5267	**0.9493**	0.5827	0.5147	**0.9440**	0.5347	0.4627	**0.9560**	0.4653	0.3693
F29	1.0000	1.0000	1.0000	**1.0000**	**1.0000**	0.9750	**1.0000**	**1.0000**	0.9833	1.0000	1.0000	1.0000
F30	**1.0000**	**1.0000**	0.9889	**1.0000**	**1.0000**	0.9722	**1.0000**	**1.0000**	0.9833	**1.0000**	**1.0000**	0.9889
Avg.	**0.9323**	0.8320	0.7903	**0.8783**	0.7777	0.7768	**0.8568**	0.7403	0.7700	**0.8013**	0.6719	0.7331

Table B.8
Influence of different accuracy levels in MENI-NCDE-LM, RADE and DR-JADE with respect to the success rate.

Prob	MENI-EA-6	RADE-6	DR-JADE-6	MENI-EA-7	RADE-7	DR-JADE-7	MENI-EA-8	RADE-8	DR-JADE-8	MENI-EA-9	RADE-9	DR-JADE-9
F1	**1.0000**	**1.0000**	0.1000	**1.0000**	**1.0000**	0.0667	**1.0000**	**1.0000**	0.1000	0.0000	**0.9333**	0.1000
F2	**0.9333**	0.4667	0.0000	**0.8667**	0.3333	0.0667	**0.9667**	0.0000	0.0333	**0.8333**	0.0000	0.0000
F3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	**1.0000**	**1.0000**	0.8000	1.0000	1.0000	0.0000
F4	**1.0000**	0.4000	**1.0000**	**1.0000**	0.0667	**1.0000**	1.0000	0.0000	1.0000	**1.0000**	0.0000	**1.0000**
F5	0.6000	**0.8333**	0.0000	**0.7667**	0.7333	0.0000	**0.6667**	0.6000	0.0000	0.0000	**0.4333**	0.0000
F6	**1.0000**	0.9667	0.0000	**1.0000**	0.5000	0.0000	**0.9333**	0.1300	0.0667	**0.9333**	0.0333	0.0333
F7	**1.0000**	0.4333	0.0000	0.0000	**0.0677**	0.0000	0.0000	0.0000	0.0333	0.0000	0.0000	0.0000
F8	**0.9333**	0.1667	0.0667	**0.9667**	0.0333	0.1333	**0.9333**	0.0000	0.1000	**0.9000**	0.0000	0.1333
F9	**1.0000**	**1.0000**	0.9667	0.0000	**1.0000**	**1.0000**	0.0000	**1.0000**	**1.0000**	0.0000	**1.0000**	**1.0000**
F10	**1.0000**	0.0000	0.9333	**1.0000**	0.0000	0.9667	**1.0000**	0.0000	0.9667	**1.0000**	0.0000	0.9667
F11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F12	0.0000	**1.0000**	0.1000	0.0000	**0.9333**	0.1333	0.0000	**1.0000**	0.1000	0.0000	**0.9000**	0.0000
F13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	**1.0000**	0.9000	**1.0000**	**1.0000**	0.1300	**1.0000**
F14	**1.0000**	0.9667	0.1333	**1.0000**	0.8333	0.1000	0.0000	**0.4667**	0.1000	0.0000	**0.1667**	0.0667
F15	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
F16	1.0000	1.0000	1.0000	**1.0000**	**1.0000**	0.9667	1.0000	1.0000	1.0000	0.0000	**1.0000**	0.9667
F17	**1.0000**	0.0667	0.0000	**1.0000**	0.0333	0.0333	**1.0000**	0.0000	0.0000	**1.0000**	0.0000	0.1000
F18	**1.0000**	0.5000	0.0000	**1.0000**	0.4667	0.0000	**1.0000**	0.9333	0.0000	**1.0000**	0.9333	0.0333
F19	**1.0000**	0.0000	**1.0000**	**1.0000**	0.0000	**1.0000**	**1.0000**	0.0000	**1.0000**	**1.0000**	0.0000	**1.0000**
F20	**1.0000**	0.7000	0.2333	**0.9667**	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
F21	**1.0000**	0.2300	0.1667	**0.9333**	0.0000	0.1667	**0.9333**	0.0000	0.1667	0.0000	0.0000	0.3000
F22	**1.0000**	**1.0000**	0.0667	**1.0000**	**1.0000**	0.0000	**1.0000**	**1.0000**	0.1000	**1.0000**	**1.0000**	0.1000
F23	**1.0000**	**1.0000**	0.9333	**1.0000**	**1.0000**	0.8667	**1.0000**	**1.0000**	0.8667	**1.0000**	**1.0000**	0.8333
F24	**0.8667**	0.0000	0.8000	**0.8667**	0.0000	0.8333	**0.9333**	0.0000	0.7333	0.0000	0.0000	**0.8333**
F25	**0.4000**	0.0333	0.0000	**0.6667**	0.0000	0.0000	**0.6333**	0.0000	0.0000	**0.4667**	0.0000	0.0000
F26	0.8667	**0.9000**	0.0333	0.6000	**0.8333**	0.0000	**0.8000**	0.6333	0.0333	**0.7333**	0.5000	0.0333
F27	**0.4667**	0.1300	0.0000	**0.2000**	0.0000	0.0000	**0.2000**	0.0000	0.0000	**0.2667**	0.0000	0.0000
F28	**0.2000**	0.0000	0.0000	**0.1000**	0.0000	0.0000	**0.1667**	0.0000	0.0000	**0.2667**	0.0000	0.0000
F28	1.0000	1.0000	1.0000	**1.0000**	**1.0000**	0.9000	**1.0000**	**1.0000**	0.9333	1.0000	1.0000	1.0000
F30	**1.0000**	**1.0000**	0.9333	**1.0000**	**1.0000**	0.8333	**1.0000**	**1.0000**	0.9000	**1.0000**	**1.0000**	0.9333
Avg.	**0.8422**	0.5931	0.4156	**0.7644**	0.4945	0.4022	**0.7056**	0.4554	0.4011	**0.5467**	0.4010	0.3811

Table B.9
Influence of different parameter values in MENI-EA in terms of the peak ratio.

Prob.	ϵ=0.1	ϵ=0.2	ϵ=0.3	ϵ=0.4	ϵ=0.5	ϵ=0.6	ϵ=0.7	ϵ=0.8
F01	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F02	**0.9956**	**0.9956**	0.9889	0.9867	0.9889	0.9867	0.9867	0.9822
F03	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F04	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F05	0.9778	0.9852	0.9852	**0.9852**	0.9778	0.9852	0.9926	0.9926
F06	**1.0000**	**1.0000**	0.9974	0.9974	**1.0000**	**1.0000**	0.9974	**1.0000**
F07	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F08	0.9905	0.9952	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.9857
F09	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F10	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F11	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F12	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F13	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F14	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F15	0.0000	0.0167	0.2500	0.4500	**0.5000**	**0.5000**	**0.5000**	**0.5000**
F16	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F17	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F18	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F19	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F20	0.9111	0.9889	0.9444	0.9778	**1.0000**	**1.0000**	**1.0000**	0.9889
F21	0.9833	0.9867	0.9733	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.9933
F22	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F23	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F24	**0.9958**	0.9917	**0.9958**	0.9833	**0.9958**	0.9917	0.9833	0.9833
F25	0.9729	0.9771	0.9854	0.9854	0.9875	0.9813	**0.9896**	**0.9896**
F26	0.9667	0.9500	0.9722	0.9778	0.9722	**1.0000**	0.9722	0.9667
F27	0.9315	0.9407	0.9556	0.9407	0.9537	**0.9667**	0.9519	0.9500
F28	0.9480	0.9707	0.9707	0.9733	**0.9827**	0.9707	0.9707	0.9827
F29	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F30	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
Avg.	0.9558	0.9599	0.9673	0.9753	0.9786	**0.9794**	0.9781	0.9772

Table B.10
Influence of different θ in MENI-EA in terms of the success rate.

Prob.	ϵ=0.1	ϵ=0.2	ϵ=0.3	ϵ=0.4	ϵ=0.5	ϵ=0.6	ϵ=0.7	ϵ=0.8
F01	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F02	**0.9333**	**0.9333**	0.8333	0.8000	0.8333	0.8000	0.8000	0.7333
F03	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F04	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F05	0.8000	0.9000	0.8667	**0.9000**	0.8000	0.8667	0.9333	0.9333
F06	**1.0000**	**1.0000**	0.9667	0.9667	**1.0000**	**1.0000**	0.9667	**1.0000**
F07	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F08	0.9333	0.9667	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	0.9000
F09	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F10	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F11	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F12	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F13	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F14	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F15	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
F16	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F17	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F18	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F19	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F20	0.7333	0.9667	0.8667	0.9333	**1.0000**	**1.0000**	**1.0000**	0.9667
F21	0.9000	0.9333	0.8667	1.0000	**1.0000**	**1.0000**	**1.0000**	0.9667
F22	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F23	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F24	**0.9667**	0.9333	**0.9667**	0.8667	**0.9667**	0.9333	0.8667	0.8667
F25	0.6333	0.7000	0.7667	0.8000	0.8000	0.7333	**0.8333**	**0.8333**
F26	0.8000	0.7000	0.8333	0.8667	0.8333	**1.0000**	0.8333	0.8000
F27	0.3000	0.4000	0.4667	0.2333	0.4000	**0.5667**	0.4333	0.3667
F28	0.1333	0.4000	0.4333	0.4667	**0.6333**	0.5000	0.5000	**0.6333**
F29	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F30	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**	**1.0000**
Avg.	0.8711	0.8944	0.8956	0.8944	0.9089	**0.9133**	0.9056	0.9000

Table B.11
Influence of different local search methods in MENI-EA in terms of the peak ratio.

Prob.	MENI-NCDE-LM	MENI-NCDE-NM	MENI-NCDE-PS	MENI-NCDE-SPX
F01	**1.0000**	**1.0000**	**1.0000**	0.9848
F02	0.9889	0.9978	0.9911	0.6822
F03	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F04	**1.0000**	**1.0000**	0.1667	0.0000
F05	**0.9778**	0.9667	0.9630	0.7000
F06	**1.0000**	**1.0000**	**1.0000**	0.9513
F07	**1.0000**	0.9958	0.7417	0.2813
F08	**1.0000**	0.9905	0.9810	0.5190
F09	**1.0000**	**1.0000**	**1.0000**	0.7667
F10	**1.0000**	**1.0000**	**1.0000**	0.5917
F11	**1.0000**	**1.0000**	**1.0000**	0.1083
F12	**1.0000**	0.0500	0.8833	1.0000
F13	**1.0000**	1.0000	0.7167	0.0000
F14	**1.0000**	1.0000	1.0000	0.6467
F15	0.5000	0.0000	0.0000	0.0000
F16	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F17	**1.0000**	**1.0000**	0.3667	0.1500
F18	**1.0000**	**1.0000**	0.5000	0.8333
F19	**1.0000**	**1.0000**	**1.0000**	0.9667
F20	**1.0000**	0.9778	0.9222	0.9444
F21	**1.0000**	0.9900	0.9767	0.4000
F22	**1.0000**	**1.0000**	**1.0000**	0.9944
F23	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F24	**0.9958**	0.9917	0.9792	0.8542
F25	**0.9875**	0.9583	0.8917	0.4063
F26	**0.9722**	0.9611	0.9556	0.8000
F27	**0.9537**	0.9407	0.9093	0.4667
F28	**0.9827**	0.9507	0.9373	0.3173
F29	**1.0000**	**1.0000**	**1.0000**	0.8250
F30	**1.0000**	**1.0000**	**1.0000**	0.4611
Avg.	**0.9786**	0.9257	0.8627	0.6217

Table B.12
Influence of different local search methods in MENI-EA in terms of the success rate.

Prob.	MENI-NCDE-LM	MENI-NCDE-NM	MENI-NCDE-PS	MENI-NCDE-SPX
F01	**1.0000**	**1.0000**	**1.0000**	0.8333
F02	0.8333	**0.9667**	0.8667	0.0000
F03	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F04	**1.0000**	**1.0000**	0.1667	0.0000
F05	**0.8000**	0.7333	0.7333	0.0000
F06	**1.0000**	**1.0000**	**1.0000**	0.5000
F07	**1.0000**	0.9333	0.0000	0.0000
F08	**1.0000**	0.9333	0.8667	0.0000
F09	**1.0000**	**1.0000**	**1.0000**	0.5333
F10	**1.0000**	**1.0000**	**1.0000**	0.0333
F11	**1.0000**	**1.0000**	**1.0000**	0.0000
F12	**1.0000**	0.0333	0.7667	**1.0000**
F13	**1.0000**	**1.0000**	0.5000	0.0000
F14	**1.0000**	**1.0000**	**1.0000**	0.1000
F15	0.0000	0.0000	0.0000	0.0000
F16	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F17	**1.0000**	**1.0000**	0.0667	0.0000
F18	**1.0000**	**1.0000**	0.0000	0.6667
F19	**1.0000**	**1.0000**	**1.0000**	0.9333
F20	**1.0000**	0.9333	0.7667	0.8333
F21	**1.0000**	0.9333	0.8333	0.0000
F22	**1.0000**	**1.0000**	**1.0000**	0.9667
F23	**1.0000**	**1.0000**	**1.0000**	**1.0000**
F24	**0.9667**	0.9333	0.8333	0.3333
F25	**0.8000**	0.5333	0.2000	0.0000
F26	**0.8333**	0.7667	0.7333	0.2333
F27	**0.4000**	0.3333	0.0333	0.0000
F28	**0.6333**	0.2333	0.0667	0.0000
F29	**1.0000**	**1.0000**	**1.0000**	0.3000
F30	**1.0000**	**1.0000**	**1.0000**	0.0000
Avg.	**0.9089**	0.8422	0.6811	0.3422

Table B.13
Influence of other niching-based DE in MENI-EA in terms of the peak ratio.

Prob	MENI-SHNCDE-LM	SHNCDE	NCDE	MENI-SHNSDE-LM	SHNSDE	NSDE
F1	**1.0000**	0.9333	0.8182	**1.0000**	0.8970	0.8697
F2	**0.9867**	0.5289	0.4022	**1.0000**	0.7778	0.7911
F3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F4	**1.0000**	0.0000	0.0000	**1.0000**	0.0667	0.1000
F5	**0.9778**	0.6667	0.5037	**0.9704**	0.8148	0.7630
F6	**1.0000**	0.9692	0.9128	**0.9846**	0.9538	0.8974
F7	**0.9958**	0.3667	0.0313	**0.9833**	0.3000	0.2500
F8	**1.0000**	0.4381	0.2667	**1.0000**	0.9238	0.8619
F9	**1.0000**	0.5333	0.6667	**1.0000**	**1.0000**	0.8500
F10	**1.0000**	0.8000	0.5083	**1.0000**	**1.0000**	0.8500
F11	**1.0000**	0.7000	0.0083	**1.0000**	**1.0000**	0.5000
F12	0.8667	**1.0000**	0.9500	0.8000	**1.0000**	**1.0000**
F13	**1.0000**	0.0000	0.0000	**1.0000**	0.0333	0.0000
F14	**1.0000**	0.6800	0.3933	**1.0000**	**1.0000**	0.8267
F15	**0.5000**	0.0000	0.0000	0.5000	0.0000	0.0000
F16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F17	**1.0000**	0.3667	0.0167	**1.0000**	0.4667	0.3167
F18	**1.0000**	0.8000	0.7500	**1.0000**	0.9000	0.8167
F19	**1.0000**	0.9333	0.8333	**1.0000**	0.9667	0.8167
F20	**1.0000**	0.9778	0.7556	**0.9111**	0.6667	0.8444
F21	**1.0000**	0.4400	0.4033	**0.9867**	0.7200	0.5933
F22	**1.0000**	**1.0000**	0.9556	**1.0000**	0.9889	0.9833
F23	**1.0000**	**1.0000**	0.9500	**1.0000**	**1.0000**	0.9500
F24	**0.9750**	0.8583	0.4875	**1.0000**	**1.0000**	0.8458
F25	**0.9875**	0.3083	0.2854	**0.9875**	0.5583	0.3792
F26	**0.9556**	0.8111	0.7389	**0.9889**	0.8222	0.8167
F27	**0.9667**	0.3815	0.2667	**0.9667**	0.6148	0.4389
F28	**0.9893**	0.4187	0.2867	**0.9926**	0.6074	0.3707
F28	**1.0000**	0.9667	0.7833	**1.0000**	0.9667	0.9417
F30	**1.0000**	0.7444	0.3278	**1.0000**	0.9889	0.6000
Avg.	**0.9734**	0.6541	0.5101	**0.9691**	0.7678	0.6758

Table B.14
Influence of other niching-based DE in MENI-EA in terms of the success rate.

Prob	MENI-SHNCDE-LM	SHNCDE	NCDE	MENI-SHNSDE-LM	SHNSDE	NSDE
F1	**1.0000**	0.3333	0.1333	**1.0000**	0.4000	0.2333
F2	**0.8000**	0.0000	0.0000	**1.0000**	0.0667	0.0333
F3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F4	**1.0000**	0.0000	0.0000	**1.0000**	0.0667	0.1000
F5	**0.8000**	0.0000	0.0000	**0.8000**	0.1333	0.1000
F6	**1.0000**	0.6667	0.2667	**0.8000**	0.4667	0.1333
F7	**0.9333**	0.0000	0.0000	**0.7333**	0.0000	0.0000
F8	**1.0000**	0.0000	0.0000	**1.0000**	0.7333	0.4333
F9	**1.0000**	0.0667	0.3333	1.0000	1.0000	0.7000
F10	**1.0000**	0.2000	0.0000	**1.0000**	**1.0000**	0.4333
F11	**1.0000**	0.3333	0.0000	**1.0000**	**1.0000**	0.0333
F12	0.7333	**1.0000**	0.9000	0.6000	**1.0000**	**1.0000**
F13	**1.0000**	0.0000	0.0000	**1.0000**	0.0000	0.0000
F14	**1.0000**	0.2000	0.0000	**1.0000**	**1.0000**	0.4333
F15	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
F16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
F17	**1.0000**	0.0000	0.0000	**1.0000**	0.0000	0.0000
F18	**1.0000**	0.6000	0.5000	**1.0000**	0.8000	0.6333
F19	**1.0000**	0.8667	0.7000	**1.0000**	0.9333	0.6333
F20	**1.0000**	0.9333	0.3000	**0.7333**	0.0000	0.5333
F21	**1.0000**	0.0000	0.0000	**0.9333**	0.0667	0.0000
F22	**1.0000**	**1.0000**	0.7333	**1.0000**	0.9333	0.9000
F23	**1.0000**	**1.0000**	0.7000	1.0000	1.0000	0.7000
F24	**0.8000**	0.2000	0.0000	**1.0000**	**1.0000**	0.1333
F25	**0.8000**	0.0000	0.0000	**0.8000**	0.0000	0.0000
F26	**0.8000**	0.2000	0.0333	**0.9333**	0.0000	0.0667
F27	**0.5333**	0.0000	0.0000	**0.6000**	0.0000	0.0000
F28	**0.7333**	0.0000	0.0000	**0.8667**	0.0000	0.0000
F28	**1.0000**	0.8667	0.1667	**1.0000**	0.8667	0.8000
F30	**1.0000**	0.0667	0.0000	**1.0000**	0.9333	0.0333
Avg.	**0.8978**	0.3511	0.2256	**0.8933**	0.5133	0.3356

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