

QUIZ 3

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- Ackermann function -

$$A(m, n) = \begin{cases} n+1 & \text{if } m=0 \\ A(m-1, 1) & \text{if } m>0 \text{ and } n=0 \\ A(m-1, A(m, n-1)) & \text{if } m>0 \text{ and } n>0 \end{cases}$$

We start by calculating base case values and values of cases until we reach $A(3, 3)$.

• For $m=0$, $A(0, n)$, where $n>0$, $= n+1$. By definition.

For $m=1$: $A(1, 0) = A(0, 1) = 2$

$$A(1, 1) = A(1-1, A(1, 1-1)) = A(0, A(1, 0)) = A(0, 2) = 3$$

$$A(1, 2) = A(0, A(1, 1)) = A(0, 3) = 4$$

$$A(1, 3) = A(0, A(1, 2)) = A(0, 4) = 5$$

$$A(1, n) = A(0, n+1)$$

For $m=2$: $A(2, 0) = A(1, 1) = A(0, 2) = 3$

$$A(2, 1) = A(1, A(2, 0)) = A(1, 3) = A(0, 4) = 5$$

$$A(2, 2) = A(1, A(2, 1)) = A(1, 5) = A(0, 6) = 7$$

$$A(2, 3) = A(1, A(2, 2)) = A(1, 7) = A(0, 8) = 9$$

$$A(2, 4) = A(1, A(2, 3)) = A(1, 9) = A(0, 10) = 11$$

$$A(2, n) = A(0, 2n+2)$$

For $m=3$: $A(3, 0) = A(2, 1) = A(0, 4) = 5$

• $A(3, 1) = A(2, A(3, 0)) = A(2, 5) = A(0, 2 \cdot 5 + 2) = A(0, 12) = 13$

$$A(3, 2) = A(2, A(3, 1)) = A(2, 13) = A(0, 2 \cdot 13 + 2) = A(0, 28) = 29$$

* $A(3, 3) = A(2, A(3, 2)) = A(2, 29) = A(0, 2 \cdot 29 + 2) = A(0, 60) = 61$ *