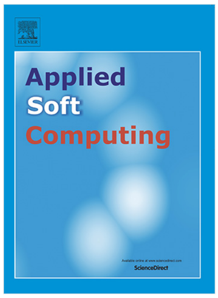
Journal Pre-proof



An intensify Harris Hawks optimizer for numerical and engineering optimization problems

Vikram Kumar Kamboj, Ayani Nandi, Ashutosh Bhadoria, Shivani Sehgal

PII: S1568-4946(19)30800-2
DOI: https://doi.org/10.1016/j.asoc.2019.106018
Reference: ASOC 106018

To appear in: *Applied Soft Computing Journal*

Received date : 29 May 2019
Revised date : 1 December 2019
Accepted date : 9 December 2019

Please cite this article as: V.K. Kamboj, A. Nandi, A. Bhadoria et al., An intensify Harris Hawks optimizer for numerical and engineering optimization problems, *Applied Soft Computing Journal* (2019), doi: https://doi.org/10.1016/j.asoc.2019.106018.

An Intensify Harris Hawks Optimizer for Numerical and Engineering Optimization Problems

Vikram Kumar Kamboj[1*], Ayani Nandi[2], Ashutosh Bhadoria[3], Shivani Sehgal[2]

[1,2]School of Electronics and Electrical Engineering, Lovely Professional University, Punjab, INDIA

[3]Department of Electrical Engineering, DAV University, Jalandhar, Punjab, INDIA

*Corresponding author's email: vikram.23687@lpu.co.in

Abstract: Recently developed Harris Hawks Optimization has virtuous behavior for finding optimum solution in search space. However, it easily get trapped into local search space for constrained engineering optimization problems. In order to accelerate the global search phase of existing Harris Hawks optimizer and to stuck it out of local search space, the proposed research aims to explore the exploration phase of the existing optimizer, the hybrid variant of Harris Hawks optimizer has been developed using sine cosine algorithm and named as *Hybrid Harris Hawks-Sine Cosine Algorithm (hHHO-SCA).* The effectiveness of the proposed optimizer has been tested for various nonlinear, non-convex and highly constrained engineering design problem. In order to validate the results of the proposed algorithm, 65 standard benchmark problems including CEC2017, CEC2018 and eleven multidisciplinary engineering design optimization problems has been taken into consideration. After verification it has been observed that the outcomes of the proposed hHHO-SCA optimization algorithm is much better than standard sine-cosine optimization algorithm, Harris Hawks Optimizer, Ant Lion Optimizer algorithm, Moth Flame Optimization algorithm, grey wolf optimizer algorithm, and others recently described meta-heuristics, heuristics and hybrid type optimization search algorithm and proposed algorithm endorses its effectiveness in multi-disciplinary design and engineering optimization problems.

Keywords: CEC2017, CEC2018, Multidisciplinary Design, Meta-Heuristics.

1. Introduction

Artificial intelligence and Machine learning are fast growing era now a days, as most of the real world problems, which are either continuous or discrete, constrained or unconstrained in nature can be easily tackled with the help of artificial intelligence and machine leaning techniques [1][2]. For these varieties of characteristics, there arose some difficulties to tackle these types of problems using conventional approaches with mathematical or numerical programming including sequential quadratic programing, quasi-Newton method, fast steepest and conjugate gradient [3][4] . There are several existing research, where it has been experimentally tested that all of these methods are not enough effective or efficient to deal with numerous types of non-differentiable, non-continuous problem and also in so many large scale real world multimodal problem [5]. Thus, meta-heuristics algorithm has been taken into consideration and has been used to tackling so many problems, which is mostly simple in nature and can be easily implemented. In optimization, inhabitants based techniques are basically used to find some solution based on sub-optimal and optimal that can be same with the exact optimal value, which situated in its nearby point or neighborhood. By generating population set of the individuals, the optimization process of the algorithm start and in the population each individuals represent candidate solution of the problem based on optimization technique. By replacing the population of the best position of the current location, the population will be changed iteratively and generate a new population using some operators, which are stochastic in nature [6][7]. This process of optimization is continued until it can satisfy the maximum iteration. Now a days, there is fast growing awareness and interest in efficient, inexpensive and successful application of such types of metaheuristics algorithm in the recent modern research.

In the proposed research, a new hybrid metaheuristics optimization technique, hHHO-SCA algorithm, which is based on nature-inspired and mathematical functions sine and cosine, has been developed to compete with the other recent types of metaheuristics optimizer. The main impression behind this kind of proposed optimization technique is encouraged from the cooperative natural behavior of the most intelligent bird, named Harris Hawks for its natural hunting behavior and the escaping or avoiding nature of the prey (rabbit) [8]. Thus, a novel mathematical model, which is stochastic and metaheuristics in nature has been developed to tackle different types of optimization problem and experimentally tested in the proposed research.

2. Literature Review

Optimization technique is enormous area of research and research is going on very fast. The researchers are doing continues research work on different problems in order to implement various types of new techniques on different problems and are capable to find the results successfully. The work is profitable on to find the new algorithms and also the algorithms with its hybrid form to mitigate any types of drawback in the present in exiting methods. In the proposed research, contemporary research papers has been selected to investigate the shortfalls of existing algorithms.

Some of these research paper includes Ant Colony Optimization (ACO) algorithm [9] was one of the nature inspired metaheuristics optimization method mimics the foraging behavior of ant species. The ants deposit pheromone on the ground to mark some favorable pathway which should be followed by other ants of the colony. This similar mechanism are used to solve optimization problems. Adaptive gbest-Guided Search Algorithm (AGG) [10] is a heuristic evolutionary search algorithm which inspired by gravitational forces between masses in nature. The Ant Lion Optimizer (ALO) [11] mimics the hunting mechanism of ant lions in nature. The main steps are such as hunting of prey like random walk of ants, building traps, entrapment of ants in traps, catching preys, and re-building traps were implemented. Artificial Neural Network [12] and an adaptive collocation strategy was implemented to solve the partial differential equation. This method raised the strength of the network estimate and the result were important for computational savings, particularly when the solution was non-smooth. Branch and Bound (BB)[13] is a new method based on branch-and-bound techniques and has been used to solve unit commitment problem. Bat Algorithm (BA) [14] is a novel optimization technique is based on Levy flights path and differential operator which is presented to increase speed of convergence. Binary Bat Algorithm (BBA) [15] is a nature-inspired feature selection technique which based on the bats behavior used to combine the exploration power of the bats considering the speed of the Optimum-Path Forest classifier to find the set of features which increased accuracy in the validating set. Biogeography Based Optimization (BBO) [16] technique motivates the application of biogeography to solve optimization problems. This unique optimization technique was used to solve the problem based on a real-world sensor selection for aircraft engine health estimation. Binary Gravitational Search Algorithm (BGSA) [17] is one of the optimization technique which based on the mass interactions and law of gravity. Bacterial Foraging Optimization Algorithm (BFOA) [18] is a global optimization algorithm inspired by the social foraging behavior of Escherichia coli which is used to solve the real-world optimization problems rising in various application domains. Bird Swarm Algorithm (BSA) [19] is a bio-inspired algorithm which was based on the swarm intelligence extracted from the social behaviors and social interactions in bird swarms. Backtracking Search Optimization (BSO) [20] is one of the stochastic new evolutionary algorithm based on the strategy to generate a trial solution with two new operators including mutation and crossover which are used to maintain the amplitude of the search-space boundaries and search-direction matrix, that improved the capabilities of the powerful exploration and exploitation. Colliding Bodies Optimization (CBO) [21] algorithm is a multi-agent metaheuristic algorithm, which was conceptualized using the one-dimensional collisions between bodies including each agent solution considered as body or object with mass. Cultural Evolution Algorithm (CEA) [22] was the population-based algorithm based on cultural evolution goal. Chaotic Krill Herd Algorithm (CKHA) [23] is a metaheuristic optimization algorithm which introduced the chaos theory into Krill Herd Optimization process with the acceleration of its global convergence speed. Cuckoo Search Algorithm (CS) [24] is one of the metaheuristic nature inspired algorithm based on the obligate brood parasitic behavior of some cuckoo species in combination with the Levy flight behavior of some birds. Dragonfly Algorithm (DA) [25] is inspired from the static and dynamic swarming behaviors of dragonflies in nature. Elephant Herding Optimization (EHO) [26] is a swarm-based metaheuristic optimization method inspired by the herding behavior of the group of elephant. Exchange Market Algorithm (EMA) [27] is a new approach of evolutionary algorithm for continuous non-linear optimization problems that inspired by the procedure of trading the shares on stock market. Electromagnetic Field Optimization (EFO) [28] is a physics inspired metaheuristic optimization method, which is based on the behavior of electromagnets with different polarities. Earthworm Optimization Algorithm (EOA) [29] is a bio-inspired metaheuristic algorithm to solve the global optimization problems. Fireworks Algorithm (FA) [30] is a swarm intelligence optimization technique based on two types of processes and the mechanisms in explosion for keeping diversity of sparks were also well designed. Firefly Algorithm (FFA) [31] is a nature inspired optimization algorithm which had been significant developed and its applications based on real-world problem. Forest Optimization Algorithm (FOA) [32] is inspired by few trees in forest which can survive for several decades, while other trees could live for a limited period. Flower Pollination Algorithm (FPA) [33] is inspired by the pollination process of flowers. Grasshopper Optimization Algorithm (GOA) [34] mimics the behavior of grasshopper swarms in nature for solving real problems. Gravitational Search Algorithm (GSA) [35] is one of the heuristic search algorithms inspired by swarm behaviors in nature. This algorithm was based on the law of gravity and mass interactions. Grey Wolf Optimizer (GWO) [36] is a meta-heuristic optimization technique inspired by grey wolves which mimics the leadership hierarchy and hunting mechanism of grey wolves in nature. Genetic Algorithm (GA) [37] motivated from the biological evolution using comparisons of based on principles mechanisms such as natural selection, genetic recombination and survival of the fittest. Lightning Search Algorithm (LSA) [38] is a novel metaheuristic optimization method which used to solve constraint optimization problems, which is based on the natural phenomenon of lightning and the mechanism of step leader proliferation using the idea of fast particles known as projectiles. Hopfield method [39] technique is used to solve ramp rate constrained for unit commitment problem. Human Group Optimizer (HGO) [40] swarm intelligence optimization method is inspired by pretending human behaviors and especially human searching/foraging behaviors with local search technique. Imperialist Competitive Algorithm (ICA) [41] is one of the evolutionary search algorithm is based on imperialist competition. Isogeometric analysis (IGA) [42] and density mapping methods for topology optimization of flexoelectric or piezoelectric materials. League Championship Algorithm (LCA) [43] was a stochastic population based algorithm which was used to solve continuous global optimization problem. Interior search algorithm (ISA) [44] is a metaheuristic algorithm inspired by interior design and decoration. Invasive Weed Optimization (IWO) [45] a numerical stochastic optimization algorithm which was inspired from colonizing weeds and this was proposed for Electromagnetic applications. Krill Herd Algorithm (KHA) [46] is based on the simulation of the herding behavior of krill individuals. Mine Blast Algorithm (MBA) [47] is a population-based optimization algorithm followed by the mine bomb explosion concept. Monarch Butterfly Optimization (MBO) [48] is a nature-inspired metaheuristic algorithm based on the updated positions of the monarch butterflies. Moth-Flame Optimization (MFO) [49] strategy depends on moths which fly in night by keeping fixed angle with respect to moon and it was a very effective mechanism for travelling in a straight line for long distances. Multi-Verse Optimizer (MVO) [50] is a nature-inspired heuristic search algorithm which was based on three concepts in Cosmology: wormhole, black hole and white hole. A multi-material level set (LS) [51] based topology optimization of flexoelectric amalgams were designed to extend the point wise density mapping technique. Optics Inspired Optimization (OIO) [52] optimization technique is based on the law of reflection. Particle Swarm Optimization (PSO) [53] is a stochastic algorithm used for the optimization problem on the global and local best particles by introducing the mutation operators and improved its performance. Runner-Root Algorithm (RRA) [54] was a metaheuristic optimization method which inspired by runners and roots of plants in nature. Simulated Annealing (SA) [55] is a deep and useful connection between statistical mechanics and multivariate or combinatorial optimization. Sine Cosine Algorithm (SCA) [56] is a population based optimization technique which created multiple initial random solutions. Stochastic Fractal Search (SFS) [57] is a metaheuristic algorithm used a mathematic concept known as fractal which explore the search space more efficiently. Shuffled Frog-Leaping Algorithm (SFLA) [58] is a population-based cooperative search metaphor inspired by natural memetics. Search Group Algorithm (SGA) [59] is a metaheuristic optimization technique to deal with the optimization of truss structures. Seeker Optimization Algorithm (SOA) [60] is based on the perception of simulating the act of humans' intelligent search with their experience, memory and uncertainty reasoning. Random Walk Grey Wolf Optimizer (RW-GWO) [61] is a new algorithm in the arena of swarm intelligence to solve continuous optimization problems and real world optimization problems. The Grey Wolf Optimizer was the only algorithm in the category of swam intelligence, which is based on leadership hierarchy. To improve the search capability by grey wolf a modified algorithm RW-GWO based on random walk. Symbiotic Organisms Search (SOS) [62] is a robust and powerful metaheuristic algorithm depend upon the strategies adopted by organisms to survive and propagate in the ecosystem. Salp Swarm Algorithm (SSA) [63] is a bio inspired heuristic search algorithm for solving optimization problems with single and multiple objectives. Tabu Search (TS)[64] is the metaheuristic optimization technique which gives good results on combinatorial optimization problems such as quadratic assignment. Teaching-Learning-Based Optimization (TLBO) [65] is based on the teaching–learning phenomenon of a classroom to solve multi-dimensional, linear and nonlinear problems with appreciable efficiency. Virus Colony Search (VCS) [66] is a nature-inspired technique which simulates diffusion and infection strategies for the host cells adopted by virus to survive and propagate in the cell environment. Water Cycle Algorithm (WCA) [67] is inspired from nature and based on the observation of water cycle process and how rivers and streams flow to the sea in the real world. Wind Driven Optimization (WDO) [68] is nature inspired optimization algorithm used to solve global optimization problems and inspired by the motion of wind in the Earth's atmosphere. Whale Optimization Algorithm (WOA) [69] is a nature-inspired meta-heuristic optimization algorithm which mimics the social behavior of humpback whales. Weighted Superposition Attraction (WSA) [70] is based on two basic mechanisms, "superposition" and "attracted movement of agents". Water Wave Optimization (WWO) [71] is inspired by the beautiful phenomena of water waves, such as propagation, refraction, and breaking. Modified Dragonfly Optimization Algorithm [72] is a nature inspired algorithm based on random flying behavior of dragonflies. Artificial Flora (AF) Optimization Algorithm [73] is a stochastic optimization method Inspired by the procedure of reproduction and migration of flora. Chicken Swarm Optimization algorithm [74] is a metaheuristic bio-inspired optimization technique, which mimics the nature of the chicken swarm. Crow Particle Optimization Algorithm [75] is the hybrid form of Particle swarm optimization and crow search algorithm. Improved Electromagnetic Field Optimization algorithm [76] was a metaheuristic physics inspired method, which mimics the performance of electromagnets considering different polarities. A significant research on multi-material level set-based topology optimization of flexoelectric composites [51], Computational Machine Learning Representation for the Flexoelectricity Effect in Truncated Pyramid Structures [77], design methodology based on a combination of isogeometric analysis (IGA), level set and point wise density mapping techniques for topology optimization of Piezoelectric/flexoelectric materials [42][78] and Sensitivity and uncertainty analysis for flexoelectric nanostructures has been reported in recent research articles [79]. Quasi-Opposition-Based Learning and Dimensional Search algorithm [80] is inspired by the attachment process of lightning in environment. Supernova Optimizer [81] is inspired by the supernova phenomena in nature. Manta ray foraging optimization [82] is a recently developed bio-inspired optimization technique, which is based on intelligent performances of manta rays. Hybrid Artificial Grasshopper Optimization (HAGOA) [83] is a metaheuristics optimization algorithm inspired by grasshopper to recover the exploration and exploitation in given search space. Harris Hawks optimization (HHO) [8] is recently proposed nature-inspired search algorithm, which is encouraged from the cooperative natural behavior of the most intelligent bird, named as Harris Hawks of its natural hunting behavior and the escaping or avoiding nature of the prey (rabbit). The brief reviews of the various meta-heuristics stochastic and heuristic search algorithms has been depicted in Table 1.

All the recently developed heuristics, metaheuristics, evolutionary and nature inspired algorithms has its own pros and cons and most of these search algorithms are not applicable to every kind of optimization problems and hence universally cannot be accepted. Further, from No Free Lunch Theorem (NFL) [84], all types of algorithms based on optimization techniques recommended and show average equivalent performance, if it is applied to all probable types of tasks based on optimization technique. According to NFL theorem, it cannot consider theoretically an algorithm as universally best type of optimizer in general purpose. Hence, NFL theorem motivates for penetrating and rising more effective algorithm based on optimization technique. Motivated from these, in the proposed research, the initiative has been taken to provide another powerful optimizer, which is based on natural hunting behavior of Harris Hawks and trigonometric functions sine and cosine and names as hybrid Harris Hawks-Sine Cosine Algorithm (hHHO-SCA).

Table 1: Review of various meta-heuristics and heuristics type search algorithms

Algorithm Name	No. of Benchmark problems	Year	Reference. No.	Author's Name	Problem Type
Animal Migration Optimization	23	2013	[85]	Li X.	NA
Artificial Bee Colony Algorithm	NA	2019	[86]	Xiao-long Chen	Economic Load Dispatch
Artificial Flora (AF) Optimization Algorithm	6	2019	[73]	Long Cheng, Xue-han Wu, Yan Wang	Standard benchmark
An upgraded Artificial Bee Colony Algorithm	5	2012	[87]	Ivona Brajevic	Constrained Optimization
Ameliorated Grey Wolf Optimization	15	2018	[88]	Diljinder Singh, J.S. Dhillon	Economic load dispatch
Barnacles Mating Optimizer	23	2018	[89]	Mohd Herwan Sulaiman	Engineering Design Optimization
Binary Gray Wolf Optimization	18	2015	[90]	E. Emary, Hossam	Feature Selection
Biogeography-Based Optimization	14	2008	[16]	Simon D.	Real World
Binary PSO-GSA	22	2014	[91]	Mirjalili S.	NA
Bird Swarm Algorithm	18	2015	[19]	Meng Bing Z.	NA
Binary whale optimization algorithm	NA	2018	[92]	Srikanth Reddy K.	Profit-Based Unit Commitment
BBO Train Multilayer perceptron	6	2014	[93]	Mirjalili S.	Bio-medical Optimization
Coyote Optimization Algorithm	40	2018	[94]	Juliano Pierezan	Engineering Design Optimization
Crow Particle Optimization Algorithm	6	2019	[75]	Ko-Wei Huang, Ze-Xue Wu	Six standard benchmark
Cultural Evolution Algorithm	7	2013	[22]	Kuo H.C.	Reliability Engineering
Chaotic Krill Herd Algorithm	14	2014	[23]	Wang G.	NA
Chicken Swarm Optimization algorithm	20	2019	[74]	Sanchari Deb, Xiao-Zhi Gao, Kari Tammi, Karuna Kalita	18 standard benchmark problems and 2 engineering problems
Competition Over Resources	8	2014	[95]	Mohseni S.	NA
Exchange Market Algorithm	12	2014	[27]	Ghorbani N.	NA
Electromagnetic Field Optimization	30	2015	[28]	Beheshti Z.	NA

Electro-Search algorithm	10	2017	[96]	Amir Tabari Arshad Ahmad	Engineering Design Optimization
Emperor Penguin Optimizer	44	2018	[97]	Gaurav Dhiman	Engineering Design
Elephant Herding optimization	15	2015	[26]	Wang G.	NA
Firework Algorithm	9	2010	[30]	Tan Ying	NA
Fuzzy Optimization Technique	NA	2015	[98]	Mohammad Shoaib Shahriar	Optimization of Unit Commitment
Forest Optimization Algorithm	4	2014	[32]	Ghaemi M.	Feature weighting
Gravitational Search Algorithm	23	2009	[35]	Rashedi E.	NA
Grid Classification Based Algorithm	NA	2016	[99]	Tripatjot Singh Panag	Wireless Sensor in Network Monitored Area
Grasshopper Optimization Algorithm	19	2017	[34]	Saremi S.	Engineering Design Optimization
Grey Wolf Optimizer	29	2014	[36]	Mirjalili S.	Engineering Design Optimization
GWO-SCA	22	2017	[100]	Singh N.	Bio-medical Optimization
Harris Hawks Algorithm	29	2019	[8]	Ali Asghar Heidari	Standard Benchmark
Hybrid Artificial Grasshopper Optimization (HAGOA)	19	2018	[83]	Brahm Prakash Dahiya, Shaveta Rania and Paramjeet Singhb	Standard benchmarks
Interior Search Algorithm	14	2014	[44]	Gandomi A.	Engineering Design Optimization
Improved Electromagnetic Field Optimization	13	2019	[76]	Alkin Yurtkuran	Standard Benchmark
Krill Herd	20	2012	[46]	Gandomi A.	NA
Lightning Search Algorithm	24	2015	[38]	Shareef H.	NA
Lion Optimization Algorithm	NA	2017	[101]	Narendrasinh B Gohil	Engineering Design Optimization
Manta ray foraging optimization	39	2019	[82]	Weiguo Zhao, Zhenxing Zhang and Liying Wang	31 Benchmark functions and 8 constrained engineering design problems
Moth-flame Optimization Algorithm	29	2015	[49]	Mirjalili S.	Engineering Design Optimization
Modified Dragonfly Optimization Algorithm	21	2019	[72]	Çi˘gdem ˙Inan Acı Hakan G¨ulcan	15 single objective and 6 multi objective problems
Multi-start methods	NA	2012	[102]	Rafael Martí	Combinatorial Optimization

Multi-verse Optimizer	19	2015	[50]	Mirjalili S.	Engineering Optimization
Oppositional Artificial Bee Colony Algorithm	NA	2016	[103]	Kamalpreet Kaur Dhaliwal	Optimization of Digital IIR Filter
Quasi-Opposition-Based Learning and Dimensional Search Algorithm	32	2019	[80]	Tongyi Zheng and Weili Luo	unimodal, multimodal, and CEC 2014 functions
Stochastic Fractal Search	23	2014	[57]	Salimi H.	Engineering Design Optimization
Supernova Optimizer	20	2019	[81]	Amjad A. Hudaib & Hussam N. Fakhouri	unimodal, multimodal, and CEC 2005 functions
Symbiotic Organism Search	26	2014	[62]	Cheng M.	Engineering Design Optimization
Sine Cosine Algorithm	19	2016	[56]	Mirjalili S.	Aircraft Wing Design
Salp Swarm Algorithm	19	2017	[63]	Mirjalili S.	Engineering Design Optimization
Self-adaptive differential artificial bee colony algorithm	28	2019	[104]	Xu Chen	global Optimization
The Sailfish Optimizer	20	2019	[105]	S. Shadravan	Engineering Design Optimization
Virus Colony Search	30	2016	[66]	Li Dond M.	Engineering Design Optimization
Volleyball Premier League Algorithm	23	2017	[106]	Reza Moghdani	Global Optimization
Variable neighborhood search	16	2007	[107]	Krzysztof Fleszar	Open Vehicle Routing
Water Cycle Algorithm	19	2012	[67]	Eskandar H.	Engineering Design Optimization
Whale Optimization Algorithm	29	2016	[69]	Mirjalili S.	Engineering Design Optimization

3. Mathematical Approach for Hybrid Harris Hawks Optimizer

In this paper, HHO algorithm includes exploratory and exploitative phases, which is encouraged by surprise attack, the nature of exploration of a prey and different strategies based on attacking phenomenon of Harris hawks. This is one of the gradient-free and inhabitants based algorithms for optimization technique, which will be useful to formulate on any types of optimization problem. HHO have some major techniques in the phase of exploration. Not only this, but also HHO have a major strategy in conversion from phase of the exploration to the phase of the exploitation.

3.1 Exploration and its conversion

In this section the mechanism of HHO is discussed. Considering the normal hunting strategy of Harris hawks, they detect the prey and track it by using their dominant eyes through which the victim cannot be realize it easily. Thus, after several hours the Harris hawks delay some time by observing and monitoring the site to detect the prey. Now for HHO, the Harris hawks are taken as best solution in each step which is taken as the projected prey and which is also in nearly optimum region. In this optimization technique

Harris hawks settle randomly on some positions and wait to detect and locate the prey based on two types of strategies. Now considering the equal chance w for each balancing strategy which is based on the locations of the another members of the family to near enough to them when attacking and the rabbit as a prey, shown in equation (1) for such condition where $w < 0.5$ or balance on random positions which is shown in equation (2) where $w \geq 0.5$ and equation (3) is determine the average location of Harris hawks.

$$H\left(iter+1\right) = H_{rand}(iter) - e_{s1}\left|H_{rand}(iter) - 2e_{s2}H(iter)\right|, w \geq 0.5 \tag{1}$$

$$H\left(iter+1\right) = H_{rabbit}(iter) - H_m(iter)) - e_{s3}(L_{Bound} + e_{s4}(U_{Bound} - L_{Bound})), w < 0.5 \tag{2}$$

$$H_m(iter) = \frac{1}{N}\sum_{i=1}^{N} H_i(iter) \tag{3}$$

where, e_{s1} and w are arbitrary numbers in between (0, 1) and these are improved in each iteration, $H_{rabbit}(iter)$ =Rabbit's position, $H_{rand}(iter)$ =Number of Harris hawks are selected from recent population, N = Total number of Harris hawks

This model is basically used to generate location randomly in between the range of upper and lower boundary. This optimization technique consist of some rules, firstly, the solution which is generated basically based on other hawks including its random location, secondly form equation (1) and (2) usually to find out the best value of different location and the component of random scale which is based on the range of the upper and the lower bound of the variables where e_{s3} is represented as coefficient scale which used to advance increase the rule of nature randomly, while the value of e_{s4} is mostly near about 1 and this pattern is distributed in similar manner including average position. According to this rule, algorithm of HHO add the movement of scale length up to lower bound L_{Bound} . After that to provide more exploration in different section of space of feature which is considered as randomization of coefficient of scaling. There is also possible to create different types of updated rules, but here we developed simple rule which has the capability to copycat the nature of the hawks. The ordinary location of the Harris Hawks is achieved by using the eqn.(3).

Conversion from the phase of exploration to the phase of exploitation is given below:

In this algorithm, based on HHO optimization technique can transference from exploration condition to exploitation condition and after that alteration between various types of nature base on exploitative behavior which is based on the avoidance energy of the prey. Due to this avoidance behavior, it decreases the energy of the prey. The equation based on the behavior of the energy of the prey is given bellow:

$$EG = 2EG_0(1 - \frac{iter}{iter_{\max}}) \tag{4}$$

where, EG=Avoidance energy of the prey, EG_0=Initial condition of the energy, $iter_{\max}$ =Maximum iteration

Here for this proposed algorithm EG_0 change randomly in between the interval (-1, 1) for each number of iteration. If the number of iteration reduces from 0 to -1, that means the nature of the physically flagging behavior of rabbit is represented and if the number of iteration rises from 0 to 1, that means the establishment nature of the rabbit is represented. The avoidance energy EG is dynamic in nature and it also has a tendency to go down in decreasing manner during the running condition of the iterations. If the avoidance energy $|EG| \geq 1$, that means to explore the location of the rabbit, the Harris hawks search various different kinds of regions, hence this algorithm can perform the phase of exploration and if, $|EG| < 1$, that means during the stage of exploitation, the nature of the algorithm is to exploit the nearby solutions. That means, we can say exploration occurs when $|EG| \geq 1$ while the exploitation occurs when $|EG| < 1$.

3.2 Exploitation strategy with Soft encircle and Hard encircle

In this section, the performance of Harris hawks is behave like the surprise attack by attacking nature upon projected prey identified in the previous phase. Though, preys often try to escape or avoid from dangerous conditions. Hence, various types of hunting styles happen in real circumstances. According to avoiding and escaping nature of the victim and the policies of hunting of Harris hawks, there are four types of probable strategies which are presented in the optimization technique based on HHO to implement the stage of attacking strategy. The main behavior of such kind of preys are try to avoid or escape from this kind of situation which is basically based on threatening conditions. Let e_s is the probability of the prey can escape successfully when $e_s < 0.5$ or cannot escape successfully when $e_s \geq 0.5$ before surprise the pounce. The Harris hawks will give their performance soft or hard encircle to catch the prey at any conditions whatever the prey does. That means, hawks will surround the prey from the different types of various directions softly or hardly which is depending upon prey's energy itself. In real time operation or for real situation by performance of the surprise attack, the Harris hawks get nearer to the projected prey to rise their probabilities in attacking and after that killing the rabbit. After some time, the avoiding or escaping prey must lose their energy, then the Harris hawks increase encircle process to smoothly catch the shattered prey. To implement this tactic and allow HHO to change over between hard and soft encircle process, the constraint EG is applied for that purpose. The soft encircle occurs for $|EG| \geq 0.5$ and hard encircle occurs for $|EG| < 0.5$. In case of Soft encircle, if $e_s \geq 0.5$ and $|EG| \geq 0.5$, that means the rabbit still has its enough energy through which it can try to avoid or escape by jumps randomly but it unable to go forward into its safe condition. Due to this softly encircle is occurred by Harris hawks, for which the victim rabbit tends to more and more exhausted and after that execute the surprise attack. This kind of natural behavior is demonstrated by some rules which is given below:

$$H(iter+1) = \Delta H(iter) - EG\left|KH_{rabbit}(iter) - H(iter)\right| \tag{5}$$

$$\Delta H(iter) = H_{rabbit}(iter) - H(iter) \tag{6}$$

Where, $\Delta H(iter)$ = Difference between the iteration based on the present location and the vector based on the position of that victim rabbit

e_s = Random number inside (0, 1), $K = 2(1-e_s)$, this signify the strength of jump randomly throughout the procedure of avoiding and escaping. The value of K randomly changes for each iteration to simulate the behavior of the motion of rabbit. In case of Hard encircle, if $e_s \geq 0.5$ and $|EG| < 0.5$, that means the rabbit is so shattered and at this condition it has small avoidance or avoidance energy. In this situation the Harris Hawks performs scarcely enclose the projected victim to do finally shock attack. At this condition, the current locations are changed and updated by using the equation no. (7)

$$H(iter+1) = H_{rabbit}(iter) - EG\left|\Delta H(iter)\right| \tag{7}$$

3.2.1 Soft encircle with advanced fast dives

If still $|EG| \geq 0.5$, but at this time $e_s < 0.5$, that means the rabbit has sufficient energy to avoid or escape successfully and before the surprise attack a soft encircle is created. This process is much more intellectual than the previous process.

The levy flight (LFT) conception is applied in HHO algorithm through which we can understand the mathematical model of the leapfrog movements and the patterns of avoiding or escaping of prey. It helps to detect the LFT based on such patterns which can be identified the activity of chasing of such kind of animals like sharks and monkeys [44, 45, 46, 47]. Hence the LFT based patterns can utilized in this technique of the phase of HHO algorithm. It is also developed to mimic the actual zigzag movements of victims (rabbits) during in the phase of escaping and asymmetrical abrupt and advanced fast dives of Harris hawks around the avoiding the prey. Basically Harris hawks performs numerous team rapidly dives round the prey (rabbit) and then try to increasingly correct their position and location as well as the directions with respect to the pretended motion of the rabbit. This type of mechanism is also maintained by observations in real world in other reasonable situations in the behavior of nature. With the help of this nature the hawks can increasingly select the best probable dive towards the rabbit at that time when they desire to catch the rabbit in the economic situations. So, for better performance of a soft encircle, the Harris hawks can decide their next movement (M_N), which is based on a rule which is given in the equation number (8):

$$M_N = H_{rabbit}(iter) - EG\left|KH_{rabbit}(iter) - H(iter)\right| \tag{8}$$

After that they can relate the probable result of such kind of movement to the earlier dives which detect that it will be better dive or not. If they saw that the performance (motion) of the prey (rabbit) was more deceptive in nature, which means they also start to accomplish abrupt, rapid and irregular dives when oncoming the rabbit. The dive based upon the LFT patterns based which follow the given rule in equation number (9) [108].

$$P = M_N + R_S \times LF_T(D_{IM}) \tag{9}$$

Where, D_{IM} = Problem's dimension, R_S = Size of random vector by size $1\times D_{IM}$

$$LF_T(x) = 0.01 \times \frac{\alpha \times \delta}{|\mu|^{\frac{1}{\gamma}}} \tag{10}$$

$$\delta = \left(\frac{\Gamma(1+\gamma) \times \sin(\frac{\pi\gamma}{2})}{\Gamma(\frac{1+\gamma}{2}) \times \gamma \times 2(\frac{\gamma-1}{2})} \right)^{\frac{1}{\gamma}} \tag{11}$$

In the proposed research, the parameters α and μ are decided heuristically between the random search space of (0, 1) and the parameter γ has been set to 1.5 as a default constant.

So, including all of these the actual and final strategy for updating the actual location of Harris hawks in the phase of soft encircle can be achieved by eqns.(12) and (13) which is given below.

$$H(iter+1) = \{M_N \; if, F(M_N) \langle F(H(iter)) \tag{12}$$

$$H(iter+1) = \{P \, if, F(P) \langle F(H(iter)) \tag{13}$$

3.2.2 Hard encircle with advanced fast dives

If $|EG| < 0.5$ and $e_s < 0.5$, at this moment the prey has not sufficient energy to escape or avoid and a hard encircle is created before the surprise attack (Fig: 1) to catch and kill the rabbit. At this situation the Harris hawks try to reduce the distance with the escaping rabbit. So this rule can be performed in basis of hard encircle condition and followed by eqns.(12) and (13), where the value of M_N and P can obtain using new rule in eqn. (14) and (15) which contain next position for next iteration.

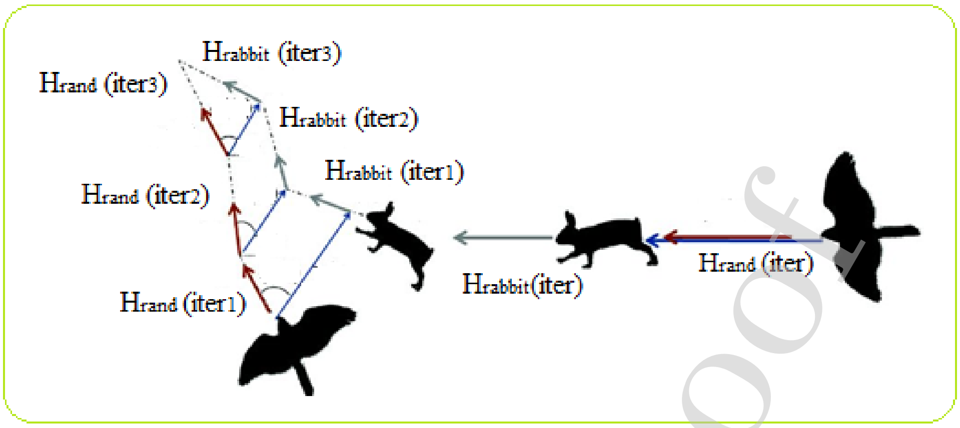


Fig.1(a): Surprise Attack

$$M_N = H_{rabbit}(iter) - EG\left|KH_{rabbit}(iter) - H_m(iter)\right| \quad (14)$$

$$P = M_N + R_S \times LF_T(D_{IM}) \quad (15)$$

Where, $H_m(iter)$ can obtain from equation number (3).

4. Sine Cosine Algorithm (SCA)

The proposed method Sine Cosine Algorithm is one of the population based optimizer which is recognized established on the mathematical functions of sine as well as cosine shown in Fig: 1(b). Similarly to the other types searching process by randomly constructing the solutions set. After that, all these types of solution attained randomly so extreme which signifies it as the point of destination and the resolutions or solutions. With the help of this optimization technique, this algorithm stores that solution which is more better and signifies it as the point of destination and the point of the solutions, which update to build a new resolution according to the function depend on sine and cosine, shown in the equation number (16) through equation number (17)

$$H_{i,iter+1} = H_{i,iter} + re_1 \times \sin(re_2) \times \left|re_3 p_{i,iter} - H_{i,iter}\right| \quad (16)$$

$$H_{i,iter+1} = H_{i,iter} + re_1 \times \cos(re_2) \times \left|re_3 p_{i,iter} - H_{i,iter}\right| \quad (17)$$

Where $H_{i,iter}$ is used to identify the position of the present solution in ith dimension at the $iter$ iteration and the random numbers are denoted as re_1, re_2 and re_3 respectively. $p_{i,iter}$ is the position of the point of the destination with ith number of dimension and absolute vale is indicated using the symbol of ||. After combining those two equations (16) and (17) are shown in equation (18).

$$H_{i,iter+1} = \left\{H_{i,iter} + re_1 \times \sin(re_2) \times \left|re_3 p_{i,iter} - H_{i,iter}\right|, re_4 \langle 0.5 \right. \quad (18)$$

$$H_{i,iter+1} = \left\{H_{i,iter} + re_1 \times \cos(re_2) \times \left|re_3 p_{i,iter} - H_{i,iter}\right|, re_4 \geq 0.5 \right. \quad (19)$$

Where re_4 is the random number [0, 1].

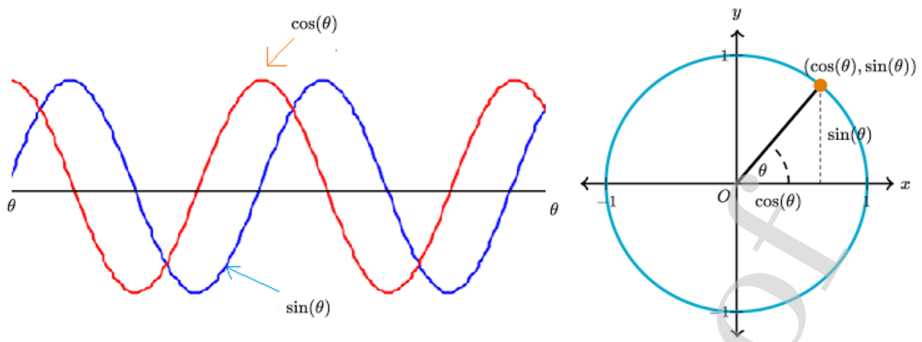


Fig.1(b): Basic principle of SCA

The optimization technique based on this algorithm can balance the phase of exploration as well as exploitation to solve the favorable regions in the search space and finally converge to the solution of global optimum. To balance the phase of exploitation and exploration, the range of cosine and sine from the eqn. (18) and (19) are improved adaptively using the eqn. (20).

$$re_1 = U - \frac{U \times iter}{iter_{\max}} \tag{20}$$

Where U is a constant and $iter_{\max}$ is maximum number of iteration. After attained the last condition, the optimization procedure of this projected algorithm stops after sustaining the maximum number of iterations. The complexity of computation of this proposed algorithm mainly be subject to on three major process, such as, at first initialization, after that evaluation of fitness and at last updating of Harris hawks. It must be noted that I number of Harris hawks, the complexity of computation of the process of initialization is $U(I)$ For the updating mechanism the complexity of computation is $U(iter_{\max} \times I) + U(iter_{\max} \times I \times D_{IM})$, which is collected for penetrating for the best position and again update the position vector of all the Harris hawks, where $iter_{\max}$ is representing the maximum iteration and the dimension of the problem is representing as D_{IM}. Thus the complexity of computation of this proposed algorithm is $U(I(iter_{\max} + iter_{\max} D_{IM} + 1))$.

In proposed hybrid HHO-SCA algorithm, the steps of SCA are applied sequentially after the HHO algorithm to improve the exploitation and exploration to further extent. The Pseudo code and the flow chart of the proposed hybrid optimizer has been shown in **Fig.1(c)** and **Fig.1(d).**

INPUTS: The population size is taken as N and maximum iteration number is taken as $iter_{max}$
OUTPUTS: The position of prey (rabbit) and its value of fitness
Initialization of random population $H_i (i = 1, 2, 3......., N)$
While (iter<iter$_{max}$)
Calculation of the fitness value of Harris hawks
Set the parameter H_{rabbit} as the best position of the prey (rabbit)
for (each Harris hawks (H_i))
Do $EG_0 = 2rand() - 1, K = 2(1 - rand())$ $\rightarrow$ Update energy at initial condition EG_0
Update EG using equation number (4)
if $|EG| \geq 1$ **then** $\rightarrow$ **Phase of Exploration**
Update the position vector using equation (1) and (2)
if $|EG| \langle 1$ **then** $\rightarrow$ **Phase of Exploitation**
1st phase of improvement of Exploration and Exploitation
if $(e_s \geq 0.5)$ and $|EG| \geq 0.5$ **then** $\rightarrow$ **Soft encircle**
Location vector updated using equation number (5)
else if $(e_s \geq 0.5)$ and $|EG| \langle 0.5$ **then** $\rightarrow$ **Hard encircle**
Location vector updated using equation number (7)
else if $(e_s \langle 0.5)$ and $|EG| \geq 0.5$ **then** $\rightarrow$ **Soft encircle with advanced fast dives**
Location vector updated using equation number (12) and (13)
else if $(e_s \langle 0.5)$ and $|EG| \langle 0.5$ **then** $\rightarrow$ **Hard encircle with advanced fast dives**
Location vector updated using equation number (14) and (15)
end
end
end
end

Initialize the starting position of the search agents using final position obtained through Harris Hawks optimizer
Do
Evaluate each of the search agents using objective functions
Update the best fitness obtained so far $(H \in X)$
Update the random numbers re_1, re_2, re_3 and re_4
if re_4<0.5
Update the position of search agents using $H_{i,iter+1} = \{H_{i,iter} + re_1 \times \sin(re_2) \times |re_3 p_{i,iter} - H_{i,iter}|$
else
Update the position of search agents using $H_{i,iter+1} = \{H_{i,iter} + re_1 \times \cos(re_2) \times |re_3 p_{i,iter} - H_{i,iter}||$
2nd stage of improvement of Exploration and Exploitation
end
While $(iter \langle iter_{max})$
Return the best optimal solution
Record the mean, best, worst fitness and standard deviation.
Save the global optimum value obtained through successive trial runs

Fig.1(c): PSEUDO code of proposed hybrid HHO-SCA algorithm

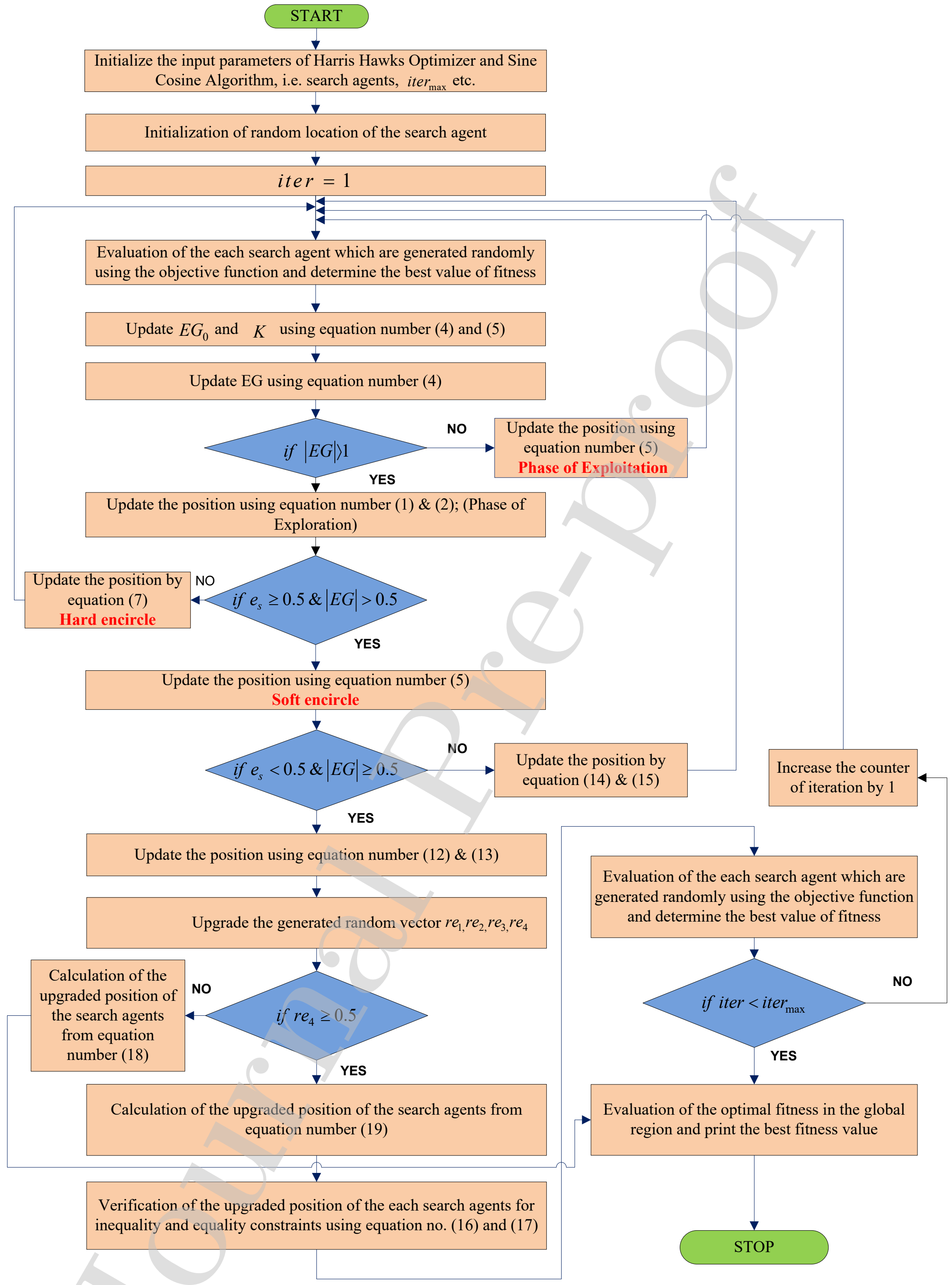


Fig.1 (d): Flow chart of proposed hHHO-SCA algorithm

5. BENCHMARK FUNCTIONS

To examine the effectiveness of the proposed hHHO-SCA optimization technique, a well-studied set of various benchmark functions are taken [109][110]. These set of benchmark are consist of three main group of benchmark function, such as, unimodal, multimodal and fixed dimension. In proposed research, the performance of the proposed hybrid optimizer has been tested for unimodal, multi-modal, fixed dimension, CEC2017 and CEC2018 benchmark functions. The mathematical formulation of unimodal, multi-modal and fixed dimension benchmarks are shown in Table 2, Table 3 and Table 4 respectively. The 3D view of unimodal and multi-modal benchmark functions is shown in Fig.1(e), Fig.1(f) and Fig.1(g) respectively.

Table 2: Unimodal benchmark function

Function	**Dim**	**Range**	$f_{\min}$
$f_1(x)=\sum_{i=1}^{n} x_i^2$	30	[-100 , 100]	0
$f_2(x)=\sum_{i=1}^{n} \lvert x_i\rvert+\prod_{i=1}^{n}\lvert x_i\rvert$	30	[-10 ,10]	0
$f_3(x)=\sum_{i=1}^{n}(\sum_{j-1}^{i} x_j)^2$	30	[-100 , 100]	0
$f_4(x)=\max_i\{\lvert x_i\rvert,1\le i\le n\}$	30	[-100 , 100]	0
$f_5(x)=\sum_{i=1}^{n-1}[100(x_{i+1}-x_i^2)^2+(x_i-1)^2]$	30	[-30 ,30]	0
$f_6(x)=\sum_{i=1}^{n}([x_i+0.5])^2$	30	[-100 , 100]	0
$f_7(x)=\sum_{i=1}^{n} ix_i^4+random[0,1]$	30	[-1.28, 1.28]	0

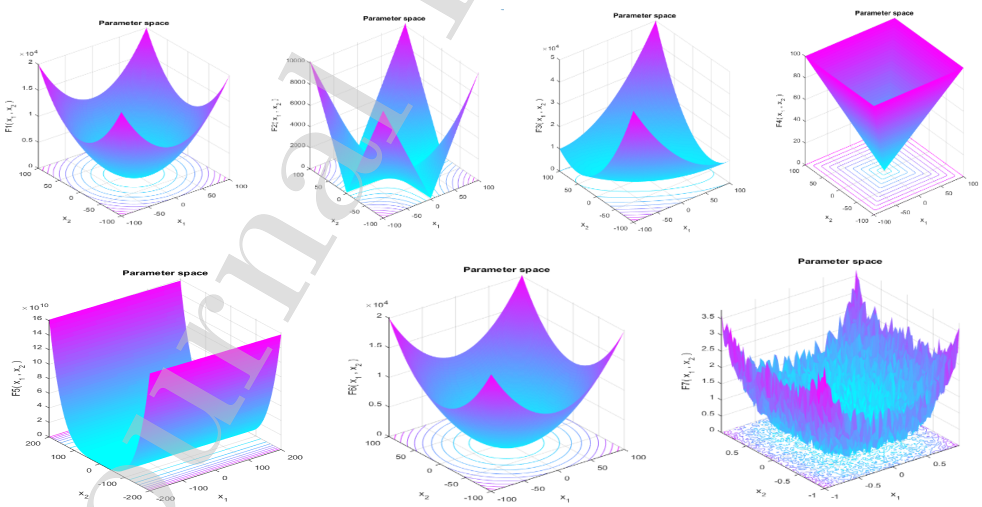


Fig.1 (e): 3D view of Unimodal Benchmark problem

Table 3: Multi-modal benchmark functions

Function	Dim	Range	$f_{\min}$
$F_8(x)=\sum_{i=1}^{n} -x_i \sin(\sqrt{\lvert x_i \rvert})$	30	[-500, 500]	-418.98295
$F_9(x)=\sum_{i=1}^{n}[x_i^2-10\cos(2\pi x_i)+10]$	30	[-5.12, 5.12]	0
$F_{10}(x)=-20\exp(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n}x_i^2})-\exp(\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i))+20+c$	30	[-32, 32]	0
$f_{11}(x)=1+\sum_{i=1}^{n}\frac{x_i^2}{4000}-\prod_{i=1}^{n}\cos\frac{(x_i)}{\sqrt{i}}$	30	[-600, 600]	0
$F_{12}(x)=\frac{\pi}{n}\left\{10\sin(\pi y_1)+\sum_{i=1}^{n-1}(y_i-1)^2[1+10\sin^2(\pi y_{i+1})]+(y_n-1)^2\right\}+\sum_{i=1}^{n}u(x_i,10,100,4)$ $y_i=1+\frac{x_i+1}{4}$ $u(x_i,a,k,m)=\begin{cases} k(x_i-a)^m & x_i>a \\ 0 & -a<x_i<a \\ k(-x_i-a)^m & x_i<-a \end{cases}$	30	[-50,50]	0

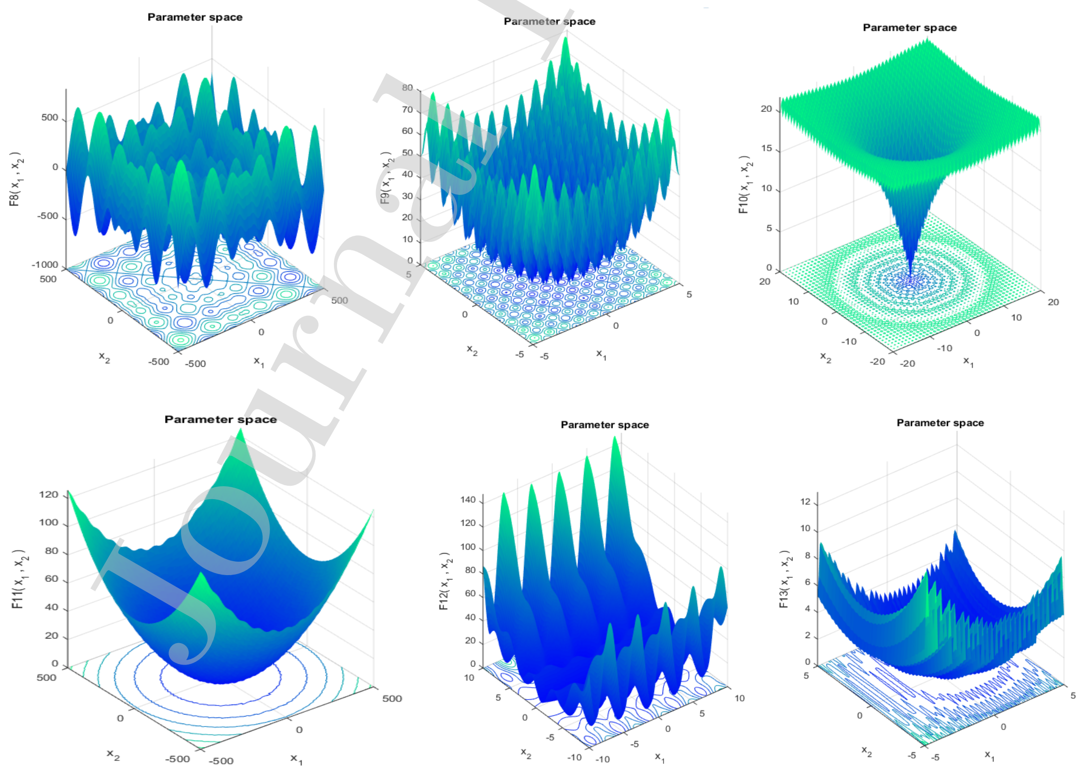


Fig.1 (f): Dimension of Multimodal Benchmark problem

Table 4: Fixed dimension benchmark functions

Function	Dim	Range	$f_{\min}$
$F_{13}(x)=0.1\left\{\sin^2(3\pi x_1)+\sum_{i=1}^{n}(x_i-1)^2\left[1+\sin^2(3\pi x_i+1)\right]+(x_n-1)^2\left[1+\sin^2(2\pi x_n)\right]\right\}+\sum_{i=1}^{n}u(x_i,5,100$	30	[-50,50]	0
$f_{14}(x)=\left[\frac{1}{500}+\sum_{j=1}^{2}5\frac{1}{j+\sum_{i=1}^{n}(x_i-a_{ij})^6}\right]^{-1}$	2	[-65.536, 65.536]	1
$F_{15}(x)=\sum_{i=11}^{11}\left[a_i-\frac{x_1\left(b_i^2+b_ix_2\right)}{b_i^2+b_ix_3+x_4}\right]^2$	4	[-5, 5]	0.00030
$F_{16}(x)=4x_1^2-2.1x_1^4+\frac{1}{3}x_1^6+x_1x_2-4x_2^2+4x_2^4$	2	[-5, 5]	-1.0316
$F_{17}(x)=\left(x_2-\frac{5.1}{4\pi^2}x_1^2+\frac{5}{\pi}x_1-6\right)^2+10\left(1-\frac{1}{8\pi}\right)\cos x_1+10$	2	[-5, 5]	0.398
$F_{18}(x)=\left[1+(x_1+x_2+1)^2\left(19-14x_1+3x_1^2-14x_2+6x_1x_2+3x_2^2\right)\right]$ $\times\left[30+\left(2x_1-3x_2\right)^2\times\left(18-32x_1+12x_1^2+48x_2-36x_1x_2+27x_2^2\right)\right]$	2	[-2,2]	3
$f_{19}(x)=-\sum_{i=1}^{4}c_i\exp\left(-\sum_{j=1}^{3}a_{ij}(x_j-p_{ij})^2\right)$	3	[1, 3]	-3.32
$f_{20}(x)=-\sum_{i=1}^{4}c_i\exp\left(-\sum_{j=1}^{6}a_{ij}(x_j-p_{ij})^2\right)$	6	[0, 1]	-3.32
$f_{21}(x)=-\sum_{i=1}^{5}\left[(x-a_i)\ (x-a_i)^T+c_i\right]^{-1}$	4	[0,10]	-10.1532
$f_{22}(x)=-\sum_{i=1}^{7}\left[(x-a_i)\ (x-a_i)^T+c_i\right]^{-1}$	4	[0, 10]	-10.4028
$f_{23}(x)=-\sum_{i=1}^{10}\left[(x-a_i)\ (x-a_i)^T+c_i\right]^{-1}$	4	[0, 10]	-10.5363

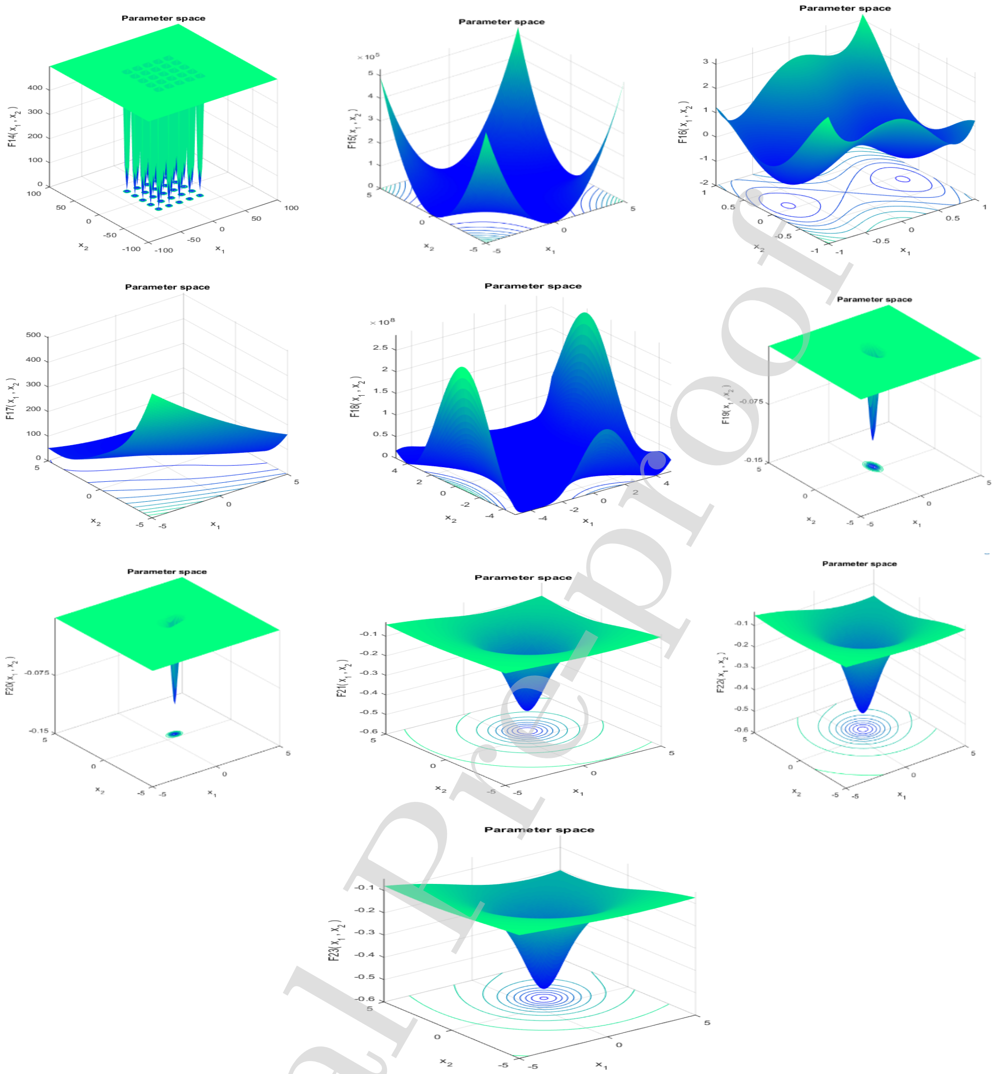
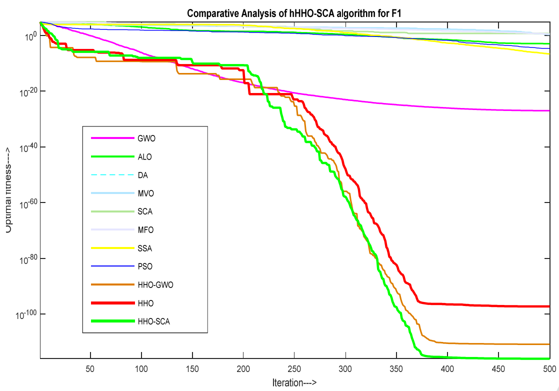


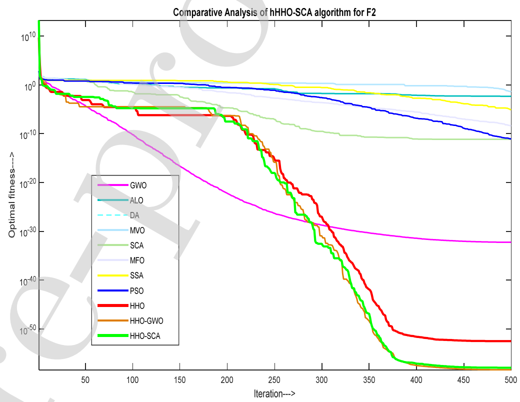
Fig.1 (g): Dimension of Fixed Dimension modal

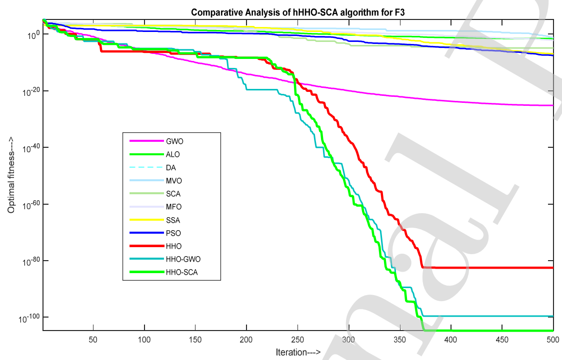
RESULTS AND DISCUSSION

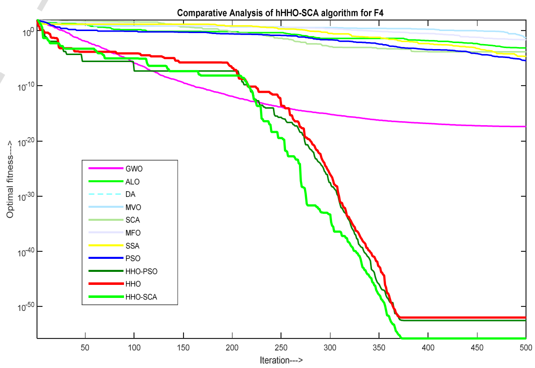
To validate the results, 30 trial runs are taken into consideration to overcome the stochastic nature of projected hHHO-SCA algorithm and each objective function has been estimated for average value, best values, standard deviation and worst value. In order to approve the phase of exploitation recommended algorithm, unimodal benchmark functions F1, F2, F3, F4, F5, F6 and F7 are taken into consideration. Table 5 shows the solution of unimodal benchmark function using hHHO-SCA algorithm. The estimate results for unimodal benchmark functions has been shown in Table 6, which are compared with others recently established metaheuristics algorithms GWO [36], GSA [35], FEP [109], ALO [11], SMS [111][112], BA [14], FPA [33], CS [113][31], FA [114], GA [115], GOA [34], MFO [49], BA [116], SMS [117], MVO [50], DA [25], BDA [25], BPSO [118], BGSA [17], SCA [56], BA [14], FPA [119], SSA [63], FEP [109] and WOA [69] in terms of average and standard deviation. The convergence curve of hHHO-SCA and its comparative analysis with GWO, ALO, DA, MVO, SCA, MFO, SSA, PSO and HHO is shown in Fig.1(h) and the trial solutions for unimodal benchmark functions are shown in Fig.2. In order to validate the phase of exploration of submitted algorithm, the multi-modal benchmark functions F8, F9, F10, F11, F12 and F13 are taken into concern, as these functions have many local optimal search with the number increasing exponentially with dimension. For effective analysis of results, similar number of iterations and runs has been taken into consideration, while comparing the results with others developed methodologies.

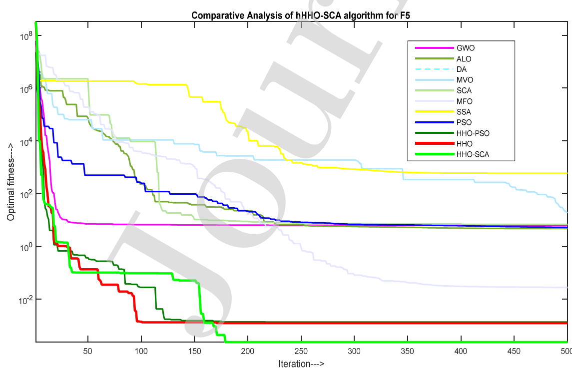
Table 5: Test results for unimodal Benchmark functions using **hHHO-SCA** algorithm						
Functions	**Mean**	**SD**	**Best**	**Worst**	**Median**	**p-Value**
F1	1.8568E-91	9.48882E-91	9.8005E-117	5.20089E-90	1.0278E-102	1.7344E-06
F2	2.46034E-51	1.11204E-50	1.1796E-58	6.03651E-50	2.01442E-54	1.7344E-06
F3	8.88008E-72	4.86382E-71	1.7853E-105	2.66402E-70	7.22098E-86	1.7344E-06
F4	8.01525E-49	2.82586E-48	1.38633E-56	1.17982E-47	8.00088E-53	1.7344E-06
F5	0.014313995	0.020188417	0.000237175	0.104630448	0.006851675	1.7344E-06
F6	0.000223702	0.000338117	3.18673E-09	0.001181842	4.05202E-05	1.7344E-06
F7	0.00012246	0.000110326	3.25707E-06	0.000469272	8.27434E-05	1.7344E-06

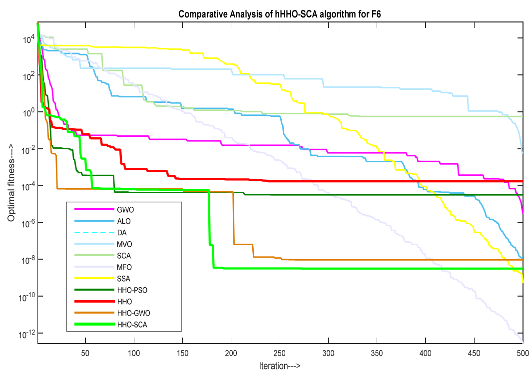












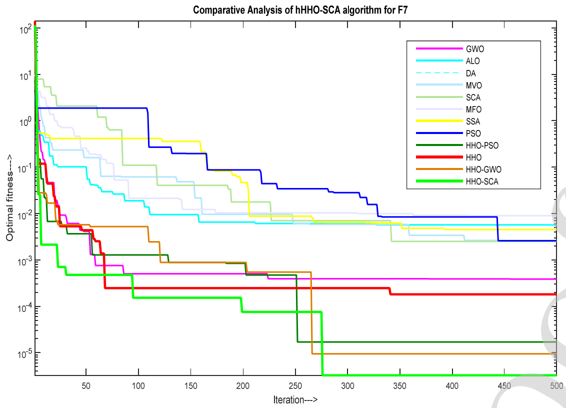
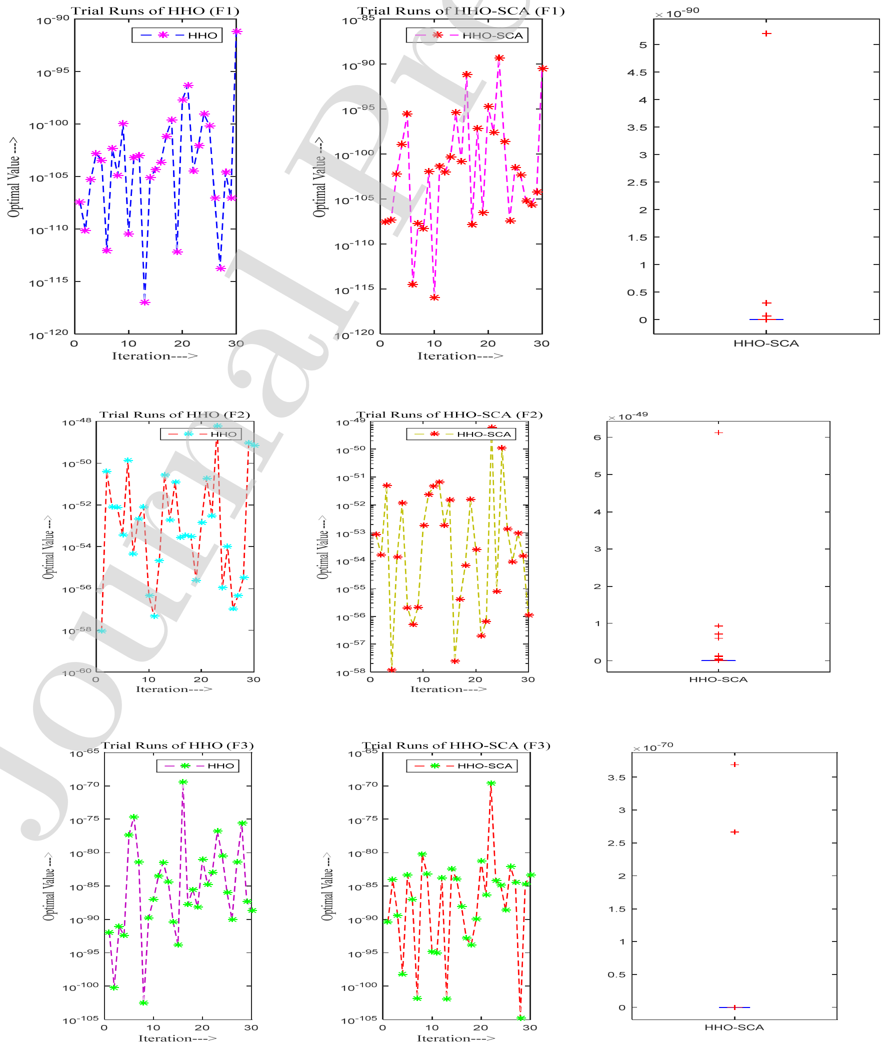


Fig.1 (h): Convergence curve of hHHO-SCA with GWO, ALO, DA, MVO, SCA, MFO, SSA, PSO, HHO and hHHO-PSO for Unimodal Benchmark functions

Table 6: Comparison of Unimodal Benchmark functions

Algorithms	Parameters	Unimodal Benchmark functions						
		F1	F2	F3	F4	F5	F6	F7
GWO [36]	**Mean**	0.00	0.00	0.00	0.00	26.81	0.82	-6120.00
	SD	0.00	0.03	79.15	1.32	69.91	0.00	-4090.00
GSA [38]	**Mean**	0.00	0.06	896.54	7.36	67.54	0.00	-2820.00
	SD	0.00	0.19	318.96	1.74	62.23	0.00	493.00
FEP [97]	**Mean**	0.00	0.01	0.02	0.30	5.06	0.00	-12600.00
	SD	0.00	0.00	0.01	0.50	5.87	0.00	52.60
ALO [12]	**Mean**	0.00	0.00	0.00	0.00	0.35	0.00	-1610.00
	SD	0.00	0.00	0.00	0.00	0.11	0.00	314.00
SMS [98]	**Mean**	0.06	0.01	0.96	0.28	0.09	0.13	-4.21
	SD	0.02	0.00	0.82	0.01	0.14	0.09	0.00
BA [14]	**Mean**	0.77	0.34	0.12	0.19	0.33	0.78	-1070.00
	SD	0.53	3.82	0.77	0.89	0.30	0.67	858.00
FPA [34]	**Mean**	0.00	0.00	0.00	0.00	0.78	0.00	-1840.00
	SD	0.00	0.00	0.00	0.00	0.37	0.00	50.40
CS [100]	**Mean**	0.01	0.21	0.25	0.00	0.01	0.00	-2090.00
	SD	0.00	0.04	0.02	0.00	0.01	0.00	0.01
FA [101]	**Mean**	0.04	0.05	0.05	0.15	2.18	0.06	-1250.00
	SD	0.01	0.01	0.02	0.03	1.45	0.01	353.00
GA [102]	**Mean**	0.12	0.15	0.14	0.16	0.71	0.17	-2090.00
	SD	0.13	0.05	0.12	0.86	0.97	0.87	2.47
GOA [35]	**Mean**	0.00	0.00	0.00	0.00	0.00	0.00	1.00
	SD	0.00	0.00	0.02	0.00	0.00	0.00	0.00
MFO [50]	**Mean**	0.00	0.00	696.73	70.69	139.15	0.00	-8500.00
	SD	0.00	0.00	188.53	5.28	120.26	0.00	726.00
MVO [51]	**Mean**	2.09	15.93	453.20	3.12	1272.13	2.30	-11700.00
	SD	0.65	44.75	177.10	1.58	1479.48	0.63	937.00
DA [26]	**Mean**	0.00	0.00	0.00	0.00	7.60	0.00	-2860.00
	SD	0.00	0.00	0.00	0.00	6.79	0.00	384.00
BDA [26]	**Mean**	0.28	0.06	14.20	0.25	23.60	0.10	-924.00
	SD	0.42	0.07	22.70	0.33	34.70	0.13	65.70
BPSO [105]	**Mean**	5.59	0.20	15.50	1.90	86.40	6.98	-989.00
	SD	1.98	0.05	13.70	0.48	65.80	3.85	16.70

BGSA [18]	**Mean**	83.00	1.19	456.00	7.37	3100.00	107.00	-861.00
	SD	49.80	0.23	272.00	2.21	2930.00	77.50	80.60
SCA [56]	**Mean**	0.00	0.00	0.04	0.10	0.00	0.00	1.00
	SD	0.00	0.00	0.14	0.58	0.00	0.00	0.00
SSA [63]	**Mean**	0.00	0.23	0.00	0.00	0.00	0.00	0.06
	SD	0.00	1.00	0.00	0.66	0.00	0.00	0.81
WOA [69]	**Mean**	0.00	0.00	0.00	0.07	27.87	3.12	-5080.00
	SD	0.00	0.00	0.00	0.40	0.76	0.53	696.00
hHHO-PSO	**Mean**	0.00	0.00	0.00	0.00	0.01	0.00	0.00
	SD	0.00	0.00	0.00	0.00	0.01	0.00	0.00
hHHO-GWO	**Mean**	0.00	0.00	0.00	0.00	0.02	0.00	0.00
	SD	0.00	0.00	0.00	0.00	0.02	0.00	0.00
hHHO-SCA	**Mean**	**0.00**	**0.00**	**0.00**	**0.00**	**0.01**	**0.00**	**0.00**
	SD	**0.00**	**0.00**	**0.00**	**0.00**	**0.02**	**0.00**	**0.00**



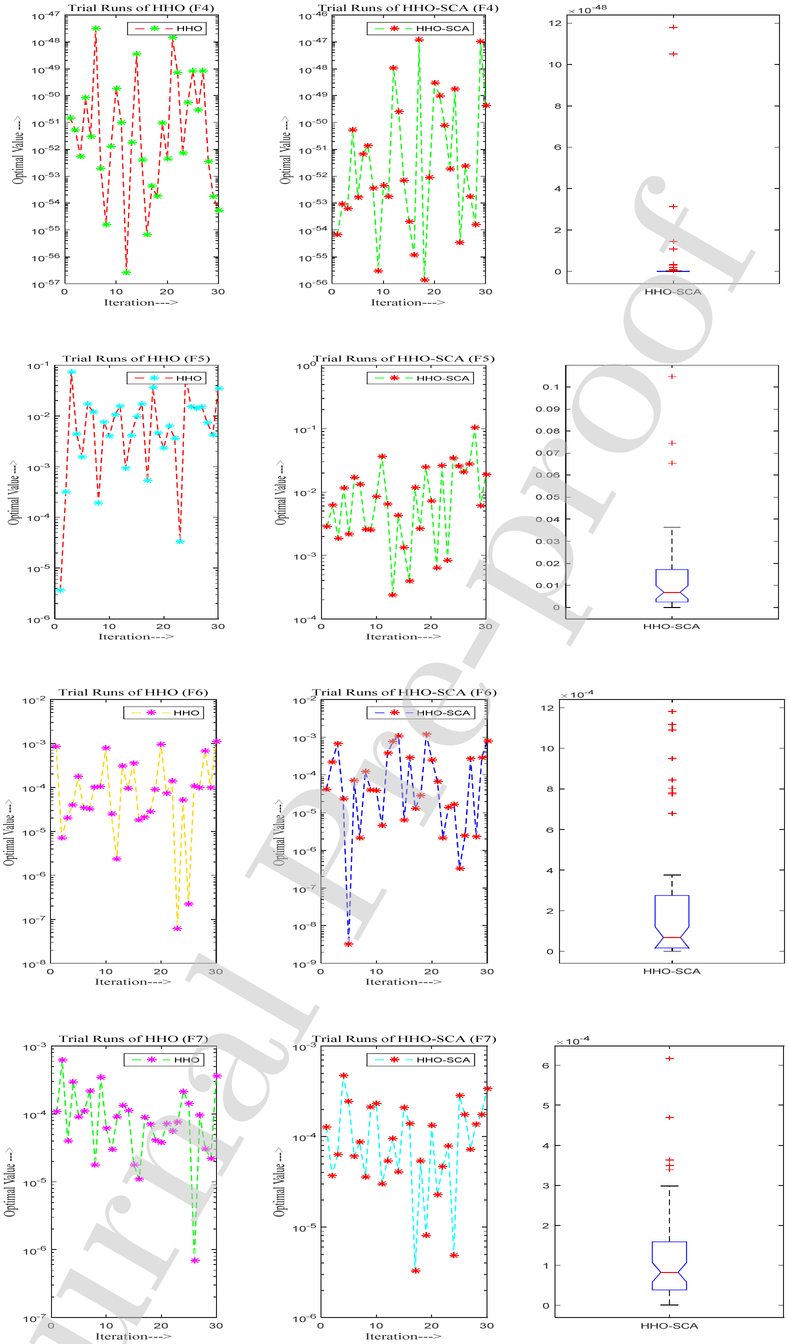


Fig.2: Trial solutions for unimodal benchmark functions

Table 7 shows the solution of multi-modal benchmark function using hHHO-SCA algorithm. The comparison results for multi-modal benchmark functions has been shown in Table 8, which are compared with others recently developed metaheuristics search algorithms GWO [36], GSA [35], FEP [109], ALO [11], SMS [111], BA [14], FPA [33], CS [113], FA [114], GA [115], GOA [34], MFO [49], BA [116], SMS [117], MVO [50], DA [25], BDA [25], BPSO [118], BGSA [17], SCA [56], BA [14], FPA [119], SSA [63], FEP [109] and WOA [69] in terms of average and standard deviation. The convergence curve of hHHO-SCA for multi-modal benchmark functions are shown in Fig.3 and corresponding their trial solutions are shown in Fig.4.

Table 7: Test results for multi-modal Benchmark functions using **hHHO-SCA** algorithm						
Functions	**Mean**	**SD**	**Best**	**Worst**	**Median**	**p-Value**
F8	-12569.08113	0.76690716	-12569.48662	-12566.13644	-12569.41349	1.7344E-06
F9	0	0	0	0	0	1
F10	8.88178E-16	0	8.88178E-16	8.88178E-16	8.88178E-16	4.32046E-08
F11	0	0	0	0	0	1
F12	1.13E-05	1.5E-05	9.73E-10	6.46E-05	4.86E-06	1.734E-06
F13	0.000113	0.000166	1.49E-06	0.000674	5.53E-05	1.734E-06

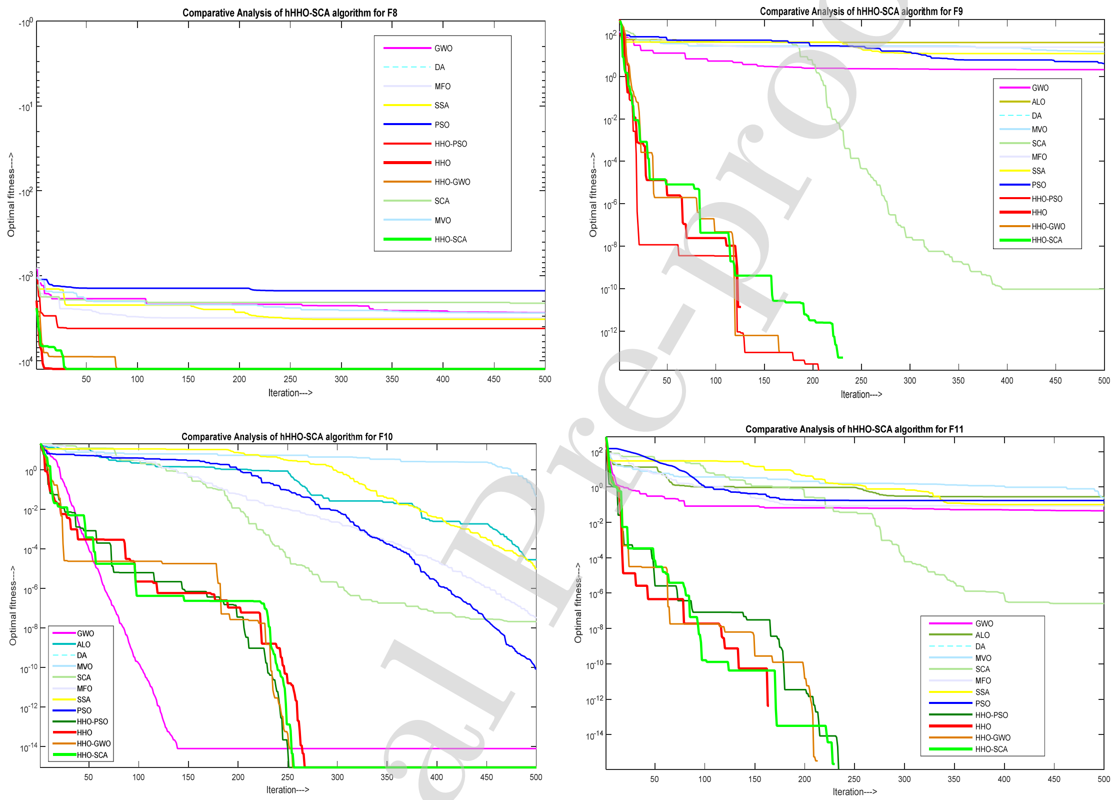


Fig.3: Convergence curve of hHHO-SCA with GWO, ALO, DA, MVO, SCA, MFO, SSA, PSO, HHO and hHHO-PSO for Multimodal Benchmark functions

Table 8: Comparison of Multi modal Benchmark functions

Algorithms	**Parameters**	**Multi modal Benchmark functions**					
		F8	**F9**	**F10**	**F11**	**F12**	**F13**
GWO [36]	**Mean**	-6.123E+224	0.310521	1.06E-13	0.004485	0.053438	0.654464
	SD	-4087.44	47.35612	0.077835	0.006659	0.654464	0.004474
PSO	**Mean**	-4841.29	46.70423	0.276015	0.009215	0.006917	0.006675
	SD	1152.814	11.62938	0.50901	0.007724	0.026301	0.008907
GSA [35]	**Mean**	-2821.07	25.97	0.06	27.70	1.80	8.90
	SD	493.04	7.47	0.24	5.04	0.95	7.13
DE[120]	**Mean**	-11080.10	69.20	0.00	0.00	0.00	0.00
	SD	574.70	38.80	0.000	0.000	0.00	0.00
FEP [109]	**Mean**	-12554.50	0.05	0.02	0.02	0.00	0.00
	SD	52.60	0.01	0.00	0.02	0.00	0.00
hHHO-PSO	**Mean**	-12568.81154	0.00	8.88178E-16	0.00	1.12558E-05	0.000113306
	SD	0.946744548	0.00	0.00	0.00	1.49725E-05	0.000166039
hHHO-GWO	**Mean**	-1.26E+04	0.00E+00	8.88E-16	0.00E+00	NA	NA
	SD	1.04E+00	0.00E+00	0.00E+00	0.00E+00	NA	NA
hHHO-SCA	**Mean**	**-12569.08113**	**0**	**8.88178E-16**	**0**	**1.13E-05**	**0.000113**
	SD	**0.76690716**	**0**	**0**	**0**	**1.5E-05**	**0.000166**

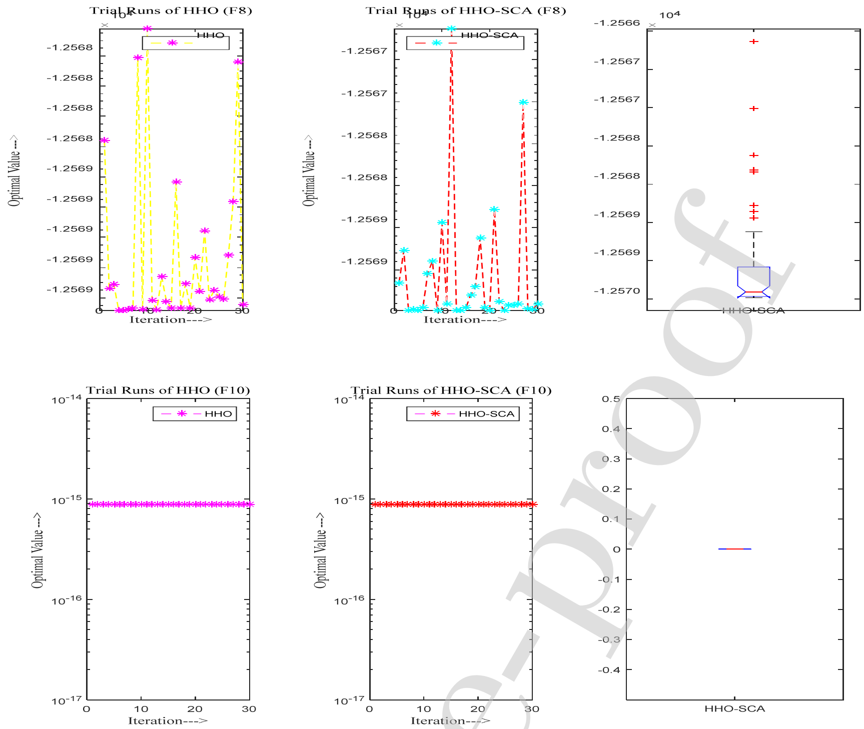


Fig.4: Trial solutions for multimodal benchmark functions

For fixed dimension benchmark problems the test results are shown in Table 9 through Table 10. The comparison results for fixed dimension benchmark functions has been shown in Table 10, which are compared with others recently developed metaheuristics search algorithms GWO [36], GSA [35], DE [120], FEP [109], ALO [11], SMS [111], BA [14], FPA [33], CS [113], FA [114], GA [115], GOA [34], MFO [49], BA [116], SMS [117], MVO [50], DA [25], SCA [56], BA [14], FPA [119], SSA [63], FEP [109] and WOA [69] in terms of standard deviation and average. The trial solutions for fixed dimension benchmark functions and their convergence curve are shown in Fig.6 and Fig.5 respectively. For statistical analysis of the proposed algorithm, the non-parametric test i.e. Wilcoxon rank-sum test has been taken into consideration at 0.05 significance level.

Table 9: Test results for fixed dimension multi-modal Benchmark functions using **hHHO-SCA** algorithm

Functions	Mean	SD	Best	Worst	Median	p-Value
F14	1.26E+00	4.47E-01	9.98E-01	1.99E+00	9.98E-01	1.73E-06
F15	3.45E-04	4.03E-05	3.07E-04	4.68E-04	3.26E-04	1.73E-06
F16	-1.03E+00	1.80E-09	-1.03E+00	-1.03E+00	-1.03E+00	1.73E-06
F17	3.98E-01	2.15E-05	3.98E-01	3.98E-01	3.98E-01	1.73E-06
F18	3.00E+00	9.98E-07	3.00E+00	3.00E+00	3.00E+00	1.73E-06
F19	-3.86E+00	3.00E-03	-3.86E+00	-3.85E+00	-3.86E+00	1.73E-06
F20	-3.09E+00	1.09E-01	-3.30E+00	-2.79E+00	-3.12E+00	1.73E-06
F21	-5.21E+00	8.95E-01	-9.95E+00	-5.03E+00	-5.05E+00	1.73E-06
F22	-5.25E+00	9.26E-01	-1.02E+01	-5.07E+00	-5.09E+00	1.73E-06
F23	-5.28E+00	8.65E-01	-9.86E+00	-5.11E+00	-5.13E+00	1.73E-06

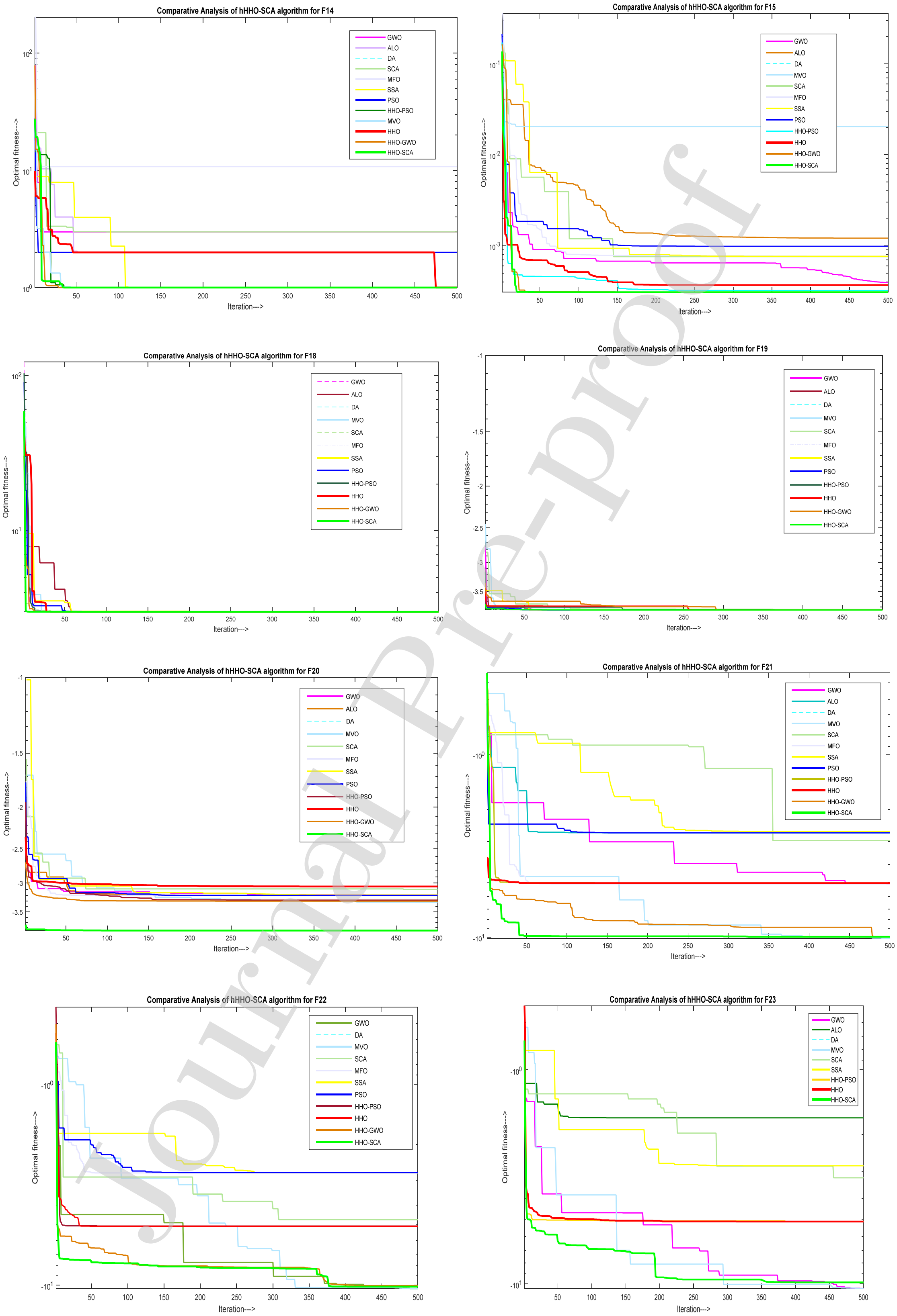
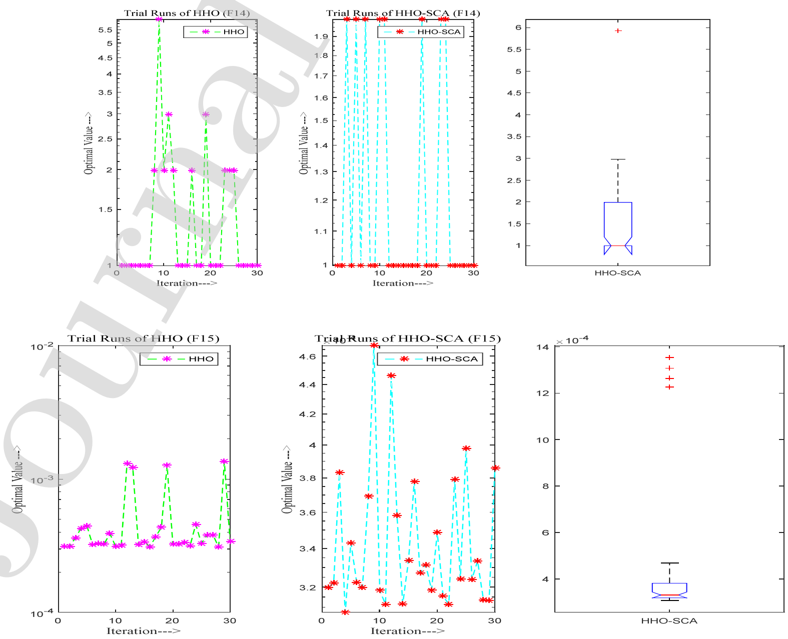
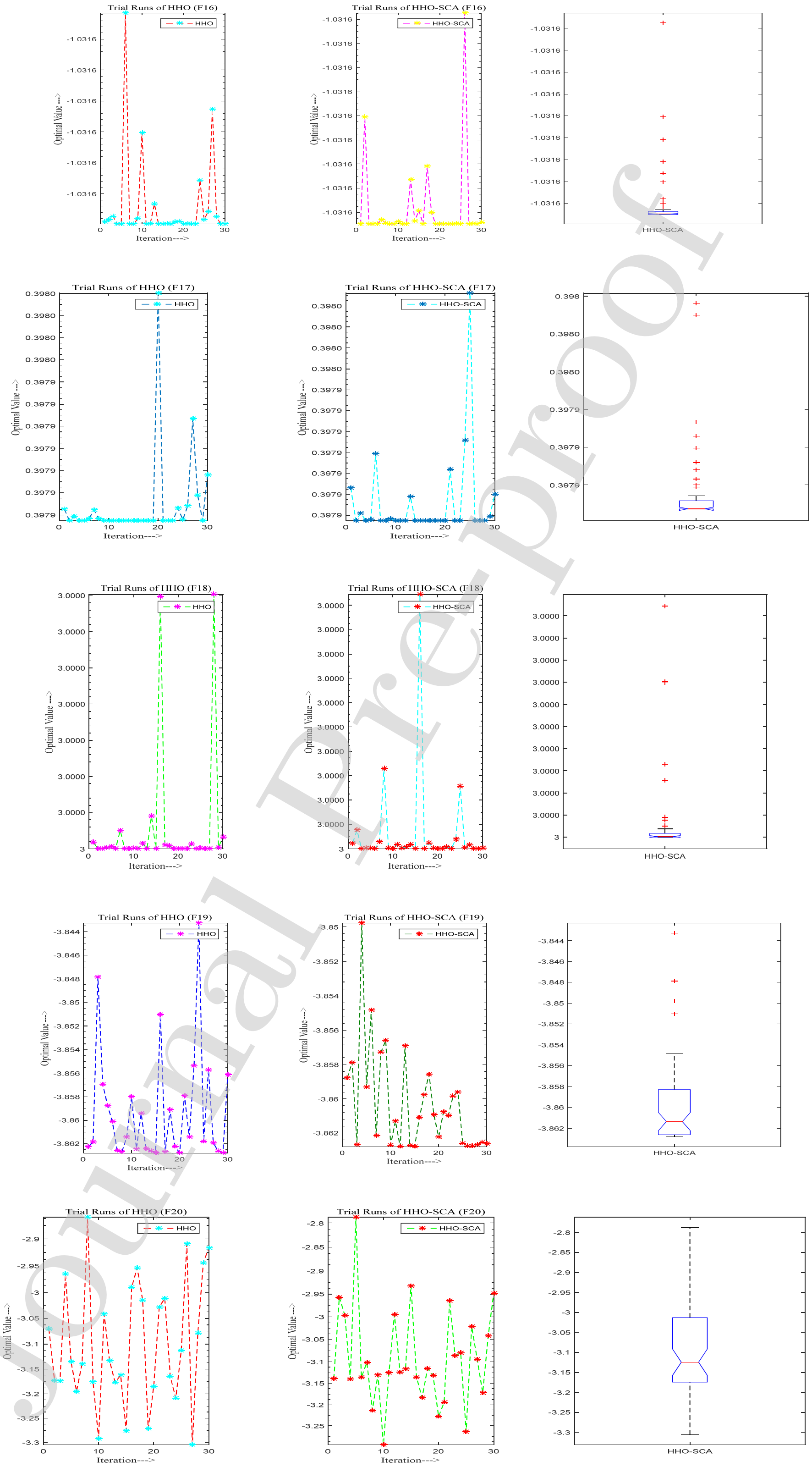


Fig.5: Convergence curve of hHHO-SCA with GWO, ALO, DA, MVO, SCA, MFO, SSA, PSO, HHO and hHHO-PSO for Fixed Dimension Benchmark functions

Algorithms	Parameters	Multi modal Benchmark functions									
		F14	F15	F16	F17	F18	F19	F20	F21	F22	F23
GWO [36]	**Mean**	4.04	0.00	-1.03	0.40	3.00	-3.86	-3.29	-10.15	-10.40	-10.53
	SD	4.25	0.00	-1.03	0.40	3.00	-3.86	-3.25	-9.14	-8.58	-8.56
PSO	**Mean**	3.63	0.00	-1.03	0.40	3.00	-3.86	-3.27	-6.87	-8.46	-9.95
	SD	2.56	0.00	0.00	0.00	0.00	0.00	0.06	3.02	3.09	1.78
GSA [35]	**Mean**	5.86	0.00	-1.03	0.40	3.00	-3.86	-3.32	-5.96	-9.68	-10.54
	SD	3.83	0.00	0.00	0.00	0.00	0.00	0.02	3.74	2.01	0.00
DE[120]	**Mean**	1.00	0.00	-1.03	0.40	3.00	N/A	N/A	-10.15	-10.40	-10.54
	SD	0.00	0.00	0.00	0.00	0.00	N/A	N/A	0.00	0.00	0.00
FEP [109]	**Mean**	1.22	0.00	-1.03	0.40	3.02	-3.86	-3.27	-5.52	-5.53	-6.57
	SD	0.56	0.00	0.00	0.00	0.11	0.00	0.06	1.59	2.12	3.14
hHHO-PSO	**Mean**	1.359860826	0.000394039	-1.031628452	0.397904414	3.000000204	-3.861232979	-3.095084545	-5.049396926	-5.083324889	-5.122724703
	SD	1.255144819	0.000217743	4.80343E-09	3.49021E-05	5.27912E-07	0.003410542	0.103220758	0.006604414	0.004116707	0.005765656
hHHO-GWO	**Mean**	1.26E+00	3.44E-04	-1.03E+00	3.98E-01	3.00E+00	-3.86E+00	-3.11E+00	-5.22E+00	-5.14E+00	-5.30E+00
	SD	9.32E-01	3.14E-05	3.97E-10	1.86E-05	2.65E-07	4.30E-03	1.21E-01	8.97E-01	1.10E+00	9.55E-01
hHHO-SCA	**Mean**	**1.26E+00**	**3.45E-04**	**-1.03E+00**	**3.98E-01**	**3.00E+00**	**-3.86E+00**	**-3.09E+00**	**-5.21E+00**	**-5.25E+00**	**-5.28E+00**
	SD	**4.47E-01**	**4.03E-05**	**1.80E-09**	**2.15E-05**	**9.98E-07**	**3.00E-03**	**1.09E-01**	**8.95E-01**	**9.26E-01**	**8.65E-01**

Table 10: Comparison of Fixed Dimension Benchmark function




Trial Runs of HHO (F16)
HHO
Trial Runs of HHO-SCA (F16)
HHO-SCA
Trial Runs of HHO (F17)
HHO
Trial Runs of HHO-SCA (F17)
HHO-SCA
Trial Runs of HHO (F18)
HHO
Trial Runs of HHO-SCA (F18)
HHO-SCA
Trial Runs of HHO (F19)
HHO
Trial Runs of HHO-SCA (F19)
HHO-SCA
Trial Runs of HHO (F20)
HHO
Trial Runs of HHO-SCA (F20)
HHO-SCA
Optimal Value --->
Iteration--->
HHO-SCA

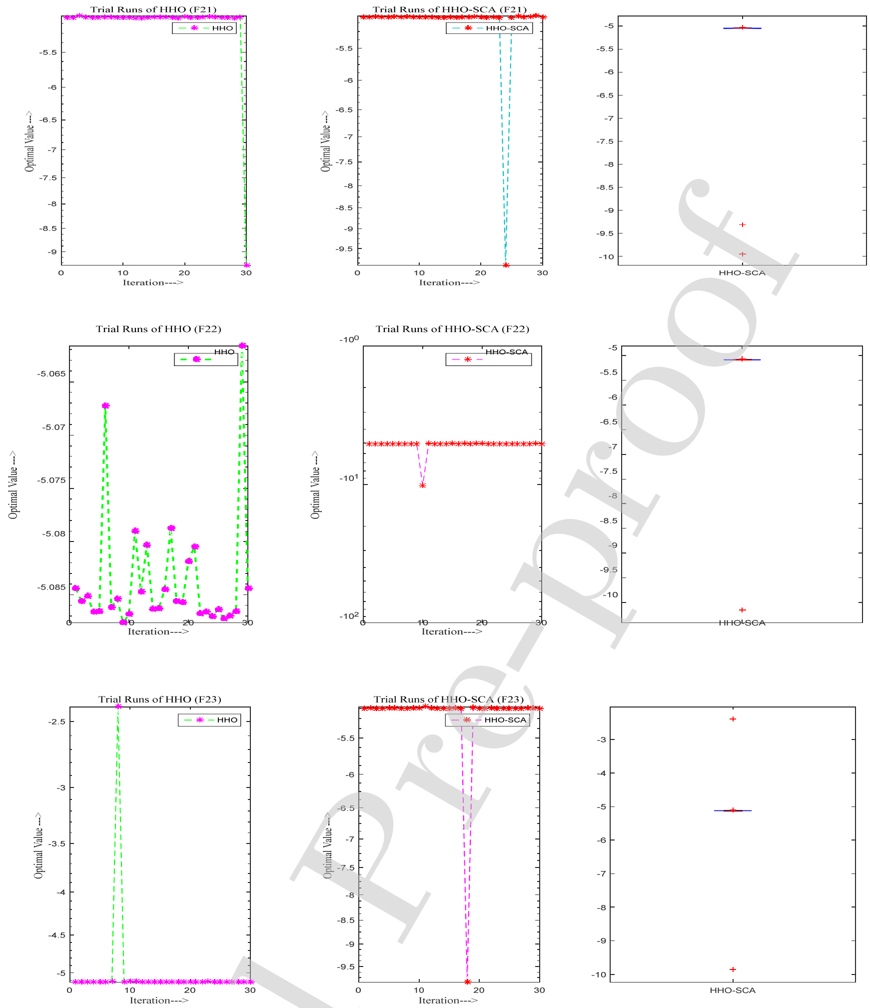


Fig.6: Trial solutions for Fixed dimension benchmark functions

For statistical analysis of the proposed algorithm, the non-parametric test i.e. Wilcoxon rank-sum test has been taken into consideration at 0.05 significance level. The p-value from Wilcoxon rank-sum has been recorded for CEC2017 multimodal Benchmark functions and shown in Table 11 and it has been experimentally found that the low p-value of Wilcoxon rank-sum validate the research study at 0.05 significant level.

Table 11: Comparison of CEC 2017 Multimodal Benchmark Functions

CEC2017 Benchmark Functions	**hHHO-SCA**	**jSO** [121]	**DES**[122]	**TLBO-FL** [123]	**PPSO** [124]
CEC2017(F1)	0	0.00E+00	0	3.50E+03	7.48E+02
CEC2017(F2)	8.9401	8.94E+00	5.88E-02	8.50E+16	5.32E+01
CEC2017(F3)	0.000002389	2.3912E–06	0	3.00E+03	1.13E+00
CEC2017(F4)	189.6287	1.90E+02	5.69E+01	9.00E+01	4.39E+01
CEC2017(F5)	43.9065	4.39E+01	4.64E+00	4.00E+01	1.12E+02
CEC2017(F6)	0.000202421	2.0244E–04	3.34E-07	4.90E-01	2.03E+01
CEC2017(F7)	144.897	1.45E+02	3.57E+01	1.40E+02	1.35E+02
CEC2017(F8)	42.151653	4.22E+01	4.55E+00	3.70E+01	8.10E+01
CEC2017(F9)	0.045903546	4.5904E–02	0	3.40E+01	1.36E+03
CEC2017(F10)	9704.36758	9.70E+03	1.39E+02	6.70E+03	3.13E+03

For statistical analysis of the proposed algorithm, the non-parametric test i.e. Wilcoxon rank-sum test has been taken into consideration at 0.05 significance level. The p-value from Wilcoxon rank-sum has been recorded for CEC2017 Hybrid Benchmark functions and shown in Table 12 and it has been experimentally found that the low p-value of Wilcoxon rank-sum validate the research study at 0.05 significant level.

Table 12: Comparison of CEC 2017 Hybrid Benchmark Functions

CEC2017 Hybrid Benchmark Functions	**hHHO-SCA**		**jSO** [121]	**DES** [122]	**TLBO-FL** [123]	**PPSO** [124]
	Mean Value	**p-Value**				
CEC2017(F11)	104.1954	0.0234	1.04E+02	2.73E+01	8.20E+01	8.43E+01
CEC2017(F12)	17033	0.1746	1.70E+04	1.21E+03	5.70E+04	2.77E+04
CEC2017(F13)	139.57953	0.3258	1.40E+02	4.87E+01	2.00E+04	3.21E+03
CEC2017(F14)	63.84568	0.477	6.39E+01	2.66E+01	7.10E+03	2.32E+03
CEC2017(F15)	164.67964	0.4282	1.65E+02	3.24E+01	2.20E+04	2.13E+03
CEC2017(F16)	1881.6784	0.3794	1.88E+03	7.64E+01	4.90E+02	8.46E+02
CEC2017(F17)	1273.1457	0.4306	1.27E+03	5.54E+01	1.40E+02	3.31E+02
CEC2017(F18)	156.92753	0.0818	1.57E+02	3.51E+01	3.70E+05	6.99E+04
CEC2017(F19)	106.34753	0.233	1.06E+02	1.64E+01	1.10E+04	1.71E+03
CEC2017(F20)	1380.9854	0.3842	1.38E+03	7.06E+01	2.20E+02	3.48E+02

In order to verify the parametric variations of hHHO-SCA algorithm, the parameters α and μ are selected heuristically between the search space rand(0,1) and two different values of α has been taken into consideration for comparative analysis for CEC2017 benchmark functions and comparative results are recorded for α=0.93 and α=0.88 in Table 14. It has been experimentally observed that the best value of α is 0.93 for proper tuning of hHHO-SCA algorithm. Also, the computational time recorded for CEC2017 benchmark functions and compared with TLBO-FL and PSO-GWO algorithm as shown in Table 13 and Table 14.

Table 13: Comparison of CEC 2017 Composite Benchmark Functions

CEC2017 Composite Benchmark Functions	**hHHO-SCA**	**jSO**[121]	**DES**[122]	**TLBO-FL**[123]	**PPSO**[124]
CEC2017(F21)	263.5574	2.64E+02	2.07E+02	2.30E+02	3.05E+02
CEC2017(F22)	10661.8734	1.07E+04	1.00E+02	1.00E+02	1.00E+02
CEC2017(F23)	571.18453	5.71E+02	3.50E+02	4.00E+02	6.81E+02
CEC2017(F24)	902.60964	9.03E+02	4.18E+02	4.70E+02	7.39E+02
CEC2017(F25)	761.62547	7.62E+02	3.87E+02	4.00E+02	3.85E+02
CEC2017(F26)	3281.0964	3.28E+03	5.74E+02	1.40E+03	2.04E+03
CEC2017(F27)	586.458356	5.86E+02	5.10E+02	5.30E+02	7.08E+02
CEC2017(F28)	523.664348	5.24E+02	3.18E+02	4.30E+02	3.27E+02
CEC2017(F29)	1237.89454	1.24E+03	4.44E+02	6.20E+02	7.80E+02
CEC2017(F30)	2317.0746	2.32E+03	2.16E+03	2.60E+04	3.32E+03

Table 14: Comparison of CEC 2017 Composite Benchmark Functions for parameters variations and computational time

CEC2017 Composite Benchmark Functions	Effect of Variations of Parameters of hHHO-SCA		Computational Time (In Seconds)		
	hHHO-SCA (α=0.93)	hHHO-SCA (α=0.88)	hHHO-SCA	TLBO-FL [123]	PSO-GWO [125]
CEC2017(F11)	263.5574	263.560	1.23E+02	1.42E+02	1.57E+02
CEC2017(F12)	10661.8734	10662.000	1.12E+02	1.31E+02	1.45E+02
CEC2017(F13)	571.1845	571.190	1.24E+01	3.10E+01	4.54E+01
CEC2017(F14)	902.6096	902.610	1.36E+02	1.55E+02	1.69E+02
CEC2017(F15)	761.6255	761.630	1.69E+02	1.87E+02	2.02E+02
CEC2017(F16)	3281.0964	3281.100	1.88E+02	2.07E+02	2.21E+02
CEC2017(F17)	586.4584	586.460	1.95E+02	2.14E+02	2.28E+02
CEC2017(F18)	523.6643	523.670	1.85E+02	2.04E+02	2.18E+02
CEC2017(F19)	1237.8945	1237.900	1.75E+03	1.77E+03	1.78E+03
CEC2017(F20)	2317.0746	2317.100	1.74E+02	1.93E+02	2.07E+02

Table 15: Comparison of CEC 2018 Benchmark function for 05, 10 and 15 Objectives problems

CEC 2018 Benchmark Functions	No. of Objectives=05		No. of Objectives=10		No. of Objectives=15	
	hHHO-SCA	NSGA-III$_{TS}$ -NL[126]	hHHO-SCA	NSGA-III$_{TS}$ -NL[126]	hHHO-SCA	NSGA-III$_{TS}$ -NL[126]
CEC2018 (F01)	9.73E-04	1.00E-03	0.0011078	1.11E-03	0.0014780	1.50E-03
CEC2018 (F02)	8.72E-04	8.72E-04	0.0018785	1.88E-03	0.0027376	2.74E-03
CEC2018 (F03)	1.63E-04	1.63E-04	0.0017698	1.77E-03	0.0020386	2.04E-03
CEC2018 (F04)	2.14E-04	2.14E-04	0.0001768	1.77E-04	0.0004609	4.61E-04
CEC2018 (F05)	1.56E-04	1.56E-04	0.0003370	3.37E-04	0.0006180	6.18E-04
CEC2018 (F06)	1.30E-04	1.21E-04	0.0026696	2.67E-03	0.0037696	3.77E-03
CEC2018 (F07)	1.43E-03	1.43E-03	0.0023589	2.36E-03	0.0023780	2.38E-03
CEC2018 (F08)	4.87E-04	4.86E-04	0.0006467	6.50E-04	0.0015287	1.53E-03
CEC2018 (F09)	2.20E-04	2.11E-04	*0.0014178*	*1.42E-03*	0.0013185	1.32E-03
CEC2018 (F10)	2.24E-03	2.24E-03	0.0020657	2.07E-03	0.0027754	2.78E-03
CEC2018 (F11)	3.62E-04	3.62E-04	0.0007388	7.40E-04	0.0013075	1.31E-03
CEC2018 (F12)	4.47E-04	4.47E-04	0.0008578	8.58E-04	*0.0020796*	*2.08E-03*
CEC2018 (F13)	4.19E-04	4.19E-04	0.0005488	5.49E-04	0.0012388	1.24E-03
CEC2018 (F14)	3.49E-02	3.49E-02	0.0980867	9.81E-02	0.0680864	6.81E-02
CEC2018 (F15)	1.67E-03	1.67E-03	0.0024560	2.50E-03	0.0080765	8.10E-03

6. MULTIDISCIPLINARY ENGINEERING DESIGN OPTIMIZATION PROBLEMS

To validate the performance of suggested hHHO-SCA algorithm in the field of multidisciplinary engineering design optimization problems, eleven types of problems of engineering design are taken into consideration in which Pressure vessel problem, Three-bar truss problem, welded beam problem, Cantilever Beam Design problem, Tension/compression spring design problem, Gear Train Design problem, Speed reducer, Belleville spring, coil compression and multidisc clutch are included shown in Table 16, Table 17 and Table 18. The complete description of those types of engineering design problems has been mentioned in the following sections.

Table 16: Abbreviation and Name of Engineering Problems

Engineering Problem Abbreviation	Problem Name	Objective Function
ENGG 1	Three-bar truss problem	@SPECIAL1
ENGG 2	Pressure Vessel	@SPECIAL2
ENGG 3	Spring Design	@SPECIAL3
ENGG 4	Welded Beam	@SPECIAL4
ENGG 5	Cantilever Beam Design	@SPECIAL5
ENGG 6	Gear Train	@SPECIAL6
ENGG 7	Speed Reducer problem	@SPECIAL7
ENGG 8	Belleville Spring	@SPECIAL8
ENGG 9	Rolling Element Bearing	@SPECIAL9
ENGG 10	Multiple Disk Clutch Brake (Discrete variables)	@SPECIAL10
ENGG 11	I beam design	@SPECIAL11

Table 17: Results of Engineering design problems using **hHHO-SCA** algorithm

ENGG PROBLEM	Mean Fitness	Std	Best fitness	Worst Fitness	Median	p-Value
ENGG 1	264.1683669	0.32426697	263.8958665	264.954366	264.035028	1.7344E-06
ENGG 2	3696.691485	659.2181603	3029.873076	5053.181732	3340.706919	1.7344E-06
ENGG 3	6960.69725	381.1146136	6393.092794	7589.688312	6846.846771	1.7344E-06
ENGG 4	0.014325517	0.001551857	0.012822904	0.017773563	0.013568707	1.7344E-06
ENGG 5	2.093154146	0.332409903	1.779032249	3.146845758	1.983147741	1.7344E-06
ENGG 6	-71372.54227	15842.67551	-84491.62424	-42057.01431	-80828.62664	1.7344E-06
ENGG 7	0.449799283	0.055402317	0.389653842	0.571013688	0.43931432	1.7344E-06
ENGG 8	0	0	0	0	0	1
ENGG 9	2.29698E+22	2.6716E+22	1.98170396	5.30072E+22	2.090597201	1.56097E-06
ENGG 10	1.307608751	0.003105711	1.30412236	1.316613773	1.307056295	1.7344E-06
ENGG 11	0.006625958	1.31572E-16	0.006625958	0.006625958	0.006625958	1.57099E-06

Table 18: Simulation time for Engineering Design Problems			
Engineering Problem	**Best Time**	**Mean Time**	**Worst Time**
ENGG 1	2.390625	2.452604167	2.5625
ENGG 2	3.34375	3.4625	3.53125
ENGG 3	2.390625	2.469791667	2.625
ENGG 4	2.546875	2.618229167	2.6875
ENGG 5	2.90625	3.106770833	3.5
ENGG 6	4.265625	4.4375	5.328125
ENGG 7	3.40625	3.477604167	3.71875
ENGG 8	1.796875	1.9578125	2.1875
ENGG 9	3.53125	3.886979167	4.21875
ENGG 10	2.125	2.402604167	2.640625
ENGG 11	2.5625	2.748958333	3.03125

7.1. Three-Bar Truss Engineering Design Problem

To verify the results of the suggested hHHO-SCA algorithm for the optimization problem based on engineering design shown in Fig.7, the problem of Three-Bar Truss engineering design has taken into consideration [47] [127]. The objective of fitness value is to decrease or minimize the weight. It consists of three types of constraints, like buckling constraints, deflection constraints and stress constraints. The numerical model for the Three-Bar Truss problem are given below in equations (21) through equation (22). The comparison results are shown in Table 19.

Consider $\vec{x} = [x_1, x_2] = [A_1, A_2]$ (21)

Minimize $f(\vec{x}) = \left(2\sqrt{2}x_1 + x_2\right) * l$, (22)

Subject to $g_1(\vec{x}) = \frac{\sqrt{2}x_1 + x_2}{\sqrt{2}x_1^2 + 2x_1x_2} P - \sigma \leq 0$ (22a)

$$g_2(\vec{x}) = \frac{x_2}{\sqrt{2}x_1^2 + 2x_1x_2} P - \sigma \leq 0 \quad (22b)$$

$$g_3(\vec{x}) = \frac{1}{\sqrt{2}x_2 + x_1} P - \sigma \leq 0 \quad (22c)$$

Variable range $0 \leq x_1$, $x_2 \leq 1$

Where l = 100cm , P= 2KN/ cm^2 , σ = 2 KN/ cm^2

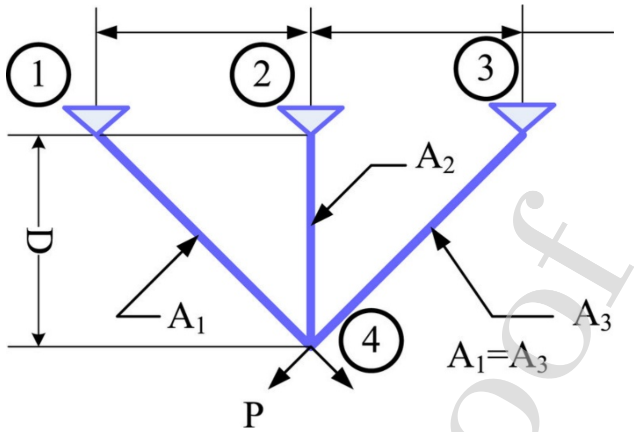


Fig.7: Three-Bar Truss Engineering Design Problem

Table 19: Comparison of Three-bar truss problem

Algorithm		**hHHO-SCA**	**CS** [128]	**Ray and Sain** [129]	**Tsa** [129]
Optimal values for variables	$x1$	**0.788498**	0.789	0.795	0.788
	$x2$	**0.40875**	0.409	0.395	0.408
Optimal weight		**263.8958665**	263.972	264.3	263.68

7.2 Pressure Vessel Engineering Design Problem

Another types of multidisciplinary engineering design problem is taken as Pressure Vessel Design problem [47] [127] shown in Fig.8. The main objective function of this engineering design problem is to decrease or minimize the total cost including the material cost, welding and forming of the vessel which is in cylindrical form, shown in Fig 11. There are four different types of variables used to design the pressure vessel problem, such as, head thickness (T_h), shell thickness (T_s), without considering head, the length of cylindrical unit (L). The both ends of the pressure vessel has capped and the head of this pressure vessel has taken as hemi spherical shape. The above mentioned engineering design problem is subjected to four types of constraints and the mathematical construction of the pressure vessel design problem are shown in equations (23) through (24d). The comparison results are shown in Table 20.

Consider,

$$\vec{x} = [x_1 x_2 x_3 x_4] = [T_s T_h R L] \tag{23}$$

Minimize $$f(\vec{x}) = 0.6224 x_1 x_3 x_4 + 1.7781 x_2 x_3^2 + 3.1661 x_1^2 x_4 + 19.84 x_1^2 x_3 \tag{24}$$

Subjected to $$g_1(\vec{x}) = -x_1 + 0.0193 x_3 \le 0 \tag{24a}$$

$$g_2(\vec{x}) = x_3 + 0.00954 x_3 \le 0 \tag{24b}$$

$$g_3(\vec{x}) = -\pi x_3^2 x_4 - \frac{4}{3}\pi x_3^3 + 1296000 \le 0 \tag{24c}$$

$$g_4(\vec{x}) = x_4 - 240 \le 0 \tag{24d}$$

Variable ranges are: $0 \le x_1 \le 99$; $0 \le x_2 \le 99$; $10 \le x_3 \le 200$; $10 \le x_4 \le 200$.

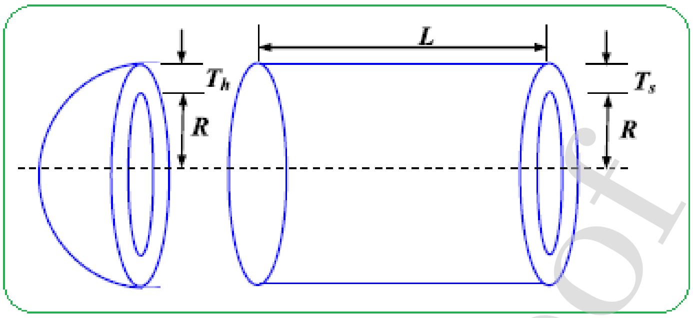


Fig.8: Pressure Vessel Engineering Design Problem

Table 20: Comparison of Pressure vessel problem

Algorithm		**hHHO-SCA**	GWO[130]	GSA[35]	PSO[131]	GA [132]	GA (Coello and Montes) [132]	GA (Deb and Gene) [133]	ES (Montes and Coello)	DE [134]	ACO [9]	Lagrangian Multiplier [135]	Branch-bound [13]
Optimum Value	*Ts*	**0.945909**	0.8125	1.125	0.8125	0.8125	0.8125	0.9375	0.8125	0.8125	0.8125	1.125	1.125
	Th	**0.447138**	0.4345	0.625	0.4375	0.4345	0.4375	0.5	0.4375	0.4375	0.4375	0.625	0.625
	R	**46.8513**	42.0892	55.9887	42.0913	40.3239	42.0974	48.329	42.0981	42.0984	42.1036	58.291	47.7
	L	**125.4684**	176.7587	84.4542	176.7465	200	176.6541	112.679	176.641	176.6377	176.5727	43.69	117.701
Optimum Cost		**6393.092794**	6051.564	8538.84	6061.078	6288.745	6059.946	6410.381	6059.75	6059.734	6059.089	7198.043	8129.1

7.3 The Tension/Compression Spring Engineering Design Problem

The Tension/Compression Spring Design optimization Problem is the part of multidisciplinary engineering optimization problem shown in Fig.9, which is a portion of mechanical engineering problem [47] [127]. The main objective of this type of problem is to decrease or minimize the spring weight. To get solution of this kind of problem the Tension/Compression Spring Design problem needs three types of design variables, such as, the diameter of the wire (d), the diameter of the mean coil (D) and the active coils number (N). The design problem is subjected to constraints depends on surge frequency, minimum deflection and constraints based on shear stress. The mathematical model of this type of engineering design problem is shown in the equations (25) through (26e). The comparison results are shown in Table 21.

Consider

$$\vec{x} = [x_1 x_2 x_3] = [dDN] \tag{25}$$

Minimize

$$f(\vec{x}) = (x_3 + 2) x_2 x_1^2 \tag{26}$$

Subjected to

$$g_1(\vec{x}) = 1 - \frac{x_2^3 x_3}{71785 x_1^4} \le 0 \, , \tag{26a}$$

$$g_2(\vec{x}) = \frac{4x_2^2 - x_1x_2}{12566\left(x_2x_1^3 - x_1^4\right)} + \frac{1}{5108x_1^2} \le 0 \tag{26b}$$

$$g_2(\vec{x}) = \frac{4x_2^2 - x_1x_2}{12566\left(x_2x_1^3 - x_1^4\right)} + \frac{1}{5108x_1^2} \le 0 \tag{26c}$$

$$g_3(\vec{x}) = 1 - \frac{140.45x_1}{x_2^2x_3} \le 0 \tag{26d}$$

$$g_4(\vec{x}) = \frac{x_1 + x_2}{1.5} - 1 \le 0 \tag{26e}$$

Variable range are: $0.005 \le x_1 \le 2.00$; $0.25 \le x_2 \le 1.30$; $2.00 \le x_3 \le 15.0$

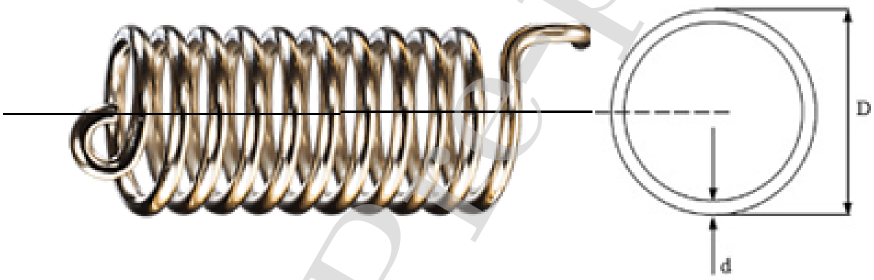


Fig.9: The Tension/Compression Spring Engineering Design Problem

Table 21: Comparison of Tension/compression spring engineering design problem

Algorithm		hHHO-SCA	GWO	GSA	PSO (Ha and Wang)	ES (Coello and Montes)	GA (Coello)	HS (Mahdavi et al.)	DE (Huang et al.)	Mathematical optimization	Constraint correction
AGASD M RW Optimum Variables	d	**0.054693**	0.0516	0.0503	0.0517	0.052	0.0515	0.0512	0.0516	0.0534	0.05
	D	**0.433378**	0.3567	0.3237	0.3576	0.364	0.3517	0.3499	0.3547	0.3992	0.3159
	N	**7.891402**	11.2889	13.5254	11.2445	10.8905	11.6322	12.0764	11.4108	9.1854	14.25
Optimum weight		**0.012822904**	0.01267	0.0127	0.01267	0.01268	0.0127	0.01267	0.01267	0.01273	0.01283

7.4 Welded Beam Engineering Design Problem

The Welded Beam Design Problem is another most important engineering design problem shown in Fig 10, which is taken into consideration [47] [127]. The main objective of this types of design problem is to decrease or minimize the cost of the fabrication of the welded beam which consists of four types of variables, such as, bar thickness (b), bar length including attached part (l), weld thickness (h) and the bar height (h). This problem is subjected to four types of constraints including Buckling constraints of bar (Pc), Side constraints, End deflection of beam (d), Bending stress of the beam (h) and stress of shear (s). The mathematical description of the above mentioned engineering design problem is discussed in the following equations (27) through (29f). The comparison results are shown in Table 22.

Consider

$$\vec{x} = [x_1 x_2 x_3 x_4] = [hltb] \tag{27}$$

Minimize

$$f(\vec{x}) = 1.10471 x_1^2 x_2 + 0.04811 x_3 x_4 (14.0 + x_2) \tag{28}$$

Subject to

$$g_1(\vec{x}) = \tau(\vec{x}) - \tau_{\max} \le 0, \tag{28a}$$

$$g_2(\vec{x}) = \sigma(\vec{x}) - \sigma_{\max} \le 0 \tag{28b}$$

$$g_3(\vec{x}) = \delta(\vec{x}) - \delta_{\max} \le 0 \tag{28c}$$

$$g_4(\vec{x}) = x_1 - x_4 \le 0 \tag{28d}$$

$$g_5(\vec{x}) = P - P_c(\vec{x}) \le 0 \tag{28e}$$

$$g_6(\vec{x}) = 0.125 - x_1 \le 0 \tag{28f}$$

$$g_7(\vec{x}) = 1.10471 x_1^2 + 0.04811 x_3 x_4 (14.0 + x_2) - 5.0 \le 0 \tag{28g}$$

Variable range $0.1 \le x_1 \le 2$; $0.1 \le x_2 \le 10$; $0.1 \le x_3 \le 10$; $0.1 \le x_4 \le 2$

Where

$$\tau(\vec{x}) = \sqrt{(\tau^{/})^2 + 2\tau^{/}\tau^{//}\frac{x_2}{2R} + (\tau^{//})^2}, \tag{29a}$$

$$\tau^{/} = \frac{P}{\sqrt{2}x_1 x_2}, \tau^{//} = \frac{MR}{J}, M = P\left(L + \frac{x_2}{2}\right), \tag{29b}$$

$$R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2}, \tag{29c}$$

$$J = 2\left\{\sqrt{2}x_1x_2\left[\frac{x_2^2}{4}+\left(\frac{x_1+x_3}{2}\right)^2\right]\right\}, \tag{29d}$$

$$\sigma(\vec{x}) = \frac{6PL}{x_4x_3^2}, \delta(\vec{x}) = \frac{6PL^3}{Ex_2^2x_4}, \tag{29e}$$

$$P_c(\vec{x}) = \frac{4.013E\frac{\sqrt{x_3^2x_4^6}}{36}}{L^2}\left(1-\frac{x_3}{2L}\sqrt{\frac{E}{4G}}\right), \tag{29f}$$

$$P = 6000lb, L = 14in, \delta_{\max} = 0.25in, E = 30\times1^6\, psi, G = 12\times10^6\, psi, \tau_{\max} = 13600\, psi, \sigma_{\max} = 3000\, psi$$

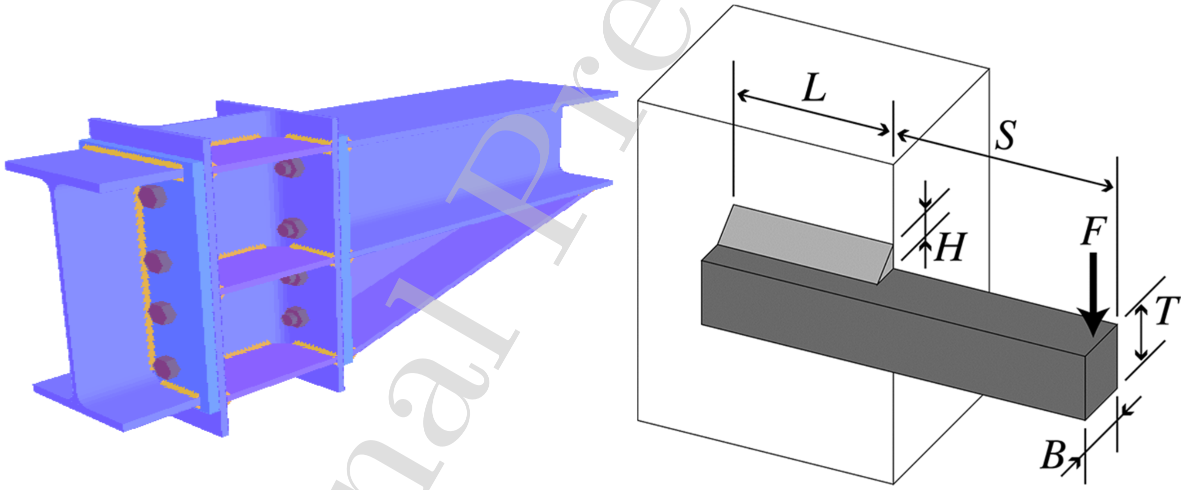


Fig.10: Welded Beam Engineering Design Problem

Table 22: Comparison of Welded beam problem

Algorithm		hHHO-SCA	GWO	GSA	GA (Coello)	GA (Deb)	GA (Deb)	HS (Lee and Geem)	Random	Simplex	David	APPROX
Optimum Variables	h	**0.190086**	0.2057	0.1821	N/A	N/A	0.2489	0.2442	0.4575	0.2792	0.2434	0.2444
	l	**3.696496**	3.4784	3.857	N/A	N/A	6.173	6.2231	4.7313	5.6256	6.2552	6.2189
	t	**9.386343**	9.0368	10	N/A	N/A	8.1789	8.2915	5.0853	7.7512	8.2915	8.2915
	b	**0.204157**	0.2058	0.2024	N/A	N/A	0.2533	0.2443	0.66	0.2796	0.2444	0.2444
Optimal Cost		**1.779032249**	1.7262	1.88	1.8245	2.38	2.4331	2.3807	4.1185	2.5307	2.3841	2.3815

7.5 Cantilever Beam Engineering Design Problem

Another engineering optimization problem is a part of civil engineering which is named as Cantilever Bream Design problem shown in Fig 11. This problem consists of five types of hollow element including its square-shaped cross section [127] . The main objective of this type of optimization is to decrease or reduce the weight of the beam shown in equation number (29). This kind of engineering design problem each elements can defined by one variable and the structure of the overall design consists of five types of structural parameters and where the beam thickness is taken constant. In this suggested design, the load due to vertical end is applied to the free end position of the beam (6^{th} node) and the side of the right portion of the beam (1^{st} node) which is supported rigidly. The comparison results are shown in Table 23. In the final design for optimal solution this types of cantilever beam design problem the displacement of vertical constraint must take into consideration which should not violate for design of final optimal solution shown on equation no (30) to equation (31). The mathematical formula is given below:

The problem formulation is as follows:

Consider $\vec{x} = [x_1 x_2 x_3 x_4 x_5]$,

Minimize $f(\vec{x}) = 0.6224(x_1 + x_2 + x_3 + x_4 + x_5),$ (30)

Subject to

$$g(\vec{x}) = \frac{61}{x_1^3} + \frac{37}{x_2^3} + \frac{19}{x_3^3} + \frac{7}{x_4^3} + \frac{1}{x_5^3} \le 1 \quad (31)$$

Variable range $0.01 \le x_1, x_2, x_3, x_4, x_5 \le 100$

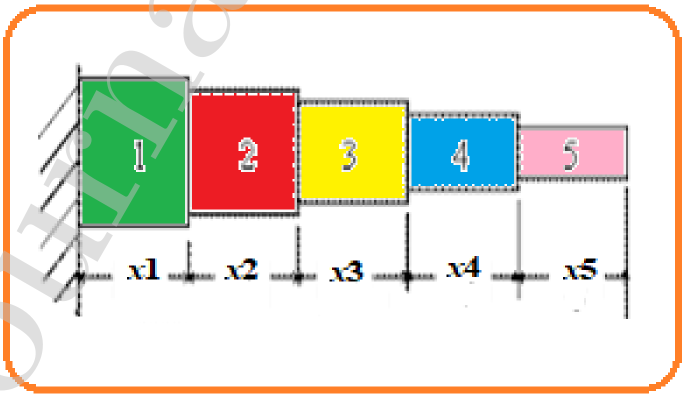


Fig.11: Cantilever Beam Engineering Design Problem

Table 23: Comparison of Cantilever Beam Engineering Design problem

Algorithm		hHHO-SCA	ALO[136]	SOS[62]	CS [128]	MMA[137]	GCA_I [137]	GCA_II [137]
Optimal values for variables	$x1$	**5.937725**	6.0181	6.0188	6.0089	6.01	6.01	6.01
	$x2$	**4.85041**	5.3114	5.3034	5.3049	5.3	5.304	5.3
	$x3$	**4.622404**	4.4884	4.4959	4.5023	4.49	4.49	4.49
	$x4$	**3.45347**	3.4975	3.499	3.5077	3.49	3.498	3.49
	$x5$	**2.089114**	2.1583	2.1556	2.1504	2.15	2.15	2.15
Optimum weight		**1.30412236**	1.33995	1.33996	1.33999	1.34	1.34	1.34

7.6 Gear Train Engineering Design Problem

Another optimization types of engineering design optimization problem is named as Gear Train Design problem, which have four types of parameters shown in the Fig.12 [47]. The actual objective of this type of design problem is that to minimize the scalar value and the teeth ratio of the gear. Thus the decision variable is included by the number of teeth on each of the gear. The comparison results are shown in Table 24. The mathematical model is given below:

Let, considering; $\vec{g} = [g_1 g_2 g_3 g_4] = [M_A M_B M_C M_D]$ (32)

Minimizing; $f(\vec{g}) = \left(\frac{1}{6.931} - \frac{g_3 g_4}{g_1 g_2}\right)^2$ (33)

Subjected to; $12 \le g_1, g_2, g_3, g_4 \le 60$

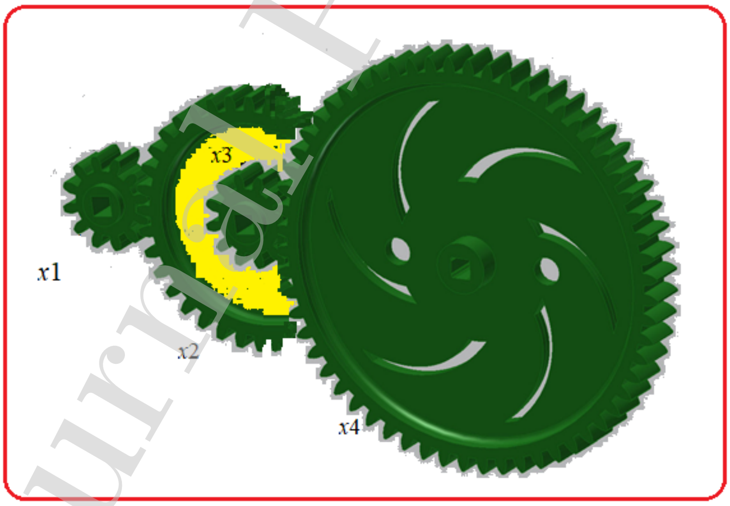


Fig.12: Gear Train engineering design Problem

Table 24: Comparison of Gear Train engineering Design problem

Algorithm		hHHO-SCA	GeneAS [133]	Kannan and Kramer [133]	Sandgren [133]
Optimal values for variables	$x1$	**58.22844**	50	41	60
	$x2$	**57.73199**	33	33	45
	$x3$	**40.41797**	14	15	22
	$x4$	**12**	17	13	18
Optimum fitness		**0**	0.144242	0.144124	0.146667

7.7 Speed Reducer Engineering Design Problem

Another most important engineering design problem is speed reducer, shown in Fig.13 [47]. Speed reducer is consists of the width of face s_1, teeth module s_2, number of the pinion teeth s_3, in between the bearings of the first shaft length s_4, in between the bearings of the second shaft length s_5, the first shaft diameter s_6 and second shaft diameter s_7; all types of variables are continuous in nature except one variable s_3, which is an integer. The main objective is to decrease or reduce the weight of the speed reducer which is subjected to some constraints which is depended on surface stress and bending stress of the teeth of the gear, shaft's transverse movement or deflection and stress of it on the shaft. The comparison results are shown in Table 25. The mathematical model is given below:

Minimizing;

$$f(\vec{s}) = 0.7854 s_1 s_2 (3.3333 s_3^2 + 14.9334 s_3 - 43.0934) - 1.508 s_1 (s_6^2 + s_7^2) + 7.4777 (s_6^3 + s_7^3) + 0.7854 (s_4 s_6^2 + s_5 s_7^2)$$

Subjected to; $g_1(\vec{s}) = \dfrac{27}{s_1 s_2^2 s_3} - 1 \le 0$ (34)

$$g_2(\vec{s}) = \frac{397.5}{s_1 s_2^2 s_3^2} - 1 \le 0 \quad (34a)$$

$$g_3(\vec{s}) = \frac{1.93 s_4^3}{s_2 s_3 s_6^4} - 1 \le 0 \quad (34b)$$

$$g_4(\vec{s}) = \frac{1.93 s_5^3}{s_2 s_3 s_7^4} - 1 \le 0 \quad (34c)$$

$$g_5(\vec{s}) = \frac{1}{110 s_6^3} \sqrt{\left(\frac{745.0 s_4}{s_2 s_3}\right)^2 + 16.9 \times 10^6} - 1 \le 0 \quad (34d)$$

$$g_6(\vec{s}) = \frac{1}{85 s_7^3} \sqrt{\left(\frac{745.0 s_5}{s_2 s_3}\right)^2 + 157.5 \times 10^6} - 1 \le 0 \quad (34e)$$

$$g_7(\vec{s}) = \frac{s_2 s_3}{40} - 1 \le 0 \quad (34f)$$

$$g_8(\vec{s}) = \frac{5 s_2}{s_1} - 1 \le 0 \quad (34g)$$

$$g_9(\vec{s}) = \frac{s_1}{12 s_2} - 1 \le 0 \quad (34h)$$

$$g_{10}(\vec{s}) = \frac{1.5 s_6 + 1.9}{12 s_2} - 1 \le 0 \quad (34i)$$

$$g_{11}(\vec{s}) = \frac{1.1 s_7 + 1.9}{s_5} - 1 \le 0 \quad (34j)$$

Where $2.6 \le s_1 \le 3.6, 0.7 \le s_2 \le 0.8, 17 \le s_3 \le 28, 7.3 \le s_4 \le 8.3, 7.8 \le s_5 \le 8.3, 2.9 \le s_6 \le 3.9 \, and \, 5 \le s_7 \le 5.5$

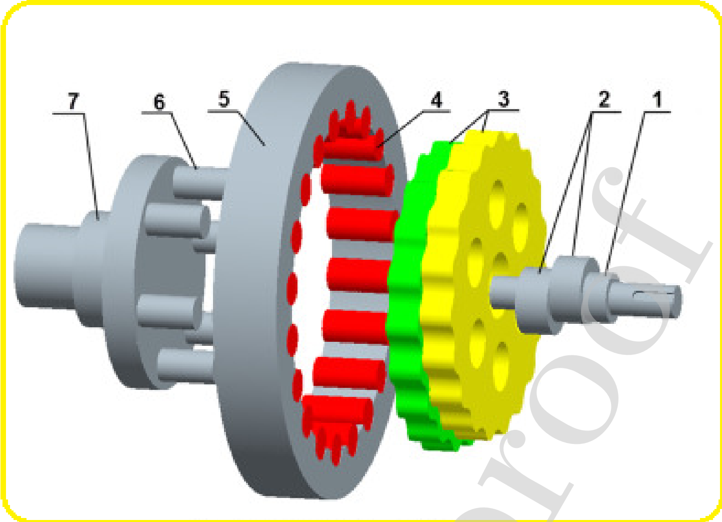


Fig.13. Speed reducer engineering design problem

Table 25: Comparison of speed reducer engineering design problem

Algorithm		hHHO-SCA	HEAA	MDE	PSO-DE	MBA
Optimal values for variables	$x1$	**3.506119**	3.500022	3.50001	3.50	3.5
	$x2$	**0.7**	0.70000039	0.7	0.7	0.7
	$x3$	**17**	17.000012	17	17	17
	$x4$	**7.3**	7.300427	7.300156	7.3	7.300033
	$x5$	**7.99141**	7.715377	7.800027	7.8	7.715772
	$x6$	**3.452569**	3.35023	3.350221	3.350214	3.350218
	$x7$	**5.286749**	5.286663	5.286685	5.2866832	5.286654
Optimum fitness		**3029.873076**	2994.49911	2996.35669	2996.34817	2994.48245

7.8 Belleville Spring Engineering Design Problem

Another types of engineering problem is Belleville spring design problem shown in Fig.14 [47]. It is a minimization of the problem through which one existing parameter in the constraints is accordingly to be selected to the designed variable ratios. The main objective function is to design Belleville spring which having the minimum weight and it should satisfy number of constraints. This type of designed problem has four types of designed variables, such as, diameter of the internal part of the Belleville spring (DIM_I), diameter of the external part of the Belleville spring (DIM_E), Spring height (S_H) and The thickness of the Belleville spring (S_T). The comparison results are shown in Table 26. The subjected constraints are concerned the deflection, compressive type of stresses, deflection height, diameter of outer and inner portion and the slope. The mathematical expressions are given below:

Minimizing; $f(x) = 0.07075\pi(DIM_E^2 - DIM_I^2)t$ (35)

Subjected to; $b_1(x) = G - \dfrac{4P\lambda_{\max}}{(1-\delta^2)\alpha DIM_E}\left[\delta(S_H - \dfrac{\lambda \max}{2}) + \mu t\right] \geq 0$ (36)

$$b_2(x) = (\frac{4P\lambda_{\max}}{(1-\delta^2)\alpha DIM_E}\left[(S_H - \frac{\lambda}{2})(S_H - \lambda)t + t^3\right])_{\lambda_{\max}} - P_{MAX} \geq 0 \quad (36a)$$

$$b_3(x) = \lambda_1 - \lambda_{\max} \geq 0 \quad (36b)$$

$$b_4(x) = H - S_H - t \geq 0 \quad (36c)$$

$$b_5(x) = DIM_{MAX} - DIM_E \geq 0 \quad (36d)$$

$$b_6(x) = DIM_E - DIM_I \geq 0 \quad (36e)$$

$$b_7(x) = 0.3 - \frac{S_H}{DIM_E - DIM_I} \geq 0 \quad (36f)$$

Where, $\alpha = \frac{6}{\pi \ln J}(\frac{J-1}{J})^2$

$$\delta = \frac{6}{\pi \ln J}(\frac{J-1}{\ln J} - 1)$$

$$\mu = \frac{6}{\pi \ln J}(\frac{J-1}{2})$$

$$P_{MAX} = 5400lb$$

$$P = 30e6psi, \lambda_{\max} = 0.2in, \delta = 0.3, G = 200Kpsi, H = 2in, DIM_{MAX} = 12.01in, J = \frac{DIM_E}{DIM_I}, \lambda_1 = f(a)a, a = \frac{S_H}{t}$$

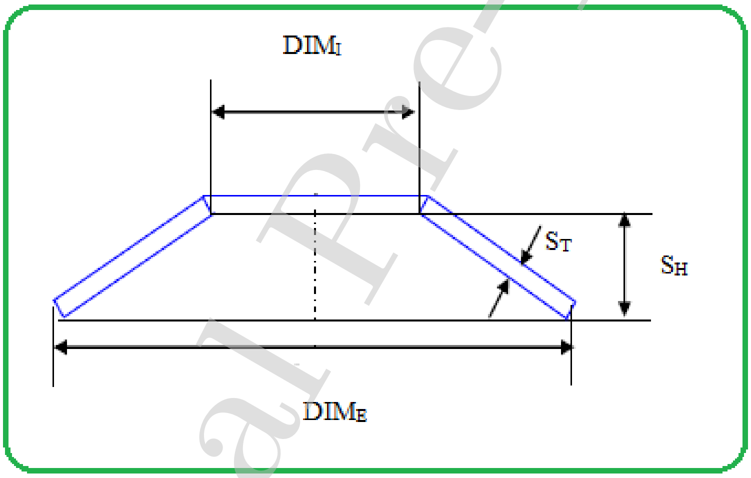


Fig.14. Belleville Spring Engineering Design

Table 26: Comparison of Belleville spring engineering design problem				
Algorithm	**hHHO-SCA**		**TLBO**	**MBA**
Optimal values for variables	$x1$	**11.98603**	12.01	12.01
	$x2$	**10.0002**	10.03047	10.030473
	$x3$	**0.204206**	0.204143	0.204143
	$x4$	**0.2**	0.2	0.2
Optimum fitness	**1.98170396**		0.198966	0.198965

7.9 Rolling Element Bearing Engineering Design Problem

Rolling element bearing design problem is another most important part of multidisciplinary engineering design problem shown in Fig. 15 [47]. The main objective of this type of engineering design problem is to increase the dynamic nature of load carrying capacity of the bearing of the rolling element as shown in the fig.18. This engineering design problem includes ten numbers of decision variable such as, diameter of the pitch (DIM_P), diameter of the ball (DIM_B), ball numbers (N), coefficient of raceway curvature for outer and inner part (R_I and R_O) etc. The five latter variables are only appeared in constraints and affect indirectly to the internal portion of the geometry. The design variable of number of the rolling balls (N) is discrete in nature and the remainder design variables are continuous in nature. In this types of design problem constraints are executed based on manufacturing and kinematic conditions.

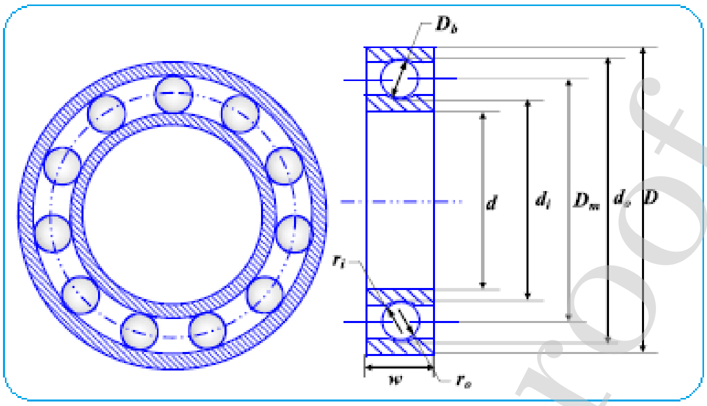


Fig.15. Rolling element bearing engineering design problem

The mathematical model is given below:

Maximizing;

$$C_D = f_c N^{2/3} DIM_B^{1.8} \text{ if } DIM \le 25.4mm \tag{37a}$$

$$C_D = 3.647 f_C N^{2/3} DIM_B^{1.4} \text{ if } DIM \ge 25.4mm \tag{37b}$$

Subjected to; $$r_1(x) = \frac{\theta_0}{2\sin^{-1}\left(\dfrac{DIM_B}{DIM_{MAX}}\right)} - N + 1 \ge 0 \tag{38}$$

$$r_2(x) = 2DIM_B - K_{DIM_{MIN}}(DIM - \dim) \ge 0 \tag{38a}$$

$$r_3(x) = K_{DIM_{MAX}}(DIM - \dim) \ge 0 \tag{38b}$$

$$r_4(x) = \beta B_W - DIM_B \le 0 \tag{38c}$$

$$r_4(x) = DIM_{MAX} - 0.5(DIM + \dim) \ge 0 \tag{38d}$$

$$r_5(x) = DIM_{MAX} - 0.5(DIM + \dim) \ge 0 \tag{38e}$$

$$r_6(x) = (0.5 + re)(DIM + \dim) \ge 0 \tag{38f}$$

$$r_7(x) = 0.5(DIM - DIM_{MAX} - DIM_B) - \alpha DIM_B \ge 0 \tag{38g}$$

$$r_8(x) = f_I \ge 0.515 \tag{38h}$$

$$r_9(x) = f_0 \ge 0.515 \tag{38i}$$

where, $$f_c = 37.91\left[1 + \left\{1.04\left(\frac{1-\varepsilon}{1+\varepsilon}\right)^{1.72}\left(\frac{f_I(2f_0 - 1)}{f_0(2f_I - 1)}\right)^{0.41}\right\}^{10/3}\right]^{-0.3} \times \left[\frac{\varepsilon^{0.3}(1-\varepsilon)^{1.39}}{(1+\varepsilon)^{1/3}}\right]\left[\frac{2f_I}{2f_I - 1}\right]^{0.41}$$

$$\theta_0 = 2\pi - 2\cos^{-1}\left(\frac{\left[\{(DIM - \dim)/2 - 3(t/4)\}^2 + \left(DIM/2 - t/4 - DIM_B\right)^2 - \{\dim/2 + t/4\}^2\right]}{2\{(DIM - \dim)/2 - 3(t/4)\}\{D/2 - t/4 - DIM_B\}}\right)$$

$$\varepsilon = \frac{DIM_B}{DIM_{MAX}}, f_I = \frac{R_I}{DIM_B}, f_0 = \frac{R_0}{DIM_B}, t = DIM - \dim - 2DIM_B$$

$DIM = 160, \dim = 90, B_W = 30, R_I = R_0 = 11.033$

$0.5(DIM + \dim) \le DIM_{MAX} \le 0.6(DIM + \dim), 0.15(DIM - \dim) \le DIM_B \le 0.45(DIM - \dim), 4 \le N \le 50$

$0.515 \le f_I$ and $f_0 \le 0.6$

$0.4 \le K_{DIM_{MIN}} \le 0.5, 0.6 \le K_{DIM_{MAX}} \le 0.7, 0.3 \le re \le 0.1, 0.02 \le re \le 0.1, 0.6 \le \beta \le 0.85$

7.10 Multidisc clutch brake engineering design problem

One of the most important engineering problem is Multidisc clutch brake design problem shown in Fig. 16 [138]. The main object of this type of optimization problem is to minimize or decrease the weight and it consists of five numbers of discrete variables, such as, radius of the inner surface (R_{in}), radius of the outer surface (R_o), disc's thickness (Th), actuating type of force (F_{ac}) and number of the surface of friction (S_f).

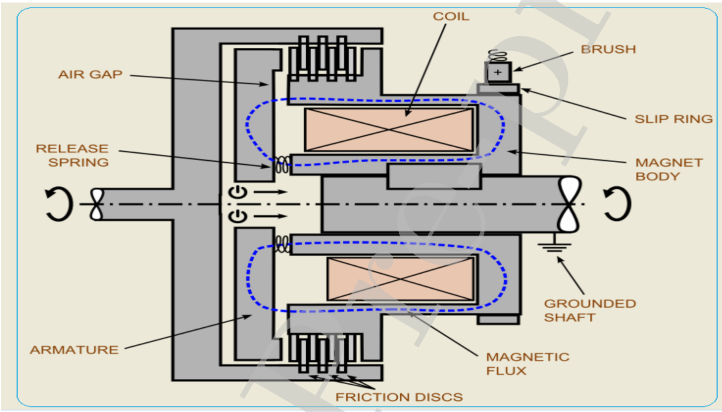


Fig.16. Multidisc clutch break design

The mathematical formulation for this engineering optimization problem is given below;

Minimizing ; $f(R_{in}, R_O, S_f, Th) = \pi Th\gamma (R_0^2 - R_{in}^2)(S_f + 1)$ (39)

Where,

$R_{in} \in 60, 61, 62....80; R_o \in 90, 91, ...110; Th \in 1, 1.5, 2, 2.5, 3; F_{ac} \in 600, 610, 620, 1000; S_f \in 2, 3, 4, 5, 6, 7, 8, 9$

Subjected to; $m_1 = R_0 - R_{in} - \Delta R \ge 0$ (40)

$m_2 = L_{MAX} - (S_f + 1)(Th + \alpha) \ge 0$ (40a)

$m_3 = PM_{MAX} - PM_\pi \ge 0$ (40b)

$m_4 = PM_{MAX} Y_{MAX} + PM_\pi Y_{SR} \ge 0$ (40c)

$m_5 = Y_{SR_{MAX}} - Y_{SR} \ge 0$ (40d)

$m_6 = t_{MAX} - t \ge 0$ (40e)

$m_7 = DC_h - DC_f \ge 0$ (40f)

$m_8 = t \ge 0$ (40g)

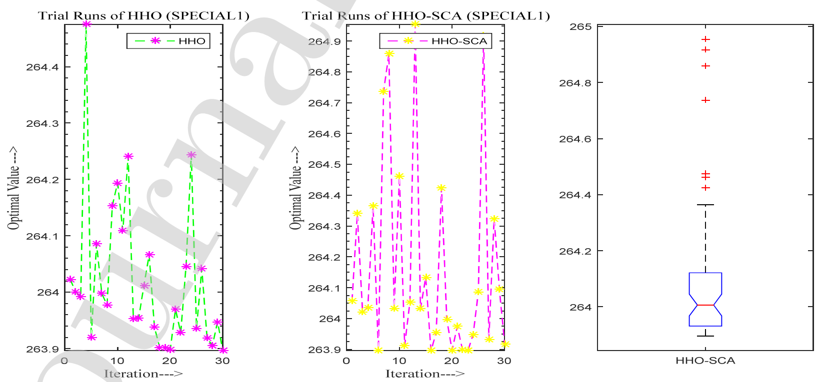
Where, $PM_\pi = \dfrac{F_{ac}}{\Pi(R_0^2 - R_{in}^2)}$

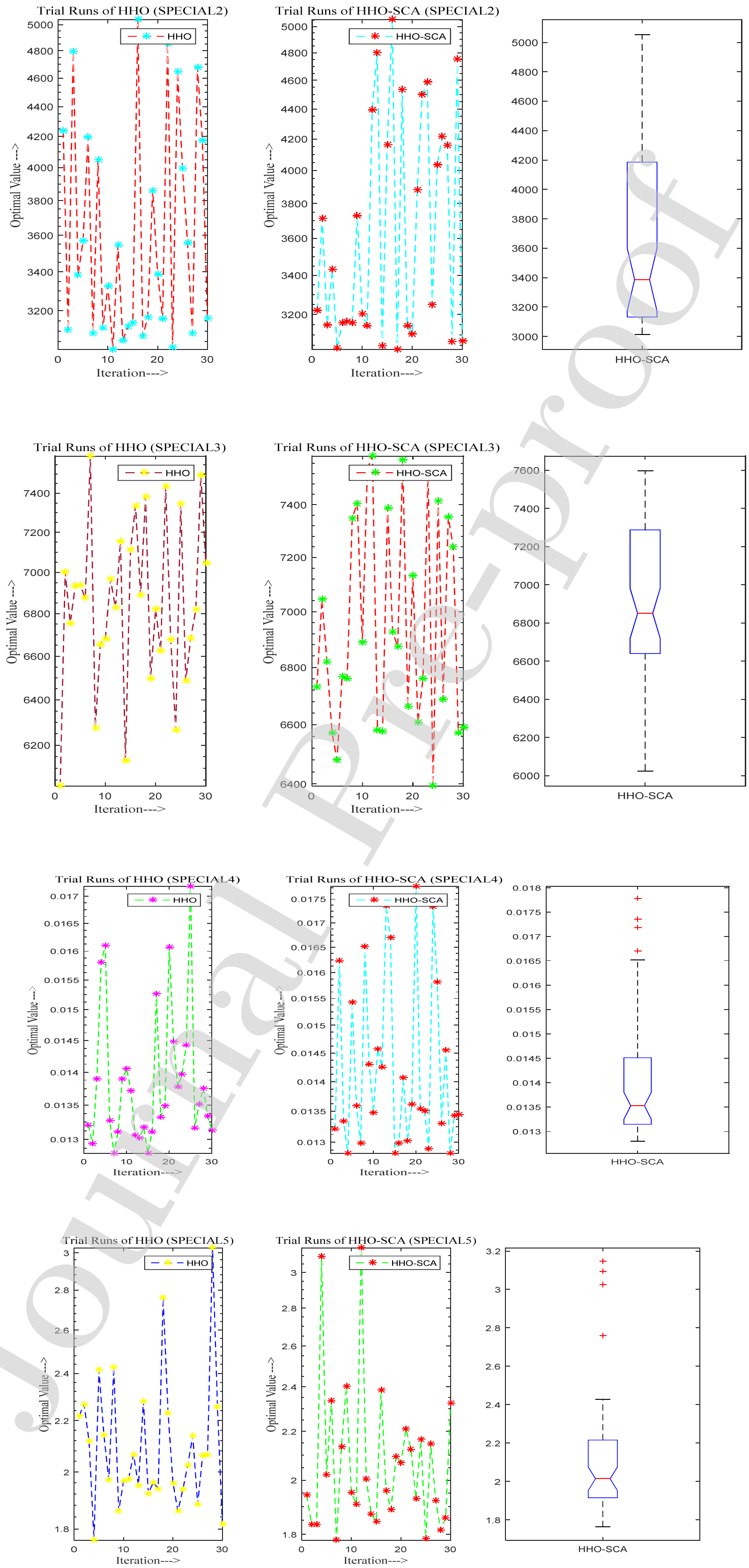
$$Y_{SR} = \frac{2\pi n\left(R_0^3 - R_{in}^3\right)}{90\left(R_0^2 - R_{in}^2\right)}$$

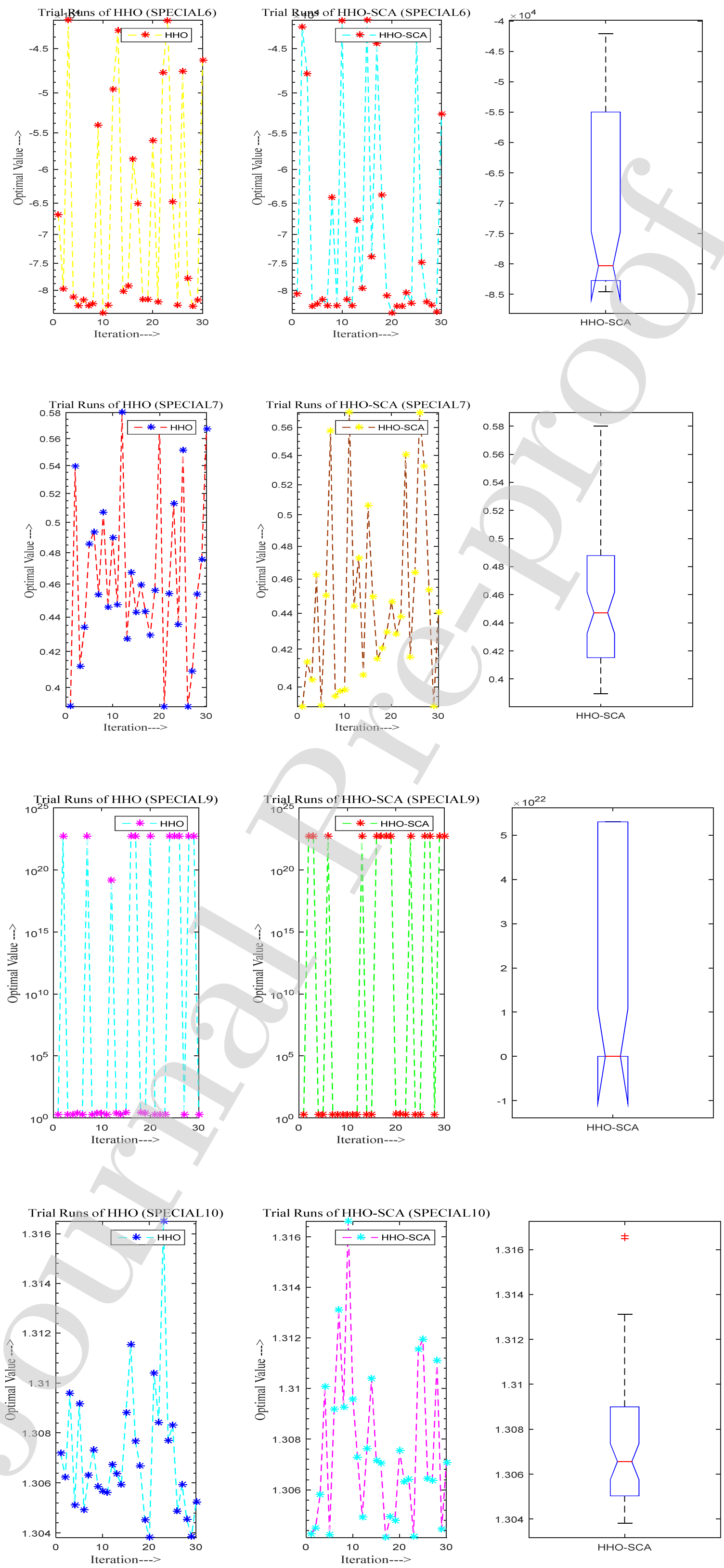
$$t = \frac{i_x \pi n}{30\left(DC_h + DC_f\right)}$$

Table 27: Comparison of Multiple disc clutch brake problem					
Algorithm		**hHHO-SCA**	**NSGA-II**	**TLBO**	**AMDE**
Optimal values for variables	$x1$	**70**	70	70	70.00
	$x2$	**90**	90	90	90
	$x3$	**2.312785**	3	3	3
	$x4$	**1.5**	1.5	1	1
	$x5$	**1000**	1000	810	810
Optimum fitness		**0.389653842**	0.4704	0.3136566	0.3136566

For the active analysis of the effect of results of hHHO-SCA are estimated for trial runs 30 and statistical hypothesis test which is non-parametric in nature i.e. Wilcoxon rank sum test which has been applied. The main consequence of that test is to observe the samples' distribution, that means weather the two types of dependent samples are selected from the number of populations which having the same number of distribution or not. The outputs of the multi-disciplinary engineering design problems are verified with the respect of the best value, worst value, p-value and standard deviation, also shown the variations of best time and worst time. The comparison of the results are also shown in the above Table 27. The results of the multi-disciplinary engineering design problems are the strongly indication of the qualities of the hHHO-SCA algorithm to solve the problems with the unknown types of search space. The output of the proposed optimization algorithm also shows the effectiveness of the problems including discrete and continuous types. The trial runs with box plot of the engineering optimization problems are shown in the Fig.17, which shows the lead of hHHO-SCA over HHO.




Trial Runs of HHO (SPECIAL2)
HHO
Trial Runs of HHO-SCA (SPECIAL2)
HHO-SCA
Trial Runs of HHO (SPECIAL3)
HHO
Trial Runs of HHO-SCA (SPECIAL3)
HHO-SCA
Trial Runs of HHO (SPECIAL4)
HHO
Trial Runs of HHO-SCA (SPECIAL4)
HHO-SCA
Trial Runs of HHO (SPECIAL5)
HHO
Trial Runs of HHO-SCA (SPECIAL5)
HHO-SCA
Optimal Value --->
Iteration--->
HHO-SCA



Trial Runs of HHO (SPECIAL6)
HHO
Optimal Value --->
Iteration--->
Trial Runs of HHO-SCA (SPECIAL6)
HHO-SCA
Optimal Value --->
Iteration--->
× 10^4
HHO-SCA
Trial Runs of HHO (SPECIAL7)
HHO
Optimal Value --->
Iteration--->
Trial Runs of HHO-SCA (SPECIAL7)
HHO-SCA
Optimal Value --->
Iteration--->
HHO-SCA
Trial Runs of HHO (SPECIAL9)
HHO
Optimal Value --->
Iteration--->
Trial Runs of HHO-SCA (SPECIAL9)
HHO-SCA
Optimal Value --->
Iteration--->
× 10^22
HHO-SCA
Trial Runs of HHO (SPECIAL10)
HHO
Optimal Value --->
Iteration--->
Trial Runs of HHO-SCA (SPECIAL10)
HHO-SCA
Optimal Value --->
Iteration--->
HHO-SCA

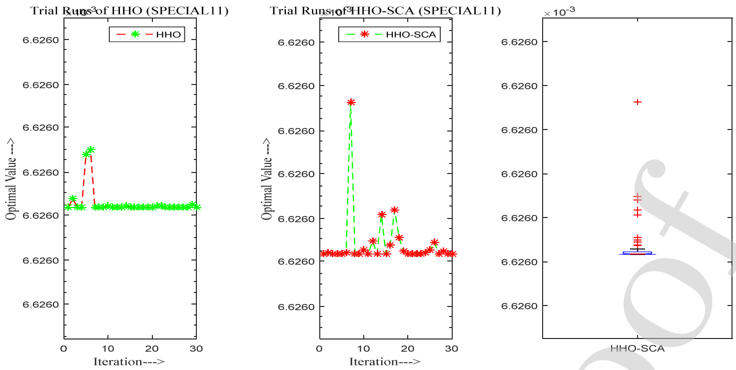


Fig.17. Trial runs with box plot for multidisciplinary engineering design problems

CONCLUSION

In the proposed research, the phase of exploitation of existing Harris Hawks Optimization has been upgraded successfully using Sine-Cosine algorithm and the developed hybrid hHHO-SCA has been successfully tested for continuous, discrete, highly constrained, nonlinear and non-convex engineering design and optimization problems include several benchmark function and engineering design problems. Experimentally, it has been found that the minimum weight for three-bar truss engineering design problem is 263.8958665, optimal cost for Pressure Vessel Design problem is 6393.092794, optimal weight for Tension/Compression Spring Design optimization Problem is 0.012822904, fabrication cost of welded beam of welded beam design problem is 1.779032249. Also, the experimental results shows that the reduced weight of Cantilever Bream Design problem and multidisc clutch brake design problem is 1.30412236 and 0.389653842 respectively. From the comparative analysis, it has been experimentally observed that the outcome of the suggested hybrid hHHO-SCA algorithm are superior than the existing Harris Hawks Optimization algorithm, sine-cosine optimizer algorithm, Ant Lion Optimization algorithm, Moth Flame Optimizer algorithm and other recently proposed meta heuristic, evolutionary, nature inspired and heuristic search algorithms and thus the proposed algorithm approves its efficiency in the research field of the meta heuristic algorithm inspired by nature for various types of optimization problems including multi-disciplinary engineering design optimization problems.

REFERENCES

[1] R. Abbassi, A. Abbassi, A. Asghar, and S. Mirjalili, "An efficient salp swarm-inspired algorithm for parameters identification of photovoltaic cell models," *Energy Convers. Manag.*, vol. 179, no. August 2018, pp. 362–372, 2019.

[2] H. Faris *et al.*, "An intelligent system for spam detection and identification of the most relevant features based on evolutionary Random Weight Networks," *Inf. Fusion*, vol. 48, pp. 67–83, 2019.

[3] J. F. McCarthy, "Block-conjugate-gradient method," *Phys. Rev. D*, vol. 40, no. 6, pp. 2149–2152, 1989.

[4] G. Wu, "Across neighborhood search for numerical optimization," *Inf. Sci. (Ny).*, vol. 329, no. 61563016, pp. 597–618, 2016.

[5] G. Wu, W. Pedrycz, P. N. Suganthan, and R. Mallipeddi, "A variable reduction strategy for evolutionary algorithms handling equality constraints," *Appl. Soft Comput. J.*, vol. 37, pp. 774–786, 2015.

[6] A. A. Heidari, R. Ali Abbaspour, and A. Rezaee Jordehi, "An efficient chaotic water cycle algorithm for optimization tasks," *Neural Comput. Appl.*, vol. 28, no. 1, pp. 57–85, 2017.

[7] M. Mafarja *et al.*, "Evolutionary Population Dynamics and Grasshopper Optimization approaches for feature selection problems," *Knowledge-Based Syst.*, vol. 145, pp. 25–45, 2018.

[8] A. A. Heidari, S. Mirjalili, H. Faris, I. Aljarah, M. Mafarja, and H. Chen, "Harris hawks optimization: Algorithm and applications," *Futur. Gener. Comput. Syst.*, 2019.

[9] M. Dorigo, M. Birattari, and T. Stutzle, "Ant colony optimization," *IEEE Comput. Intell. Mag.*, vol. 1, no. 4, pp. 28–39, 2006.

[10] S. Mirjalili and A. Lewis, "Adaptive gbest-guided gravitational search algorithm," *Neural Comput. Appl.*, vol. 25, no. 7–8, pp. 1569–1584, 2014.

[11] S. Mirjalili, "The ant lion optimizer," *Adv. Eng. Softw.*, vol. 83, pp. 80–98, 2015.

[12] C. Anitescu, E. Atroshchenko, N. Alajlan, and T. Rabczuk, "Artificial Neural Network Methods for the Solution of Second Order Boundary Value Problems," vol. 59, no. 1, pp. 345–359, 2019.

[13] A. I. Cohen and M. Yoshimura, "A Branch-and-Bound Algorithm for Unit Commitment," *IEEE Trans. Power Appar. Syst.*, vol. 102, no. 2, pp. 444–451, 1983.

[14] A. Yang X-s., "new metaheuristic bat-inspired algorithm," in *Nature inspired cooperative strategies for optimization (NICSO 2010)*, ; p. 65-74: Springer, 2010.

[15] R. Y. M. Nakamura, L. A. M. Pereira, K. A. Costa, D. Rodrigues, J. P. Papa, and X. S. Yang, "BBA: A binary bat algorithm for feature selection," *Brazilian Symp. Comput. Graph. Image Process.*, pp. 291–297, 2012.

[16] D. Simon, "Biogeography-Based Optimization," *IEEE Trans. Evol. Comput.*, vol. 12, no. 6, pp. 702–713, Dec. 2008.

[17] E. Rashedi, H. Nezamabadi-Pour, and S. Saryazdi, "BGSA: Binary gravitational search algorithm," *Nat. Comput.*, vol. 9, no. 3, pp. 727–745, 2010.

[18] S. Das, A. Biswas, S. Dasgupta, and A. Abraham, "Bacterial foraging optimization algorithm: Theoretical foundations, analysis, and applications," *Stud. Comput. Intell.*, vol. 203, pp. 23–55, 2009.

[19] X. B. Meng, X. Z. Gao, L. Lu, Y. Liu, and H. Zhang, "A new bio-inspired optimisation algorithm: Bird Swarm Algorithm," *J. Exp. Theor. Artif. Intell.*, vol. 28, no. 4, pp. 673–687, 2016.

[20] P. Civicioglu, "Backtracking Search Optimization Algorithm for numerical optimization problems," *Appl. Math. Comput.*, vol. 219, no. 15, pp. 8121–8144, 2013.

[21] A. Kaveh and V. R. Mahdavi, "Colliding bodies optimization: Extensions and applications," *Colliding Bodies Optim. Extensions Appl.*, pp. 1–284, 2015.

[22] H. C. Kuo and C. H. Lin, "Cultural evolution algorithm for global optimizations and its applications," *J. Appl. Res. Technol.*, vol. 11, no. 4, pp. 510–522, 2013.

[23] G. G. Wang, L. Guo, A. H. Gandomi, G. S. Hao, and H. Wang, "Chaotic Krill Herd algorithm," *Inf. Sci. (Ny).*, vol. 274,

pp. 17–34, 2014.

[24] A. H. Gandomi, X.-S. Yang, and A. H. Alavi, “Cuckoo search algorithm: a metaheuristic approach to solve structural optimization problems,” *Eng. Comput.*, vol. 29, no. 1, pp. 17–35, Jan. 2013.

[25] S. Mirjalili, “Dragonfly algorithm: a new meta-heuristic optimization technique for solving single-objective, discrete, and multi-objective problems,” *Neural Comput. Appl.*, vol. 27, no. 4, pp. 1053–1073, 2016.

[26] G. G. Wang, S. Deb, and L. D. S. Coelho, “Elephant Herding Optimization,” *Proc. - 2015 3rd Int. Symp. Comput. Bus. Intell. ISCBI 2015*, pp. 1–5, 2016.

[27] N. Ghorbani and E. Babaei, “Exchange market algorithm,” *Appl. Soft Comput. J.*, vol. 19, no. April, pp. 177–187, 2014.

[28] H. Abedinpourshotorban, S. Mariyam Shamsuddin, Z. Beheshti, and D. N. A. Jawawi, “Electromagnetic field optimization: A physics-inspired metaheuristic optimization algorithm,” *Swarm Evol. Comput.*, vol. 26, pp. 8–22, 2016.

[29] G. G. Wang, S. Deb, and L. D. S. Coelho, “Earthworm optimization algorithm: a bio-inspired metaheuristic algorithm for global optimization problems,” *Int. J. Bio-Inspired Comput.*, vol. 1, no. 1, p. 1, 2015.

[30] Y. Tan, Y. Tan, and Y. Zhu, “Fireworks Algorithm for Optimization Fireworks Algorithm for Optimization,” no. December, pp. 355–364, 2015.

[31] X. S. Yang, “Firefly algorithm,” *Eng. Optim. pp*, vol. 221, 2010.

[32] M. Ghaemi and M. R. Feizi-Derakhshi, “Forest optimization algorithm,” *Expert Syst. Appl.*, vol. 41, no. 15, pp. 6676–6687, 2014.

[33] Y. X-s., “Flower pollination algorithm for global optimization,” in *Unconventional computation and natural computation*, ; p. 240-49: Springer, 2012.

[34] S. Saremi, S. Mirjalili, and A. Lewis, “Grasshopper Optimisation Algorithm: Theory and application,” *Adv. Eng. Softw.*, vol. 105, pp. 30–47, 2017.

[35] E. Rashedi, H. Nezamabadi-Pour, and S. Saryazdi, “GSA: a gravitational search algorithm,” *Inf Sci*, vol. 179, p. 2232, 2009.

[36] S. M. S. M. Mirjalili, S. M. S. M. Mirjalili, and A. Lewis, “Grey Wolf Optimizer,” vol. 69, 2014.

[37] S. A. Kazarlis, “A genetic algorithm solution to the unit commitment problem, IEEE Transactions on Power Systems, pp. 83 -92,” 1996.

[38] H. Shareef, A. A. Ibrahim, and A. H. Mutlag, “Lightning search algorithm,” *Appl. Soft Comput. J.*, 2015.

[39] V. N. Dieu and W. Ongsakul, “Ramp rate constrained unit commitment by improved priority list and augmented Lagrange Hopfield network,” *Electr. Power Syst. Res.*, vol. 78, no. 3, pp. 291–301, 2008.

[40] C. Dai, W. Chen, L. Ran, Y. Zhang, and Y. Du, “Human Group Optimizer with Local Search,” pp. 310–320, 2011.

[41] E. Atashpaz-Gargari and C. Lucas, “Imperialist competitive algorithm: An algorithm for optimization inspired by imperialistic competition,” *2007 IEEE Congr. Evol. Comput. CEC 2007*, pp. 4661–4667, 2007.

[42] H. Ghasemi, H. Park, and T. Rabczuk, “A level-set based IGA formulation for topology optimization of flexoelectric materials,” *Comput. Methods Appl. Mech. Engrg.*, 2016.

[43] A. H. Kashan, “League Championship Algorithm (LCA): An algorithm for global optimization inspired by sport championships,” *Appl. Soft Comput. J.*, vol. 16, pp. 171–200, 2014.

[44] A. H. Gandomi, “Interior search algorithm (ISA): A novel approach for global optimization,” *ISA Trans.*, vol. 53, no. 4, pp. 1168–1183, 2014.

[45] S. Karimkashi, S. Member, and A. A. Kishk, “Invasive Weed Optimization and its Features in Electromagnetics,” vol. 58, no. 4, pp. 1269–1278, 2010.

[46] A. H. Gandomi and A. H. Alavi, “Krill herd: A new bio-inspired optimization algorithm,” *Commun. Nonlinear Sci. Numer. Simul.*, vol. 17, no. 12, pp. 4831–4845, 2012.

[47] A. Sadollah, A. Bahreininejad, H. Eskandar, and M. Hamdi, “Mine blast algorithm: A new population based algorithm for solving constrained engineering optimization problems,” *Appl. Soft Comput. J.*, 2012.

[48] G.-G. G. Wang, S. Deb, and Z. Cui, "Monarch butterfly optimization," *Neural Comput Applic DOI*, vol. 10, no. 1007, pp. 15–521, 2015.

[49] S. Mirjalili, "Knowledge-Based Systems Moth-flame optimization algorithm : A novel nature-inspired heuristic paradigm," *Knowledge-Based Syst.*, vol. 89, pp. 228–249, 2015.

[50] S. Mirjalili, S. M. Mirjalili, and A. Hatamlou, "Multi-Verse Optimizer: a nature-inspired algorithm for global optimization," *Neural Comput. Appl.*, vol. 27, no. 2, pp. 495–513, 2016.

[51] H. Ghasemi, H. S. Park, and T. Rabczuk, "A multi-material level set-based topology optimization of flexoelectric composites," *Comput. Methods Appl. Mech. Engrg.*, 2017.

[52] A. Husseinzadeh Kashan, "A new metaheuristic for optimization: Optics inspired optimization (OIO)," *Comput. Oper. Res.*, vol. 55, pp. 99–125, 2014.

[53] J. Kennedy and (1995) Eberhart R. C., "Particle Swarm Optimization," in *Proceedings of the IEEE International Conference on Neural Networks*, pp. 1942–1948.

[54] F. Merrikh-Bayat, "The runner-root algorithm: A metaheuristic for solving unimodal and multimodal optimization problems inspired by runners and roots of plants in nature," *Appl. Soft Comput. J.*, vol. 33, pp. 292–303, 2015.

[55] S. Kirkpatrick, C. D. Gelatt, and M. P. Vecchi, "Optimization by simulated annealing.," *Science*, vol. 220, no. 4598, pp. 671–80, May 1983.

[56] S. Mirjalili, "SCA: A Sine Cosine Algorithm for solving optimization problems," *Knowledge-Based Syst.*, vol. 96, pp. 120–133, 2016.

[57] H. Salimi, "Stochastic Fractal Search: A powerful metaheuristic algorithm," *Knowledge-Based Syst.*, vol. 75, pp. 1–18, 2015.

[58] J. M. Anita and I. J. Raglend, "Shuffled Frog Leaping Algorithm," *Int. Conf. Comput. Electron. Electr. Technol.*, pp. 109–115, 2012.

[59] M. S. Gonçalves, R. H. Lopez, L. Fleck, and F. Miguel, "Search group algorithm : A new metaheuristic method for the optimization of truss structures," vol. 153, pp. 165–184, 2015.

[60] C. Dai, Y. Zhu, W. Chen, and C. Engineering, "Seeker Optimization Algorithm," pp. 167–176, 2007.

[61] S. Gupta and K. Deep, "A novel Random Walk Grey Wolf Optimizer," *Swarm Evol. Comput.*, Jan. 2018.

[62] M. Y. Cheng and D. Prayogo, "Symbiotic Organisms Search: A new metaheuristic optimization algorithm," *Comput. Struct.*, vol. 139, pp. 98–112, 2014.

[63] S. Mirjalili, A. H. Gandomi, S. Z. Mirjalili, S. Saremi, H. Faris, and S. M. Mirjalili, "Salp Swarm Algorithm: A bio-inspired optimizer for engineering design problems," *Adv. Eng. Softw.*, vol. 114, pp. 163–191, 2017.

[64] F. Glover, "Tabu search—Part I," *ORSA J. Comput.*, vol. 1, p. 190, 1989.

[65] S. C. Satapathy, A. Naik, and K. Parvathi, "A teaching learning based optimization based on orthogonal design for solving global optimization problems," pp. 1–12, 2013.

[66] M. D. Li, H. Zhao, X. W. Weng, and T. Han, "Advances in Engineering Software A novel nature-inspired algorithm for optimization : Virus colony search," *Adv. Eng. Softw.*, vol. 92, pp. 65–88, 2016.

[67] H. Eskandar, A. Sadollah, A. Bahreininejad, and M. Hamdi, "Water cycle algorithm – A novel metaheuristic optimization method for solving constrained engineering optimization problems," *Comput. Struct.*, vol. 110–111, pp. 151–166, 2012.

[68] Z. Bayraktar, M. Komurcu, and D. H. Werner, "Wind Driven Optimization (WDO): A Novel Nature-Inspired Optimization Algorithm and its Application to Electromagnetics (1)," no. 1, pp. 0–3, 2010.

[69] S. Mirjalili and A. Lewis, "The Whale Optimization Algorithm," *Adv. Eng. Softw.*, vol. 95, pp. 51–67, 2016.

[70] A. Baykasoğlu and Ş. Akpinar, "Weighted Superposition Attraction (WSA): A swarm intelligence algorithm for optimization problems – Part 1: Unconstrained optimization," *Appl. Soft Comput. J.*, vol. 56, pp. 520–540, 2017.

[71] Y. J. Zheng, "Water wave optimization: A new nature-inspired metaheuristic," *Comput. Oper. Res.*, vol. 55, pp. 1–11, 2015.

[72] A. License, “A Modified Dragonfly Optimization Algorithm for Single- and Multiobjective Problems Using Brownian Motion,” vol. 2019, 2019.

[73] L. Cheng, “applied sciences Artificial Flora (AF) Optimization Algorithm,” 2018.

[74] S. Deb, X. G. Kari, T. Karuna, and P. Mahanta, “Recent Studies on Chicken Swarm Optimization algorithm : a review (2014 – 2018),” *Artif. Intell. Rev.*, 2019.

[75] K. Huang, Z. Wu, and T. Csa, “CPO : A Crow Particle Optimization Algorithm,” 2019.

[76] A. Yurtkuran, “An Improved Electromagnetic Field Optimization for the Global Optimization Problems,” vol. 2019, 2019.

[77] K. M. Hamdia, H. Ghasemi, X. Zhuang, N. Alajlan, and T. Rabczuk, “Computational machine learning representation for the flexoelectricity effect in truncated pyramid structures,” *Comput. Mater. Contin.*, vol. 59, no. 1, pp. 79–87, 2019.

[78] H. Ghasemi, H. S. Park, and T. Rabczuk, “A level-set based IGA formulation for topology optimization of flexoelectric materials,” *Comput. Methods Appl. Mech. Eng.*, vol. 313, no. October, pp. 239–258, 2017.

[79] K. M. Hamdia, H. Ghasemi, X. Zhuang, N. Alajlan, and T. Rabczuk, “Sensitivity and uncertainty analysis for flexoelectric nanostructures,” *Comput. Methods Appl. Mech. Eng.*, vol. 337, pp. 95–109, 2018.

[80] T. Zheng and W. Luo, “An Enhanced Lightning Attachment Procedure Optimization with Quasi-Opposition-Based Learning and Dimensional Search Strategies,” vol. 2019, 2019.

[81] A. A. Hudaib and H. N. Fakhouri, “Supernova Optimizer : A Novel Natural Inspired Meta-Heuristic,” vol. 12, no. 1, pp. 32–50, 2018.

[82] W. Zhao, Z. Zhang, and L. Wang, “Engineering Applications of Artificial Intelligence Manta ray foraging optimization : An effective bio-inspired optimizer for engineering applications,” *Eng. Appl. Artif. Intell.*, vol. 87, no. October 2019, p. 103300, 2020.

[83] B. P. Dahiya, “A Hybrid Artificial Grasshopper Optimization (HAGOA) Meta- Heuristic Approach : A Hybrid Optimizer For Discover the Global Optimum in Given Search Space,” vol. 4, no. 2, pp. 471–488, 2019.

[84] D. H. Wolpert and W. G. Macready, “No free lunch theorems for optimization,” *IEEE Trans. Evol. Comput.*, vol. 1, no. 1, pp. 67–82, 1997.

[85] X. Li, J. Zhang, and M. Yin, “Animal migration optimization: An optimization algorithm inspired by animal migration behavior,” *Neural Comput. Appl.*, vol. 24, no. 7–8, pp. 1867–1877, 2014.

[86] X. Chen, P. W. Qian, W. Y. Dong, and X. Chen, “A Two-Stage strategy to handle equality constraints in ABC-based power economic dispatch problems,” *Soft Comput.*, 2019.

[87] I. Brajevic and M. Tuba, “An upgraded artificial bee colony (ABC) algorithm for constrained optimization problems,” pp. 729–740, 2013.

[88] D. Singh and J. S. Dhillon, “Ameliorated Grey Wolf Optimization for Economic Load Dispatch,” *Energy*, 2018.

[89] R. Rq and R. Dqg, “% DUQDFOHV 0DWLQJ 2SWLPL] HU $ Q (YROXWLRQDU \ $ OJRULWKP IRU 6ROYLQJ 2SWLPL] DWLRQ.”

[90] B. Gray and W. Optimization, “Author ’ s Accepted Manuscript Binary Gray Wolf Optimization Approaches for Feature Selection,” *Neurocomputing*, 2015.

[91] S. M. G. Wang, S. Mirjalili, G. G. Wang, and L. dos S. Coelho, “Binary optimization using hybrid particle swarm optimization and gravitational search algorithm,” *Neural Comput. Appl.*, vol. 25, no. 6, pp. 1423–1435, 2014.

[92] S. Reddy K., L. Panwar, B. K. Panigrahi, and R. Kumar, “Binary whale optimization algorithm: a new metaheuristic approach for profit-based unit commitment problems in competitive electricity markets,” *Eng. Optim.*, vol. 0, no. 0, pp. 1–21, 2018.

[93] S. Mirjalili, S. M. Mirjalili, and A. Lewis, “Let a biogeography-based optimizer train your Multi-Layer Perceptron,” *Inf. Sci. (Ny).*, vol. 269, pp. 188–209, Jun. 2014.

[94] J. Pierezan, “Coyote Optimization Algorithm : A new metaheuristic for global optimization problems,” *2018 IEEE Congr. Evol. Comput.*, pp. 1–8, 2018.

[95] S. Mohseni, R. Gholami, N. Zarei, and A. R. Zadeh, “Competition over resources: A new optimization algorithm based on animals behavioral ecology,” *Proc. - 2014 Int. Conf. Intell. Netw. Collab. Syst. IEEE INCoS 2014*, pp. 311–315, 2014.

[96] A. Tabari and A. Ahmad, “Ac ce pt e us cr t,” *Comput. Chem. Eng.*, 2017.

[97] G. Dhiman and V. Kumar, “PT,” *Knowledge-Based Syst.*, 2018.

[98] M. S. Shahriar, J. Rana, M. A. Asif, and M. Hasan, “Optimization of Unit Commitment Problem for Wind-Thermal Generation using Fuzzy Optimization Technique,” pp. 88–92, 2015.

[99] T. S. Panag and J. S. Dhillon, “Two Stage Grid Classification Based Algorithm for the Identification of Fields Under a Wireless Sensor,” *Wirel. Pers. Commun.*, 2016.

[100] N. Singh and S. B. Singh, “A novel hybrid GWO-SCA approach for optimization problems,” *Eng. Sci. Technol. an Int. J.*, vol. 20, no. 6, pp. 1586–1601, 2017.

[101] N. B. Gohil and V. V. Dwivedi, “A Review on Lion Optimization : Nature Inspired Evolutionary Algorithm,” vol. 7, no. 12, pp. 340–352, 2017.

[102] R. Martí, M. G. C. Resende, and C. C. Ribeiro, “Multi-start methods for combinatorial optimization q,” vol. 226, pp. 1–8, 2013.

[103] K. K. Dhaliwal, “On the Design and Optimization of Digital IIR Filter using Oppositional Artificial Bee Colony Algorithm,” 2016.

[104] X. Chen, H. Tianfield, and K. Li, “SC,” *Swarm Evol. Comput. BASE DATA*, 2019.

[105] S. Shadravan, H. R. Naji, and V. K. Bardsiri, “Engineering Applications of Artificial Intelligence The Sailfish Optimizer : A novel nature-inspired metaheuristic algorithm for solving constrained engineering optimization problems ☆,” *Eng. Appl. Artif. Intell.*, vol. 80, no. July 2018, pp. 20–34, 2019.

[106] R. Moghdani and K. Salimifard, “Volleyball Premier League Algorithm,” *Appl. Soft Comput. J.*, vol. 64, pp. 161–185, 2018.

[107] K. Fleszar, I. H. Osman, and K. S. Hindi, “A variable neighbourhood search algorithm for the open vehicle routing problem,” *Eur. J. Oper. Res.*, vol. 195, no. 3, pp. 803–809, 2020.

[108] X. Yang, *Nature-Inspired Metaheuristic Algorithms Second Edition*, no. July 2010. 2010.

[109] X. Yao, Y. Liu, and G. Lin, “Evolutionary programming made faster,” *IEEE Trans Evol Comput*, vol. 3, p. 82, 1999.

[110] J. G. Digalakis and K. G. Margaritis, “On benchmarking functions for genetic algorithms,” *Int. J. Comput. Math.*, vol. 77, no. 4, pp. 481–506, 2007.

[111] E. Cuevas, A. Echavara, and M. A. Ramrez-Ortegn, “An optimization algorithm in- spired by the States of Matter that improves the balance between exploration and exploitation,” *Appl Intel*, vol. 40, p. 256, 2014.

[112] E. Cuevas, A. Echavarra, D. Zaldvar, and M. A. Prez-Cisneros, “novel evolutionary al- gorithm inspired by the states of matter for template matching,” in *Expert Syst Appl 2013;40:635973*, 27] Yang X-S . Flower pollination algorithm for global optimization. In: Unconven- tional computation and natural computation. Springer; p. 240-49, 2012.

[113] Y. X-s and S. Deb, “Cuckoo search via Lеvy flights,” *World Congr. Nat. Biol. inspired Comput.*, vol. 2009, 2009.

[114] Y. X. F. algorithm, “Levy flights and global optimization,” in *Research and development in intelligent systems XXVI*, ; p. 209-18: Springer, 2010.

[115] H. John, *Holland, adaptation in natural and artificial systems*. Cambridge: MIT Press, 1992.

[116] X. -s. Yang, “A new metaheuristic bat-inspired algorithm, in: Nature Inspired Cooperative Strategies for Optimization (NICSO 2010),” *Springer pp*, vol. 65, 2010.

[117] E. Cuevas, A. Echavarra, and R.-O. Ma., “An optimization algorithm inspired by the States of Matter that improves the balance between exploration and exploitation,” *Appl Intell*, vol. 40, p. 256, 2014.

[118] J. Kennedy and R. C. Eberhart, “A discrete binary version of the particle swarm algorithm,” *1997 IEEE Int. Conf. Syst. Man, Cybern. Comput. Cybern. Simul.*, vol. 5, pp. 4104–4108, 1997.

[119] “X.-S,”*ang,M.Karamanoglu, X.He, Flower pollination algorithm: a novel ap-proach for multi objective optimization,*

Eng. Optim., vol. 4612, 2014.

[120] R. Storn and K. Price, “Differential evolution-a simple and efficient heuristic for global optimization over continuous spaces,” *J. Glob. Optim.*, vol. 11, no. 4, pp. 341–359, 1997.

[121] J. Brest and M. S. Mauˇ, “Single Objective Real-Parameter Optimization : Algorithm jSO,” pp. 1311–1318, 2017.

[122] D. Jagodzi, “A Differential Evolution Strategy,” vol. 1, no. 3, pp. 1872–1876, 2017.

[123] R. Kommadath, “Teaching Learning Based Optimization with Focused Learning and its Performance on CEC2017 functions,” no. 1, pp. 2397–2403, 2017.

[124] A. Tangherloni, L. Rundo, and M. S. Nobile, “Proactive Particles in Swarm Optimization : a Settings-Free Algorithm for Real-Parameter Single Objective Optimization Problems,” pp. 1940–1946, 2017.

[125] V. K. Kamboj, “A novel hybrid PSO – GWO approach for unit commitment problem,” *Neural Comput. Appl.*, 2015.

[126] J. N. Kuk, R. A. Gonc, C. P. Almeida, S. M. Venske, A. T. Pozo, and A. C.-B. Functions, “A New Adaptive Operator Selection for NSGA-III Applied to CEC 2018 Many-Objective Benchmark,” pp. 7–12, 2018.

[127] A. Bhadoria and V. K. Kamboj, “Optimal generation scheduling and dispatch of thermal generating units considering impact of wind penetration using hGWO-RES algorithm,” *Appl. Intell.*, 2018.

[128] K. Shankar and P. Eswaran, “RGB-Based Secure Share Creation in Visual Cryptography Using Optimal Elliptic Curve Cryptography Technique,” *J. Circuits, Syst. Comput.*, vol. 25, no. 11, p. 1650138, Nov. 2016.

[129] S. Mohanty, B. Subudhi, and P. K. Ray, “A New MPPT Design Using Grey Wolf Optimization Technique for Photovoltaic System Under Partial Shading Conditions,” *IEEE Trans. Sustain. Energy*, vol. 7, no. 1, pp. 181–188, Jan. 2016.

[130] I. A. Hameed, R. T. Bye, and O. L. Osen, “Grey wolf optimizer (GWO) for automated offshore crane design,” in *2016 IEEE Symposium Series on Computational Intelligence (SSCI)*, 2016, pp. 1–6.

[131] V. Ariables, “The Butterfly-Particle Swarm Optimization (BUTTERFLY-PSO / BF-PSO) Techniques and Its,” vol. 4, no. 3, pp. 23–39, 2015.

[132] L. C. Cagnina, S. C. Esquivel, U. Nacional, D. S. Luis, S. Luis, and C. A. C. Coello, “Solving Engineering Optimization Problems with the Simple Constrained Particle Swarm Optimizer 1 Introduction 2 Literature review 3 Our proposed approach : SiC-PSO,” vol. 32, pp. 319–326, 2008.

[133] K. Deb, “A Combined Genetic Adaptive Search (GeneAS) for Engineering Design,” vol. 26, pp. 30–45, 1996.

[134] I. J. Raglend, R. Kumar, and S. P. Karthikeyan, “Deregulated Environment.”

[135] S. Virmani, E. Adrian, … K. I.-I. T. on, and undefined 1989, “Implementation of a Lagrangian relaxation based unit commitment problem,” *ieeexplore.ieee.org*.

[136] E. Cuevas and A. Echavarría, “An optimization algorithm inspired by the States of Matter that improves the balance between exploration and exploitation,” 2013.

[137] H. CHICKERMANE and H. C. GEA, “Structural Optimization Using a New Local Approximation Method,” *Int. J. Numer. Methods Eng.*, vol. 39, no. 5, pp. 829–846, 2002.

[138] H. Abderazek, D. Ferhat, and A. Ivana, “Adaptive mixed differential evolution algorithm for bi-objective tooth profile spur gear optimization,” *Int. J. Adv. Manuf. Technol.*, 2016.

HIGHLIGHTS:

- A hybrid variant of Harris Hawks optimizer i.e. *Hybrid Harris Hawks-Sine Cosine Algorithm (hHHO-SCA)* has been developed.
- The exploration phase of the existing Harris Hawks optimizer has been improved.
- The proposed hybrid variant has been tested for nonlinear, non-convex and highly constrained engineering design problem.
- Validity of the developed hybrid optimizer has been tested for 65 standard benchmark problems including uni-modal, multi-modal, fixed dimensions, CEC2017, and CEC2018.
- Proposed hHHO-SCA has been tested 11 multidisciplinary engineering design optimization problems.
- The outcomes of the proposed hHHO-SCA algorithm is much better than standard sine-cosine optimization algorithm, Harris Hawks Optimizer, Ant Lion Optimizer algorithm, Moth Flame Optimization algorithm, grey wolf optimizer algorithm, and others recently described meta-heuristics, heuristics and hybrid optimization search algorithm.

GRAPHICAL ABSTRACT

INPUTS: The population size is taken as N and maximum iteration number is taken as $iter_{max}$
OUTPUTS: The position of prey (rabbit) and its value of fitness
Initialization of random population $H_i (i = 1, 2, 3......., N)$
While (iter<$iter_{max}$)
Calculation of the fitness value of Harris hawks
Set the parameter H_{rabbit} as the best position of the prey (rabbit)
for (each Harris hawks (H_i))
Do $EG_O = 2rand() - 1, K = 2(1 - rand())$ →Update energy at initial condition EG_O
Update EG using equation number (4)
if $|EG| \geq 1$ **then** → **Phase of Exploration**
Update the position vector using equation (1) and (2)
if $|EG| \langle 1$ **then** → **Phase of Exploitation**
1st phase of improvement of Exploration and Exploitation
if $(e_s \geq 0.5)$ and $|EG| \geq 0.5$ **then** → **Soft encircle**
Location vector updated using equation number (5)
else if $(e_s \geq 0.5)$ and $|EG| \langle 0.5$ **then** → **Hard encircle**
Location vector updated using equation number (7)
else if $(e_s \langle 0.5)$ and $|EG| \geq 0.5$ **then** → **Soft encircle with advanced fast dives**
Location vector updated using equation number (12) and (13)
else if $(e_s \langle 0.5)$ and $|EG| \langle 0.5$ **then** → **Hard encircle with advanced fast dives**
Location vector updated using equation number (14) and (15)
end
end
end
end

Initialize the starting position of the search agents using final position obtained through Harris Hawks optimizer
Do
Evaluate each of the search agents using objective functions
Update the best fitness obtained so far $(H \in X)$
Update the random numbers re_1, re_2, re_3 and re_4
if re_4<0.5
Update the position of search agents using $H_{i,iter+1} = \{H_{i,iter} + re_1 \times \sin(re_2) \times |re_3 p_{i,iter} - H_{i,iter}|$
else
Update the position of search agents using $H_{i,iter+1} = \{H_{i,iter} + re_1 \times \cos(re_2) \times |re_3 p_{i,iter} - H_{i,iter}||$
2nd stage of improvement of Exploration and Exploitation
end
While ($iter \langle iter_{max}$)
Return the best optimal solution
Record the mean, best, worst fitness and standard deviation.
Save the global optimum value obtained through successive trial runs

DECLARATION OF INTEREST STATEMENT

We wish to confirm that there are no known conflicts of interest associated with this publication and there has been no significant financial support for this work that could have influenced its outcome.

To
The Editor
Applied Soft Computing

This is to confirm you that every authors has significant contribution for preparing the revised manuscript.
We are thankful to the reviewers for going through our manuscript thoroughly and making constructive comments. On the basis of their comments, we have revised the manuscript. The lists of changes made in the revised manuscript are given below as an Annexure I.

Yours sincerely,

Dr. Vikram Kumar Kamboj

(Corresponding Author)

Tel: +91-87288-87287

E-mail: vikram.23687@lpu.co.in