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# A novel heuristic, based on a new robustness concept, for multi-objective project portfolio optimization

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ABSTRACT

In this paper, multi-objective project portfolio optimization problem, with interval uncertainties, is addressed. Due to the existence of internal and external uncertainties, the robust optimization approach is selected. We have defined and examined the robustness in terms of resources, in which, uncertainty can be the cause of wasted resources. On the other hand, the existing methods in the literature measure robustness in terms of the objective functions. These methods, which implicitly suppose equal objective weights, use the measure of the diameter of sensitivity region of a solution as robustness index. This is in contradiction to the philosophy of the posterior preferences decision-making approach. Therefore, we have proposed a novel robustness index based on the new concept of *preferential weights* of each portfolio and deduced a formula based on the normal vectors of bounding hyperplanes of the Pareto frontier. Then, the relationship between the two newly defined robustness indexes is investigated. Finally, an efficient heuristic is developed and applied in a case study as well as a numerical example. In addition, our proposed robustness concept is compared to an existing concept in the literature. Results show the effectiveness and efficiency of the presented concept and the proposed algorithm.

## 1. Introduction

In regards to intensive global competition, companies must take vital strategic decisions, namely project selection, to maintain and develop the market share, and thus, the survival of the company (Tofighian & Naderi, 2015). The project portfolio is a set of candidate projects competing to get resources (Korotkov & Wu, 2019). The project portfolio selection is defined as the problem of selecting and funding a portfolio that maximizes objectives aligned with the goals of a company, and by using available recourses and not violating other necessary constraints (Li, Fang, Guo, Deng, & Qi, 2016). Due to the strategic nature of project portfolio decisions, objectives' payoffs, availability of recourses (constraints) and decision-makers (DMs) preferences are subject to substantial uncertainty (Fliedner & Liesiö, 2016). The goal of this research is to develop a robustness index for partially funded project portfolio optimization problem with interval uncertainties in objective functions coefficients (external uncertainty) and decision variables (internal uncertainty).

Uncertainty in input data and the existence of multiple and contradictory goals are two important features of the real world project portfolio optimization problems that make the use of optimization techniques challenging. Researches have proposed the multi-objective robust optimization approach as a powerful tool for dealing with uncertainty in real-world problems. Existing multi-objective robust optimization methods in the literature consider only objective robustness and define robustness as the sensitivity of objectives to changes in uncertain parameters. These methods, in fact, implicitly assume that the available resources, which in fact contribute to the constraints of mathematical model, can be consumed in full or in part. In addition, the criterion for evaluating a solution is merely the value of objective functions; and thus, the efficiency of utilizing the resources is not studied.

In some real-world problems, especially in innovative projects, DMs consider the total amount of resources available to be allocated to a portfolio of candidate projects dependent on the degree of uncertainty. In other words, when there is considerable uncertainty about the outcome of a decision, DMs may prefer to use fewer resources for their first priority portfolio of projects and use the rest of the resources that are exposed to less uncertainty in other lower priority portfolio of projects. The logic behind this strategy is that the DMs prioritize investment in different portfolios of projects. Each portfolio of projects, which is assigned with a priority, is composed of some candidate projects that compete to get resources. From this strategy, the focus is on the level of resources applied to produce a targeted level of objectives. Therefore, if uncertainty is considerable enough to cause excessive waste of resources by producing objectives that could be produced by applying less

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resources in the absence of uncertainty, only a part of the resources, rather than all resources, are allocated to the first priority class of projects and the remaining recourses are preserved for consumption by other lower priority classes of projects. In fact, DMs decide the trade-off between the level of *resource waste degree* and objectives achievement. This practical issue is our main motivation for developing a resource-wise robustness measure. Thus, the main objective of the presented paper, motivated by a real-world project portfolio optimization problem in the port industry (Section 5), is to develop a resource-wise robustness measure for project portfolio optimization problems, based on the definitions of the concepts of *resource consumption level* and *resource waste degree*.

Other novelties of this research are as follows: we proposed a new objective-wise robustness index based on a new concept of *preferential weights*. The new robustness index is similar to the robustness measures proposed by Deb and Gupta (2006) and Zhou et al. (2018), however, with the difference that we use preferential weights in calculating the diameter of sensitivity region, while, existing methods use the *p-norm* (e.g., Euclidian distance) of difference between the worst-case scenario and the point representing nominal objective values. By using the *p-norm*, implicitly, equal weights of objectives are assumed, which is in contradiction to the philosophy of posterior preferences decision-making approach. To address this problem, we have used the preferential weights in the *p-norm* function. We also investigated the relationship between the two new resource-wise and objective-wise robustness measures. After that, a novel heuristic is developed to calculate and incorporate the proposed robustness index into the multi-objective project portfolio optimization procedure.

The remainder of the paper proceeds as follows. In Section 2, the literature on robust multi-objective optimization concepts and techniques is studied. In Section 3, a new concept of robustness for project portfolio optimization problems is proposed. Section 4 describes the proposed solution method in details. Then, the algorithm is applied to a case study in the port industry and two numerical example, including a nonlinear problem. Results are represented in Section 5. Finally, concluding remarks are summarized in Section 6.

## 2. Literature review

Many researchers have considered uncertainty in project portfolio optimization problems. Fuzzy portfolio selection methods are prevailing in recent publications, which address both uncertainty and multiple objectives. Mohagheghi, Mousavi, and Vahdani (2016) applied interval-valued fuzzy sets to take into account uncertainty. Wu, Xu, Ke, Li, and Li (2019) used interval type-2 fuzzy weighted averaging to calculate strategic alignment indexes of each portfolio. Some researchers applied the fuzzy goal programming approach to aggregate multiple objectives into a single function, and by doing this, they converted the multi-objective problem into a single-objective problem (W. Chen, Li, & Liu, 2018; Hocine, Kouaissah, Bettahar, & Benbouziane, 2018). Saborido, Ruiz, Bermúdez, Vercher, and Luque (2016) presented a new algorithm based on the Mean-Downside Risk-Skewness model. Real options valuation is another powerful tool for dealing with uncertainty in the project portfolio selection problem. Guo, Wang, Li, Chen, and Cheng (2018) considered uncertainty with fuzzy real options. Other works on the fuzzy portfolio selection are based on the multi-objective credibilistic model (García, González-Bueno, Oliver, & Tamošiūnienė, 2019; Jalota, Thakur, & Mittal, 2017).

Other methods found in the literature are those that have applied robust optimization methods. Some methods used additive portfolio function and then consider a single-objective robust problem (Fliedner & Liesiö, 2016; Mild, Liesiö, & Salo, 2015). Others used an extension of single-objective robust optimization methods into multi-objective context. Chen and Zhou (2018) have considered multi-objective minimax robustness. In the method proposed by Mavrotas, Figueira, and Siskos (2015) judging the robustness of a Pareto solution is delegated to DMs by visualizing robustness. Crespi, Kuroiwa, and Rocca (2018) used sensitivity analysis by evaluating a possible decrease in uncertainty. A different work by Mavrotas, Pechak, Siskos, Doukas, and Psarras (2015) addressed uncertainty in objectives' weights. They used Mont Carlo simulation to calculate robustness against perturbations on objectives' weights.

This paper aims to provide an index for measuring the robustness of portfolios. By adding this index to the constraints, the non-deterministic model will be turned into a deterministic model. Therefore, in the following review, general multi-objective robustness literature is reviewed.

There has been a lot of work on dealing with uncertainty in the single-objective optimization domain. The presented methods in the literature can be categorized into two groups: stochastic optimization and robust optimization. The choice between methods of each of these two groups depends on the type of uncertainty and the type of problem under investigation. In stochastic optimization, it is assumed that there is some information about the probability distribution of uncertain parameters of the problem. For a comprehensive overview of the literature, refer to the works of Abdelaziz (2012) and Prékopa (2013). The second category of methods developed for dealing with uncertainty is called robust optimization. These methods apply to the problems in which there isn't enough information about the probability distribution of uncertain parameters. The uncertainty of these parameters is mainly defined as changes over an interval or a finite set of different scenarios. The goal of robust optimization is to find optimal solutions that are resistant to changes in uncertain parameters. Since these methods usually consider the worst-case scenario, they adopt a conservative approach to decision-making. Due to this feature, robust optimization has attracted many researchers interested in developing practical methods to deal with uncertainty, thus, having a wide application in real-world problems. To study various methods of single-objective robust optimization techniques, see Ben-Tal and Nemirovski (2008), Ben-Tal, El Ghaoui, and Nemirovski (2009), Bertsimas, Brown, and Caramanis (2011) and Goerigk and Schöbel (2016). Robust optimization approaches can be put into two different categories. The first category includes methods in which a measure is used to quantify the robustness of a solution. This measure quantifies the size of the objective function variation over different scenarios. The variance of the objective function and the distance between the worst-case and the best case of the objective function over different scenarios are examples of a robustness measure.

By replacing the sensitivity region of each solution with nominal value or the mean value of its objectives and adding the robustness measure as a new constraint or objective function into the mathematical model, the uncertain optimization problem transforms into a new deterministic one. These methods use information regarding all scenarios. The opposite are methods in the second category. These methods summarize all values of the objective functions under different scenarios in only one representative value (e.g., the worst-case scenario) and solve the resulting deterministic problem. It is evident that the first approach offers greater flexibility compared to the second approach. In other words, the first approach proposes adjustable robustness.

As noted, each of the multi-objective optimization and robust optimization concepts have been deeply explored by various researchers. However, the combination of these two areas, as robust multi-objective optimization, has not been extensively studied.

One of the reasons for this is the complexity of the multi-objective context, which makes it difficult to directly extend the concepts of the robustness from single-objective to multi-objective space. Although robust multi-objective optimization is still in its infancy, many researchers have tried to define the concept of robustness in multi-objective space (Goberna, Jeyakumar, Li, & Vicente-Pérez, 2018; Ide & Schöbel, 2016). The literature on multi-objective robustness is reviewed below with the same classification as for single-objective robustness.

The first category, as mentioned above, includes the methods of applying a robustness measure. Initial works on extending the concept of single-objective robustness into multi-objective context generally fall into this category. Deb and Gupta (2006) introduced the concept of robustness in multi-objective context. After that, Gunawan and Azarm (2005) introduced the concept of sensitivity region. Li, Azarm, and Aute (2005) presented a genetic algorithm, based on the concept of the sensitivity region, to find robust solutions in multi-objective space. In their algorithm, which is applied to solve an engineering design problem, the diameter of the sensitivity region is used as a robustness index and considered as a new objective function. Deb and Gupta (2006) generalized the work of Branke (1998) to multi-objective space. They proposed two different approaches. In the first approach, which belongs to the second category, the uncertain objective functions are represented by the vector of the mean value of objectives over all scenarios. In the second approach, which falls under the first category, the distance between the two points representing the objective vector in the worst-case scenario and the nominal objective vector is regarded as the robustness measure; in addition, it is added as a new constraint to the problem.

Ferreira, Fonseca, Covas, and Gaspar-Cunha (2008) also approached the uncertainty problem in a way similar to the Deb and Gupta (2006) approach. They provided various definitions of expected values and variation measures in multi-objective space. Moreover, Saha, Ray, and Smith (2011) provided a new algorithm based on the definition of robustness presented by Deb and Gupta (2006), reducing the number of objective functions evaluations, thus increasing the speed of the algorithm.

As mentioned above, the second group includes methods in which a solution is replaced with a point (usually, the worst-case objective values) representing the whole sensitivity region. By generalizing this concept to multi-objective context, a set of points (mostly the worst-case frontier), which is a subset of objective values over all scenarios, represents a solution; works of Avigad and Branke (2008), Soares, Parreiras, Jaulin, Vasconcelos, and Maia (2009), Kuroiwa and Lee (2012), Bokrantz and Fredriksson (2017), Ehrgott, Ide, and Schöbel (2014), Goberna, Jeyakumar, Li, and Vicente-Pérez (2015) and Schmidt, Schöbel, and Thom (2019) all fall into this category.

In the single-objective robustness literature, the two concepts of the minimax robustness (Soyster, 1973) and maximum regret robustness (Kouvelis & Yu, 2013) are well-known and widely used. Considering that these robustness concepts are non-flexible and very conservative, relatively new concepts such as adjustable robustness (Ben-Tal, Goryashko, Guslitzer, & Nemirovski, 2004), light robustness (Fischetti & Monaci, 2009), recovery robustness (Erera, Morales, & Savelsbergh, 2009; Liebchen, Lübbecke, Möhring, & Stiller, 2009) and soft robustness (Ben-Tal, Bertsimas, & Brown, 2010) have been proposed.

More recently, Ide and Schöbel (2016) have reviewed new concepts and works in the field of multi-objective robustness and defined new robustness concepts by generalizing the concepts outlined above to multi-objective space. In addition, Khosravi, Borst, and Teich (2018) developed a multi-objective specific robustness concept called probabilistic dominance. Other works in the field of robust multi-objective optimization are based on the same concepts. Some researchers such as Meneghini, Guimarães, and Gaspar-Cunha (2016), Kuhn, Raith, Schmidt, and Schöbel (2016) and Xie et al. (2018) have proposed new algorithms for producing robust solutions based on well-known concepts. Some scholars have also applied existing robustness concepts in various areas and applications (e.g., Wang, Li, Zhang, Ding, & Sun, 2018; Peng, Hou, Che, Xu, & Li, 2018; Xidonas, Mavrotas, Hassapis, & Zopounidis, 2017; Habibi-Kouchaksaraei, Paydar, & Asadi-Gangraj, 2018; Sun, Zhang, Fang, Li, & Li, 2018; Doolittle, Kerivin, & Wiecek, 2018; Tabrizi & Ghaderi, 2016(.

By reviewing the literature of robust multi-objective optimization, it can be concluded that this area is still at an early stage. Still, the single-objective robustness concepts and methods are not well-suited to the multi-objective context, and new concepts consistent with multi-objective space have not been well defined. In addition, there is a need to develop efficient algorithms for producing robust solutions. Many of the available algorithms are designed to solve a particular category of problems such as linear or convex structures. Another disadvantage of the existing algorithms is their inefficiency in terms of run-time.

In the present research, the focus is on the first category of robust optimization approaches. In other words, this study seeks to propose a robustness index to measure the degree of robustness of a portfolio by adding this index as a new constraint to the mathematical model; thus, transforming the non-deterministic project portfolio selection problem into a deterministic one.

The main contribution of the presented paper, motivated by a real-world project portfolio optimization problem in the port industry, is to propose a resource-wise robustness concept for linear project portfolio optimization problems. In order for this concept to be applicable, the proposed resource-wise robustness measure is translated into the objective-wise robustness measure (Theorems 1,2). Then, the developed heuristic applies the new objective-wise robustness measure to generate robust solutions. In Section 5.4, we will demonstrate that the proposed method, based on the objective-wise robustness can also be applied to general nonlinear multi-objective problems. Innovations and contributions of this research can be summarized as follows:

- Definition of a new resource-wise robustness measure of portfolios based on the *resource waste degree* of portfolios;
- Definition of a new objective-wise robustness index of a portfolio based on the *preferential weights* of that portfolio;
- Deducing a formula for the new objective-wise robustness index of portfolios, based on the normal vectors of bounding hyperplanes of Pareto frontier (PF);
- Deducing the relationship between the two newly defined resource-based and objective-wise robustness index of portfolios;
- Providing a new and accurate algorithm for generating robust portfolios, including a heuristic, based on a simulated annealing algorithm to estimate the normal vector of bounding hyperplanes of PF.

## 3. Preliminaries

The linear multi-objective model for projects portfolio optimization can be defined as follows:

$$\begin{aligned} &\max\,(\widetilde{C}^1X,\dots,\widetilde{C}^mX)\\ &\quad \textit{subject to}:\\ &\quad AX \leqslant B,\\ &0 \leqslant x_i \leqslant 1, \quad 1 \leqslant i \leqslant n \end{aligned} \tag{1}$$

where $X = (x_1,\dots,x_n)'$, the vector of decision variables, represents the decided fraction of projects to be funded. In this model, objective functions are assumed to be non-negative. Vector $B$ is the amount of resources available and matrix $A$ is the constraint matrix; each coefficient $a_j^i$ represents the amount of resource $i$ needed to fully execution of the project $j$. Also, $\widetilde{C}^i = (\widetilde{c}_1^i,\dots,\widetilde{c}_n^i),\ 1 \leqslant i \leqslant m$ represents the vector of objective function $i$ coefficients, where $\widetilde{c}_j^i \in [c_j^i - \Delta_j^i,\ c_j^i + \Delta_j^i],\ 1 \leqslant i \leqslant m,\ 1 \leqslant j \leqslant n$ are random parameters. If the random coefficient $\widetilde{c}_j^i$ is replaced by its nominal value $c_j^i$, then the uncertain problem will turn into a deterministic one. For the resulting deterministic problem (2), $D$ indicates the feasible region in the decision space and $\Theta$ represents the feasible objective region, which is, in fact, the image of $D$ in the objective space.

$$\begin{aligned} &\max\,(C^1X,\dots,C^mX)\\ &\quad \textit{subject to}:\\ &\quad AX \leqslant B,\\ &0 \leqslant x_i \leqslant 1, \quad 1 \leqslant i \leqslant n \end{aligned} \tag{2}$$

Changes in the coefficients of objective functions, which are due to uncertainty, will change the objective values within a region, which is called the sensitivity region. The sensitivity region for a solution like $X$ is defined as follows:

$$S_X = \{\tilde{Y} = (\tilde{Y}^1, ..., \tilde{Y}^m) \in \mathfrak{R}^m \mid (\tilde{Y}^1, ..., \tilde{Y}^m) = (\tilde{C}^1\tilde{X}, ..., \tilde{C}^m\tilde{X})\} \tag{3}$$

**Definition 1** (*Resource consumption level*). Let $X$ be a feasible solution (portfolio) of problem (2), then the resource consumption level of this portfolio, denoted by $\beta_X$, is calculated as follows:

$$\beta_X = \max_{1\leqslant i\leqslant r}\left\{\frac{a^iX}{b_i}\right\} \tag{4}$$

where $b_i$ is $i$-th component of vector B ($i$-th resource), $a^i$ is $i$-th row in the matrix A, and $r$ is the total number of constraints (resources).

**Definition 2** (*Required level of resources*). Let $Y \in \Theta$ be a point in the feasible objective region of problem (2), then the required level of resources for the point, denoted by $\beta_Y$, is calculated as follows:

$$\beta_Y = \min_{X\in X_Y^C} \beta_X, \quad X_Y^C = \{X \mid CX = Y\} \tag{5}$$

**Definition 3** (*Resource waste degree (RWD)*). Let $X$ be a feasible solution of problem (2) and $S_X$ its sensitivity region, then the resource waste degree of the portfolio, denoted by $\gamma_X$, is calculated as follows:

$$\gamma_X = \frac{\beta_X - \beta_W}{\beta_X}, \quad \beta_W = \min_{Y'\in S_X} \beta_{Y'} \tag{6}$$

where $\beta_W$ represents the minimum level of resources required to generate the objective value $Y' \in S_X$ in the absence of uncertainty. However, due to the uncertainty, by employing $\beta_X$ level of resources, the objective value $Y'$ is generated rather than the objective value $Y = CX$. Therefore, the waste of resources equals $\gamma_X$. To calculate $\gamma_X$, the equation $Y' = C.X'$ must be solved for each point $Y' \in S_X$, which is time consuming. In the following, a new concept and method for measuring the robustness of a solution will be presented and its relationship with the concept of *resource waste degree*, as defined above, will be examined.

The size of the sensitivity region of a solution (portfolio) defined above is in some way indicative of the degree of robustness of that portfolio against variations of uncertain parameters. The smaller the region is, the more robust it will be. Therefore, the problem of measuring the robustness is in fact the question of measuring the size of the sensitivity region.

**Definition 4** (*Diameter of the sensitivity region*). The diameter of the sensitivity region, denoted by $d(X^0)$, is defined as follows:

$$d(X^0) = \max_{\tilde{Y}\in S(X^0)} U(Y^0 - \tilde{Y}) \tag{7}$$

where $U(v)$: $\mathfrak{R}^m \to \mathfrak{R}$ is a function that measures the magnitude of the vector $v$. Defining the function $U$ in a single-objective space is straightforward. In fact, the function $U$ is the length of the line segment $(Y^0, \tilde{Y})$. Vector norm is the direct extension of the length of a line segment in a single-objective space to multi-objective space. M. Li et al. (2005) used the p-norm function to measure size of the sensitivity region:

$$U(v) = \|v\|_p = \left(\sum_{i=1}^{m} |v_i|^p\right)^{\frac{1}{p}} = \left(\sum_{i=1}^{m} |\tilde{Y}_i - Y_i^0|^p\right)^{\frac{1}{p}} \tag{8}$$

Using the p-norm function to measure the diameter of the sensitivity region in the multi-objective space has two fundamental flaws. First, given that in this method no weight is assigned to the objectives, it is implicitly assumed that all objectives have equal importance for the DMs. This is in contradiction to the philosophy of the posterior preferences decision-making approach. Second, when the absolute value of the difference between the two vectors is applied, then, the positive and negative changes are considered to be the same. In this paper, we introduce the concept of *preferential weights* to build a new $U$ function, which doesn't have the two above-mentioned weaknesses:

$$U(\tilde{Y} - Y^0) = W^*(X^0).\,(\tilde{Y} - Y^0) = \sum_{i=1}^{m} w_i^*(X^0) \times (\tilde{Y}_i - Y_i^0) \tag{9}$$

where $W^*(X^0)$ is the vector of objective's preferential weights for the solution $X^0$. As in our method, we use the posterior preferences decision-making approach, we assume that the decision-makers' preferences are not known to us at the outset. In fact, a set of non-dominated solutions must be produced and presented to the DMs in order to choose between them. As we see later, preferential weights will be used only to calculate the size of the sensitivity region rather than evaluating solutions; therefore, it does not represent the preferences of the DMs.

**Definition 5** (*Preferential weights of a solution*). Let $X^0 \in D$ be a feasible solution of problem (2) and $Y^0 \in \Theta$ its nominal objective values, then, the vector of preferential weights, denoted by $W^*(X^0)$ is obtained by solving the following mathematical programming problem:

$$\begin{aligned} &\min_W \frac{W.Y_W^* - W.Y^0}{W.Y_W^*} \\ &\quad \textit{subject to:} \\ &W.\,Y \leqslant W.\,Y_W^*, \quad Y \in \Theta,\ Y_W^* \in \Theta \end{aligned} \tag{10}$$

where $Y_W^*$ is the optimal solution of problem (2) when the multiple objectives have aggregated into one objective via objective weights $W$. According to this definition, preferential weights is a weight vector, based on which, the utility of a portfolio has the least difference from the most desirable portfolio. In other words, the preferential weights vector of a portfolio is a DMs' preference by whom the portfolio is most likely to be chosen.

Now, let $W.\,Y_W^* = \varkappa$ and $u = \frac{W}{\varkappa}$, then, model (10) transforms to the following model:

$$\begin{aligned} &\max_u h_0 = u.\,Y^0 \\ &\quad \textit{subject to:} \\ &\quad u.\,Y \leqslant 1 \end{aligned} \tag{11}$$

The concept of preferential weights is schematically illustrated in Fig. 1. The figure represents objective space of a problem with PF comprising three line segments of $PF_1$, $PF_2$ and $PF_3$. Let $Y = C.\,X$ be an arbitrary point in the feasible objective region. The sensitivity region $S_X$ is represented by a rectangle. Suppose three DMs with different preferences weights $w^1$, $w^2$ and $w^3$. Let $\eta$ be the maximum allowable difference from the most desirable solutions i.e., $Y^*_{w^i}$, $1 \leqslant i \leqslant 3$.

Since $PF_1$ is the nearest line segment among the line segments $PF_1$, $PF_2$ and $PF_3$ to the point $Y$, then, by increasing $\eta$, the first DM that accepts $Y$ is the DM with the preferences weight of $w^1$. Therefore, the diameter of $S_X$, as a robustness measure, must be judged based on the

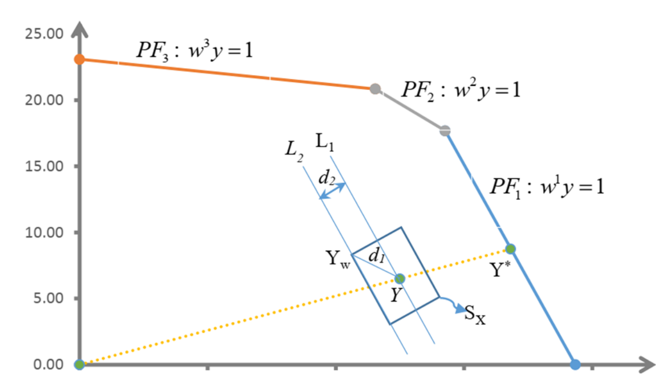


**Fig. 1.** Graphical illustration of the concept of preferential weights, used to measure the diameter of sensitivity region.

weight vector $w^1(d_2 = w^1.(Y - Y_w)$ in Fig. 1, rather than the Euclidian distance $d_1(d_1 = \|Y - Y_w\|_2$ in Fig. 1). It can be seen from Fig. 1 that $d_2 < d_1$.

We are now ready to introduce an index for measuring the robustness of a portfolio:

**Definition 6.** Giving $X^0 \in D$ a feasible solution (portfolio) of problem (2) and $Y^0 \in \Theta$ its nominal objective values, the robustness index of $X^0$, denoted by $r(X^0)$ is defined as:

$$r(X^0) = 1 - \max_{\tilde{Y} \in S_{X^0}} \frac{U(Y^0 - \tilde{Y})}{U(Y^0)} = 1 - \max_{\tilde{Y} \in S_{X^0}} \frac{W^*(X^0).(Y^0 - \tilde{Y})}{W^*(X^0).Y^0} \tag{12}$$

Given that the objective functions and constraints of problem (2) are linear, the resulting PF is comprised of some $(m-1)$-polytopes that are located on bounding hyperplanes $P^i = \{Y \in \Re^m \mid \alpha^i.Y = 1\}$, $1 \leqslant i \leqslant q$. Therefore, let $x^0 \in D$ be a feasible solution of problem (2) and $y^0$ its vector of objective values. Then, for every $i$, $1 \leqslant i \leqslant q$ we have $\alpha^i.y^0 \leqslant 1$, also for some $j$, $1 \leqslant j \leqslant q$, $\alpha^j.y^0 = 1$, if and only if $x^0$ is a non-dominated solution. We define $H^0 = \{y \in \Re^m \mid \alpha^i.y \leqslant 1,\ y \geqslant 0,\ 1 \leqslant i \leqslant q\}$ as a $m$-polytope including the feasible objective region: $\Theta \subseteq H^0$. Let $P'^{\,j} = \{y \in \Re^m \mid y_j = 0\}$, $1 \leqslant j \leqslant m$ then, $P^i, P'^{\,j}$, $1 \leqslant i \leqslant q$, $1 \leqslant j \leqslant m$ are bounding hyperplanes of $H^0$. Let $F^i$, $1 \leqslant i \leqslant q$ denote the facet of $H^0$ that is located on hyperplane $P^i$. Therefore, $F^i$ is a $(m-1)$-polytope with the set of vertices represented by $B^i$. It is evident that $H^0$ can be partitioned into hyper-pyramids $H^i$, $1 \leqslant i \leqslant q$ with $F^i$ as base and origin as apex, Therefore, we have $H^0 = \bigcup_{i=1}^{q} H^i$.

**Theorem 1.** *Let $X^0 \in D$ be a feasible solution (portfolio) of problem (2) and $y^0 \in H^t$ for $t \in \{1, ..., q\}$ its objective values, then, the vector of preferential weights of $X^0$ is equal to the normal vector of hyperplane $P^t$. In other words, $W^*(x^0) = \alpha^t$.*

**Proof.** Appendix A.

**Theorem 2.** *Let $y^0 \in H^t$ be a point in feasible objective region of problem (2) and $x^0 \in X^C_{Y^0}$ a feasible portfolio that $\beta_{y^0} = \beta_{x^0}$, so if $S_{x^0} \subseteq H^t$, then $r(x^0) = 1 - \gamma_{x^0}$.*

**Proof.** Appendix B.

## 4. Proposed solution method

In this section, our proposed *robust multi-objective project portfolio optimization* (RMOPO) algorithm to find the robust optimal portfolios will be presented in detail.

### 4.1. RMOPO algorithm to find the robust optimal portfolios

The RMOPO algorithm (Pseudocode 1) consists of several steps. In the first step, the deterministic case of the problem (1) (i.e., problem (2)) is considered and the set of Pareto solutions $PF$ is generated using the NSGA-III (Deb & Jain, 2013). The input parameters of the algorithm are $N_p$, the population size and $N_g$, the maximum number of generations (iterations). It should be noted that any other efficient algorithm may be considered to generate the set of Pareto solutions. In this paper, we use a metaheuristic algorithm, such as NSGA-III, to produce the PF for two reasons: (1) some large-sized linear and nonlinear problems can only be efficiently solved with metaheuristic algorithms. The results and discussion section will show that even for the problems that can be solved by exact methods, the proposed algorithm is highly accurate and efficient. Indeed, the potential of applying the proposed algorithm to a variety of problems is demonstrated by showing that the algorithm can produce quality robust solutions even if using an estimated PF as an input; (2) even distribution of solutions over the PF are of great significance to solving multi-objective optimization problems (Mirjalili & Lewis, 2015). NSGA-III was chosen since it was one of the most recommended algorithms for generating well-distributed Pareto solutions (Deb & Jain, 2013). This will be verified in the results and discussion section by using the spacing metric, first proposed by Schott (1995).

In the next step, using the set of Pareto solutions, $PF$ as input, the normal vectors $\alpha^i$, $1 \leqslant i \leqslant q$ are approximated by using a sub-algorithm for finding the Pareto frontier's bounding hyper-planes (PFHP). In step 3, the set of vertices $B^i$, $1 \leqslant i \leqslant q$ are calculated using a vertex enumeration algorithm (see Avis & Fukuda, 1992; Dyer, 1983). In step four, new MOP models $\wp^i$, $1 \leqslant i \leqslant q$ are formulated as follows:

$$\begin{aligned}
&\wp^t,\ 1 \leqslant t \leqslant q:\\
&\max\,(C^1X, ..., C^mX)\\
&\quad \textit{subject to}:\\
&\quad AX \leqslant B,\\
&\quad P^tX \leqslant 0,\\
&\quad r(X) \geqslant \eta\\
&0 \leqslant x_i \leqslant 1, \quad 1 \leqslant i \leqslant n
\end{aligned} \tag{13}$$

In the above model, the first and last constraints are the same as the constraints of the main problem (2). The second constraint specifies the hyper-pyramid $H^t$ in the objective space. The hyper-pyramid $H^t$ is in fact the convex hull of points $\{B^t \cup O\}$ where, $O$ represents the origin. Therefore, matrix $P^t$ can be calculated by a convex hull algorithm (see Barber, Dobkin, & Huhdanpaa, 1996; Avis, Bremner, & Seidel, 1997). The robustness index $r(X)$ is incorporated into the model as the third constraint, where, parameter $\eta$ is the minimum allowable robustness of a portfolio to be considered as a robust decision by the DMs.

After solving the problems $\wp^t$, $1 \leqslant t \leqslant q$ using the NSGA-III algorithm, the sets of Pareto solutions $RF^i$, $1 \leqslant i \leqslant q$ are obtained. Let $RF^0 = \bigcup_{i=1}^{q} RF^i$, therefore, all members of $RF^0$ are compared together and dominated ones are eliminated. The remaining set represents the robust Pareto portfolios.

***Pseudocode 1 – RMOPO Algorithm***

1: Generate $PF$, the set of non-dominated solutions of the deterministic problem (2) via NSGA-III,
2: Approximate $\alpha^i$, $1 \leqslant i \leqslant q$, the normal vectors of $P^i$, $1 \leqslant i \leqslant q$, bounding hyperplanes of the PF, via PFHP algorithm and using the set $PF$ as input,
3: Find $B^i$, $1 \leqslant i \leqslant q$, the set of vertices of $F^i$, the $(m-1)$-polytopes located on hyperplane $P^i$, $1 \leqslant i \leqslant q$ via a vertex enumeration algorithm,
4: Construct new deterministic multi-objective problems $\wp^i$, $1 \leqslant i \leqslant q$ by adding the linear constraints, witch specify the hyper-pyramids $H^i$, $1 \leqslant i \leqslant q$ in the objective space, to the problem (2) and adding the robustness constraint,
5: Generate sets of Pareto solutions $RF^i$, $1 \leqslant i \leqslant q$ of problems $\wp^i$, $1 \leqslant i \leqslant q$ via NSGA-III.
6: $RF^0 \leftarrow \bigcup_{i=1}^{q} RF^i$
7: Eliminate all dominated solutions from $RF^0$. The result is robust Pareto frontier (RPF) of problem (1).

#### 4.1.1. PFHP algorithm for approximating the normal vector of bounding hyperplanes of Pareto frontier

In this algorithm (Pseudocode 2), the normal vector of bounding hyperplanes of the PF, generated by the NSGA-III algorithm, are approximated. For this purpose, an arbitrary point like $y^\kappa$ is chosen from the set of Pareto solutions $PF$. Then, all the members of the set $PF$ are sorted by Euclidean distance from the selected point $y^\kappa$. Let $\mathfrak{I}$ denote the set of $n_R$ closest points to point $y^\kappa$. Now, we want to fit a hyperplane $P(\vartheta)$, with $\vartheta$ as the normal vector, to the points $y \in \mathfrak{I}$. Knowing the fact that fitting a hyperplane to a set of points in $m$-dimension space is equivalent to estimating the coefficients of multiple linear regression model with $(m-1)$ independent predictor variables, the least square estimator of multiple linear regression model (see Montgomery, Peck, & Vining, 2012) is used to fit a hyperplane like $P(\vartheta)$ to the points within the set $\mathfrak{I}$. For any $y \in \mathfrak{I}$ let $y_m$ be the response variable and $y_2, \ldots, y_{m-1}$ be the predictor variables, therefore, the multiple linear regression model will be as follows:

$$y_m = \beta_0 + \beta_1 y_1 + \ldots + \beta_{m-1} y_{m-1} + \varepsilon \tag{14}$$

Let hyperplane $P(\vartheta)$ be defined as $\vartheta \cdot y = 1$ then:

$$\sum_{i=1}^{m} \vartheta_i y_i = 1 + \varepsilon(\vartheta, y) \Rightarrow y_m = \frac{1}{\vartheta_m} - \frac{\vartheta_1}{\vartheta_m} y_1 - \cdots - \frac{\vartheta_{m-1}}{\vartheta_m} y_{m-1} + \frac{1}{\vartheta_m}\varepsilon(\vartheta, y), \tag{15}$$

where $\varepsilon(\vartheta, y)$ is the estimation error. By setting $\vartheta_m = \frac{1}{\beta_0}$, $\vartheta_{m-1} = -\frac{\beta_{m-1}}{\beta_0}$, ..., $\vartheta_1 = -\frac{\beta_1}{\beta_0}$ the approximate normal vector $\vartheta$ can be obtained from the estimation of regression coefficients. Obviously, if all of the points in the set $\mathfrak{I}$ are located on the same hyperplane $P^t$(i.e., $\mathfrak{I} \subset F^t$), then the vector $\vartheta$ will be in fact an estimate for the normal vector $\alpha^t$. Therefore, if the *R-squared* value is less than the predetermined parameter $R_{\min}$, then it is concluded that the points are not located on the same hyperplane, and thus, $y^\kappa$ is replaced with another point from the set *PF* and the steps above are repeated. Otherwise, the set of points $\mathfrak{I}$ is regarded to be located on the hyperplane $P^t$ and is used by a simulated annealing algorithm to improve the accuracy of estimated normal vectors (SANV). The threshold *s* is defined as multiplication of the mean squared error and *sc,* a parameter of the algorithm. If the estimation error of a point is less than the threshold *s* then the point is regarded to be on the fitted hyperplane.

***Pseudocode 2 – PFHP Algorithm***

**Input:** $\{PF,\ n_R,\ R_{\min},\ sc\}$
**Output:** $\alpha^t,\ 1 \leqslant t \leqslant q$
Initialize variables: $\Omega \leftarrow PF,\ \kappa \leftarrow 1,\ n \leftarrow n_0,\ i \leftarrow 1$
**While** $(\kappa \leqslant n\ \&\ n \geqslant n_R)$ **do**
  Sort $\Omega$ by Euclidian distance from the point $y^\kappa \in \Omega$ and put number of $n_R$ nearest points to $y^\kappa$ into the set $\mathfrak{I}$
  Fit hyperplane $P(\vartheta)$ to the points within the set $\mathfrak{I}$, via multiple linear regression model. Outputs are normal vector $\vartheta$ and $R-square$ value.
  **If** $R-square \geqslant R_{\min}$ **then**
    $s \leftarrow sc \times \sqrt{\left(\sum_{y\in\mathfrak{I}} \varepsilon(\vartheta, y)^2/(n_R - 1)\right)} = sc \times \sqrt{\left(\sum_{y\in\mathfrak{I}} (\vartheta.\ y - 1)^2/(n_R - 1)\right)}$
    Run the sub-algorithm SANV for the inputs $(\Omega, \vartheta, s)$ to improve the accuracy of estimated normal vector of hyperplane $P(\vartheta)$. The results are the set of points $\mathfrak{I}^*$ and the normal vector $\vartheta^*$.
    $\alpha^i \leftarrow \vartheta^* / \max_{y\in\Omega} \vartheta^*.\ y.$
    Remove the points belonging to $\mathfrak{I}^*$ from $\Omega$. $(\Omega \leftarrow \Omega - \mathfrak{I}^*)$
    $i \leftarrow i + 1,\ n \leftarrow n - |\mathfrak{I}^*|$
  **Else**
    $\kappa \leftarrow \kappa + 1$
  **End if**
**End while**

In the next step, accuracy of the estimation of normal vector $\vartheta$ will be improved via an efficient simulated annealing metaheuristic (Pseudocode 3). The objective function to be maximized in this algorithm is the number of points that are on the fitted hyperplane, or, in other words, the number of points with estimation error less than the threshold *s*. Let $\Omega \subset PF$ denote the set of unassigned Pareto solutions and $\mathfrak{I}^* = \{y \in \Omega \mid |\varepsilon(\vartheta, y)| \leqslant s\}$ represent the set of unassigned Pareto solutions on the fitted hyperplane $P(\vartheta)$. Then, the objective function denoted by $\ell(\vartheta)$ is equal to $|\mathfrak{I}^*|$. In the following, how to move from one solution to another solution in the neighborhood is described.

First, $i, j,\ i \neq j$ are chosen randomly from the set $\{1, \ldots, m\}$. Then, the $i$-th and $j$-th components of vector $\vartheta$ are changed as follows: a random number like $\Delta_i \in (0, \vartheta_i \times \delta)$ is generated ($\delta$ is a parameter of the algorithm) and added to $\vartheta_i$, therefore, a new normal vector $\vartheta'$ is resulted. Then, $\varepsilon(\vartheta', y)$ is calculated for all $y \in \Omega$. Now, suppose that the vector $\vartheta''$ is the same as the vector $\vartheta'$ except that $\Delta_j \geqslant 0$ is subtracted from the $j$-th component of the normal vector $\vartheta'$:

$$\vartheta''_k = \begin{cases} \vartheta'_k, & k \neq j \\ \vartheta'_k - \Delta_j, & k = j \end{cases} \tag{16}$$

So, for any $y \in \Omega$ we have:

$$\varepsilon(\vartheta'', y) = \vartheta''.\ y - 1 = (\vartheta'.\ y - 1) - \Delta_j \times y_j = \varepsilon(\vartheta', y) - \Delta_j \times y_j \tag{17}$$

By definition, the point $y \in \Omega$ is regarded to be on the hyperplane $P(\vartheta'')$ if and only if:

$$\begin{gathered} |\varepsilon(\vartheta'', y)| \leqslant s, \Rightarrow -s \leqslant \varepsilon(\vartheta'', y) \leqslant s \Rightarrow -s \leqslant \varepsilon(\vartheta', y) - \Delta_j \times y_j \leqslant s \\ \Rightarrow \varepsilon(\vartheta', y) - s \leqslant \Delta_j \times y_j \leqslant \varepsilon(\vartheta', y) + s \Rightarrow ((\varepsilon(\vartheta', y) - s)/y_j) \leqslant \Delta_j \leqslant \\ ((\varepsilon(\vartheta', y) + s)/y_j) \end{gathered} \tag{18}$$

Let define $\psi^j_{\min}(\vartheta', y) = ((\varepsilon(\vartheta', y) - s)/y_j)$ and $\psi^j_{\max}(\vartheta', y) = ((\varepsilon(\vartheta', y) + s)/y_j)$, then, we have:

$$\psi^j_{\min}(\vartheta', y) \leqslant \Delta_j \leqslant \psi^j_{\max}(\vartheta', y) \tag{19}$$

Now, the goal is to find $\Delta_j$ that maximize the objective function $\ell(\vartheta'')$. For this purpose, for any $y' \in \Omega$ the set $\zeta(\vartheta', y, j)$ is defined as follows:

$$\zeta(\vartheta', y, j) = \{y' \in \Omega \mid \psi^j_{\min}(\vartheta', y') \leqslant \mu < \psi^j_{\max}(\vartheta', y')\},\ \mu = \psi^j_{\min}(\vartheta', y) \tag{20}$$

Let $y^*$ be the optimum solution of the problem $\underset{y\in\Omega}{Max}\ |\zeta(\vartheta', y, j)|$ then, set $\Delta_j$ as below:

$$\Delta_j = \left(\underset{y\in\zeta(\vartheta', y^*, j)}{Max} \psi^j_{\min}(\vartheta', y) + \underset{y\in\zeta(\vartheta', y^*, j)}{Min} \psi^j_{\max}(\vartheta', y)\right)/2 \tag{21}$$

**Theorem 3 (.).**

$\ell(\vartheta'') = |\zeta(\vartheta', y^*, j)|.$

**Proof.** By definition, the inequality $\psi^j_{\min}(\vartheta', y') \leqslant \Delta_j \leqslant \psi^j_{\max}(\vartheta', y')$ holds for any $y' \in \zeta(\vartheta', y^*, j)$. On the other hand, if there exist some $y'$, that for which the above inequality holds, but $y' \notin \zeta(\vartheta', y^*, j)$, then by definition is:

$$\psi^j_{\min}(\vartheta', y^*) < \psi^j_{\min}(\vartheta', y') \Rightarrow \forall\ y \in \zeta(\vartheta', y^*, j),\ \psi^j_{\min}(\vartheta', y) < \psi^j_{\min}(\vartheta', y') \tag{22}$$

On the other hand, according to the definition of $\Delta_j$ we have:

$$\Delta_j < \min_{y\in\zeta(\vartheta', y^*, j)} \psi^j_{\max}(\vartheta', y) \tag{23}$$

Since $\psi^j_{\min}(\vartheta', y') \leqslant \Delta_j$ then:

$$\forall\ y \in \zeta(\vartheta', y^*, j),\ \psi^j_{\min}(\vartheta', y') < \psi^j_{\max}(\vartheta', y) \tag{24}$$

$$\begin{gathered} \Rightarrow \forall\ y \in \zeta(\vartheta', y^*, j),\ \psi^j_{\min}(\vartheta', y) < \psi^j_{\min}(\vartheta', y') < \psi^j_{\max}(\vartheta', y) \Rightarrow y \\ \in \zeta(\vartheta', y', j) \end{gathered} \tag{25}$$

$$\Rightarrow \zeta(\vartheta', y^*, j) \subset \zeta(\vartheta', y', j) \Rightarrow |\zeta(\vartheta', y^*, j)| < |\zeta(\vartheta', y', j)| \tag{26}$$

That is in contradiction to the assumption that $y^*$ is the optimum solution of $\max_{y\in\Omega} |\zeta(\vartheta', y, j)|$. Therefore, there is no $y' \notin \zeta(\vartheta', y^*, j)$ for which inequality $\psi^j_{\min}(\vartheta', y') \leqslant \Delta_j \leqslant \psi^j_{\max}(\vartheta', y')$ holds, thus, $\ell(\vartheta'') = |\zeta(\vartheta', y^*, j)|$.

***Pseudocode 3 – SANV Sub-algorithm***

**Input:** $\{\Omega, \vartheta, s, T_0, T_{\min}, \Delta\tau, \eta, \delta\}$
**Output:** $\{\mathfrak{I}^*, \vartheta^*\}$
Initialize variables: $\vartheta^* \leftarrow \vartheta,\ n \leftarrow |\Omega|,\ t \leftarrow T_0$
$\mathfrak{I}^* \leftarrow \{y \in \Omega \mid |\varepsilon(\vartheta^*, y)| \leqslant s\},\ \varepsilon(\vartheta^*, y) = \vartheta^*.\ y - 1$
$\ell^* \leftarrow |\mathfrak{I}^*|,\ \ell \leftarrow |\mathfrak{I}^*|$
**While** $t > T_{\min}$ **do**
  $N \leftarrow 0$
  $\vartheta' \leftarrow \vartheta$

***Pseudocode 3 – SANV Sub-algorithm***

**While** $N \leqslant \eta$ **do**
  Select $i, j,\ i \neq j,\ i, j \in \{1, \ldots, m\}$ randomly
  Generate random number $\Delta_i \in (0, \vartheta_i' \times \delta)$
   $\vartheta_i' \leftarrow \vartheta_i' + \Delta_i$
     For all $y \in \Omega$ compute $\varepsilon(\vartheta', y) = \vartheta'.y - 1$
   For all $y \in \Omega$ compute:
 $\psi_{\min}^{j}(\vartheta', y) = (\varepsilon(\vartheta', y) - s)/y_j,\ \psi_{\max}^{j}(\vartheta', y) = (\varepsilon(\vartheta', y) + s)/y_j$
     For all $y \in \Omega$ compute:
 $\zeta(\vartheta', y, j) = \{y' \in \Omega |\ \psi_{\min}^{j}(\vartheta', y') \leqslant \mu < \psi_{\max}^{j}(\vartheta', y')\},\ \mu = \psi_{\min}^{j}(\vartheta', y)$
     Find $y^*$, the optimal solution of $\max_{y \in \Omega}\ \ell' = |\zeta(\vartheta', y, j)|$
   $\Delta_j \leftarrow (\max_{y \in \zeta(\vartheta', y^*, j)} \psi_{\min}^{j}(\vartheta', y) + \min_{y \in \zeta(\vartheta', y^*, j)} \psi_{\max}^{j}(\vartheta', y))/2$
     $\vartheta_j' \leftarrow \vartheta_j' - \Delta_j$
     **If** $\ell' > \ell$ **then**
      $\Im \leftarrow \zeta(\vartheta', y^*, j),\ \vartheta \leftarrow \vartheta',\ \ell \leftarrow \ell'$
       $N \leftarrow N + 1$
    **If** $\ell' > \ell^*$ **then**
        $\Im^* \leftarrow \zeta(\vartheta', y^*, j),\ \vartheta^* \leftarrow \vartheta',\ \ell^* \leftarrow \ell'$
       **End if**
     **Else if** $r < \exp((\ell' - \ell)/t)$ ($r$ is a random number between 0 and 1)
       $\Im \leftarrow \zeta(\vartheta', y^*, j),\ \vartheta \leftarrow \vartheta',\ \ell \leftarrow \ell'$
       $N \leftarrow N + 1$
    **End if**
  **End While**
    $t \leftarrow t - \Delta\tau$
**End While**

## 5. Results and discussion

In this section, using a case study in port industry and two other numerical example, including a nonlinear problem, the efficiency of the proposed algorithm is examined.

### *5.1. A numerical example for PFHP sub-algorithm*

The most important part of the RMOPO algorithm for finding robust Pareto portfolios is the PFHP sub-algorithm, a heuristic based on the well-known simulated annealing metaheuristic, which approximates the normal vectors of bounding hyperplanes of the PF of a deterministic problem. The details of this algorithm were described in the previous section. In the following, using a numerical example, the efficiency of the sub-algorithm is evaluated.

Consider a linear programming problem with two decision variables and two objective functions as follows:

$$\begin{aligned} &\max\ f_1 = x_1 \\ &\max\ f_2 = x_2 \\ &\textit{Subject to:} \\ &0.5x_1 + x_2 \leqslant 1 \\ &1.83x_1 + 0.92x_2 \leqslant 1 \\ &2.79x_1 + 0.8x_2 \leqslant 1 \\ &3.54x_1 + 0.64x_2 \leqslant 1 \\ &4.12x_1 + 0.46x_2 \leqslant 1 \\ &4.64x_1 + 0.19x_2 \leqslant 1 \\ &x_1, x_2 \leqslant 1 \\ &x_1, x_2 \geqslant 0 \end{aligned} \tag{27}$$

In Fig. 2, the exact PF of problem (27) and the PF obtained through solving the problem by NSGA-III are compared. As shown in Fig. 2, the PF consists of six line segments (hyperplanes). In the next step of the algorithm, the Pareto solutions are used as inputs of the PFHP sub-algorithm to approximate the coefficients $\alpha_1$, $\alpha_2$(normal vector) of the six line segments defined by equation $\alpha_1 y_1 + \alpha_2 y_2 = 1$.

The results of the PFHP algorithm for the numerical example are summarized in Table 1. As shown in Table 1, the proposed algorithm has a very high accuracy in the approximation of the coefficients.

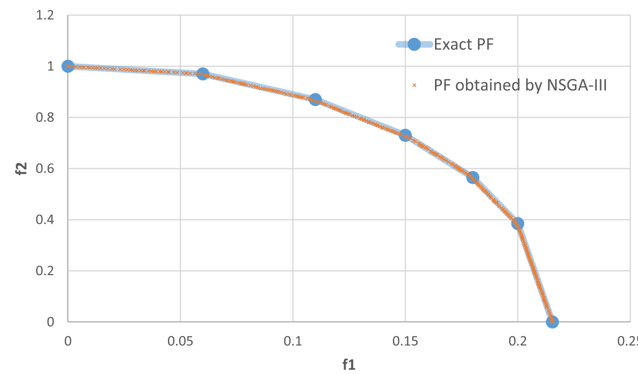


**Fig. 2.** . The PFs of example 1, obtained by exact method and NSGA-III.

### *5.2. Case study*

A company, as an operator of a medium-sized container terminal, is required to improve the efficiency of terminal operations and service quality to be granted a long-term concession. A container terminal is a place for loading and unloading containers to/from ships and storage and other value-added logistics services. Container terminal consists of two main parts: docks and container yard.

In the quay, the loading and unloading of the containers to/from a ship is carried out using gantry cranes. Storage and value-added services are done in the container yard. According to Tongzon (2009), port efficiency (speed and reliability of port services) is the most important factor influencing port choice from the Southeast Asian freight forwarders' perspective. The speed and quality of port operations reduce the ship turnaround time, which subsequently reduces the total shipping time, thus, reducing the shipping costs (see Clark, Dollar, & Micco, 2004; Sánchez et al., 2003).

Numerous indicators have been proposed to measure port efficiency. Tongzon and Ganesalingam (1994) have classified the indicators of port efficiency into two groups of operational efficiency measures and customer-oriented measures. One of the most important indicators for measuring the speed and efficiency of terminal operations is *ship rate* or *lifts per hour*. This measure is equal to the average number of containers loaded and unloaded to/from a container-ship in one hour.

One of the primary ways to increase *ship rate* is to increase the length of quays, the number of gantry cranes and other related equipment. Developing port infrastructure and superstructure requires very high investment. Therefore, if the market share of a terminal does not develop in proportion to its capacity expansion, it will reduce the return on capital and resource utilization index.

Therefore, the rational way to increase the efficiency and effectiveness, and consequently, the competitiveness of the terminal, is to increase productivity through improving resource utilization. Concerning the case study, there are three different approaches available for the top management to improve productivity:

- Project I: Upgrading the terminal operating system (TOS),
- Project II: Integrated operational planning through data interchange with other nodes of the supply chain, and
- Project III: Implementing a comprehensive repair and maintenance system for terminal equipment.

DMs consider two different objectives as criteria for funding the projects: net present value of the investment as a financial measure and ship rate as an operational measure. Limitations imposed on the case study can be categorized in three general constraints: total investment up to \$4.1 million; first-year budget up to \$1.45 million; and lack of specialist staff required to run the project; namely a maximum of six work teams of three people is needed. All projects can be partially

**Table 1**
Coefficients of the six line segments of the PF of example 1.

| Line coefficients $\alpha_1$, $\alpha_2$, obtained by exact method | | Line coefficients $\alpha_1$, $\alpha_2$, obtained by PFHP algorithm | | Estimation error | | The number of solutions found on each line segment using the PFHP algorithm-$\ell(\vartheta)$ | The exact number of solutions on each line segment |
|---|---|---|---|---|---|---|---|
| $\alpha_1$ | $\alpha_2$ | $\alpha_1$ | $\alpha_2$ | $\alpha_1$ | $\alpha_2$ | | |
| 0.5000 | 1.0000 | 0.5027 | 0.9999 | 0.5% | 0.0% | 42 | 45 |
| 1.8349 | 0.9174 | 1.8428 | 0.9166 | 0.4% | 0.1% | 57 | 50 |
| 2.7888 | 0.7968 | 2.7917 | 0.7963 | 0.1% | 0.1% | 47 | 50 |
| 3.5370 | 0.6431 | 3.5376 | 0.6430 | 0.0% | 0.0% | 50 | 43 |
| 4.1190 | 0.4577 | 4.1102 | 0.4617 | 0.2% | 0.9% | 35 | 42 |
| 4.6425 | 0.1857 | 4.6426 | 0.1856 | 0.0% | 0.0% | 66 | 70 |

**Table 2**
Resources required for full implementation of projects I, II, and III.

| Resource/Project | Project I | Project II | Project III |
|---|---|---|---|
| Total Investment – M$ | 1.2 | 3.8 | 2 |
| first-year budget – M$ | 1.1 | 0.9 | 1.05 |
| Number of teams | 6 | 2 | 4 |

**Table 3**
Objective payoffs for projects I, II, and III.

| Objective/Project | Project I | Project II | Project III |
|---|---|---|---|
| NPV – M$ | 0.6 | 2.8 | 1.9 |
| Ship rate - Lifts/Hour | 22.5 | 5 | 6.5 |

funded. Tables 2 and 3, respectively, represent resources required for full implementation of the projects and objective payoffs.

Now, we focus on the main subject of this research, uncertainty. For this purpose, we first define the uncertainties. In this case, there are two sources of uncertainty: uncertainty of coefficients of the objective functions and uncertainty of implementation of the projects. It is assumed that coefficients of the objective functions vary within intervals. In other words, if the random variable $\tilde{c}_{ij}$, $1 \leqslant i \leqslant 2$, $1 \leqslant j \leqslant 3$ represents the coefficient of the $j$th project in the $i$th objective function, then, we have: $\tilde{c}_{ij} \in (c_{ij} - d_{ij}c_{ij}, c_{ij} + d_{ij}c_{ij})$. Where, $d_{ij}$, $1 \leqslant i \leqslant 2$, $1 \leqslant j \leqslant 3$ are elements of matrix $D$ (Table 4), which is calculated using the Delphi method.

Thus, the radius of the intervals denoted by matrix $\Delta_C$ is calculated as follows:

$$\Delta_C = [d_{ij}c_{ij}],\ \ 1 \leqslant i \leqslant 2,\ \ 1 \leqslant j \leqslant 3 \tag{28}$$

The second source of uncertainty is uncertainty about the success rate of executing the planned level of the projects. $x_i$, $1 \leqslant i \leqslant 3$ as decision variable, represents a fraction of the project $i$ that is funded and planned to be implemented. It is expected that the objective payoffs will be gained in proportion to this fraction. However, in practice, since these projects are innovative, the results are subject to uncertainty; thus, the actual achieved level of the projects, denoted by $\widehat{x_i}$ are different from the planned level $x_i$. It is supposed that $\widehat{x_i} \in (x_i - \Delta_i, x_i + \Delta_i)$ in which $\Delta_i = \hbar_i \times x_i^2$ . In other words, the more a project is planned to be implemented, the greater the uncertainty will be. For the case study, we have $\hbar = (20\%, 15\%, 10\%)^t$.

Based on the assumptions of Tables 2 and 3, the deterministic MOLP model of the case study is as follows:

**Table 4**
Uncertainty matrix of coefficients of objective functions.

| Objective/Project | Project I | Project II | Project III |
|---|---|---|---|
| $f_1$ | 3% | 6% | 4% |
| $f_2$ | 3% | 4% | 3% |

$$\begin{aligned}
\max f_1 &= 0.6x_1 + 2.8x_2 + 1.9x_3\\
\max f_2 &= 22.5x_1 + 5x_2 + 6.5x_3\\
&\textit{Subject to:}\\
1.2x_1 &+ 3.8x_2 + 2x_3 \leqslant 4.1\\
1.1x_1 &+ 0.9x_2 + 1.05x_3 \leqslant 1.45\\
6x_1 &+ 2x_2 + 4x_3 \leqslant 6\\
&x_1, x_2 \leqslant 1\\
&x_1, x_2 \geqslant 0
\end{aligned} \tag{29}$$

Based on our experience in a local container terminal operator company and the shipping industry, the number of candidate innovative projects to be optimized is usually less than 20 projects (e.g., Pozzi, Noè, Lazzarotti, & Rossi, 2015; Oktavera & Saraswati, 2012). Nevertheless, to evaluate the performance of the proposed algorithm for larger-sized problems, we extend the case study problem into a larger problem with 30 projects. Thus, we have replaced each of the three projects with 10 hypothetical projects. In fact, each of the three projects is the weighted sum of the 10 new projects, with the sum of the weights equal to one. The input matrix of the new larger-sized problem is given in Appendix D. In the remainder of this sub-section, the results of solving the two problems are distinguishable by the titles '$n$ = 3' and '$n$ = 30'.

In Fig. 3, the PF of the case study (29) obtained by NSGA-III is depicted (step 1 of the RMOPO algorithm). We also generated the PF by the ε - constraint method (Haimes, Lasdon, & Wismer, 1971), based on MATLABs *Linprog* routine that uses linear programming techniques. We compared the uniformity of the two PFs based on the spacing metric. The spacing metric, which was proposed by Schott (1995), is a coverage performance indicator that measures the uniformity of the PF produced by an algorithm based on the variance of the Pareto-optimal solutions. With the same number of solutions, the lower values of the metric indicate the better distributions of solutions over the PF. The spacing metric values of the two PFs generated by NSGA-III and the exact method are 0.046 and 0.114, respectively. Thus, we can conclude that the PF of NSGA-III is more uniformly distributed than the PF of the exact method. As shown in Fig. 3, the PF consists of three line segments

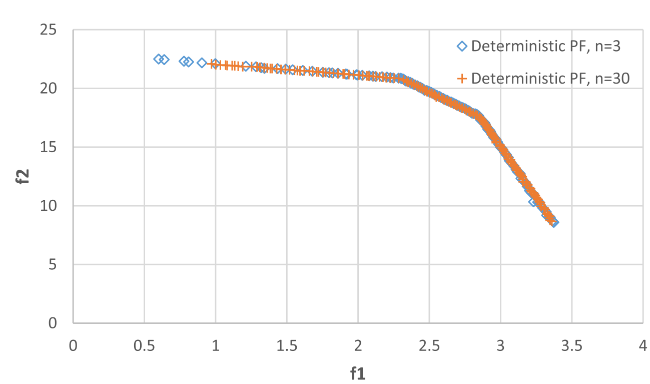


**Fig. 3.** . Deterministic PF of the case study (problem (29)) and the equivalent larger-sized problem, obtained by NSGA-III.

**Table 5**
The coefficients of line segments of PF of the problem (29) and the equivalent larger-sized problem, estimated by PFHP algorithm.

| Line segment | Exact value | | $n = 3$ | | | | $n = 30$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | Estimation | | Error-% | | Estimation | | Error-% | |
| | $\alpha_1$ | $\alpha_2$ | $\alpha_1$ | $\alpha_2$ | $\alpha_1$ | $\alpha_2$ | | | | |
| 1 | 0.0417 | 0.0433 | 0.0418 | 0.0433 | 0.41% | 0.04% | 0.0416 | 0.0433 | 0.17% | 0.02% |
| 2 | 0.1695 | 0.0292 | 0.1695 | 0.0292 | 0.00% | 0.00% | 0.1713 | 0.0290 | 1.02% | 0.64% |
| 3 | 0.2584 | 0.0149 | 0.2585 | 0.0149 | 0.06% | 0.18% | 0.2573 | 0.0151 | 0.41% | 1.43% |

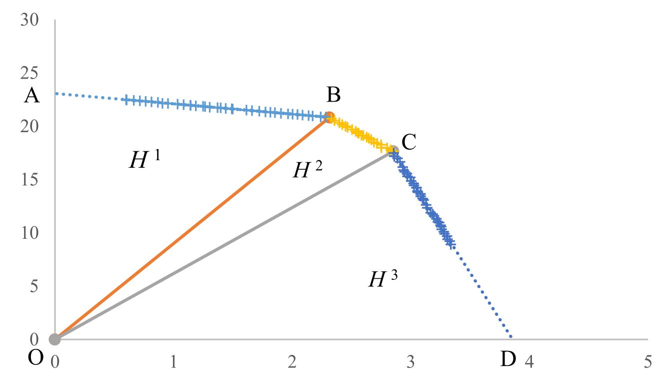


**Fig. 4.** . Partitioning of the extended feasible objective region of the problem (29) into three hyper-pyramids (triangle) of $H^1$, $H^2$, $H^3$, with bases located on the bounding hyperplanes of the PF.

defined by equation $\alpha_1 y_1 + \alpha_2 y_2 = 1$. The exact value of the coefficients $\alpha_1$, $\alpha_2$, as well as their estimated values by the PFHP sub-algorithm (Step 2 of the RMOPO algorithm), are summarized in Table 5.

According to the results presented in Tables 1 and 5, we conclude that the PFHP algorithm has been able to accurately estimate the coefficients $\alpha_1$, $\alpha_2$(Maximum estimation errors for '$n = 3$' and '$n = 30$' are 0.41% and 1.43%, respectively). As these estimations are used to calculate the robustness index, the accuracy is a critical component of the efficiency of the proposed algorithm.

As it can be seen in Fig. 4, the extended feasible objective region of the problem (29) is partitioned into three triangles of (2- dimensional hyper-pyramids) $H^1$, $H^2$ and $H^3$ with the bases $F^1$, $F^2$ and $F^3$ (Table 6) on which, the Pareto solutions are located. Based on step 3 of the RMOPO algorithm, the set of vertices $B^1 = \{A, B\}$, $B^2 = \{B, C\}$ and $B^3 = \{C, D\}$ are calculated by a vertex enumeration algorithm (Fig. 4). These triangles specify the feasible region of three multi-objective problems $\wp^i$, $1 \leqslant i \leqslant 3$ (step 4 of the RMOPO algorithm).

According to the definition of robustness index (Definition 6), robustness of a portfolio like $X = (x_1, x_2, x_3)$ is equal to:

$$r(X) = 1 - \max_{\tilde{Y} \in S(X)} \frac{\alpha^t (CX - \tilde{C}\tilde{X})}{\alpha^t CX} = 1 - \frac{\alpha^t (CX - (C - \Delta_C)(X - \Delta_x))}{\alpha^t CX} = 1 - \frac{\alpha^t (C\Delta_x + \Delta_C X - \Delta_C \Delta_x)}{\alpha^t CX} \quad (30)$$

Therefore, the portfolio $X$ is regarded as a robust portfolio if and only if:

$$r(X) \geqslant \eta, \Rightarrow 1 - \frac{\alpha^t (C\Delta_x + \Delta_C X - \Delta_C \Delta_x)}{\alpha^t CX} \geqslant \eta \quad (31)$$

$$\Rightarrow \frac{\alpha^t (C\Delta_x + \Delta_C X - \Delta_C \Delta_x)}{\alpha^t CX} \leqslant (1 - \eta) \quad (32)$$

$$\Rightarrow \alpha^t (C\Delta_x + \Delta_C X - \Delta_C \Delta_x) \leqslant (1 - \eta)\alpha^t CX \quad (33)$$

$$\Rightarrow \alpha^t (\Delta_C - (1 - \eta)C)X + \alpha^t (C - \Delta_C)\Delta_x \leqslant 0 \quad (34)$$

The nonlinear (quadratic) constraint (34) is added as the robustness constraint to the models $\wp^i$, $1 \leqslant i \leqslant 3$. In this case, the parameter $\eta$ is set to 0.85 and the models $\wp^i$, $1 \leqslant i \leqslant 3$ are formulated as models (C.1), (C.2), and (C.3) in Appendix C, and each one is solved by NSGA-III. Therefore, three sets of PF are generated and aggregated into one set. Then, dominated portfolios are eliminated, and remaining portfolios comprise the so-called RPF (Fig. 5). RPF of the case study for different values of the robustness threshold $\eta$ is plotted in Fig. 6. In addition, Fig. 7 shows the fraction of each project in the portfolio to be funded (decision space), which corresponds to the portfolios belonging to the RPF, as depicted in Fig. 6 (objective space).

According to Theorem 2, we know that the robustness index has an inverse relationship with the resource waste degree. For example, if the robustness index of a portfolio is equal to 0.85, then the resource waste degree of that portfolio will be 0.15 ($0.15 = 1 - 0.85$). Therefore, since the DMs have more insight into the concept of resource waste degree than the objective robustness, subsequently, they have clearly understood the meaning of robustness threshold $\eta$. Accordingly, through the robustness threshold $\eta$, they can incorporate their risk preferences into the decision-making process more confidently.

Minimum and maximum values of decision variables of robust Pareto solutions of the case study for different values of the robustness threshold $\eta$ is presented in Table 7. By increasing the robustness threshold $\eta$, two trends can be deduced from the results represented in Table 7. The first trend is about the range of the objectives variations over the PF. In other words, if the DMs are stricter with robustness (larger values of $\eta$), the minimum and maximum values of projects

**Table 6**
Coordinates (objective values) of the set of vertices $F^1$, $F^2$ and $F^3$ of the PF of problem (29).

| Segment | Vertex | $f_1$ | $f_2$ |
|---|---|---|---|
| 1 | A | 0.00 | 23.09 |
| | B | 2.31 | 20.84 |
| 2 | B | 2.31 | 20.84 |
| | C | 2.85 | 17.68 |
| 3 | C | 2.85 | 17.68 |
| | D | 3.87 | 0.00 |

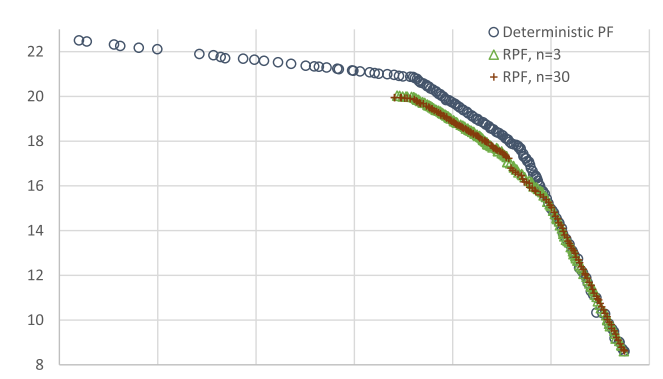


**Fig. 5.** . The deterministic PF and the RPF of the case study (problem (29)) and the equivalent larger-sized problem, obtained respectively by NSGA-III and RMOPO algorithm for the robustness threshold $\eta = 0.85$.

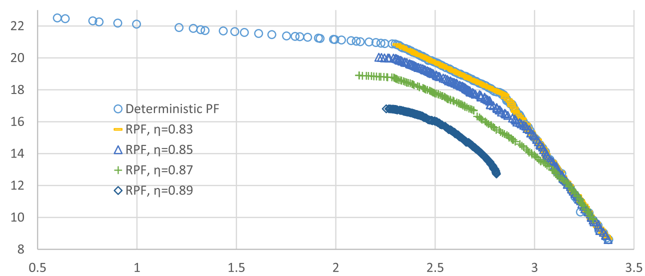


**Fig. 6.** . Comparison of the RPFs of the case study for different values of the robustness threshold $\eta$.

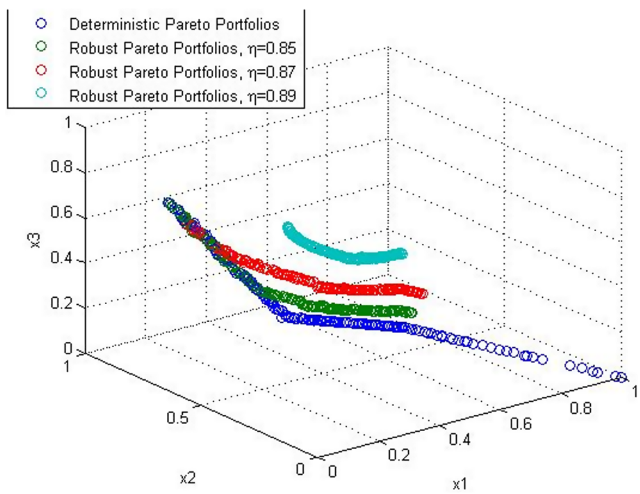


**Fig. 7.** . Robust Pareto portfolios of the case study in the decision space, for different values of the robustness threshold $\eta$.

**Table 7**
Minimum and maximum values of decision variables of robust Pareto solutions of the case study for different values of the robustness threshold $\eta$

| Robustness Threshold | $x_1$ | | $x_2$ | | $x_3$ | |
|---|---|---|---|---|---|---|
| | *Min* | *Max* | *Min* | *Max* | *Min* | *Max* |
| $\eta = 0.80$ | 0.02 | 0.92 | 0.23 | 0.89 | 0.00 | 0.80 |
| $\eta = 0.81$ | 0.00 | 0.89 | 0.34 | 0.89 | 0.00 | 0.82 |
| $\eta = 0.82$ | 0.00 | 0.84 | 0.49 | 0.89 | 0.00 | 0.83 |
| $\eta = 0.83$ | 0.00 | 0.78 | 0.64 | 0.89 | 0.00 | 0.82 |
| $\eta = 0.84$ | 0.00 | 0.76 | 0.61 | 0.86 | 0.02 | 0.83 |
| $\eta = 0.85$ | 0.00 | 0.73 | 0.55 | 0.83 | 0.10 | 0.83 |
| $\eta = 0.86$ | 0.00 | 0.70 | 0.49 | 0.80 | 0.17 | 0.82 |
| $\eta = 0.87$ | 0.08 | 0.66 | 0.41 | 0.74 | 0.26 | 0.70 |
| $\eta = 0.88$ | 0.14 | 0.60 | 0.37 | 0.64 | 0.36 | 0.75 |
| $\eta = 0.89$ | 0.26 | 0.51 | 0.30 | 0.50 | 0.50 | 0.72 |

$x_1$, $x_2$, and $x_3$ are closer and subsequently, the choices for the DMs are more limited. The second trend is about the fraction of the projects to be funded. In short, to be more robust, less fraction of the projects have to be funded. For example, based on Theorem 2, for $\eta = 0.89$, the maximum allowable recourse waste degree is up to 0.11 $(1 - 0.89 = 0.11)$. Therefore, according to Table 7, the maximum fraction of implementation is up to 72% (project III); thus, the remaining resources can be allocated to the next priorities in other terminals.

To solve the two mentioned problems above, by using the algorithm RMOPO, the input parameters of the algorithm are set in Table 8. This algorithm is coded in MATLAB 7.9.0, and all computational experiments are performed on a PC Intel® Core™ i7@ 3.6 GHz processor with 4 GB RAM. For the case study and the equivalent larger-sized problem the average run time of the algorithm with the settings mentioned above were 67 and 353 s, respectively.

**Table 8**
Input parameters of RMOPO algorithm.

| Parameter | $N_p$ | $N_g$ | $n_R$ | $R_{\min}$ | $sc$ | $T_0$ | $T_{\min}$ | $\Delta\tau$ | $\eta$ | $\delta$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $n = 3$ | 150 | 250 | 10 | 0.8 | 2 | 5 | 0.3 | 0.2 | 30 | 0.1 |
| $n = 30$ | 150 | 1500 | 10 | 0.8 | 2 | 5 | 0.3 | 0.2 | 30 | 0.1 |

### 5.3. Comparative analysis

In this paper, we suggested two robustness indexes, namely resource-wise and objective-wise robustness. Latter is mainly based on the extension of concept of "*robust solution type II*" to multi-objective context, proposed by Deb and Gupta (2006). They define a solution $x$ as robust solution type II, if it is a global minimum solution of the following problem:

$$\begin{aligned} &\min\ f(x)\\ \textit{subject to}\quad &\frac{\| \tilde{f}(x) - f(x) \|_p}{\| f(x) \|_p} \leqslant 1 - \eta\\ &x \in D \end{aligned} \tag{35}$$

where $f = (f_1, \ldots, f_M)$ and $\eta$ is a threshold determined by DMs to exert their direct control to the extent of desired robustness. In fact, Deb and Gupta (2006) have used the *p-norm* function to directly extend the concept of robust solution type II from single-objective to multi-objective context. By this definition, implicitly, equal weights for all objectives $f_i$, $1 \leqslant i \leqslant M$ are supposed. The main contribution of our proposed objective-wise robustness index is to apply preferential weights, instead of equal weights, to extend the single-objective robustness concept to multi-objective context (Eq. (12)) and to generate the set of robust project portfolios. It is essential to note that we set $p = 1$ in the *p-norm* function.

In this section, performance of our proposed robustness concept (RMOPO algorithm) is compared with MORST2 concept (multi-objective robust solution type II) for the case study problem. MORST2 is implemented via NSGA-III. To do a sensible comparison of the RPFs resulting from the two methods, we need a reference robust frontier (RRPF). As both methods try to extend the same single-objective robustness concept (single-objective robust solution type II) to multi-objective context, RRPF is generated using weighted-sum scalarization of the multi-objective problem and then applying the concept of single-objective robust solution type II. Resulting single-objective problem is solved for 150 objective weights (chosen uniformly) by the "*fmincon*" routine in MATLAB. RPFs obtained by RMOPO and MORST2 methods are compared with RRPF and deterministic PF for different robustness thresholds in Fig. 8. The figure clearly shows that RPF found by RMOPO is closer to RRPF than RPF found by MORST2. The run-time of RMOPO and MORST2 algorithms were 67 and 62 s, respectively.

We analyze conformity of the two methods RMOPO and MORST2 with RRPF using two measures: *hyper-volume metric* (HV) and *robust convergence metric* (RCM). Hyper-volume indicator is designed by Zitzler and Thiele (1999) to evaluate the convergence behavior of multi-objective evolutionary algorithms (MOEA). RCM is a metric proposed by Mirjalili and Lewis (2015) to quantify the performance of MOEAs. For both metrics, RRPF is used as a reference Pareto set. Furthermore, we also calculated *average resource waste degree* (ARWD) for each RPF based on definition 3. To investigate success of the two methods RMOPO and MORST2, we define *robustness rate* (RR) of each RPF, which is calculated as percentage of portfolios in a RPF that really are robust. A portfolio $x$ is regarded as a *really robust portfolio* if its RWD is not greater than the threshold determined by DMs, in other words: $RWD(x) \leqslant (1 - \eta) \times (1 + 0.05)$. In fact, 5% tolerance is allowed to consider the approximation errors. To compute $RWD(x)(\gamma_x$ in definition 3), we have solved the minimization problem of Eq. (6) for all the

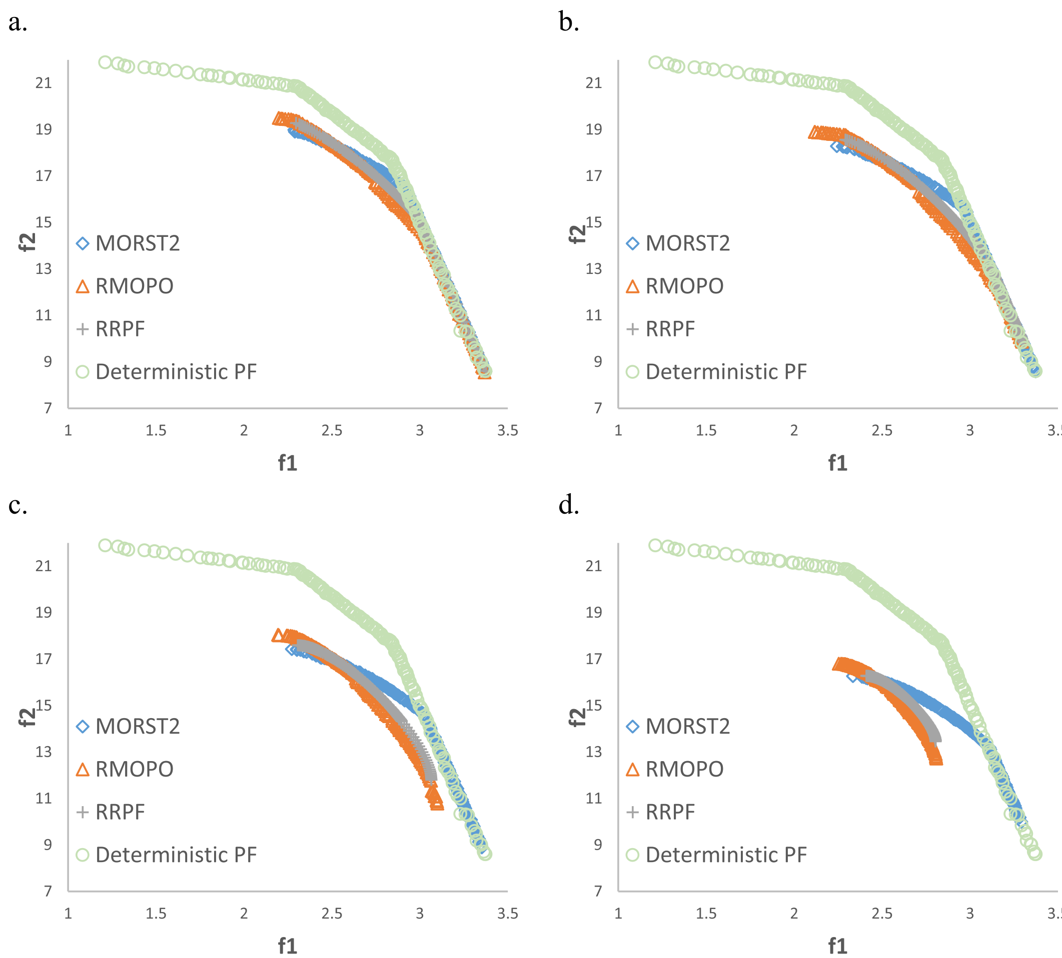


**Fig. 8.** . Comparison of RPFs obtained by RMOPO and MORST2 methods with RRPF and deterministic PF for different values of the robustness threshold: a. $\eta = 0.86$, b. $\eta = 0.87$, c. $\eta = 0.88$, d. $\eta = 0.89$.

**Table 9**
Comparison of RPFs obtained by RMOPO and MORST2 with RRPF, based on hyper-volume indicator

| $\eta$ | *RMOPO* | | *MORST2* | | *RRPF* |
|---|---|---|---|---|---|
| | *HV* | *Dif-%* | *HV* | *Dif-%* | *HV* |
| 0.850 | 0.8270 | 0.3% | 0.8114 | −1.6% | 0.8244 |
| 0.855 | 0.8171 | 0.5% | 0.8052 | −1.0% | 0.8132 |
| 0.860 | 0.8051 | 0.5% | 0.7926 | −1.0% | 0.8009 |
| 0.865 | 0.7937 | 0.8% | 0.7799 | −1.0% | 0.7877 |
| 0.870 | 0.7706 | 0.8% | 0.7662 | 0.3% | 0.7641 |
| 0.875 | 0.7410 | 1.9% | 0.7502 | 3.2% | 0.7273 |
| 0.880 | 0.7057 | 2.5% | 0.7317 | 6.2% | 0.6888 |
| 0.885 | 0.6641 | 2.9% | 0.7084 | 9.7% | 0.6457 |
| 0.890 | 0.6100 | 2.5% | 0.6801 | 14.3% | 0.5949 |

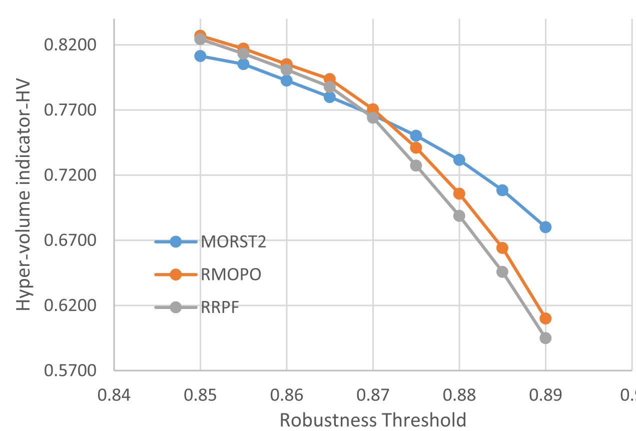


**Fig. 9.** . Hyper-volume values of RPFs obtained by RMOPO and MORST2 algorithms and hyper-volume value of RRPF plotted against different values of the robustness threshold

portfolios in the RPF, using the MATLAB "*fmincon*" function. Hyper-volume indicator of RPFs obtained by RMOPO and MORST2 algorithms for different values of robustness threshold $\eta$ are presented in Table 9. The two columns in Table 9, titled "*Dif-%*", represent the percentage difference between HV of corresponding RPF and HV of RRPF. In addition, to better understand the trend, HV values of RPFs obtained by RMOPO and MORST2 algorithms are graphically depicted in Fig. 9. We conclude from Table 9 and Fig. 9 that HV behavior of RMOPO is closer to HV behavior of RRPF than that of MORST2.

Comparison of RMOPO and MORST2, based on the three above mentioned metrics including RCM, ARWD and RR is summarized in Table 10. RWD of all portfolios in RPFs found by RMOPO and MORST2 algorithms for four different values of robustness threshold are plotted in Fig. 10. Results show that in terms of the three performance metrics, our proposed RMOPO algorithm outperforms the MORST2 robustness concept, especially for the greater values of the threshold $\eta$. In other words, our proposed robustness concept measures the robustness of a portfolio in the multi-objective context better and more precisely than the MORST2 method measures.

**Table 10**
Comparison of RMOPO and MORST2 algorithms, based on three performance metrics

| $\eta$ | RMOPO | | | MORST2 | | |
|---|---|---|---|---|---|---|
| | RCM | ARWD | RR | RCM | ARWD | RR |
| 0.850 | 0.0036 | 0.1440 | 100% | 0.0037 | 0.1435 | 87% |
| 0.855 | 0.0042 | 0.1411 | 100% | 0.0037 | 0.1416 | 84% |
| 0.860 | 0.0048 | 0.1381 | 100% | 0.0044 | 0.1397 | 79% |
| 0.865 | 0.0069 | 0.1347 | 100% | 0.0048 | 0.1374 | 73% |
| 0.870 | 0.0084 | 0.1308 | 100% | 0.0193 | 0.1351 | 69% |
| 0.875 | 0.0119 | 0.1268 | 100% | 0.0550 | 0.1329 | 49% |
| 0.880 | 0.0177 | 0.1224 | 100% | 0.0677 | 0.1306 | 22% |
| 0.885 | 0.0193 | 0.1181 | 100% | 0.0826 | 0.1285 | 17% |
| 0.890 | 0.0206 | 0.1142 | 98% | 0.1023 | 0.1260 | 11% |

### *5.4. A nonlinear example*

In this section, we will demonstrate the applicability of the proposed algorithm to general nonlinear multi-objective problems. Although we have developed the RMOPO algorithm to solve linear problems, it can also be used as an approximation method for specific nonlinear problems (e.g., problems with convex PF). In other words, the RMOPO algorithm can approximate the convex Pareto surface using a number of hyperplanes. To illustrate, the proposed algorithm is applied for a test problem DTLZ2 (Deb, Thiele, Laumanns, & Zitzler, 2005). To convert the test problem to an uncertain maximization problem with convex PF, some modifications to the original DTLZ2 are made as follows:

$$
\begin{aligned}
&\max\ \tilde{f}_1(\mathrm{X}) = [\cos(x_1\pi/2) + \varepsilon_{11}(x_1)]/[1 + g(\mathrm{X}_2) + \varepsilon_{12}(\mathrm{X}_2)],\\
&\max\ \tilde{f}_2(\mathrm{X}) = [\sin(x_1\pi/2) + \varepsilon_{21}(x_1)]/[1 + g(\mathrm{X}_2) + \varepsilon_{22}(\mathrm{X}_2)],\\
&\qquad with\ \ g(\mathrm{X}_2) = \textstyle\sum_{x_j\in \mathrm{X}_2}(x_j - 0.5)^2,\\
&\qquad \varepsilon_{11}(x_1) \in (-0.2x_1\,,\ 0.2x_1),\ \ \varepsilon_{21}(x_1) \in (-0.1x_1\,,\ 0.1x_1),\\
&\qquad \varepsilon_{12}(x_1) \in (-0.1\varepsilon_g\,,\ 0.1\varepsilon_g),\ \ \varepsilon_{22}(x_1) \in (-0.9\varepsilon_g\,,\ 0.9\varepsilon_g),\\
&\qquad \varepsilon_g = \cos([g(\mathrm{X}_2)/(n-1)] \times \pi),\\
&\qquad 0 \leqslant x_i \leqslant 1,\quad for\ i = 1, 2, \ldots, n.
\end{aligned}
\tag{36}
$$

The deterministic Pareto-optimal solutions ($\varepsilon_{11} = \varepsilon_{12} = \varepsilon_{21} = \varepsilon_{22}=0$) correspond to $x_i = 0.5$ for all $x_i \in \mathrm{X}_2$. Thus, all deterministic Pareto-optimal solutions satisfy $f_1^2(\mathrm{X}) + f_2^2(\mathrm{X}) = 1$, and therefore, the solutions lie on the quarter arc of a circle of radius 1. Fig. 11a and b shows the deterministic PFs obtained by solving problem (36) with NSGA-III and ε- constraint method. We have used MATLABs *fmincon* routine to find the Pareto-optimal solutions in each iteration of the ε- constraint method. The figure shows that the solutions obtained by NSGA-III are better distributed over the PF, compared with the solutions obtained by the ε-constraint method. This can be verified by the spacing metric values of 0.0024 and 0.0035, respectively for the PFs obtained by the NSGA-III and the ε- constraint method. In the next step, the PFHP algorithm approximates the arc of the circle (i.e., the PF) by 20 line segments, using the PFs obtained in the first step. In Fig. 11c and d, it can be seen that a better distribution of the solutions in the first step can result in a better approximation in the second step.

Fig. 12 compares the performance of the proposed RMOPO algorithm in finding robust solutions of the nonlinear problem defined by the equation (36) with the performance of the MORST2 method. To

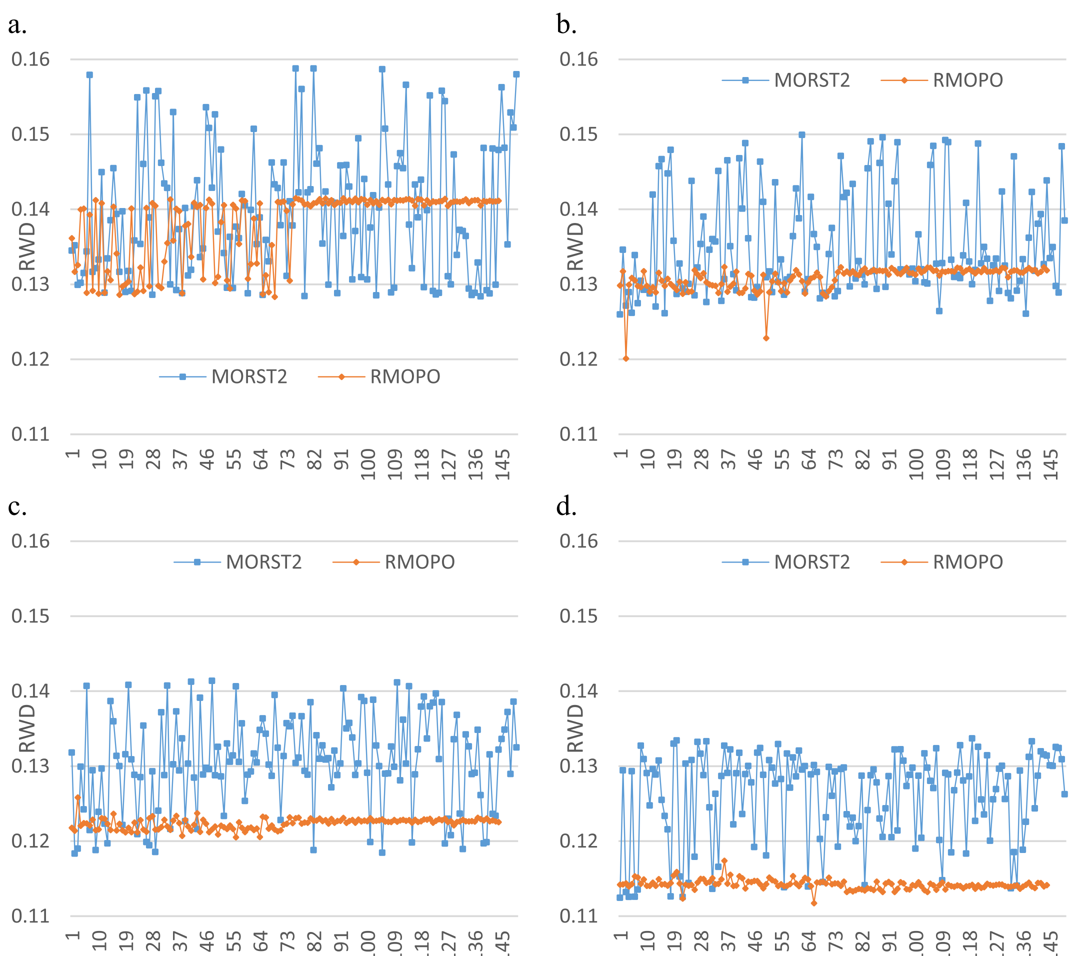


**Fig. 10.** . RWD of all portfolios in RPFs obtained by RMOPO and MORST2 algorithms for different values of the robustness threshold: a. $\eta = 0.86$, b. $\eta = 0.87$, c. $\eta = 0.88$, d. $\eta = 0.89$.

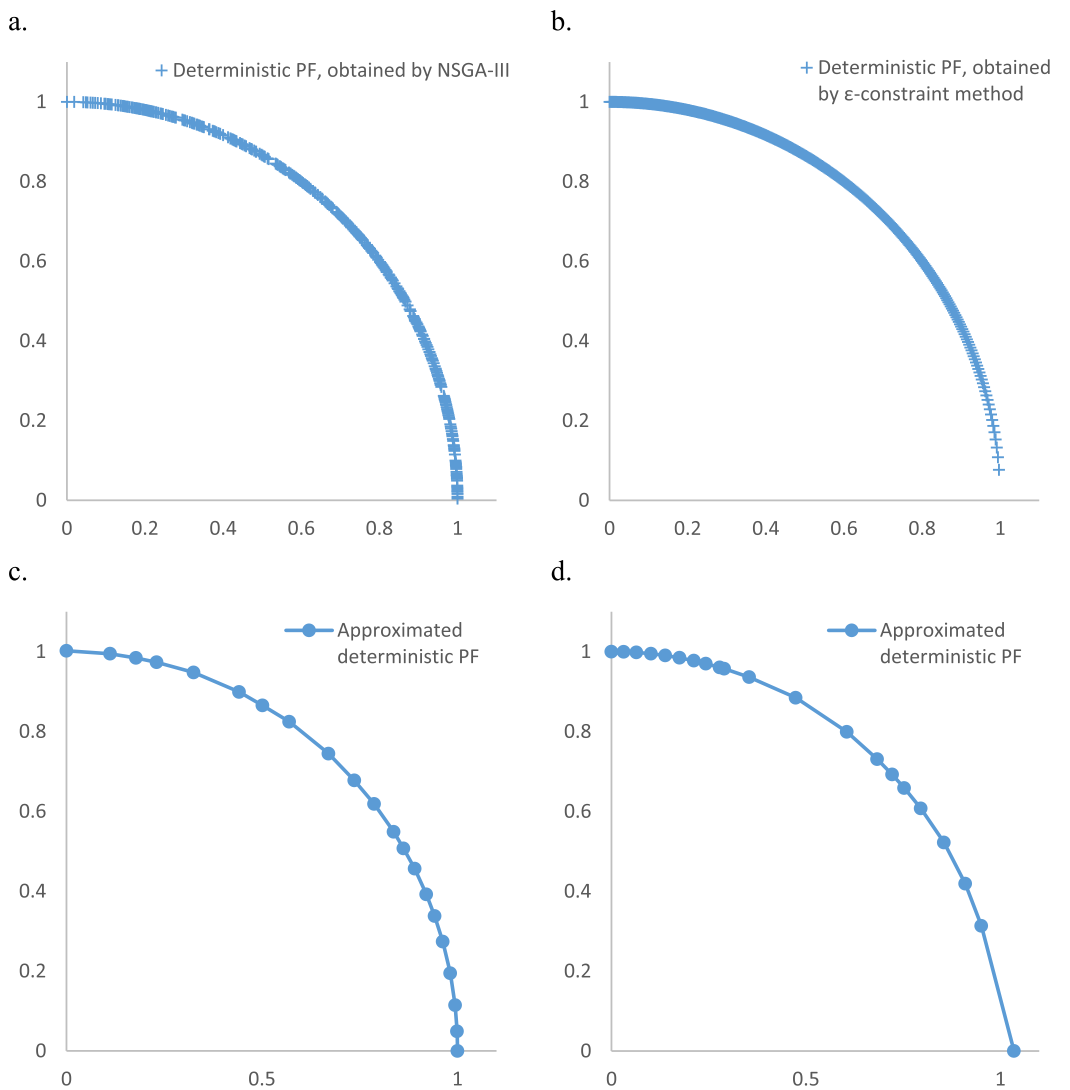


**Fig. 11.** . Comparison of deterministic PFs with PFs approximated by the PFHP algorithm: a. Deterministic PF, obtained by NSGA-III; b. Deterministic PF, obtained by ε-constraint method; c. PF approximated by the PFHP algorithm, based on the PF obtained by NSGA-III; d. PF approximated by the PFHP algorithm, based on the PF obtained by the ε-constraint method.

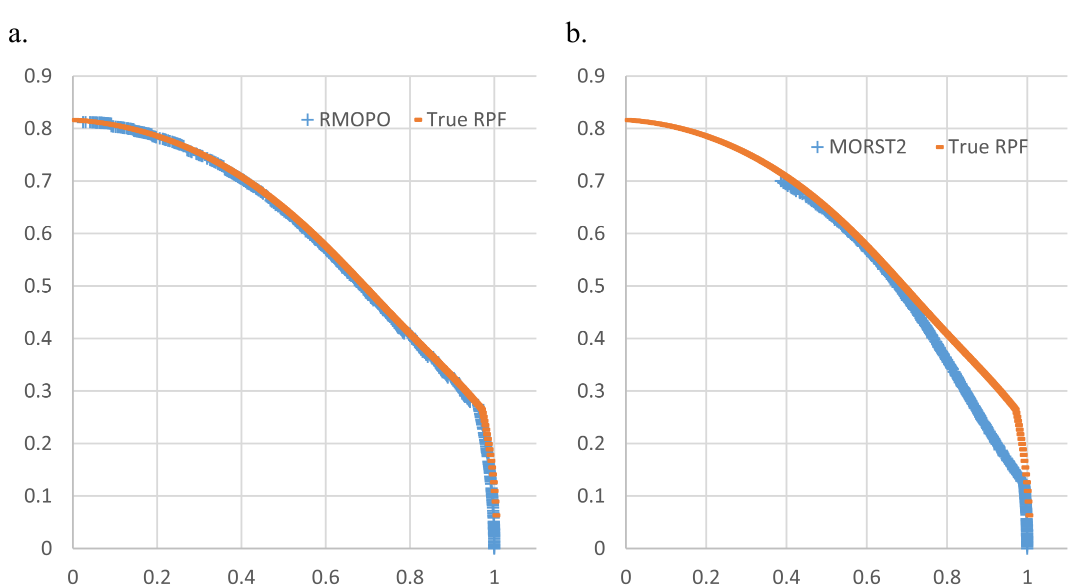


**Fig. 12.** . Comparison of RPFs obtained by RMOPO and MORST2 methods with True RPF for problem (36).

obtain the true RPF, one can easily deduce $W^* = (f_1/[f_1 + f_2], f_2/[f_1 + f_2])$ by manipulating equations (10) and (36). Thus, the exact objective-wise robustness measure of equation (12) can be calculated using the exact preferential weights $W^*$. Then, the true RPF is generated through solving the problem (13) by the ε-constraint method. It should be noted that, since the objective functions are nonlinear, theorem 2 is not applicable here, and therefore, we only rely on the objective-wise robustness measure. Fig. 12 shows that the MORST2 method has not been successful in finding the RPF. Instead, our proposed RMOPO algorithm has been able to produce a well-distributed RPF very close to the true RPF. In addition, the run-times of the RMOPO and MORST2 algorithms were 76 and 210 s, respectively. Our proposed RMOPO algorithm performs much better than the MORST2 algorithm in terms of run-time. This has been achieved by taking advantage of partitioning the decision space into 20 parts, and thus, reducing the number of solutions generated in each implementation of the NSGA-III (step 5 of the RMOPO).

## 6. Conclusion

In this research, robust projects portfolio optimization problem was investigated. Two types of external and internal uncertainty, which respectively, affect the objective payoffs and project portfolios, were identified. We considered a situation in which, because of uncertainty, DMs don't consume all resources available to achieve better objective values. If uncertainty causes vital resources to waste, they limit the level of resources applied to a portfolio and therefore, save the remaining resources to be applied in other lower priority programs associated with less uncertainty. In other words, DMs decide the trade-off between getting more objective values of portfolios and the level of resources is sacrificed in front of uncertainty. Therefore, for the purpose of modeling these conditions, the new concept of resource-wise robustness of project portfolios was introduced.

On the other hand, the concept of preferential weights of each portfolio was defined, based on which, a new objective-wise robustness index of portfolios was developed. The new proposed objective-wise robustness index is calculated based on the diameter of the sensitivity region. Contrary to the methods proposed in the literature, which implicitly consider equal objective weights, in our method of measuring the diameter of sensitivity region, the preferential weights of each portfolio was used as objective weights. In addition, the relationship between the proposed robustness index and the normal vectors of bounding hyperplanes of the PF was investigated and the normal vectors were used to calculate the robustness index. It has also been shown that for linear problems, the objective-wise robustness is in fact a translation of resource-wise robustness into objectives space. Finally, a new and efficient heuristic, based on the NSGA-III and simulated annealing metaheuristic, was proposed to find the robust Pareto portfolios.

The proposed algorithm was applied to solve a numerical example and a case study in port industry. The case study is extended to a larger-sized problem with 30 projects. In addition, the efficiency of the proposed algorithm for solving a nonlinear problem was also investigated; results were compared with exact solutions. This comparison represents the high accuracy of the proposed algorithm. We also compared performance of the introduced method (RMOPO), in terms of three performance indicators with the method developed based on the literature (MORST2) for the case study problem. Results show that our method better performs to find the right robust portfolios for the case study and we concluded that our proposed robustness concept better and more precisely extends the single-objective robustness into the multi-objective context than the MORST2 concept does.

The two important advantages of the proposed robustness index, from a practical point of view are the possibility of controlling the desired level of robustness through the robustness threshold, and clear and understandable decision-making process so that, DMs have enough insight to select among the portfolios generated by the proposed algorithm. It should be noted that preferences of the DMs should include the desired level of robustness. In this regard, as it was done for the case study, robust portfolios for different values of robustness threshold should be presented to the DMs. In addition, due to the existence of a relationship between the resource waste degree and the proposed robustness index, DMs can gain a thorough understanding of the threshold they would determine; this understanding comes from the concept of resource waste degree which has a clear meaning for them; and consequently they can correctly incorporate their desired level of robustness into the algorithm.

The results presented in the previous section show the very high speed and accuracy of the proposed algorithm, which can be used as an efficient algorithm for real-world project portfolio selection problems. Nevertheless, the proposed method has some limitations. The first limitation is the necessity of the linearity of the objectives and the constraints of the problem. This limitation could be the basis for future research by researchers in order to investigate the relationship between resource-wise and objective-wise robustness for nonlinear models. Another limitation is the runtime of the algorithm. In fact, the running time increases with the number of objectives and constraints. Time complexity of the algorithm depends on the time complexity of vertex enumeration and the convex hull algorithms used. This can overshadow the efficiency of the proposed algorithm by the problems with large number of objectives and constraints, however, it is quite appropriate for the problems with a limited number of objectives and constraints. In the case of very large problems with a large number of objectives and constraints, steps 4 and 5 of the RMOPO algorithm can be replaced by a heuristic that divides the decision space into an optimal number of partitions as well as efficiently examines the preferential weights found in step 2 (i.e., normal vectors of the bounding hyperplanes) for each candidate solution located in each partition. This study shows that the definition of robustness in multi-objective space is still not mathematically rigorous and precise. Therefore, in future studies, the researchers should try to present a more integrated definition for the multi-objective robustness, compare the various methods in the literature, and develop efficient algorithms.

## Acknowledgement

We gratefully thank our colleagues from Sina Port and Marine Company who provided insight and expertise that greatly assisted the research.

## Appendix A. Proof of theorem 1.

**Proof.** By definition 5, the preferential weights of the feasible solution $x^0$ with nominal objective values $y^0$ is $u^*$, the optimal solution of following LP:

$$\begin{aligned} &\max_u \; h_0 = u.\, y^0 \\ &\quad \textit{subject to:} \\ &\forall\, y \in B^0,\;\; u.\, y \leqslant 1 \end{aligned} \tag{A.1}$$

where $B^0 = \bigcup_{i=1}^{q} B^i$. On the other hand, given that $\alpha^t$ is the normal vector of the hyperplane $P^t$ that is part of the PF, then $\forall\, y \in \Theta,\;\; \alpha^t.\, y \leqslant 1$. Therefore,

$\alpha^t$ is a feasible solution to the problem (A.1). So $h_0(\alpha^t) = \alpha^t.\, y^0$. Now consider the dual of problem (A.1):

$$\min_z \ \theta_0 = I.\, z$$
$$\textit{subject to}:$$
$$Y^B.\, z \geqslant y^0 \tag{A.2}$$

Where $I = (1, ..., 1)_{n_0}$ and $n_0 = |B^0|$. $Y^B$ is a $m \times n_0$ matrix:

$$Y^B = [y^1 \dots y^{n_0}],\ y^i \in B^0,\ 1 \leqslant i \leqslant n_0 \tag{A.3}$$

Given that the m-dimensional hyper-pyramid $H^t$ is a convex hull of the set $\{B^t \cup O\}$, where O represents for the origin, so if $y^0 \in H^t$ then $y^0$ is convex combination of the points $\{B^t \cup O\}$:

$$y^0 = \textstyle\sum_{i=1}^{n_t} \lambda^{it} y^{it} + \lambda^{(n_t+1)t} O = \sum_{i=1}^{n_t} \lambda^{it} y^{it}$$
$$\textstyle\sum_{i=1}^{n_t} \lambda^{it} + \lambda^{(n_t+1)t} = 1$$
$$\lambda^{it} \geqslant 0, \quad 1 \leqslant i \leqslant n_t + 1 \tag{A.4}$$

Let define the vector $z$ as below:

$$z^0 = (z_1, ..., z_{n_0})',\ \forall\ i,\ 1 \leqslant i \leqslant n_0,\ z_i^0 = \begin{cases} 0 & y^i \notin B^t \\ \lambda^{jt} & y^i \in B^t,\ y^i = y^{jt} \end{cases} \tag{A.5}$$

$z^0$ is a feasible solution of the dual problem (A.2):

$$Y^B.\, z^0 = [\dots y^{1t}, \dots y^{n_t t}, \dots] \begin{bmatrix} 0 \\ \vdots \\ \lambda^{1t} \\ \vdots \\ \lambda^{n_t t} \\ \vdots \\ 0 \end{bmatrix} = \sum_{i=1}^{n_t} \lambda^{it} y^{it} = y^0 \tag{A.6}$$

The objective value of the dual problem (A.2) is then:

$$\theta_0(z^0) = I.\, z^0 = \sum_{i=1}^{n_0} z_i^0 = \sum_{i=1}^{n_t} \lambda^{it} \tag{A.7}$$

On the other hand, giving that for all $y^{it} \in B^t,\ 1 \leqslant i \leqslant n_t$ we have $\alpha^t.\, y^{it} = 1$, the objective value of the problem (A.1) by using Eq. (A.4) for the solution $\alpha^t$ is equal to:

$$h_0(\alpha^t) = \alpha^t.\, y^0 = \alpha^t.\, \sum_{i=1}^{n_t} \lambda^{it} y^{it} = \sum_{i=1}^{n_t} \lambda^{it} \alpha^t.\, y^{it} = \sum_{i=1}^{n_t} \lambda^{it} \tag{A.8}$$

We conclude from Eqs. (A.7) and (A.8) that the objective value of the primal and dual problems for two feasible solutions $\alpha^t$ and $z^0$ are equal (i.e., $h_0 = \theta_0$), thus, these two feasible solutions are respectively the optimal solutions of the primal and dual problems and therefore $W^*(X^0) = u^* = \alpha^t$.

## Appendix B. Proof of theorem 2

**Lemma.** *Let $y^* \in PF$ then $\beta_{y^*} = 1$*

**Proof.** As $y^* \in PF$ and $PF \subset \Theta$ so $y^* \in \Theta$. Therefore, there exist a feasible solution $x$ so that $Cx = y^*$. Since $x$ is feasible then, $Ax \leqslant B$, thus, according to definition 1, $\beta_x \leqslant 1$, and therefore, it follows from definition 2 that $\beta_{y^*} \leqslant 1$. If $\beta_{y^*} < 1$, then according to definition 2, there exist a solution like $x$ so that $Cx = y^*$ and $\beta_x < 1$. Therefore, letting $x' = x/\beta_x$, it is concluded from definition1:

$$\beta_x = \max_{1 \leqslant i \leqslant r} \left\{ \frac{a^i x}{b_i} \right\} \Rightarrow \forall\ i,\ 1 \leqslant i \leqslant r,\ \frac{a^i x}{b_i} \leqslant \beta_x \tag{B.1}$$

$$\Rightarrow \forall\ i,\ 1 \leqslant i \leqslant r,\ a^i x/\beta_x \leqslant b_i, \Rightarrow a^i x' \leqslant b \tag{B.2}$$

Thus, $x'$ is a feasible solution to the problem (2). Let $y'$ be the vector of objective values of $x'$, then:

$$y' = Cx' = Cx/\beta_x = y^*/\beta_x \tag{B.3}$$

$$\beta_x < 1 \Rightarrow y' > y^* \tag{B.4}$$

Accordingly, $y^*$ is dominated by the feasible objective value $y'$, which is in contradiction with the assumption that $y^*$ is a non-dominated Pareto solution. Consequently, the assumption $\beta_{y^*} < 1$ couldn't be valid and then we conclude $\beta_{y^*} = 1$. □

**Proof of theorem 2.** Eq. (A.4) follows:

$$y^0 = \sum_{i=1}^{n_t} \lambda^{it} y^{it} = \sum_{i=1}^{n_t} \lambda^{it} C x^{it} = C \sum_{i=1}^{n_t} \lambda^{it} x^{it} \tag{B.5}$$

$$x^0 = \sum_{i=1}^{n_t} \lambda^{it} x^{it} \Rightarrow y^0 = Cx \tag{B.6}$$

Let $x^* = \sum_{i=1}^{n_t} \lambda^{it} x^{it} / \sum_{i=1}^{n_t} \lambda^{it}$ and $y^* = \sum_{i=1}^{n_t} \lambda^{it} y^{it} / \sum_{i=1}^{n_t} \lambda^{it}$. Obviously $y^* = Cx^*$. Then, since $x^*$ is a convex combination of the feasible points $x^{it},\ 1 \leqslant i \leqslant n_t$, it is therefore a feasible point. Also, since $y^*$, the objective vector of $x^*$ is a convex combination of the feasible objective points $y^{it} \in F^t,\ 1 \leqslant i \leqslant n_t$, then $y^* \in F^t$, and therefore, it is located on the hyperplane $H^t$ and subsequently is a non-dominated Pareto solution. So, we have:

$$\forall\ i,\ 1 \leqslant i \leqslant r,\ a^i x^* \leqslant b_i,\ 1 \leqslant i \leqslant r,\ \exists\ p,\ 1 \leqslant p \leqslant r,\ a^p x^* = b_p \tag{B.7}$$

From Eq. (A.7) we have $\sum_{i=1}^{n_t} \lambda^{it} = \theta_0$. Then, $x^* = x^0/\theta_0$ and $y^* = y^0/\theta_0$. So, we have:

$$\forall\ i,\ 1 \leqslant i \leqslant r,\ a^i x^0/\theta_0 \leqslant b_i,\ \exists\ p,\ 1 \leqslant p \leqslant r,\ a^p x^0/\theta_0 = b_p \tag{B.8}$$

$$\Rightarrow \forall\ i,\ 1 \leqslant i \leqslant r,\ a^i x^0/b_i \leqslant \theta_0,\ \exists\ p,\ 1 \leqslant p \leqslant r,\ a^p x^0/b_p = \theta_0 \tag{B.9}$$

$$\Rightarrow \max_{1 \leqslant i \leqslant r} \left\{ \frac{a^i x^0}{b_i} \right\} = \theta_0 \Rightarrow \beta_{x^0} = \theta_0 \tag{B.10}$$

Since $y^0 = Cx^0$, Then, $x^0 \in X^C_{y^0}$ so, by definition 2:

$$x^0 \in X^C_{y^0} \Rightarrow \beta_{x^0} \geqslant \beta_{y^0} \Rightarrow \theta_0 \geqslant \beta_{y^0} \tag{B.11}$$

Now, let $x' \in X^C_{y^0}$ so that $\beta_{x'} = \min_{x \in X^C_{y^0}} \beta_x = \beta_{y^0}$. Then by definition 1:

$$\beta_{x'} = \max_{1 \leqslant i \leqslant r} \left\{ \frac{a^i x'}{b_i} \right\} = \beta_{y^0} \tag{B.12}$$

$$\Rightarrow \forall\ i,\ 1 \leqslant i \leqslant r,\ a^i x'/b_i \leqslant \beta_{y^0},\ \exists\ q,\ 1 \leqslant q \leqslant r,\ a^q x'/b_q = \beta_{y^0} \tag{B.13}$$

Let define $x^{**} = x'/\theta_0$, then:

$$\Rightarrow \forall\ i,\ 1 \leqslant i \leqslant r,\ a^i x^{**}\theta_0/b_i \leqslant \beta_{y^0},\ \exists\ q,\ 1 \leqslant q \leqslant r,\ a^q x^{**}\theta_0/b_q = \beta_{y^0} \tag{B.14}$$

$$\Rightarrow \forall\ i,\ 1 \leqslant i \leqslant r,\ a^i x^{**}/b_i \leqslant \beta_{y^0}/\theta_0,\ \exists\ q,\ 1 \leqslant q \leqslant r,\ a^q x^{**}/b_q = \beta_{y^0}/\theta_0 \tag{B.15}$$

$$\Rightarrow \beta_{x^{**}} = \beta_{y^0}/\theta_0 \tag{B.16}$$

$$x^{**} = x'/\theta_0 \Rightarrow Cx^{**} = Cx'/\theta_0 = y^0/\theta_0 = y^* \tag{B.17}$$

$$\Rightarrow x^{**} \in X^C_{y^*} \Rightarrow \beta_{x^{**}} \geqslant \beta_{y*} \tag{B.18}$$

$$\Rightarrow \beta_{y^0}/\theta_0 \geqslant \beta_{y*} \tag{B.19}$$

Since $y^*$ is a Pareto solution, then according the lemma $\beta_{y^*} = 1$. Then:

$$\beta_{y^0}/\theta_0 \geqslant 1 \Rightarrow \beta_{y^0} \geqslant \theta_0 \tag{B.20}$$

It is concluded from the inequalities (B.11) and (B.20) that $\beta_{y^0} = \theta_0$. Also, according to Eqs. (A.7) and (A.8):

$$\theta_0 = \alpha^t.\ y^0 \Rightarrow \beta_{y^0} = \alpha^t.\ y^0 = W^*(x^0).\ y^0 \tag{B.21}$$

Now, given that $S_{x^0} \subseteq H^t$, then for each $y \subseteq S_{x^0}$ we have $\beta_y = W^*(x^0).\ y$; therefore, $\beta_W = \min_{\tilde{Y} \in S_{x^0}} W^*(x^0).\ \tilde{Y}$ and by definition 6 we have $r(x^0) = 1 - \gamma_{x^0}$.

## Appendix C. Problems $\wp^1$, $\wp^2$ and $\wp^3$ of the case study.

$$\begin{gathered}
\textit{MOOP } \wp^1: \\
\max f_1 = 0.6x_1 + 2.8x_2 + 1.9x_3 \\
\max f_2 = 22.5x_1 + 5x_2 + 6.5x_3 \\
\textit{Subject to}: \\
1.2x_1 + 3.8x_2 + 2x_3 \leqslant 4.1 \\
1.1x_1 + 0.9x_2 + 1.05x_3 \leqslant 1.45 \\
6x_1 + 2x_2 + 4x_3 \leqslant 6 \\
-2.12x_1 + 2.53x_2 + 1.33x_3 \leqslant 0 \\
0.09x_1^2 + 0.11x_2^2 + 0.06x_3^2 - 0.06x_1 - 0.07x_2 - 0.07x_3 \leqslant 0 \\
x_1, x_2 \leqslant 1 \\
x_1, x_2 \geqslant 0
\end{gathered} \tag{C.1}$$

$$\begin{aligned}
&\textit{MOOP } \wp^2: \\
&\max f_1 = 0.6x_1 + 2.8x_2 + 1.9x_3 \\
&\max f_2 = 22.5x_1 + 5x_2 + 6.5x_3 \\
&\textit{Subject to}: \\
&1.2x_1 + 3.8x_2 + 2x_3 \leqslant 4.1 \\
&1.1x_1 + 0.9x_2 + 1.05x_3 \leqslant 1.45 \\
&6x_1 + 2x_2 + 4x_3 \leqslant 6 \\
&3.62x_1 - 2.38x_2 - 1.01x_3 \leqslant 0 \\
&0.19x_1^2 + 0.05x_2^2 + 0.03x_3^2 - 0.12x_1 - 0.03x_2 - 0.04x_3 \leqslant 0 \\
&x_1, x_2 \leqslant 1 \\
&x_1, x_2 \geqslant 0
\end{aligned} \tag{C.2}$$

$$\begin{aligned}
&\textit{MOOP } \wp^3: \\
&\max f_1 = 0.6x_1 + 2.8x_2 + 1.9x_3 \\
&\max f_2 = 22.5x_1 + 5x_2 + 6.5x_3 \\
&\textit{Subject to}: \\
&1.2x_1 + 3.8x_2 + 2x_3 \leqslant 4.1 \\
&1.1x_1 + 0.9x_2 + 1.05x_3 \leqslant 1.45 \\
&6x_1 + 2x_2 + 4x_3 \leqslant 6 \\
&-3.62x_1 + 2.38x_2 + 1.01x_3 \leqslant 0 \\
&2.12x_1 - 2.53x_2 - 1.33x_3 \leqslant 0 \\
&0.15x_1^2 + 0.09x_2^2 + 0.05x_3^2 - 0.09x_1 - 0.06x_2 - 0.06x_3 \leqslant 0 \\
&x_1, x_2 \leqslant 1 \\
&x_1, x_2 \geqslant 0
\end{aligned} \tag{C.3}$$

## Appendix D. An example project portfolio model with 30 projects.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **A** | **X1** | **X2** | **X3** | **X4** | **X5** | **X6** | **X7** | **X8** | **X9** | **X10** |
| | 0.1116 | 0.1116 | 0.1953 | 0.0279 | 0.1116 | 0.0279 | 0.1674 | 0.1674 | 0.0279 | 0.2512 |
| | 0.1023 | 0.1023 | 0.1791 | 0.0256 | 0.1023 | 0.0256 | 0.1535 | 0.1535 | 0.0256 | 0.2302 |
| | 0.5581 | 0.5581 | 0.9767 | 0.1395 | 0.5581 | 0.1395 | 0.8372 | 0.8372 | 0.1395 | 1.2558 |
| | **X11** | **X12** | **X13** | **X14** | **X15** | **X16** | **X17** | **X18** | **X19** | **X20** |
| | 0.4984 | 0.4361 | 0.1869 | 0.5607 | 0.3738 | 0.5607 | 0.0623 | 0.4984 | 0.4984 | 0.1246 |
| | 0.1180 | 0.1033 | 0.0443 | 0.1328 | 0.0885 | 0.1328 | 0.0148 | 0.1180 | 0.1180 | 0.0295 |
| | 0.2623 | 0.2295 | 0.0984 | 0.2951 | 0.1967 | 0.2951 | 0.0328 | 0.2623 | 0.2623 | 0.0656 |
| | **X21** | **X22** | **X23** | **X24** | **X25** | **X26** | **X27** | **X28** | **X29** | **X30** |
| | 0.1154 | 0.0385 | 0.2308 | 0.1154 | 0.3077 | 0.3462 | 0.1923 | 0.0385 | 0.3846 | 0.2308 |
| | 0.0606 | 0.0202 | 0.1212 | 0.0606 | 0.1615 | 0.1817 | 0.1010 | 0.0202 | 0.2019 | 0.1212 |
| | 0.2308 | 0.0769 | 0.4615 | 0.2308 | 0.6154 | 0.6923 | 0.3846 | 0.0769 | 0.7692 | 0.4615 |
| **C** | **X1** | **X2** | **X3** | **X4** | **X5** | **X6** | **X7** | **X8** | **X9** | **X10** |
| | 0.0558 | 0.0558 | 0.0977 | 0.0140 | 0.0558 | 0.0140 | 0.0837 | 0.0837 | 0.0140 | 0.1256 |
| | 2.0930 | 2.0930 | 3.6628 | 0.5233 | 2.0930 | 0.5233 | 3.1395 | 3.1395 | 0.5233 | 4.7093 |
| | **X11** | **X12** | **X13** | **X14** | **X15** | **X16** | **X17** | **X18** | **X19** | **X20** |
| | 0.3672 | 0.3213 | 0.1377 | 0.4131 | 0.2754 | 0.4131 | 0.0459 | 0.3672 | 0.3672 | 0.0918 |
| | 0.6557 | 0.5738 | 0.2459 | 0.7377 | 0.4918 | 0.7377 | 0.0820 | 0.6557 | 0.6557 | 0.1639 |
| | **X21** | **X22** | **X23** | **X24** | **X25** | **X26** | **X27** | **X28** | **X29** | **X30** |
| | 0.1096 | 0.0365 | 0.2192 | 0.1096 | 0.2923 | 0.3288 | 0.1827 | 0.0365 | 0.3654 | 0.2192 |
| | 0.3750 | 0.1250 | 0.7500 | 0.3750 | 1.0000 | 1.1250 | 0.6250 | 0.1250 | 1.2500 | 0.7500 |
| **D** | **X1** | **X2** | **X3** | **X4** | **X5** | **X6** | **X7** | **X8** | **X9** | **X10** |
| | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 |
| | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 |
| | **X11** | **X12** | **X13** | **X14** | **X15** | **X16** | **X17** | **X18** | **X19** | **X20** |
| | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 |
| | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 |
| | **X21** | **X22** | **X23** | **X24** | **X25** | **X26** | **X27** | **X28** | **X29** | **X30** |
| | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 | 0.04 |
| | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 |
| $\hbar$ | **X1** | **X2** | **X3** | **X4** | **X5** | **X6** | **X7** | **X8** | **X9** | **X10** |
| | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 |
| | **X11** | **X12** | **X13** | **X14** | **X15** | **X16** | **X17** | **X18** | **X19** | **X20** |
| | 0.15 | 0.15 | 0.15 | 0.15 | 0.15 | 0.15 | 0.15 | 0.15 | 0.15 | 0.15 |
| | **X21** | **X22** | **X23** | **X24** | **X25** | **X26** | **X27** | **X28** | **X29** | **X30** |
| | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 |

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