Image Contrast Enhancement using Selfish Herd Optimizer

Ritam Guha[a], Imran Alam[a], Suman Kumar Bera[a], Kushal Kanti Ghosh[a], Neeraj Kumar[b,*], Ram Sarkar[a]

[a] *Dept. of Computer Science and Engineering, Jadavpur University, Kolkata, India - 700032*
[b] *Dept. of Computer Science and Engineering, Thapar Institute of Engineering and Technology, Punjab, India*

Abstract

Contrast enhancement is an important pre-processing task in any Image Analysis (IA) system. In this paper, we formulate the image contrast enhancement problem as an optimization problem where the goal is to optimize the pixel intensity values of an input image to obtain a contrast enhanced version of the same. This optimization task is executed by suitably customizing a nature-inspired optimization algorithm called Selfish Herd Optimizer (SHO). The optimization problem is solved using two different solution representations: pixel wise optimization (SHO(direct)) and transformation function based optimization (SHO(transformation)). Moreover, an ablation study is performed to select the most appropriate parameters which can be used in fitness measure for this optimization problem. On experimenting over the popular Kodak image dataset, it has been observed that the proposed methods outperform many existing methods published recently. Further comparisons indicate that the direct approach performs better than its transformation counterpart. This paper further investigates the robustness of SHO(direct) approach by applying it to enhance the degraded document images of H-DIBCO 2018.

Keywords: Image contrast enhancement, Selfish Herd Optimizer, Meta-heuristic, Transformation function, Evolutionary algorithm, Kodak dataset, H-DIBCO 2018

☆This document is a collaborative effort.

*Corresponding author

Email addresses: ritamguha16@gmail.com (Ritam Guha), imranalam5555@gmail.com (Imran Alam), berasuman007@gmail.com (Suman Kumar Bera), kushalkanti1999@gmail.com (Kushal Kanti Ghosh), neeraj.kumar@thapar.edu (Neeraj Kumar), ramjucse@gmail.com (Ram Sarkar)

1. Introduction

Degradation in digital images may arise due to several reasons starting from the image acquisition to processing of the images. It also suffers from the aging, uneven illumination of light and dark creases. Thus, an Image Analysis (IA) system always needs a precise contrast enhancement technique to handle all the said complexities. A contrast enhancement technique actually tries to improve the contrast (i.e. intensity) level of an image so that a computer system or a human being can easily retrieve the accurate information present therein. Image contrast can be defined as the partition-factor between the darkest and brightest pixels (spots) in an image [1] where a smaller partition-factor indicates lower contrast and a larger partition factor indicates higher contrast. Since image contrast enhancement (ICE) has become a crucial part of any kind of image processing task, numerous methods have already been proposed in the literature addressing the challenges of the contrast enhancement problem. In spatial domain, histogram equalization (HE) is the preliminary image enhancement method. It is further improved by the adaptive histogram equalization developed by Santhi and Banu [2], where the magnitude of probability density function of an input image is scaled up prior to actual contrast enhancement using HE. Poddar et al. [3] present a modified HE method in a non-parametric way to reduce the distortion in smooth regions on gray images. The contrast and sharpness measures can also be applied to adjust the image intensity values [4]. But most of them suffer where there is combined degradation in the images.

Recently, it has been observed that many researchers apply evolutionary algorithms [5, 6, 7, 8] to process the low quality images. In this way of contrast enhancement, each pixel intensity gets altered to its optimal value, and this mapping is decided by using the evolutionary algorithms [9]. In medical image processing, many bionic algorithms have been utilized for enhancing the contrast as well as reducing noises. In 2016, Daniel and Anitha [10] have combined optimum wavelet and Cuckoo Search Algorithm (CSA) for medical image enhancement where CSA is used to find the optimal scale value of wavelet. In next year, Singh et al. [11] develop a method to enhance the magnetic resonance (MR) images, where Bat algorithm is combined with neural network to tune the model parameters so that estimated losses between two competitive images get minimized. In the same year, Gong et al. [12] have designed a MR imaging-clonal algorithm using the improved immune algorithm to enhance the MRI brain images. In this method, the immune algorithm has been improved by using the real coding approach and mutation process is better controlled by adding mutation distance into mutation operator. In 2018, Artificial Bee Colony (ABC) algorithm has been applied by Chen et al. [13] to enhance the pixel level clarity in scene images. In this method, the Incomplete Bate Function (IBF) is adopted to show its effectiveness in parametric image enhancement. Particle Swarm Optimization (PSO) algorithm has been applied for contrast enhancement technique in [14] where PSO is used to find the optimal gamma value of the gray-level intensity space.

Although the existing methods bring out the hidden details of a degraded image, most of them are unable to provide any explanation for proper fitness measures which can guide the candidate solutions to the optimal solution. In order to overcome such shortcomings in the domain, this paper proposes a new ICE method based on Selfish Herd Optimizer (SHO) which also adapts fitness functions based on ablation studies. The contributions of this paper are mainly threefold:

1) Application of a nature-inspired algorithm called SHO for the first time in the domain of ICE. SHO is inspired from the interactions between a group of preys and a pack of predators, which guides its solutions to the optimal one by using the concept of survival of preys based on their relative positions in the herd. This unique concept of SHO has been used after required modifications to make it applicable for the problem of ICE.

2) Use of two different ways to perform optimization in ICE domain: (a) direct application of optimization algorithm over image intensity values, and (b) application of optimization algorithm on transformation function which in turn converts the input intensity values to output intensity values. SHO has been applied in both of these ways to show the flexibility of the algorithm to adapt to any kind of scenario.

3) An ablation test has been conducted to choose the image related properties which are most appropriate for contrast enhancement. Many image properties are being used in the literature for ICE, but almost none of the existing methods has provided reasons for using them. But, in this work, we have used this ablation study to first extract the parameters which are more useful in the present domain, and then use those to perform the contrast enhancement by applying the SHO variant. The final algorithm with the selected set of parameters, found from the ablation study, has been applied over 24 images of Kodak dataset to assess the performance of the same. The better performing variant i.e. the direct approach has been further applied over H-DIBCO 2018 datasets to prove its robustness.

2. Proposed Work

In this work, we have formulated ICE as an optimization problem. There are mainly two ways in which we can use the optimization for enhancing the contrast of the images-

1. Optimization of every pixel intensity value.
2. Optimization of parameters of a transformation function which maps every pixel intensity to a new value.

Both the ways are used in this work. The optimization is done using an existing evolutionary algorithm known as SHO. In this section, we have provided the details of SHO algorithm and described the way we have tuned it to make it applicable for ICE problem.

2.1. Selfish Herd Optimization

SHO is a nature-inspired swarm optimization algorithm used for solving the global optimization problems. It was proposed by observing and imitating the interaction between a group of preys, also known as Selfish herd, and a pack of predators. Predation is a biological interaction where the predator hunts and kills its prey. In the phase of predation, the preys which are situated near the centre of the herd have a higher probability of survival (also known as herd's leader) while those near the periphery have a risk of being hunted down by the predators. The relative position of the individual plays an important factor to decide the survival chance. Hence, the individual prey adopts a selfish move to increase their chance of survival. The position vectors of individuals represent the possible solutions of the problem under consideration given the search space. The movements are defined for the two types of agents. The steps performed in the SHO algorithm are presented below.

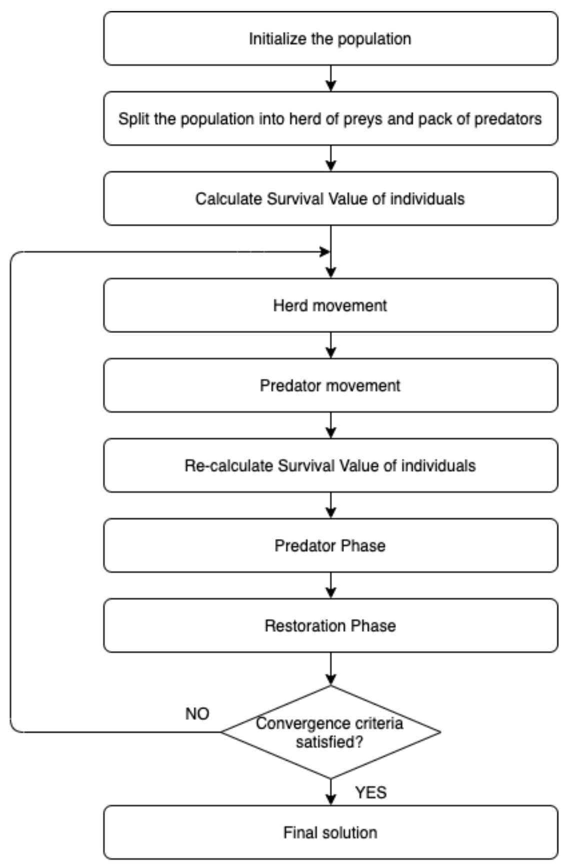

Figure 1: Flowchart describing the internal procedures of SHO.

2.1.1. Population initialization

The population is initialized with size N and each individual is represented by a D-dimensional vector representing the possible solution of the optimization problem. These vectors are initialized by considering the random uniform distribution between the given lower and upper bounds of the D-dimensional space.

$$a_i = lb + \mathrm{rand}(0,1) \cdot (ub - lb) \; where \; (i = 1, \ldots, N) \tag{1}$$

2.1.2. Population splitting

The entire population is divided into two groups, a group of preys or herd aggregation, $H = \{h_1, h_2, \ldots, h_{N_h}\}$ and a pack of predators $P = \left\{p_1, p_2, \ldots, p_{N_p}\right\}$, where N_h and N_p denote the number of members belonging to the herd and pack of the predators respectively. The size of herd N_h is set as 70-90% of the total population size and the remaining are labeled as predators.

$$\begin{aligned} N_h &= \lfloor N \cdot \mathrm{rand}(0.7, 0.9) \rfloor \\ N_p &= N - N_h \end{aligned} \tag{2}$$

2.1.3. Survival value assignment

The survival capability, i.e. chance of survival or success of an individual (prey or predator) is defined by a survival value. The survival value is dependent on the current best and worst solutions found by SHO. The survival value of individual can be calculated as:

$$\mathrm{SV}_{a_i} = \frac{f\left(a_i\right) - f_{\text{best}}}{f_{\text{best}} - f_{\text{worst}}} \tag{3}$$

Where $f(a_i)$ is the fitness value of the individual a_i obtained by the application of the objective function f. f_{best} and f_{worst} denote the fitness values of the current best and worst solutions. They are defined as,

$$\begin{aligned} f_{\text{best}} &= \max_{j \in \{0,1,\ldots,k\}} \left(\left(\max_{i \in \{1,2,\ldots,N\}} \left(f\left(a_i\right)\right) \right)_j \right) \\ f_{\text{worst}} &= \min_{j \in \{0,1,\ldots,k\}} \left(\left(\min_{i \in \{1,2,\ldots,N\}} \left(f\left(a_i\right)\right) \right)_j \right) \end{aligned} \tag{4}$$

2.1.4. Movements of selfish herds

An important aspect of the SHO algorithm is to increase the herd members' chances of surviving i.e. avoiding the predation attack. There are three characteristic operations in selfish herd as discussed below.

Herd leader movement

. The herd's leader is the strongest member of the herd i.e. its survival value is greater than the rest. At every iteration, the herd's leader is chosen as:

$$h_L = \{h_i \in H | SV_{h_i} = \max\left(SV_H\right)\} \tag{5}$$

The leader helps in guiding the herd. It shows two types of behavior namely seemingly cooperative leadership and openly selfish leadership. These behavior are reflected in the leader's movements. These leadership movements are dependent on the survival value of the herd's leader and can be determined by

$$\vec{h}_L = \begin{cases} \vec{h}_L + \vec{c} \text{ if } SV_{h_L} = 1 \\ \vec{h}_L + \vec{s} \text{ if } SV_{h_L} < 1 \end{cases} \tag{6}$$

Where c and s are the movement vectors with which the herd's leader move depending on its survival value. They are calculated as

$$\begin{aligned} \vec{c} &= 2 \cdot \alpha \cdot (-SV_{p_M}) \cdot e^{-\|p_M - h_L\|_2^2} \cdot (p_M - h_L) \\ \vec{s} &= 2 \cdot \alpha \cdot e^{-\|x_{best} - h_L\|_2^2} \cdot (x_{best} - h_L) \end{aligned} \tag{7}$$

Where x_{best} is the best position found so far, and $\alpha \in [0,1]$ chosen randomly. SV_{P_M} denotes the survival value of the predator's centre of mass p_M, given as

$$p_M = \frac{\sum_{j=1}^{N_p} SV_{p_j} \cdot p_j}{\sum_{j=1}^{N_p} SV_{p_j}} \tag{8}$$

The first type of movement is known as seemingly cooperative movement. This occurs only when the leader has the best position among the aggregation. The second type is openly selfish movement and this occurs when leader is not at the best position and in order to increase its survival value, it tries to move to the safest position.

Herd following and desertion movements

. In this step, the herd members except the leader is divided into two groups - herd followers H_F, which follow the aggregation and herd deserters H_D, which desert from aggregation and move independently. The groups are formed as follows.

$$\begin{aligned} H_{\mathrm{F}} &= \{h_i \neq h_L | SV_{h_i} \geq \mathrm{rand}(0,1)\} \\ H_{\mathrm{D}} &= \{h_i \neq h_L | SV_{h_i} < \mathrm{rand}(0,1)\} \end{aligned} \tag{9}$$

The members of corresponding groups move as below:

$$h_i = \begin{cases} h_i + f_i \text{ if } h_i \in H_{\mathrm{F}} \\ h_i + d_i \text{ if } h_i \in H_{\mathrm{D}} \end{cases} \tag{10}$$

where f_i and d_i are movement vectors made by the followers and deserters respectively. The herd followers H_F are further divided into dominant members H_d and subordinate members H_s, based on the individual's survival value with respect to the mean survival value of the entire herd members. They are defined as below:

$$\begin{aligned} H_{\mathrm{d}} &= \left\{h_i \in H_{\mathrm{F}} | SV_{h_i} \geq SV_{H_\mu}\right\} \\ H_{\mathrm{s}} &= \left\{h_i \in H_{\mathrm{F}} | SV_{h_i} < SV_{H_\mu}\right\} \end{aligned} \tag{11}$$

where SV_{H_μ} is the mean survival value of the herd H. The herd follower's movement f_i is thus calculated, depending on whether it belongs to the set of dominant or subordinate members, as:

$$f_i = \begin{cases} 2 \cdot \left(\beta \cdot \psi_{h_i, h_i} \cdot (h_L - h) + \gamma \cdot \psi_{h_i, h_{i_i}} \left(h_b - h_i\right)\right) & \text{if } h_i \in H_{\mathrm{d}} \\ 2 \cdot \delta \cdot \psi_{h_i, h_{\mathrm{M}}} \cdot (h_M - h_i) & \text{if } h_i \in H_{\mathrm{s}} \end{cases} \tag{12}$$

where β, γ, and δ are random real numbers within the interval [0,1]. ψ_{h_i,h_L}, ψ_{h_i,h_b}, and ψ_{h_i,h_M} are the selfish attraction experienced by the herd member h_i towards the herd leader h_L, its nearest best neighbor h_b, and the centre of mass h_M, respectively. They are given as,

$$\begin{aligned} \psi_{h_i,h_L} &= SV_{h_L} \cdot e^{-\|h_i - h_L\|_2^2} \\ \psi_{h_i,h_\mathrm{b}} &= SV_{h_b} \cdot e^{-\|h_i - h_b\|_2^2} \\ \psi_{h_i,h_M} &= SV_{h_M} \cdot e^{-\|h_i - h_M\|_2^2} \end{aligned} \tag{13}$$

where h_b and h_M are defined as below:

$$\begin{aligned} h_b &= \left\{ h_n \in H, h_n \notin \{h_i, h_L\} \,|\, r_{i,n} = \min_{n \in \{1,2,\ldots,N_h\}} (\|h_i - h_n\|_2) \right\} \\ h_M &= \frac{\sum_{i=1}^{N_h} SV_{h_i} \cdot h_i}{\sum_1^{N_h} SV_{h_i}} \end{aligned} \tag{14}$$

where $r_{i,n}$ is the Euclidean distance between two herd members h_i and h_n. The movement vector d_i is defined for the herd deserters H_D, irrespective of the other individuals, as follows:

$$d_i = 2 \cdot (\beta \cdot \psi_{h_i,x_\mathrm{best}} \cdot (x_{best} - h_i) + \gamma \cdot (1 - SV_{h_i}) \cdot \hat{r}) \tag{15}$$

where $\hat{r}$ is the unit vector pointing to a random direction within the given D-dimensional search space, and $\psi_{h_i,x_\mathrm{best}}$ is the selfish attraction experienced by the herd deserter member towards the current global best position, given by

$$\psi_{h_i,x_{best}} = e^{-\|h_i - x_{best}\|_2^2} \tag{16}$$

2.1.5. Predator movement

The predation risk is directly related to the survival value of the individuals within the herd aggregation and their positions relative to the attacking predators. The pursuit probability is presented to model the predator's movement, by considering the predator risk property. From the pursuit probability, the predator determines which specific prey will be chased by it. The pursuit probability is calculated as,

$$\mathcal{P}_{p_j,h_i} = \frac{\omega_{p_j,h_i}}{\sum_{i=1}^{N_h} \omega_{p_j,h_i}} \tag{17}$$

where ω_{p_j,h_i} denotes the prey attractiveness considering the survival value of individual h_i in the herd aggregation and the distance between the individual h_i and attacking predator p_j. It is given as,

$$\omega_{p_j,h_i} = (1 - SV_{h_i}) \cdot e^{-\|p_j - h_i\|_2^2} \tag{18}$$

SHO algorithm also models the movement of individual predator p_j in the pack of predators P as follows:

$$p_j = p_j + 2 \cdot \rho \cdot (h_r - p_j) \tag{19}$$

where ρ is a random number generated within the range [0,1], h_r (r $\in$ 1, ..., N_h) is a herd member chosen randomly from the herd H by applying the Roulette wheel selection considering the individual pursuit probabilities P_{p_j,h_I}.

2.1.6. Re-calculate survival values

After the herd's and predator's movements, the survival value for each individual is re-calculated as described in section 2.1.3.

2.1.7. Predator phase

The interaction between the group of predators and prey in which the preys are hunted by the predators is known as Predation. Usually such interaction results in the death of the individual preys. A set of circular regions defined around the individual preys is known as Domain of Danger. If a prey is present inside the Domain of Danger of a predator, then there is a risk of it being hunted. The Domain of Danger is defined as follows:

$$R = \frac{\|ub - lb\|_1}{2D} \tag{20}$$

where D is number of variables, and . denotes l_1-norm.

If the predator p_j has higher survival value than the herd individual h_i, and h_i is inside the Domain of Danger of p_j, then the herd individual h_i is said to be under the threat by p_j. Such herd members are selected by considering the previous two criteria, also given as:

$$T_{p_j} = \left\{h_i \in H | SV_{h_i} < SV_{p_j}, \|p_j - h_i\|_2 < R\right\} \tag{21}$$

where T_{p_j} is a set containing all the herd individuals that are under the threat of predator p_j, and $|p_j - h_i|_2$ indicates the Euclidean distance between h_i and p_j. Based on the probabilities of being hunted by p_j, the Roulette wheel selection chooses the herd member killed by the predator p_j. The probability is given as,

$$\mathcal{H}_{p_j,h_i} = \frac{\omega_{p_j,h_i}}{\sum_{\left(h_i \in T_{p_j}\right)} \omega_{p_j,h_i}}, h_i \in T_{p_j} \tag{22}$$

where ω_{p_j,h_i} denotes the attraction between the pair of prey and predator. After the predation phase, the set K contains all the herd members that are killed by the predators.

2.1.8. Restoration phase

In this phase, all the herd members $h_i \in K$, which were hunted by the predators, are replaced with new ones to maintain the total population size intact. A new herd member is produced by the following mating operation:

$$h_i = h_{new} = \text{mix}\left([h_{m_1,1}, \ldots, h_{m_1,l}, \ldots, h_{m_D,D}]\right), h_i \in K \tag{23}$$

where D is the dimension of the solution space and mix(.) is an operator that creates a new solution h_{new} by setting the $l-th$ index $h_{m_l,l}$ of candidate herd member h_{m_l} to the $l-th$ index of h_{new}. The mating candidates $\mathcal{M}$ are selected from the group of surviving herd members by applying Roulette wheel selection with reference to their mating probability:

$$\mathcal{M}_{h_i} = \frac{SV_{h_i}}{\sum_{(h_i \in H \cap K)} SV_{h_i}}, h_i \in \overline{H \cap K} \tag{24}$$

where K denotes the set containing all the herd members that were killed , and $\overline{H \cap K}$ denotes the set containing all the surviving individuals within the herd.

2.2. Application of SHO in ICE

As discussed previously, we have considered ICE as an optimization problem which has made us intrigued to apply SHO algorithm to this interesting problem. The intuitive similarity of SHO algorithm with the requirement of ICE has motivated us to apply it to this problem. In SHO, there is a herd of preys and a pack of predators. The predators are constantly eating out weak preys, thereby eliminating weak solutions and forcing surviving individuals (i.e. solutions) to improve their quality. In ICE, our main concern is to form an image having better contrast than raw input image without any significant loss of information in the image. So, while performing ICE, if the preys get weak, the predator solutions overwhelm them. Thus it leads to the elimination of bad solutions and acceptance of new, better solutions. As there is no perfect solution in ICE, our aim is to find an optimal solution within a reasonable time duration. With most of the other optimizing algorithms like Genetic Algorithm [15], PSO [16], Ant Colony Optimization [17] etc., there is no forcing criteria. So, it may happen that the algorithms keep running even without any significant improvement. But, the forcing criteria in SHO leads to proper convergence of the solutions. For this reason, we have been highly interested in applying SHO to solve ICE problems. There are two major concerns in any evolutionary algorithm: the fitness function and the formation of candidate solutions. We have addressed both of these in SHO to make it applicable to ICE.

2.2.1. Solution Formation

Evolutionary algorithms work by using information learnt by some intelligent agents which are called candidate solutions. These candidate solutions traverse various paths and check several combinations of values to find out which combination is the optimal one. The formation of such candidate solutions varies depending on the context where the optimization algorithm is being applied. In ICE, there are two procedures by which we can solve the problem: pixel-wise optimization and transformation function optimization.

Pixel-wise Optimization (SHO(direct)):

In this procedure, the intensity value of each and every pixel gets optimized by the application of an optimization algorithm[18, 19, 20]. At first, the input image is analyzed and the unique intensity values are detected. Then we arrange these unique values in increasing order to form the initial solutions. The final motive is to find optimal values for every position of the initial solution and replace the intensity values of the input image to form the enhanced image. Consider a 3×3 image matrix. Suppose this is an input image at a certain case.

5	45	120
45	30	100
1	220	5

The initial solution of the image can be easily found to be the following vector which contains the unique intensity values obtained from the input image matrix.

1	5	30	45	100	120	220

This initial solution is known as the base solution of the optimization algorithm. Suppose during optimization, one of the candidate solution becomes the following vector.

3	9	23	60	110	125	210

This solution will have a position-wise correspondence with the base solution. So, the final output or enhanced image matrix will become a new matrix obtained by replacing the base intensity values with the new intensity values.

9	60	125
60	23	110
3	210	9

This is how pixel-wise optimization is performed. In this paper, we have used this approach as one of the ways to perform ICE using the SHO algorithm, and it is named as SHO (direct).

Transformation Function Optimization (SHO(transformation)):

Some implementations use a transformation function to map the pixel intensity values of an input image to the corresponding intensity values of the enhanced image [21, 22, 23]. Then the job of the optimization algorithm becomes to optimize the parameters of the transformation function so that it works properly. In the proposed approach, we have used IBF as the transformation function for ICE. Tubbs et al. proposed this function in [24]. IBF is continuous in nature and can be represented as

$$B(y) = \frac{1}{\int_0^1 x^{\alpha-1}(1-x)^{\beta-1}dx} \times \int_0^y x^{\alpha-1}(1-x)^{\beta-1}dx \tag{25}$$

where β and α are parameters of the function and x is the integration variable. In order to improve the quality of the enhanced image, the values of α and β should be optimized. In the proposed model, this optimization is performed by SHO as the underlying optimization algorithm. This approach is named as SHO (transformation).

2.2.2. Fitness Function

After the agents or candidate solutions are formed, we need some measure to understand how good a particular solution is. Fitness function is used to serve this purpose. Usually, fitness function takes a candidate solution as input and outputs the quality of the solution. It, thus, provides a way to assess and compare different candidate solutions. The optimization algorithms then guide the candidate solutions based on their fitness values where the motive is to improve the fitness of each and every candidate solution. In the proposed SHO variants, we have performed an ablation study to select the parameters which can be used to perform ICE efficiently and exactly. There are four important parameters which are used widely to assess the quality of contrast enhanced images - intensity sum, count of edges, entropy and dissimilarity, After experimentation, we have chosen the most useful combination of parameters for ICE. A brief overview of these parameters is given as follows:

Log sum intensity(LSI)

. It is double logarithmic scaling of the sum of input image intensities. This value should increase for high-contrast images when compared to the low-contrast ones. First, an edge detection technique (like Sobel [25] or Canny [26]) is applied to the image and then the corresponding edge intensities are summed to get the intensity sum. Finally double logarithmic scaling is used to normalize the value.

Edge count(EC)

. A more contrast image should be sharper than its lower contrast counterpart. So, logically the number of edges in an image should increase through ICE. This is the second parameter we have considered as the fitness measure in the SHO variants. To get the edge count of the image, an edge detection technique is used and then the count is divided by the total number of pixels in the image.

Image entropy(IE)

. The image entropy uses the histogram of the image to get how good the tonal distribution of the image is. After contrast enhancement, the image intensities should be properly distributed for the high contrast image. The term can be mathematically represented in equation 26.

$$IE(Img_{enhanced}) = \sum_{i=0}^{255} P_i log(P_i) \tag{26}$$

where P_i is the probability of occurrence of intensity value $'i'$ $(i = 0, \ldots, 255)$ in the enhanced image.

Dissimilarity(D)

. A statistical measure independent of the viewing conditions has been mentioned in [27]. This is known as image dissimilarity which measures the statistical difference between the input image and output image. The dissimilarity value can be obtained using the following equation.

$$D = \frac{4\sigma_{pq}\bar{p}\bar{q}}{({\sigma_p}^2 + {\sigma_q}^2)[(\bar{p}^2) + (\bar{q}^2)]} \tag{27}$$

where $\bar{p}$ and $\bar{q}$ represent mean intensity values for input and output image respectively, σ_p and σ_q represent variation of the input and output image intensities. σ_{pq} is the covariance of the image intensity values of the original image and the enhanced image.

From the ablation study, it becomes evident that when optimization is performed using transformation function (reported in Table 1), the combination of *log sum intensity* and *image entropy* parameters give the best results, while for direct pixel based optimization (reported in Table 2), combination of *log sum intensity*, *edge count* and *image entropy* is proved to be best. The above study is performed on the Kodak lossless true color image suit [28] and average performance is contrasted. As evaluation metric, we have used Peak Signal-to-Noise Ratio (PSNR) [29].

Table 1: Ablation study of image parameters in case of SHO(transformation) approach.

LSI	EC	IE	D	avg PSNR
✓				5.314982
	✓			14.943709
		✓		17.709927
			✓	12.216786
✓	✓			12.289722
✓		✓		**20.252124**
✓			✓	12.740231
	✓	✓		15.998355
	✓		✓	12.552212
		✓	✓	13.722193
✓	✓	✓		15.569136
✓	✓		✓	13.888251
✓		✓	✓	16.09796
	✓	✓	✓	14.487564
✓	✓	✓	✓	12.815473

Table 2: Ablation study of image parameters in case of SHO(direct) approach.

LSI	EC	IE	D	avg PSNR
✓				24.541615
	✓			28.611198
		✓		30.98864
			✓	25.305023
✓	✓			29.005712
✓		✓		27.982442
✓			✓	24.71918
	✓	✓		29.929087
	✓		✓	26.011489
		✓	✓	25.822101
✓	✓	✓		**31.911131**
✓	✓		✓	25.979185
✓		✓	✓	25.331229
	✓	✓	✓	26.944138
✓	✓	✓	✓	25.075357

The final form of the fitness functions for direct and transformation approach can be represented as stated in equations 28 and 29. The final objective is to increase the fitness values of the candidate solutions.

$$fitness_{transformation}(c) = LSI(I_c) \times IE(I_c) \tag{28}$$

where c is a candidate solution which is a combination of α and β values for IBF and I_c is the image enhanced by the application of IBF with the α, β values of c.

$$fitness_{direct}(c) = LSI(I_c) \times IE(I_c) \times EC(I_c) \tag{29}$$

where c is a candidate solution which is a combination of unique pixel intensity values and I_c is the image enhanced by implanting the intensity values of c to the input image. The process of this enhancement is mentioned in Section 2.2.1. The experimentation and its results are explained in detail in the following section.

3. Results and Discussion

In this section. we have presented the experimental outcomes obtained via testing the proposed approaches over different images. The results have been further evaluated against some well-known state-of-the-art algorithms to confirm the applicability of SHO in the ICE domain.

3.1. Dataset and evaluation metrics

For extensive experiment, we have applied our proposed algorithms on Kodak test image dataset [28]. It consists of 24 color images which serves as the ground truth (GT). The contrast of these images are degraded by 50 percent and then their contrasts are enhanced using SHO(direct) and SHO(transformation) algorithms. The ultimate goal of these algorithms is to obtain enhanced images while maintaining their naturality and information. During entire experimentation, both maximum iteration and number of agents for SHO variants have been set as 50 respectively following the format mentioned in [21].

The quantitative comparison for the scene images of Kodak dataset is carried out using three well known image quality assessment parameters, (i) Peak Signal-to-Noise Ratio (PSNR), (ii)Structural Similarity Index Measure (SSIM) [30] and (iii) Visual Information Fidelity (VIF) [31].

3.2. Performance evaluation

3.2.1. Kodak dataset

Finally after getting the results for SHO(direct) and SHO(transformation) over 24 Kodak images, the PSNR, SSIM and VIF values for each image have been tabulated in Table 3. From the table, we can see that in terms of all three evaluation parameters, SHO(direct) performs better than SHO(transformation). In terms of PSNR, SHO(direct) outperforms SHO(transformation) over 22 out of 24 images, but in terms of SSIM, SHO(direct) achieves better results for 15 images and in case of VIF, SHO(direct) performs better for 21 images.

Table 3: Comparison between SHO(direct) and SHO(transformation) results with respect to PSNR, SSIM and VIF values for 24 Kodak images (Kodim01,. . . ,Kodim24).

Image Code	SHO(transformation)			SHO(direct)		
	PSNR	SSIM	VIF	PSNR	SSIM	VIF
Kodim01	14.900244	0.91489	0.8431	31.993281	0.964551	0.9621
Kodim02	10.367487	0.76718	0.8045	32.230312	0.992897	0.873
Kodim03	17.015928	0.92652	0.8044	27.673301	0.911035	0.7918
Kodim04	13.088375	0.866652	0.8007	32.356303	0.979908	0.8714
Kodim05	14.529461	0.847661	0.7892	33.753897	0.975477	0.857
Kodim06	29.490634	0.985596	0.8167	36.303673	0.98397	0.8831
Kodim07	11.930575	0.854175	0.8072	35.391147	0.978114	0.8933
Kodim08	29.116586	0.978161	0.8459	26.959204	0.966742	0.7866
Kodim09	20.391209	0.960129	0.9301	30.637907	0.866091	0.9362
Kodim10	20.677836	0.897695	0.8127	30.914831	0.923624	0.8489
Kodim11	15.942182	0.892751	0.7310	31.889882	0.963769	0.7481
Kodim12	30.289411	0.973251	0.7947	32.910699	0.975787	0.8192
Kodim13	29.864875	0.981211	0.8134	28.366674	0.944139	0.8189
Kodim14	16.911521	0.912009	0.8164	31.285666	0.975383	0.8354
Kodim15	26.885472	0.938779	0.7538	30.803987	0.962005	0.8294
Kodim16	21.772791	0.93042	0.8272	29.002666	0.836267	0.8651
Kodim17	14.271381	0.794954	0.793	33.041173	0.922703	0.8867
Kodim18	12.908285	0.77271	0.7682	36.289192	0.966909	0.8546
Kodim19	30.512041	0.988551	0.8248	33.838364	0.951837	0.883
Kodim20	23.031899	0.914796	0.8757	33.362864	0.968421	0.9018
Kodim21	15.982692	0.927178	0.9005	27.763862	0.947139	0.8875
Kodim22	17.197154	0.936458	0.7824	33.755149	0.95391	0.8707
Kodim23	25.084859	0.979808	0.746	31.140358	0.974381	0.832
Kodim24	23.888087	0.961797	0.8082	34.202752	0.954617	0.8673
Average	20.25212438	0.912638833	0.812109	31.911131	0.951653167	0.8584625

For visual comparison, the outputs (i.e. enhanced images) obtained by SHO(direct) and SHO(transformation) are presented in Figures 2, 3 and 4 along with the corresponding GT and low-contrast input images. Figure 2 contains the results over first 8 images (Kodim01,. . . , Kodim08), Figure 3 represents the results for the next 8 images (Kodim09,. . . , Kodim16) and Figure 4 holds the result for final 8 images (Kodim17,. . . , Kodim24). From the visual comparison also, we can see that SHO(direct) is better at preserving the naturality of the images when compared to SHO(transform).

Figure 2: Comparison of visual appearances of the enhanced images produced by SHO(direct) and SHO(transform). The first row contains a set of 8 ground truth images (Kodim01,. . . ,Kodim08), second row contains the corresponding low-contrast input images (50% reduction in contrast), third and fourth row represents the enhanced images produced by SHO(direct) and SHO(transformation) respectively.

Figure 3: Comparison of visual appearances of the enhanced images produced by SHO(direct) and SHO(transform). The first row contains a set of 8 ground truth images (Kodim09,. . . ,Kodim16), second row contains the corresponding low-contrast input images, third and fourth row represents the enhanced images produced by SHO(direct) and SHO(transformation) respectively.

Figure 4: Comparison of visual appearances of the enhanced images produced by SHO(direct) and SHO(transform). The first row contains a set of 8 ground truth images (Kodim17,. . . ,Kodim24), second row contains the corresponding low-contrast input images, third and fourth row represents the enhanced images produced by SHO(direct) and SHO(transformation) respectively.

The results obtained by SHO(direct) and SHO(transformation) for 24 Kodak images have been compared with some of the state-of-the-art ICE procedures [21, 32, 33, 34, 35, 36, 37, 38, 39] in Table 4. The comparison is based on the average PSNR, SSIM and VIF values computed for the 24 images. From the results and corresponding comparison, we can clearly see that both SHO(direct) and SHO (transformation) have outperformed most of their contemporaries. SHO(direct) performs better than SHO(transformation) but it comes at the cost of time requirement. As SHO(direct) is an approach which performs optimization over all unique pixel intensities, it requires more time compared to SHO(transformation) which on the other hand always performs optimization over only two variables of the transformation function. In terms of average PSNR, SSIM and VIF values, SHO(diect) is able to achieve the best result out of all the algorithms. On the other hand, although SHO(transformation) performs well but it has not surpassed the results of ABC algorithm presented in [21] in terms of PSNR and SSIM. So, it can be considered to be the third best algorithm with respect to the results tabulated in Table 4.

Table 4: Comparison of the proposed SHO variants with other ICE methods. The performance is averaged over all the images in Kodak dataset.

Method	PSNR	SSIM[40]	VIF
[32]	16.26	0.79	0.41
[33]	19.88	0.87	0.51
[34]	17.78	0.72	0.63
[35]	18.87	0.88	0.51
[36]	15.46	0.86	0.52
[37]	19.84	0.91	0.53
[38]	13.56	0.85	0.60
[39]	13.79	0.86	0.69
[21]	24.66	0.95	0.75
SHO(transformation)	20.25	0.913	0.81
SHO(direct)	**31.91**	**0.9517**	**0.86**

3.2.2. Test for Robustness

Enhancement of degraded document images becomes very challenging as it needs pixel level clarity. Thus, traditional binarization methods need an effective pre-processing that enhances the picture quality by reducing uneven illumination and background variations. From the application of SHO over Kodak dataset, we have seen that SHO(direct) is able to outperform SHO(transformation). So, we have applied SHO(direct) method over degraded documents to prove the robustness of our proposed algorithm. For this purpose, we have selected the H-DIBCO 2018 document image dataset [41]. It consists of 10 handwritten images which are naturally very degraded. ICDAR competition organizer also provides the GT binary images for these 10 handwritten pages.

Four evaluation metrics are used to check the performance of SHO(direct) over H-DIBCO 2018 dataset. These are: (i) F-Measure (FM) [42], (ii) pseudo-F Measure (Fps) [43], (iii) PSNR, and (iv) Distance Reciprocal Distortion (DRD) [44]. The quantitative comparison between before and after enhancement is tabulated in Table 5. From the table, it is clear that SHO(direct) is able to improve the image quality significantly. To further validate the outcomes, the enhanced images are binarized using the winning method of H-DIBCO 2019 competition [45], and compared with their corresponding GT images. Three sample images and their enhanced images along with their binarized version (both before and after the enhancement) are shown in Figure 5. The figure reveals that, there has been significant reduction of background illumination which can be visually perceived over their binarized images. Therefore, we may safely claim that, proposed SHO(direct) method has the ability to enhance the image quality for both the scene and document images.

Table 5: Comparison of quality of images before and after enhancement by SHO(direct) in terms of FM, Fps, PSNR and DRD over H-DIBCO 2018 dataset.

H-DIBCO 2018 dataset								
Image	Before Enhancement				After Enhancement			
	FM	Fps	PSNR	DRD	FM	Fps	PSNR	DRD
1	82.255	91.266	19.561	6.011	85.21	93.568	19.994	5.098
2	72.25	79.025	11.294	10.849	73.152	78.901	13.201	9.6
3	80.154	81.01	19.547	7.054	81.955	83.057	21.922	6.58
4	64.118	78.541	9.581	13.25	73.25	79.01	11.054	12.066
5	63.002	75.217	8.184	14.89	72.185	76.821	10.52	11.575
6	80.25	86.254	20.288	5.624	81.022	88.45	21.055	4.521
7	81.601	86.044	22.698	6.35	83.254	89.66	22.254	6.215
8	79.322	85.211	8.44	11.54	81.054	87.251	10.254	10.449
9	83.154	90.148	21.988	7.218	85.247	96.005	23.244	6.998
10	78.254	86.254	15.24	13.655	79.01	88.944	17.654	12.502
Average	76.436	83.897	15.6821	9.6441	79.5339	86.1667	17.1152	8.5604

Figure 5: Three sample images of H-DIBCO 2018 dataset and their corresponding enhanced and binarized images.

4. Conclusion

Degradation in digital images may occur during image acquisition or during application of image processing algorithms. Image contrast enhancement is considered as an important but challenging pre-processing step for various image analysis tasks. In this work, we have framed the contrast enhancement problem as an optimization problem, and adjusted the pixel intensity values in order to obtain a contrast enhanced image. In doing so, we have tailor-made a recently proposed nature-inspired optimization algorithm called SHO to fit it into ICE problem. We have solved the optimization problem using two different solution representations: pixel wise optimization and transformation function optimization. Besides, we have done an ablation study to choose the useful image properties used in the fitness measure of the optimization problem. Evaluation is done on a publicly available standard image database of Kodak, and it has been found that proposed methods have performed better than many existing methods published recently. Moreover, SHO(direct) approach provides better results than SHO(transformation) which proves that pixel-wise optimization has better applicability in ICE operations. Addtional experimentation has been performed on H-DIBCO 2018 dataset with SHO(direct) algorithm and it results significant improvement while binarizing which further proves the robustness of the algorithm. As a future study, we can apply this approach for other domains like medical image and document image, may be after suitable modifications. SHO algorithm can be hybridized with other meta-heuristic algorithms to get better performance. Also other image properties can be applied in the fitness measure of the optimization problem.

References

[1] K. Gu, G. Zhai, W. Lin, M. Liu, The analysis of image contrast: From quality assessment to automatic enhancement, IEEE transactions on cybernetics 46 (1) (2015) 284–297 (2015).

[2] K. Santhi, R. W. Banu, Adaptive contrast enhancement using modified histogram equalization, Optik-International Journal for Light and Electron Optics 126 (19) (2015) 1809–1814 (2015).

[3] S. Poddar, S. Tewary, D. Sharma, V. Karar, A. Ghosh, S. K. Pal, Non-parametric modified histogram equalisation for contrast enhancement, IET Image Processing 7 (7) (2013) 641–652 (2013).

[4] J. Tian, L. Chen, L. Ma, W. Yu, Multi-focus image fusion using a bilateral gradient-based sharpness criterion, Optics communications 284 (1) (2011) 80–87 (2011).

[5] S. Hashemi, S. Kiani, N. Noroozi, M. E. Moghaddam, An image contrast enhancement method based on genetic algorithm, Pattern Recognition Letters 31 (13) (2010) 1816–1824 (2010).

[6] S. Agrawal, R. Panda, An efficient algorithm for gray level image enhancement using cuckoo search, in: International Conference on Swarm, Evolutionary, and Memetic Computing, Springer, 2012, pp. 82–89 (2012).

[7] J. Tian, L. Chen, Image noise estimation using a variation-adaptive evolutionary approach, IEEE Signal Processing Letters 19 (7) (2012) 395–398 (2012).

[8] Y. Xue, J. Jiang, B. Zhao, T. Ma, A self-adaptive artificial bee colony algorithm based on global best for global optimization, Soft Computing 22 (9) (2018) 2935–2952 (2018).

[9] D. Yugandhar, S. Nayak, A comparative study of evolutionary based optimization algorithms on image quality enhancement, Int. J. Appl. Eng. Res 10 (15) (2015) 35247–35252 (2015).

[10] B. Akay, D. Karaboga, A survey on the applications of artificial bee colony in signal, image, and video processing, Signal, Image and Video Processing 9 (4) (2015) 967–990 (2015).

[11] M. Singh, A. Verma, N. Sharma, Bat optimization based neuron model of stochastic resonance for the enhancement of mr images, Biocybernetics and Biomedical Engineering 37 (1) (2017) 124–134 (2017).

[12] E. Daniel, J. Anitha, Optimum wavelet based masking for the contrast enhancement of medical images using enhanced cuckoo search algorithm, Computers in biology and medicine 71 (2016) 149–155 (2016).

[13] T. Gong, T. Fan, L. Pei, Z. Cai, Magnetic resonance imaging-clonal selection algorithm: An intelligent adaptive enhancement of brain image with an improved immune algorithm, Engineering Applications of Artificial Intelligence 62 (2017) 405–411 (2017).

[14] A. Rajasekhar, N. Lynn, S. Das, P. N. Suganthan, Computing with the collective intelligence of honey bees–a survey, Swarm and Evolutionary Computation 32 (2017) 25–48 (2017).

[15] R. Guha, M. Ghosh, S. Kapri, S. Shaw, S. Mutsuddi, V. Bhateja, R. Sarkar, Deluge based genetic algorithm for feature selection, Evolutionary Intelligence (Mar. 2019). `doi:10.1007/s12065-019-00218-5`.
URL `https://doi.org/10.1007/s12065-019-00218-5`

[16] M. Ghosh, R. Guha, I. Alam, P. Lohariwal, D. Jalan, R. Sarkar, Binary genetic swarm optimization: A combination of GA and PSO for feature selection, Journal of Intelligent Systems 29 (1) (2019) 1598–1610 (Sep. 2019). `doi:10.1515/jisys-2019-0062`.
URL `https://doi.org/10.1515/jisys-2019-0062`

[17] M. Ghosh, R. Guha, R. Sarkar, A. Abraham, A wrapper-filter feature selection technique based on ant colony optimization, Neural Computing and Applications (Apr. 2019). `doi:10.1007/s00521-019-04171-3`.
URL `https://doi.org/10.1007/s00521-019-04171-3`

[18] M. Ghosh, S. K. Bera, R. Guha, R. Sarkar, Contrast enhancement of degraded document image using partitioning based genetic algorithm (2019).

[19] S. Hashemi, S. Kiani, N. Noroozi, M. E. Moghaddam, An image contrast enhancement method based on genetic algorithm, Pattern Recognition Letters 31 (13) (2010) 1816–1824 (Oct. 2010). `doi:10.1016/j.patrec.2009.12.006`.
URL `https://doi.org/10.1016/j.patrec.2009.12.006`

[20] H. Deborah, A. M. Arymurthy, Image enhancement and image restoration for old document image using genetic algorithm, in: 2010 Second International Conference on Advances in Computing, Control, and Telecommunication Technologies, IEEE, 2010, pp. 108–112 (Dec. 2010). `doi:10.1109/act.2010.24`.
URL `https://doi.org/10.1109/act.2010.24`

[21] J. Chen, W. Yu, J. Tian, L. Chen, Z. Zhou, Image contrast enhancement using an artificial bee colony algorithm, Swarm and Evolutionary Computation 38 (2018) 287–294 (Feb. 2018). `doi:10.1016/j.swevo.2017.09.002`.
URL `https://doi.org/10.1016/j.swevo.2017.09.002`

[22] Z. Ling, Y. Wang, H. Shen, Y. Liang, X. Lu, Adaptive extended piecewise histogram equalisation for dark image enhancement, IET Image Processing 9 (11) (2015) 1012–1019 (Nov. 2015). `doi:10.1049/`

iet-ipr.2014.0580.
URL https://doi.org/10.1049/iet-ipr.2014.0580

[23] F. Russo, Piecewise linear model-based image enhancement, EURASIP Journal on Advances in Signal Processing 2004 (12) (Sep. 2004). doi:10.1155/s1110865704404041.
URL https://doi.org/10.1155/s1110865704404041

[24] J. Tubbs, A note on parametric image enhancement, Pattern Recognition 20 (6) (1987) 617–621 (1987). doi:10.1016/0031-3203(87)90031-8.
URL https://doi.org/10.1016/0031-3203(87)90031-8

[25] J. Kittler, On the accuracy of the sobel edge detector, Image and Vision Computing 1 (1) (1983) 37–42 (Feb. 1983). doi:10.1016/0262-8856(83)90006-9.
URL https://doi.org/10.1016/0262-8856(83)90006-9

[26] P. Bao, L. Zhang, X. Wu, Canny edge detection enhancement by scale multiplication, IEEE Transactions on Pattern Analysis and Machine Intelligence 27 (9) (2005) 1485–1490 (Sep. 2005). doi:10.1109/tpami.2005.173.
URL https://doi.org/10.1109/tpami.2005.173

[27] Z. Ye, M. Wang, Z. Hu, W. Liu, An adaptive image enhancement technique by combining cuckoo search and particle swarm optimization algorithm, Computational Intelligence and Neuroscience 2015 (2015) 1–12 (2015). doi:10.1155/2015/825398.
URL https://doi.org/10.1155/2015/825398

[28] Kodak lossless true color image suite (1999).
URL http://r0k.us/graphics/kodak/

[29] S. Winkler, P. Mohandas, The evolution of video quality measurement: From PSNR to hybrid metrics, IEEE Transactions on Broadcasting 54 (3) (2008) 660–668 (Sep. 2008). doi:10.1109/tbc.2008.2000733.
URL https://doi.org/10.1109/tbc.2008.2000733

[30] H. Sheikh, A. Bovik, Image information and visual quality, IEEE Transactions on Image Processing 15 (2) (2006) 430–444 (Feb. 2006). doi:10.1109/tip.2005.859378.
URL https://doi.org/10.1109/tip.2005.859378

[31] Z. Wang, A. Bovik, H. Sheikh, E. Simoncelli, Image quality assessment: From error visibility to structural similarity, IEEE Transactions on Image Processing 13 (4) (2004) 600–612 (Apr. 2004). doi:10.1109/tip.2003.819861.
URL https://doi.org/10.1109/tip.2003.819861

[32] K. Santhi, R. W. Banu, Adaptive contrast enhancement using modified histogram equalization, Optik - International Journal for Light and Electron Optics 126 (19) (2015) 1809 – 1814 (2015). doi:https://doi.org/10.1016/j.ijleo.2015.05.023.
URL http://www.sciencedirect.com/science/article/pii/S0030402615003198

[33] H.-J. Kim, J.-M. Lee, J.-A. Lee, S.-G. Oh, W.-Y. Kim, Contrast enhancement using adaptively modified histogram equalization, in: Advances in Image and Video Technology, Springer Berlin Heidelberg, Berlin, Heidelberg, 2006, pp. 1150–1158 (2006).

[34] S. Poddar, S. Tewary, D. Sharma, V. Karar, A. Ghosh, S. K. Pal, Non-parametric modified histogram equalisation for contrast enhancement, IET Image Processing 7 (7) (2013) 641–652 (10 2013). doi:10.1049/iet-ipr.2012.0507.

[35] S. Hashemi, S. Kiani, N. Noroozi, M. Ebrahimi Moghaddam, An image enhancement method based on genetic algorithm, Pattern Recognition Letters 31 (2010) 1816–1824 (10 2010). doi:10.1016/j.patrec.2009.12.006.

[36] S. Agrawal, R. Panda, An efficient algorithm for gray level image enhancement using cuckoo search, in: Proceedings of the Third International Conference on Swarm, Evolutionary, and Memetic Computing, SEMCCO'12, Springer-Verlag, Berlin, Heidelberg, 2012, p. 82–89 (2012). doi:10.1007/978-3-642-35380-2_11.
URL https://doi.org/10.1007/978-3-642-35380-2_11

[37] G. Qinqing, C. Dexin, Z. Guangping, H. Ketai, Image enhancement technique based on improved pso algorithm, in: 2011 6th IEEE Conference on Industrial Electronics and Applications, 2011, pp. 234–238 (06 2011). doi:10.1109/ICIEA.2011.5975586.

[38] A. Draa, A. Bouziz, An artificial bee colony algorithm for image contrast enhancement, Swarm and Evolutionary Computation 16 (06 2014). doi:10.1016/j.swevo.2014.01.003.

[39] P. Joshi, S. Prakash, An efficient technique for image contrast enhancement using artificial bee colony, IEEE International Conference on Identity, Security and Behavior Analysis (ISBA 2015) (2015) 1–6 (2015).

[40] Zhou Wang, A. C. Bovik, H. R. Sheikh, E. P. Simoncelli, Image quality assessment: from error visibility to structural similarity, IEEE Transactions on Image Processing 13 (4) (2004) 600–612 (04 2004). doi:10.1109/TIP.2003.819861.

[41] I. Pratikakis, K. Zagori, P. Kaddas, B. Gatos, ICFHR 2018 competition on handwritten document image binarization (h-DIBCO 2018), in: 2018 16th International Conference on Frontiers in Handwriting Recognition (ICFHR), IEEE, 2018, pp. 489–493 (Aug. 2018). doi:10.1109/icfhr-2018.2018.00091.
URL https://doi.org/10.1109/icfhr-2018.2018.00091

[42] S. Srihari, S. Shetty, S. Chen, H. Srinivasan, C. Huang, G. Agam, O. Frieder, Document image retrieval using signatures as queries, in: Second International Conference on Document Image Analysis for Libraries (DIAL'06), IEEE, 2006, pp. 6 pp.–203 (2006). `doi:10.1109/dial.2006.17`.
URL `https://doi.org/10.1109/dial.2006.17`

[43] F. Westphal, N. Lavesson, H. Grahn, Document image binarization using recurrent neural networks, in: 2018 13th IAPR International Workshop on Document Analysis Systems (DAS), IEEE, 2018, pp. 263–268 (Apr. 2018). `doi:10.1109/das.2018.71`.
URL `https://doi.org/10.1109/das.2018.71`

[44] H. Lu, A. Kot, Y. Shi, Distance-reciprocal distortion measure for binary document images, IEEE Signal Processing Letters 11 (2) (2004) 228–231 (Feb. 2004). `doi:10.1109/lsp.2003.821748`.
URL `https://doi.org/10.1109/lsp.2003.821748`

[45] I. Pratikakis, K. Zagoris, X. Karagiannis, L. Tsochatzidis, Icdar 2019 competition on document image binarization (dibco 2019), in: 2019 International Conference on Document Analysis and Recognition (ICDAR), IEEE, 2019, pp. 1547–1556 (2019). `doi:10.1109/ICDAR.2019.00249`.