

# Facial expression recognition using histogram of oriented gradients based transformed features

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**Abstract** Facial expression recognition has been an emerging and long standing research problem in last two decades. Histograms of oriented gradients (HOGs) have proven to be an effective descriptor for preserving the local information using orientation density distribution and gradient of the edge. A robust powerful approach of HOG features has been investigated in this paper. In particular, this paper highlights that the transformation of HOG features to frequency domain can make this descriptor one of the most suitable to characterize illumination and orientation invariant facial expressions. Discrete cosine transform (DCT) is applied to transform the features into frequency domain and obtain the most important discriminant features. Finally, these features are fed to the well-known classifier to determine the underlying emotions from expressive facial images. To validate the proposed framework, we used MMI, Extended Cohn-Kanade dataset (CK+) and cross dataset. The results indicate that the proposed framework is better as compared to other methods in terms of classification accuracy rate with utilization of minimum number of features.

**Keywords** Facial expression recognition · Histograms of oriented gradients · Discrete cosine transform · Cross dataset

## 1 Introduction

Facial expression recognition has been extensively studied in the areas of computer vision and pattern recognition over the past decades due to its potential applications like human robot interaction and intelligent affective computing. It contains the cues of the non-verbal communication to help in understanding the intended meaning of the verbal communication.

Various frameworks and different styles of data are used to recognize facial expressions. Generally, the data uses from static images and image sequences [1, 2] for facial expression recognition. We chose the peak image frames for experiments because they contain sufficient information of the specific expressions. Facial expression recognition system can be divided into three steps: (1) Face acquisition (2) Feature extraction and representation (3) Facial expression classification.

Face portion is extracted in face acquisition step. Feature extraction technique is grouped into two categories i.e. (1) appearance based features and (2) geometric based features. Different face components including nose, mouth and eyebrows are used to represent the geometric features of face. HOG [3] and Local binary pattern [4] descriptors are used as appearance features. Finally, in classification step, different classifiers like K-nearest neighbor (KNN), (SMO) [5] and Random forest (RF) [6] are used to recognize facial expressions.

Most of the previous studies focus on recognizing face expressions from still facial images. However, in real world application it is more convenient to use videos as it provide more information as compared to expression image. The key issue of facial expression recognition in video is how to extract illumination and orientation invariant features which more clearly represent face structure. Similarly, low

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resolution and large data dimensions makes this task even harder.

We have applied two main algorithms in facial expression recognition work. Histogram of gradient orientation [3] is first used to store the local information of face image and discrete cosine transform is performed to select the prominent face features and reduce the data dimensions. Gradient of the HOG features corresponds to the first derivative of the image and it also reflects the shape information of the image which makes this descriptor robust to illumination and orientation. To the best of our knowledge, HOG has been used for facial expression recognition in few studies.

Kar et al. [7] used HOG descriptor to extract the robust face features. Linear discriminant analysis (LDA) and principal component analysis (PCA) are applied to obtain the important features. Back-propagation neural network (BPNN) classifier is trained and tested on the extracted features. In order to extract more efficient features, Salient areas on the faces is first defined in [8] and then HOG and LBP descriptors are used to extract the face features. Next, fusion of LBP and HOG feature is performed to achieve better results and then PCA is utilized to reduce the data dimensions. Experiments are performed on JAFFE and CK+ datasets. Shape and appearance-based features are combined in [9] to generate hybrid feature vector. LBP is utilized to extract the appearance features and pyramid of histogram of gradients (PHOG) is used for shape features. To capture the information of major changes during different expressions, active facial patches are located and then hybrid features are extracted from these facial patches. Linear discriminant analysis is applied to select the more discriminative face features and then fed these features to support vector machine for classification. Wang et al. [10] proposed a hybrid scheme by combining the weber local descriptor and HOG features. First the image is divided into different regions and then features are extracted from these regions. Nearest neighbor and chi-square distance is used to classify the weighted fused histograms for different expressions. Average classification accuracy rate of 95.86% has been obtained for CK+ database. Facial components like mouth, nose and eyebrows are first located in [11], and then HOG is used to encode these components. To perform different experiments, cell size is set to  $8 \times 8$  and histogram bins to 9. The effectiveness of the proposed framework is tested on CK+ and JAFFE database. Donia et al. [1,2] performed experiments on both static and video database for facial expression classification. HOG is utilized to perform the micro expression analysis. The face image is first divided into six basic components which are the most representative regions for expression recognition. SVM is used for training and testing and the average accuracy rate of 95 and 80% is reported on static and videos respectively. Fan and Tjahjadj [12] formulated the 3-dimensional facial features by extending the histogram of gradients to spatio-temporal domain

and then integrate with dense optical flow which contribute to both dynamic and spatial motion of facial expressions. Experiments are performed on MMI and CK+ database with leave-one-out cross validation strategy and multi-class support vector machine is used for classifying different facial expressions. A novel framework has been presented in [13] to handle low resolution images. The local binary pattern (LBP) has been extended to preserve the texture and spatial layout of an image. The extended version of the LBP is known as pyramid of local binary pattern (PLBP). PLBP was used to extract the face features from only salient region and thus computationally efficient. PLBP equally work well on high and low resolution images. Similarly, a novel framework known as Weber local binary image based cosine transform features (WLBI-CT) has been presented in [14] to handle the multi-scaling and multi-orientation problem.

Previous recognition methods tend to focus only on movement of facial landmarks, not on the analysis of the effect of multi-scaling, illumination and variations in the facial shape. In this paper we utilize HOG to extract the dynamic and orientation features from an image and then discrete cosine transform (DCT) is used to select the high variance features from HOG image.

The main contributions of the proposed framework are summarized as follows:

- Proposed framework is reliable for multi scale images and work adequately on videos.
- The selected features using DCT maximize the between class variations and minimize the within-class variation of expressions.
- Proposed framework is computationally efficient and can be used for real time applications.
- Proposed framework is robust to illumination and orientations.

This article has been organized as follows: In Sect. 2, we provide an overview of the proposed method. Empirical results and related discussion are provided in Sect. 3. We conclude our work with Sect. 4.

## 2 Overview of the proposed technique

The steps of the proposed contrast enhancement framework are outlined below:

- (1) Take input human facial image representing facial expressions.
- (2) Face Detection using Viola and Jones Algorithm.
- (3) Apply HOG for large gradient features and then normalize the features
- (4) Image is formulated from the normalized features.

- (5) Transformation of feature image into frequency domain using DCT
- (6) High variance features are extracted from the frequency domain in a zigzag manner to form a feature vector.
- (7) Feature vector is fed to KNN, SMO and MLP for classification of expressions.

Figure 1 illustrates the proposed framework steps.

Algorithmic steps of the proposed framework are given below;

Gradients thus obtained are then utilized to calculate gradient magnitude and angular orientations using the Eqs. 3 and 4.

$$m(x, y) = \sqrt{g_v^2(x, y) + g_h^2(x, y)} \quad (3)$$

$$\theta(x, y) = \tan^{-1} \left( \frac{g_v(x, y)}{g_h(x, y)} \right) \quad (4)$$

It divides the image into cells. A block is formed consisting of various cells and features are constituted by convolving

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#### ALGORITHM

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1. **Input:** Facial images  $I(I_1, I_2, I_3, \dots, I_N)$  representing facial expressions
2.  $I_f$  = Face Detection using Viola and Jones Algorithm from  $I$
3. Compute  $HOG_I(I_f)$  using equation(1-4).
4.  $[f_1, f_2, f_3, \dots, f_M] = HOG_I(I_f)$  where  $M$  = number of HOG features calculated using equation (5).
5. Compute feature set  $OGDCT_{fs} = DCT[HOG_{img}]$  high variance feature extraction in zigzag manner from HOG-image equations (6-7).
6. Store  $OGDCT_{fs}$  features
7. Repeat steps (2-6) for  $N$  input images
8. Train classifiers for all expressions
9. Perform Classification

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Symbol	Description
$I(I_1, I_2, I_3, \dots, I_N)$	$N$ input images.
$I_f$	Facial Image extracted using Viola and Jones Algorithm.
HOG	Histogram Oriented Gradient function.
$f_1, f_2, f_3, \dots, f_M$	m-number of Features formed using HOG.
$HOG_{img}$	Image constructed after applying HOG.
$OGDCT_{fs}$	Resultant feature set extracted using DCT for HOG image.

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## 2.1 Materials and methods

Histogram of oriented gradients (HOG) and discrete cosine transform (DCT) are presented in this section.

### 2.1.1 Histogram of oriented gradients (HOG)

HOG descriptor was developed by Dalal and Triggs [3] and has been successfully utilized by many researchers working in the domain of computer vision. Mostly it has been used for human detection, object identification and pedestrian identification. HOG is computed using magnitude and orientation. Horizontal and vertical gradients of the input image are computed using the following Eqs. 1 and 2.

$$G_x = I_f * [-1, 0, 1] \quad (1)$$

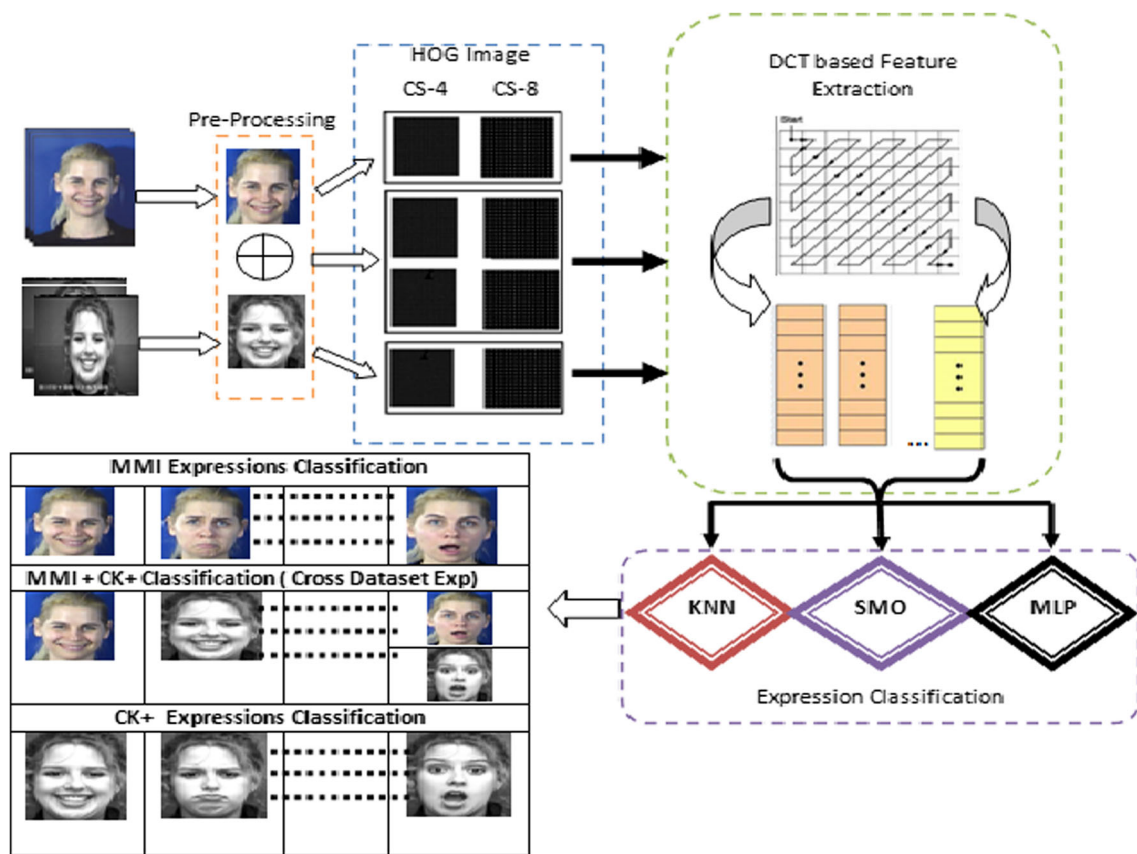
$$G_y = I_f * [-1, 0, 1]^T \quad (2)$$

the blocks. These blocks are overlapped. Orientations related to the same cell are quantized and are integrated into final histogram bins. These histogram bins are sorted and are combined into final histogram. Let  $T_{hog\_fs}$  represents the total number of features computed using HOG descriptor.  $B_{img}$  denotes blocks per image, block size is represented  $B_s$  and  $N_b$  denotes the number of bins used. Then total number of features can be computed using the following formula given in Eq. 5.

$$T_{hog\_fs} = B_{img} * B_s * N_b \quad (5)$$

### 2.1.2 Discrete cosine transform (DCT)

DCT is a well-known feature extraction technique being used by computer vision research community for solving various problems. We can take advantage of its discrimination capabilities in the areas of energy compaction, orthogonality, decorrelation and high separability. These properties make



**Fig. 1** Proposed framework

it robust in solving complex problems like facial expression recognition. We have applied DCT on the image formed after applying normalized HOG using the following Eq. 6. Features having high variations lie near top left corner and carry most significant information and realizes the effectiveness of the DCT for feature extraction. These features are extracted in zigzag manner starting from top left corner leaving the mean value. DCT of an input HOG image  $f(x, y)$  of size  $N \times M$  can be calculated using the Eqs. 6 and 7.

$$F(u, v) = \alpha(u) \alpha(v) \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} \cos \left[ \frac{\pi u}{2N} (2x+1) \right] \times \cos \left[ \frac{\pi v}{2M} (2y+1) \right] f(x, y) \quad (6)$$

where  $f(x, y)$  represents the pixel intensities at point  $(x, y)$  and  $u = \{0, 1, \dots, N-1\}$  and  $v = \{0, 1, \dots, M-1\}$ . The function  $\alpha(u)$  and  $\alpha(v)$  are defined as;

$$\alpha(u), \alpha(v) = \begin{cases} \sqrt{\frac{1}{N}} & \text{for } u, v=0 \\ \sqrt{\frac{2}{N}} & \text{for } u, v \neq 0 \end{cases} \quad (7)$$

### 2.1.3 Classification

K-nearest neighbor(KNN) classifier is considered to be the simplest but an efficient algorithm and has been applied successfully by research community for solving both supervised and unsupervised learning problems. It classifies an input sample among its  $k$ -nearest neighbors in the training data. Distance measure used for classification should be selected carefully in order to get the improved performance. Task of classification using KNN is done by taking a feature vector  $f_v$  and calculating the distance between  $f_v$  and feature vectors of known training samples. The distance between the training set and the test set can be calculated by using one of the distance metrics i.e. Euclidean distance, the Manhattan distance or cosine distance measure. For classification purpose the dataset is divided into training set and test set using different train to test ratios. Let the training set be represented by  $T = \{(s_1, c_1), (s_2, c_2), \dots, (s_n, c_n)\}$ , where  $n$  is the number of input feature vectors with their output class labels  $c_i (i = 1, 2, \dots, n)$ . The test sample  $t_i$  as feature vector  $f_v$  is assigned the class label with majority of  $k$ -neighbors having least distance. We have used the Euclidean distance ( $D_e$ ) using the formula given in Eq 8 defined as follows [15].

$$D_e(s, t) = \sqrt{\sum_i^l (s^2(i) - t^2(i))} \quad (8)$$

Here  $s_i$  is the number of feature set used for training and  $t_i$  is the feature set used for testing.  $l$  is length of the feature vector.

### 3 Experiments

Three set of experiments are performed on CK+, MMI and cross dataset including both MMI and CK+ images. 10-fold cross validation scheme is employed to all experiments. The original sequences are down sample to obtain the low resolution images. Three different resolutions ( $128 \times 128$ ,  $64 \times 64$  and  $32 \times 32$ ) are used to test the proposed framework. Table 1 shows the parameter settings for KNN, SMO and Random forest classifier.

AdaBoost face detection algorithm proposed by Viola and Jones [16] is firstly used to detect all the face images. After detection of face image the original image of size  $256 \times 256$  is reduced to  $128 \times 128$  sizes. All the extracted face images are passed to HOG descriptor for feature extraction. The HOG divides the image into  $16 \times 16$  block size and overlap of 50% is used. It results into total  $15 \times 15 = 225$  block. Each block consists of  $2 \times 2$  cells each of size  $8 \times 8$ . Gradient orientations range is set between  $[0-180]$  and is quantized using histograms of 9 bins. In this way, the total number of HOG features for input image of size  $128 \times 128$  becomes 8100.  $\text{HOG features} = 15 \times 15 \times 2 \times 2 \times 9$

Normally two main parameters are used to describe characteristics of the HOG descriptor i.e. orientation bins and cell size. The dimension of the patch involved in the single histogram computation is represented by cell size. Appropriate selection of parameters plays a vital role and should be selected with care for better performance of classification algorithm. Features extracted using HOG are affected by varying cell size. In case of using larger cell size, Spatial information of certain region from facial image is squeezed into the unit cell histogram, and the contribution becomes less significant. High resolution analysis on the other hand can be done by using smaller cell size, but it results in more detailed information and in order get most significant information from larger features becomes computationally expensive and

also can be contributed towards reduced accuracy. Similarly, while calculating HOG features, selection of number of orientation bins can play important role. It is related to the quantization levels of the gradient information. Significant information can be lost by using less number of orientation bins. Whereas, using more quantization levels, classification performance get affected because the information can be scattered along the bins. We have analyzed the classification performance on the dataset both with respect to cell size and orientation bins. Figures 2, 3 and 4 shows the happy face image with varying cell size and different image resolutions. After detailed experiments, we have found maximum performance on cell size  $8 \times 8$  and bin size is set to be 9 with block size  $2 \times 2$ .

HOG used gradient histogram which is not scale invariant [17]. Choosing the scale invariant coefficient for reliable facial expression recognition is a challenging task. The proposed framework integrating the DCT provides scale invariance for input image having different resolutions. DCT is applied to HOG image to select the high variance features of different length in zigzag manner. DCT method is computationally efficient as it did not require complex arithmetic.

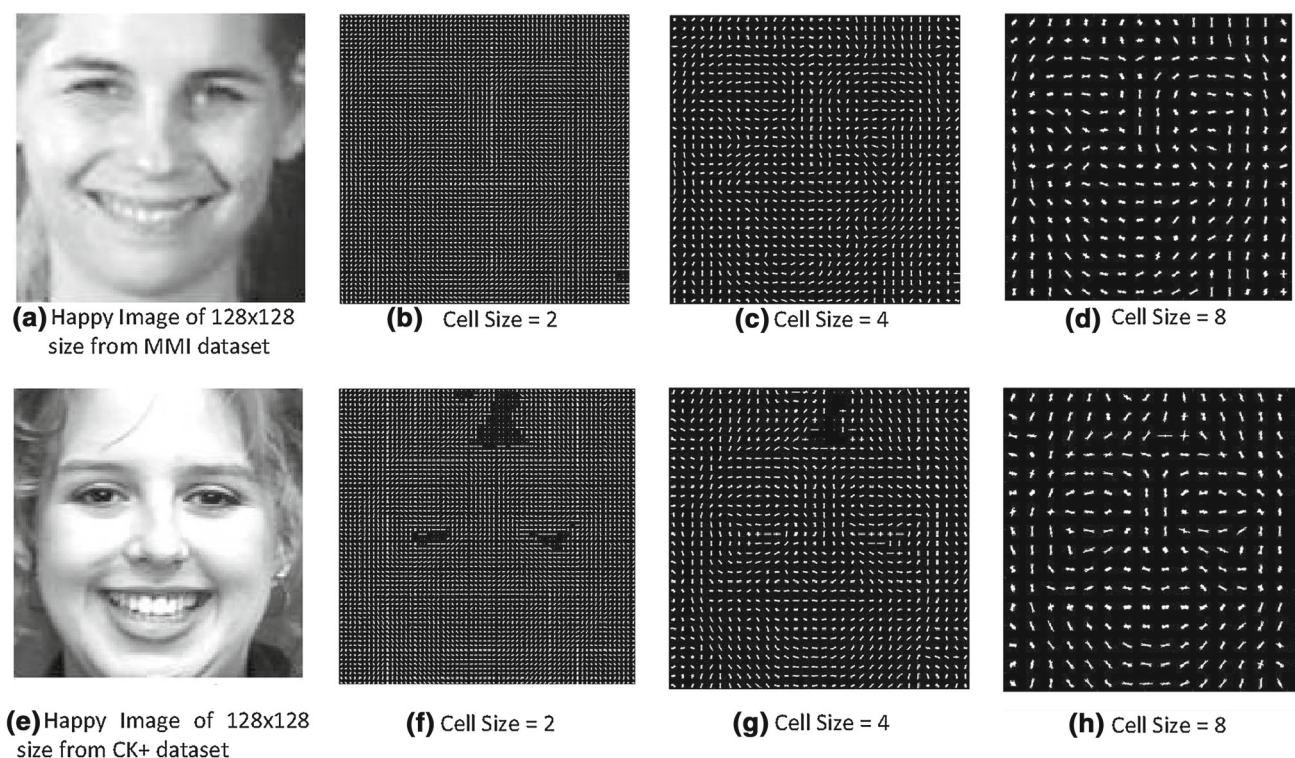
#### 3.1 Experiments on CK+ database

In order to evaluate the performance of the proposed algorithm extended CK+ dataset (CK+) [18] has been applied. In most of the work done in this area CK+ is being used extensively and is publicly available. The dataset consists of 593 image sequences taken from 120 subjects. Ages of the subjects used in the dataset ranges from 18 to 30 years. One of the major characteristic of this dataset is that the subjects in the dataset belong to different gender and races. Which makes it versatile and algorithm trained become more generalized. Ratio of different subjects consists of approximately 65% female, 81% Euro-American, 13% Afro-American, and 6% of other racial groups. Seven basic facial expressions, namely Anger, Contempt, Disgust, Fear, Happiness, Sadness and Surprise are given in the dataset. Images with resolution  $640 \times 480$  or  $640 \times 490$  pixels having 8-bit gray scale are used. We used 540 image sequences of six basic expressions. Neutral expression has been excluded. Sample of the images used is shown in the Fig. 5 and the number of images used per expression is illustrated by Table 2.

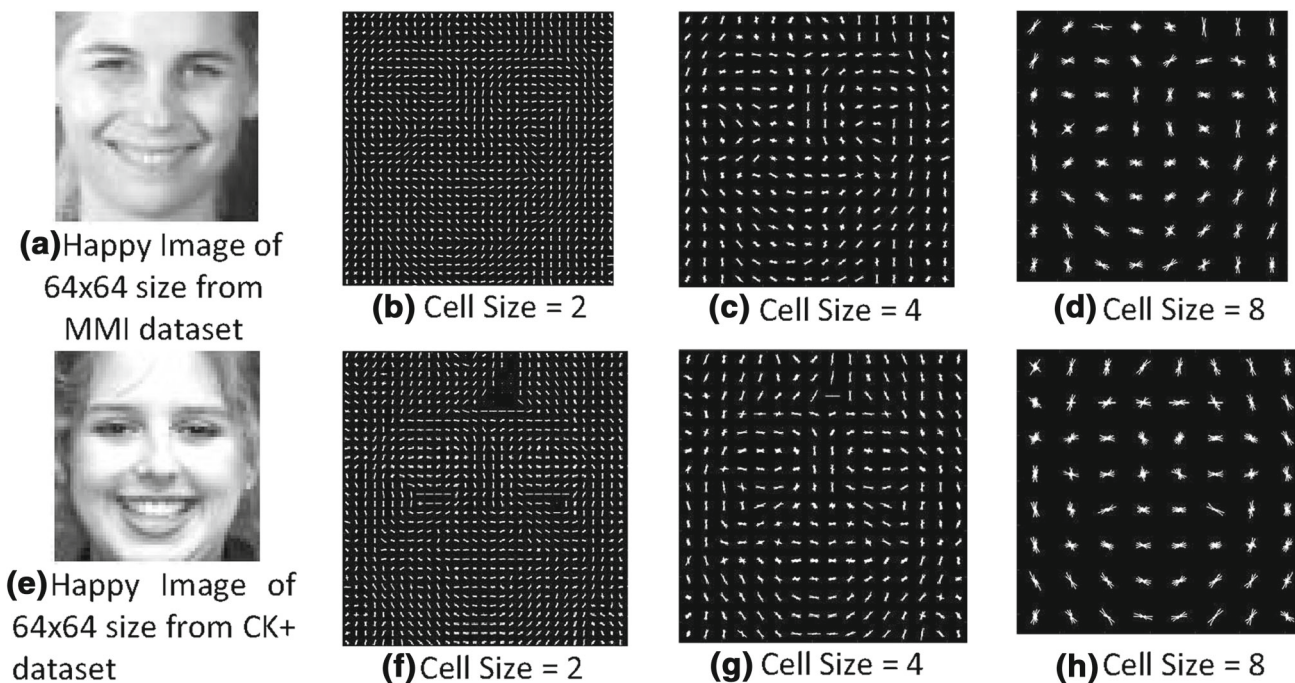
**Table 1** Classifiers parameter settings

Parameter settings for KNN		Parameter settings for SMO		Parameter settings for random forest	
Set of nearest neighbors	2	Complexity parameter 'C'	1.0	Number of iterations	100
nearest neighbor search algorithm	Euclidean distance	Kernal	Poly kernel	Bag size	100
batch size	100	Number of folds	-1	Seed	1





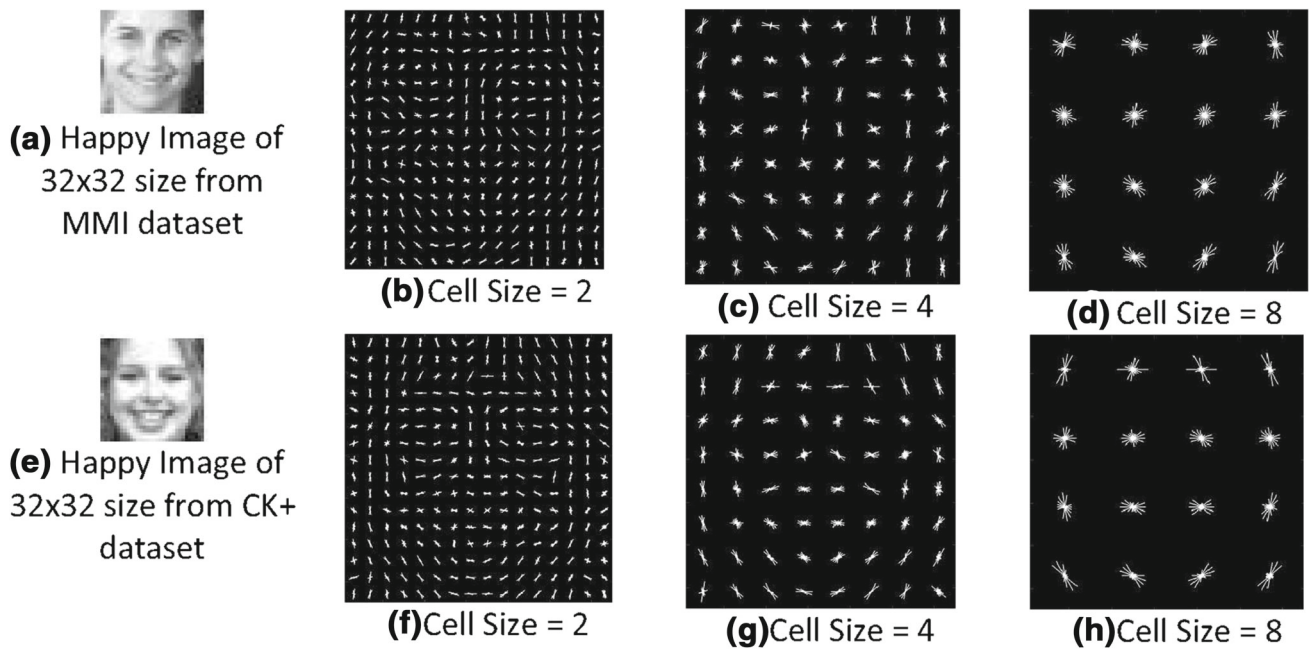
**Fig. 2** HOG processed CK+ and MMI images using different cell size and  $128 \times 128$  image size



**Fig. 3** HOG processed CK+ and MMI images using different cell size and  $64 \times 64$  image size

Figure 6 illustrates the average recognition accuracy of the proposed framework with different feature vector size (FV-32, FV-48, FV-64, FV-80, FV-96, FV-112).

It has been observed that the proposed framework work amicably on low resolution images by utilizing less number of features as shown in Fig. 6. We stop increasing the



**Fig. 4** HOG processed CK+ and MMI images using different cell size and  $32 \times 32$  image size

**Fig. 5** The Sample images of CK+ database

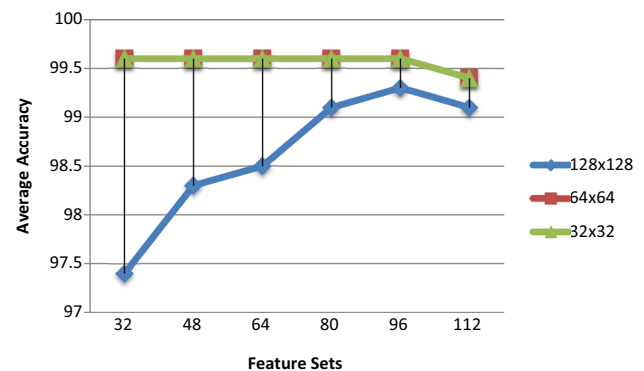


**Table 2** Number of CK+ image per expression

Expression	No. of images
Happy	90
Sad	90
Surprised	90
Angry	90
Disgust	90
Fear	90
Total	540

feature vector size after getting the feature vector of size FV-112 because adding more features will increase the computational time and there is minor improvement in accuracy. The maximum average accuracy of 99.6% has been obtained for images of size  $64 \times 64$  and  $32 \times 32$  using feature vector of size 32. Due to the lower feature vector dimensions the proposed framework can be utilized for real time applications.

In general, the proposed framework also achieved high accuracy using other classifiers like Sequential minimal optimization (SMO) and Random forest (RF) which shows the strength of the extracted feature irrespective to the classifiers. Figure 7 shows the average recognition accuracy rate

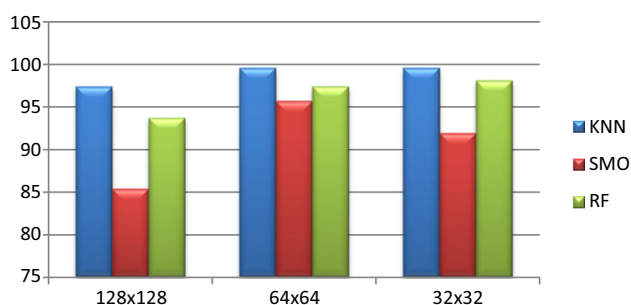


**Fig. 6** Average recognition rate with increasing number of features and with different resolution

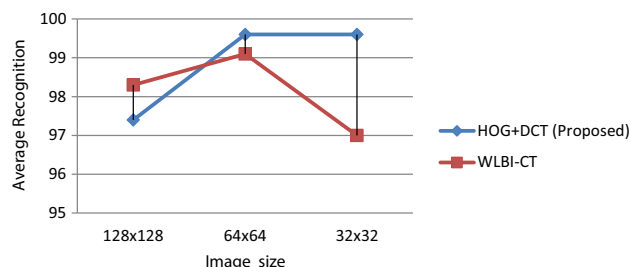
of fixed feature vector size (FV-32) using KNN, SMO and RF classifiers.

To compare with Khan et al. [14] work, we tested our proposed framework on three different image resolution (i.e.  $128 \times 128$ ,  $64 \times 64$  and  $32 \times 32$ ) on CK+ database. Facial image resolutions used in [14] are  $144 \times 192$ ,  $72 \times 96$  and  $36 \times 48$  which is comparable to the resolutions used in our experiments. Referenced Fig. 8 shows the supremacy of the





**Fig. 7** Classification accuracy using different classifiers



**Fig. 8** Proposed framework accuracy rate comparison with other technique

proposed framework on different image resolutions as compared to other methods.

Khan et al. [14] reported the accuracy using feature vector of size 64 which indicate that our proposed framework data dimension cost is low as compared to [14].

### 3.2 Experiments on MMI database

The MMI dataset [19] was specially designed in 2002 to evaluate the performance of research techniques in the area of facial expression classification. The specialty of this dataset is that it contains full temporal pattern of complete facial expressions along with all existing action units. Subjects used in this dataset also belong to different race and ethnicity and age ranging from 19 to 62. The subjects performed six basic facial expressions. For experimental purpose, we have transformed the original frames into 8-bit grayscale images and extracted 273 image sequences labelled as one of the six basic facial expressions are selected from the MMI dataset. Sample of the images used is shown in the Fig. 9 and Table 3 illustrates the number of expression per image.

**Fig. 9** The Sample images of MMI database



Detailed experiments are performed on MMI dataset as shown in Table 4. In order to demonstrate the effectiveness of the proposed technique on different resolutions we have used dataset with different resolutions of size  $128 \times 128$ ,  $64 \times 64$  and  $32 \times 32$ . The pattern shown in the Table 4 clearly shows the performance of the methodology.

The accuracy of the technique increased with increase in number of features initially. It keeps on increasing till 112 features and then it becomes almost stable with further increase in number of features. It is evident from the results shown in Table 4 that proposed techniques perform equally better on images of all sizes. It proves the robustness of the methodology against varying resolutions thus exhibiting multi resolution capability. We have achieved 96% accuracy on dataset size  $64 \times 64$  by using only 48 features.

In the Table 5 results generated using proposed methodology on MMI dataset are shown by applying different classifiers. Consistency of the technique is obvious with all the three classifiers. We have presented these results by

**Table 3** Number of images from each expression of MMI

Expression	No. of images
Happy	39
Sad	34
Surprised	39
Angry	45
Disgust	39
Fear	41
Neutral	36
Total	273

**Table 4** MMI images accuracy rate with varying feature vectors and image size

Feature vector (FV)	Images resolutions		
	$128 \times 128$	$64 \times 64$	$32 \times 32$
16	97.4	98.7	99.3
32	97.4	99.6	99.6
48	98.3	99.6	99.6
64	98.5	99.6	99.6
80	99.1	99.6	99.6
96	99.3	99.6	99.6
112	99.1	99.4	99.4



**Table 5** MMI images accuracy rate with varying feature vectors and image size

Classifier	Images resolutions		
	128 × 128	64 × 64	32 × 32
KNN	93.8	96	95.2
SMO	79.9	93.8	93
RF	88.6	93	94.1

using only 48 features which are considerable as far as the number of features is concerned. It results in reduced computational cost. KNN performs better in all sizes. The results at low resolutions are comparatively better. Maximum accuracy achieved is by using KNN classifier and is 96% at size  $64 \times 64$ .

### 3.3 Experiments on MMI and CK+ database (cross database)

Another milestone set in this manuscript was to check the effectiveness of the methodology by using cross dataset. For this purpose, we have combined both datasets and have performed experiments using the proposed framework. Results produced in this way are promising and have boosted the confidence with enhanced accuracy relative to the recently developed techniques in this domain. In this combined dataset, we have used six basic expressions and total number of images used was 777. Results of the dataset used are shown in the Table 6.

The dataset MMI and CK+ have been developed using different environments and with subjects belonging to different regions, gender, race and ethnicity. Using combined dataset results, presented in the Table 6 are quite impres-

**Table 6** Cross dataset images accuracy rate with varying feature vector size

Feature vector (FV)	Accuracy rate (%)
32	96.1
48	96.9
64	97.4
80	97.8
96	97.7
112	97.9
128	97.7
144	97.9
160	98.1
176	98.2
192	98.1
208	98.2

**Table 7** Comparison with other methods on CK+ database

References	Technique	Accuracy (%)
2017 [8]	LBP+HOG	96.6
2016 [7]	HOG+PCA+LDA	99.2
2015 [9]	PHOG+LBP	94.6
2013 [10]	HOG+WLD	95.86
2012 [11]	HOG+SVM	88.7
2014 [1,2]	HOG+SVM	80
2015 [12]	PHOG_TOP	83.7
Proposed	HOG+DCT	99.6

sive. It depicts the generalizability of the proposed technique. With the increase in number of features accuracy increases and reaches to its climax 98.2% by using only 208 features which is a significant contribution. The main point in these results is that 96.1% accuracy has been achieved by using only 32 features.

### 3.4 Comparisons with other state-of-the-art techniques

We chose to compare the average recognition accuracy rate with other state-of-the-art techniques using the same database (i.e. Cohn-Kanade database). It can be observed that the recognition accuracy of the proposed framework is better than other techniques presented in the literature as shown in Table 7.

## 4 Conclusion

Facial expression classification is a vigorous research topic with amplified number of applications. Highly reliable and accurate solution is required to develop such a system. In this paper a hybrid feature extraction technique is presented which is accurate, reliable and equally adept in handling multi-scale and illumination variation problem. Variety of results has been presented to prove the effectiveness of the methodology. System employs only face components extracted using Viola and Jones method. It utilizes the power of Histogram of Oriented Gradients descriptor and is further refined by using high variance features extracted using Discrete cosine transform. System is tested using MMI and CK+ datasets and achieved high accuracy of 99.6% by using only 32 features with KNN as a classifier. Results using other classifiers are also presented. Experiments have also been performed using combined dataset and results are even better on the combined dataset. Comparison with the state-of-the-art techniques has also been presented and robustness of the technique is evident from the results and discussion.

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