Thermodynamical description of the ghost dark energy model

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In this paper, we point out thermodynamical description of ghost dark energy and its generalization to the early universe. Thereinafter, we find expressions for the entropy changes of these dark energy candidates. In addition, considering thermal fluctuations, thermodynamics of the dark energy component interacting with a dark matter sector is addressed. We will also find the effects of considering the coincidence problem on the mutual interaction between the dark sectors, and thus the equation of state parameter of dark energy. Finally, we derive a relation between the mutual interaction of the dark components of the universe, accelerated with the either ghost dark energy or its generalization, and the thermodynamic fluctuations.

I. INTRODUCTION

Big bang, as the cornerstone of standard cosmology, is the primary generator of the Universe expansion. Bearing the Einstein equations in mind, the rate of the universe expansion is determined by a fluid which has significant contribution in filling the universe, and is called dominated fluid. Since it seems that the universe is homogeneous and isotropic in scales larger than 100 Mpc, the universe expansion is modeled by the so-called Friedmann-Robertson-Walker (FRW) metric,

$$ds^2 = dt^2 - a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right], \tag{1}$$

where $a(t)$ is the scale factor, and $k = -1, 0, 1$ is the curvature parameter corresponding to open, flat and closed universes respectively [1]. It is also shown that the conformal form of this metric can be used to explain the inhomogeneity of cosmos in scales smaller than 100 Mpc [2].

Thermodynamic description of the Einstein equations is pointed out by Jacobson [3]. In order to generalize the Jacobson's results to the cosmological setup, we need to find a causal boundary for the FRW spacetime. In fact, it was argued that apparent horizon can be considered as a causal boundary for dynamical spacetimes [4–6]. For this propose, the apparent horizon is defined as the marginally trapped surface located at

$$\tilde{r}_A = \frac{1}{\sqrt{H^2 + k/a^2}}, \tag{2}$$

where $H = \dot{a}/a$ is the Hubble parameter [7, 8]. It was shown that the apparent horizon can be considered as a causal boundary for the FRW spacetime associated with gravitational entropy and surface gravity, and makes the establishment of the first law of thermodynamics. [9–16].

Recent observations indicate that our Universe is currently undergoing a phase of accelerated expansion with $\dot{a}(t) \geq 0$ and $\ddot{a}(t) \geq 0$ [17–20]. It implies either a non-baryonic dominated fluid named dark energy (DE) and fills about 70 percent of the universe [21, 22] or modifying the Einstein equations [23]. Although the nature of the dominated fluid needed to support this phase of expansion is mysterious [1], but this phase of expansion is compatible with the generalized second law of thermodynamics [24–27]. In fact, the tendency of the universe to rise its entropy can be interpret as an origin for the gravity and thus the Einstein equations [28]. This hypothesis is generalized to various theories of gravity and cosmological setups [29–43]. Therefore, the tendency of the cosmos, as a closed system, to increase its entropy is compatible with the current accelerated phase of the expansion. Finally, we should note that the thermodynamic analysis of the Universe helps us to get a more better understanding from the gravity, the Universe, its origin and evolution, and thus the nature of the mysterious dominated fluid supporting the current accelerated expansion.

Considering either a new degree of freedom or a new parameter leads to explanations of DE in the Einstein relativity framework [44–49]. In another approach, based on the Quantum Chromodynamics (QCD), new particles known as Veneziano ghost are guaranteed for the DE source [50]. These particles explain the $U(1)$ problem in QCD [50–52],

and yield a reasonable answer for the time-varying cosmological constant in a universe with non-trivial geometry [53]. The energy density of the Veneziano ghost DE (GDE) model is proportional to $\Lambda_{QCD}^3 H$, where Λ_{QCD} is the QCD mass scale and thus $\rho_D = \alpha H$ [54]. Although the Veneziano GDE model suffers the coincidence problem, but this shortcoming can be eliminated by choosing proper value for Λ_{QCD} [53, 54]. From thermodynamics point of view, it was shown that GDE satisfies the generalized second law of thermodynamics which is in agreement with the current accelerated phase of the Universe expansion [24]. Instability of GDE against perturbations is studied in [55]. More studies on the GDE properties can be found in Refs. [56–63]. In the standard cosmology, dark matter (DM) fills about 26 percent of the universe which influences the structure formation and the galaxies rotating curves [1]. Since the nature of DM is unknown and its gravitational effects differ from DE, people usually study DM and DE separately.

Recently, observational evidences of interaction between DE and DM are presented [64–72]. Moreover, such interactions between the dark sectors of the cosmos may lead to solve the coincidence problem [72–79]. In addition, bearing the thermal fluctuation theory in mind [80], one can show that the fluctuation theory may lead to logarithmic correction to the entropy of event horizon [81]. Its generalization to the cosmological setup helps us to explain thermal fluctuation of the cosmos components as the result of the mutual interaction between the dark sectors of the cosmos [82–86]. Therefore, it seems that DM should interact with GDE, as a candidate for DE. Such interactions between DM and GDE are studied in Refs. [87, 88]. Indeed, in order to solve the coincidence problem, DE should decay into DM [82]. Finally, we should note that one may find a relation for the mutual interaction between the dark sectors by using the thermal fluctuations of the universe components.

In general, the vacuum energy of the Veneziano ghost field in QCD is of the form $H + O(H^2)$ [89]. This indicates that in the previous works on the GDE model, only the leading term H has been considered. Indeed, the density of Veneziano ghost is in the form $\rho_D = \alpha H + \beta H^2$ [89]. It was first pointed out by Maggiore et al., [90] that the second term (βH^2) might play the crucial role in the early universe when $\tilde{r}_A \ll 1$. Therefore, one should use the generalized Veneziano ghost density (GGDE) in order to get a more comprehensive look-out from the effects of considering the Veneziano ghost as the source of the DE on the universe expansion and thus related topics. In fact, it was shown that GGDE leads to better agreement with observational data against GDE [91]. The GGDE interaction with DM and its relation with observational data is studied in Ref. [92].

Here, we are going to study thermodynamics of the Universe filled with GDE/GGDE and DM and in the absence of interaction between DM and GDE/GGDE. The associated entropy to GDE/GGDE models is addressed. Thereinafter, we will study thermodynamics of the GDE and GGDE models when the interaction with DM is considered. We derive thermodynamic interpretations for such interactions. Since the WMAP data indicates a flat universe [1], we restrict ourselves to the $k = 0$ case. This option leads to $\Omega_k = 0$, where $\Omega_k = k/(a^2 H^2)$ is the dimensionless energy density parameter induced by the spacetime curvature [1]. For the sake of simplicity, we take $c = 1$ throughout this paper.

The paper is organized as follows. In the next section, we consider the GDE model which does not interact with DM and study its thermodynamics. Section III includes a thermodynamical description of the GDE interacting with DM. We investigate the thermodynamics of the non-interacting GGDE in section IV. In section V, we provide a thermodynamical interpretation for the GGDE interacting with DM. Last section is devoted to summary and concluding remarks.

II. THERMODYNAMICAL DESCRIPTION OF THE NON-INTERACTING GDE

Here, we consider the flat FRW Universe filled by the GDE and the pressureless DM, and study the thermodynamics of the GDE whereas GDE does not interact with DM. For the flat FRW Universe, the first Friedmann equation is

$$H^2 = \frac{1}{3M_p^2} \left(\rho_m + \rho_D \right). \tag{3}$$

In this equation, $M_p^2 = (8\pi G)^{-1}$ is the reduced Plank mass, ρ_m is the energy density of the pressureless DM and ρ_D is the GDE density

$$\rho_D = \alpha H, \tag{4}$$

where α is a constant of order Λ_{QCD}^3. With $\Lambda_{QCD} \sim 100$ MeV and $H \sim 10^{-33}$ eV, $\Lambda_{QCD}^3 H$ gives the right order of magnitude for the observed DE density [54]. For the dimensionless energy density parameters, we have

$$\begin{aligned} \Omega_m &= \frac{\rho_m}{\rho_{cr}} = \frac{\rho_m}{3m_p^2 H^2}, \\ \Omega_D &= \frac{\rho_D}{\rho_{cr}} = \frac{\alpha}{3m_p^2 H}, \end{aligned} \tag{5}$$

where $\rho_c = 3M_p^2 H^2$ is critical energy density. Therefore, the Friedmann equation (3) can be written as

$$\Omega_m + \Omega_D = 1. \tag{6}$$

Since GDE does not interact with DM, the energy-momentum conservation implies

$$\dot{\rho}_m + 3H\rho_m = 0, \tag{7}$$

and

$$\dot{\rho}_D + 3H\rho_D \left(1 + \omega_D^0\right) = 0. \tag{8}$$

Here $p_D = \omega_D^0 \rho_D$ where ω_D^0 is the equation of state parameter of the non-interacting GDE, and superscript (0) is used to remember that the GDE does not interact with DM. Taking the derivative of Eqs. (3) and (4) with respect to time, we arrive at

$$2H\dot{H} = \frac{1}{3M_p^2} \left(\dot{\rho}_m + \dot{\rho}_D\right), \tag{9}$$

and

$$\dot{\rho}_D = \alpha \dot{H}, \tag{10}$$

respectively. Combining Eqs. (7), (8) with Eq. (9), we get

$$\dot{H} = -\frac{\rho_D}{2M_p^2} \left(1 + u + \omega_D^0\right), \tag{11}$$

where

$$u = \frac{\rho_m}{\rho_D} = \frac{\Omega_m}{\Omega_D} = \frac{1 - \Omega_D}{\Omega_D}. \tag{12}$$

Using Eq. (11), one can rewrite Eq. (10) as

$$\dot{\rho}_D = \alpha \dot{H} = -\frac{\alpha}{2M_p^2}\rho_D \left(1 + u + \omega_D^0\right). \tag{13}$$

Inserting this equation into Eq. (8), we arrive at

$$-\frac{\alpha}{2M_p^2}\rho_D \left(1 + u + \omega_D^0\right) + 3H\rho_D \left(1 + \omega_D^0\right) = 0, \tag{14}$$

which yields

$$\left(1 + \omega_D^0\right)\left(6M_p^2 H - \alpha\right) = \alpha u. \tag{15}$$

Inserting Eq. (4) into the Eq. (3) leads to

$$3M_p^2 H = \alpha \left(1 + u\right). \tag{16}$$

Combining this equation with Eq. (15), we get

$$\omega_D^0 = \frac{u}{2u + 1} - 1. \tag{17}$$

Finally, we use Eq. (12) to obtain

$$\omega_D^0 = -\frac{1}{2 - \Omega_D^0}, \tag{18}$$

Since the apparent horizon is the causal boundary of the FRW spacetime, we write the first law of thermodynamics on the apparent horizon in order to find an expression for the entropy of the GDE as

$$TdS_D = dE_D + p_D dV. \tag{19}$$

In this equation, S_D is the entropy associated to the GDE while $V = \frac{4\pi}{3}\tilde{r}_A^3$ and $E_D = \rho_D V$ are, respectively, the volume of the flat FRW Universe and the total energy of the DGE. Note that in flat FRW Universe the apparent horizon radius is indeed the Hubble radius, $\tilde{r}_A = 1/H$. The temperature T of the GDE which will be equal to the temperature of the apparent horizon when thermodynamic equilibrium is supposed, and therefore we have

$$T = \frac{1}{2\pi\tilde{r}_A^0} = \frac{H_0}{2\pi}. \tag{20}$$

Therefore, for the volume and the total energy we have

$$V = \frac{4\pi}{3}(\tilde{r}_A^0)^3 = \frac{4\pi}{3}H_0^{-3}, \tag{21}$$

$$E_D = \alpha H_0 \times \frac{4\pi}{3}H_0^{-3} = \frac{4\pi}{3}\alpha H_0^{-2} = \frac{4\pi}{3}\alpha(\tilde{r}_A^0)^2. \tag{22}$$

Differentiating relations (21) and (22) and substituting the results into Eq. (19), we get

$$dS_D^{(0)} = 2\pi\tilde{r}_A^0 \left[\frac{8\pi}{3}\alpha\tilde{r}_A^0 d\tilde{r}_A^0 + 4\pi\alpha\omega_D^0 \tilde{r}_A^0 d\tilde{r}_A^0 \right], \tag{23}$$

where we have used $p_D = \rho_D \omega_D^0 = \alpha H_0 \omega_D^0 = \frac{\alpha\omega_D^0}{\tilde{r}_A^0}$. Combining (23) with (18), we obtain

$$dS_D^{(0)} = 8\pi^2 \alpha \left(\tilde{r}_A^0 \right)^2 d\tilde{r}_A^0 \left[\frac{2}{3} - \frac{1}{2 - \Omega_D^0} \right]. \tag{24}$$

We should note again that the subscript/superscript (0) indicates that the GDE does not interact with the DM in our model. Therefore, we find an expression for the entropy changes of the non-interacting GDE confined by the apparent horizon when the DM sector does not interact with the GDE sector.

III. THERMODYNAMICAL DESCRIPTION OF THE INTERACTING GDE

In this section we study the case where the GDE and the pressureless DM interact with each other. We look at this interaction as a generator for thermodynamic fluctuations around equilibrium state. Therefore, bearing the thermodynamic fluctuations in mind, we try to find an expression for the interaction. The energy-momentum conservation leads to

$$\dot{\rho}_T + 3H(\rho_T + p_T) = 0, \tag{25}$$

where $\rho_T = \dot{\rho}_m + \dot{\rho}_D$ and $p_T = p_m + p_D = p_D$. In addition, H is the Hubble parameter of the universe filled with the interacting GDE, and differs from the H_0 introduced in the previous section. Also, Eq. (2) is valid for the both of H and H_0 with radii $\tilde{r}_A$ and $\tilde{r}_A^0$, respectively. Since the dark components interact with each other, one can decompose Eq. (25) into the

$$\dot{\rho}_m + 3H\rho_m = Q, \tag{26}$$
$$\dot{\rho}_D + 3H\rho_D (1 + \omega_D) = -Q, \tag{27}$$

Here, $Q = 3b^2 H (\rho_m + \rho_D)$ denotes the interaction term and b^2 is a coupling constant [79]. In addition, $\omega_D = \frac{p_D}{\rho_D}$ is the equation of the state parameter of the interacting GDE. Using Eq. (12), we get

$$Q = 3b^2 H\rho_D (1 + u). \tag{28}$$

Inserting Eqs. (13) and (28) into Eq. (27), and doing simple calculations yield

$$\alpha \left[(1 + \omega_D) + u \right] = 6M_p^2 H \left[b^2 (1 + u) + (1 + \omega_D) \right]. \tag{29}$$

Since Eq. (16) is independent of the interaction, we use it and reach at

$$\omega_D = -\frac{2b^2 (1 + u)^2 + (u + 1)}{(2u + 1)}. \tag{30}$$

Using relation (12), we get

$$\omega_D = -\frac{1}{2-\Omega_D}\left[\frac{2b^2}{\Omega_D}+1\right], \tag{31}$$

as the equation of the state parameter of the interacting GDE. Inserting Eqs. (4) and (12) into the Eq. (28), we find

$$b^2 = \frac{Q\Omega_D}{3\alpha H^2}, \tag{32}$$

which yields

$$\omega_D = -\frac{1}{2-\Omega_D}\left[\frac{2b^2}{\Omega_D}+1\right] = -\frac{1}{2-\Omega_D}\left[\frac{2Q}{3\alpha H^2}+1\right]. \tag{33}$$

Bearing the flat FRW spacetime and Eq. (2) in mind, one can rewrite this equation as

$$\omega_D = -\frac{1}{2-\Omega_D}\left[\frac{2Q\tilde{r}_A^2}{3\alpha}+1\right]. \tag{34}$$

In order to study thermodynamics of interaction term, Q, we consider the first law of thermodynamics

$$TdS_D = dE_D + p_D dV, \tag{35}$$

where E_D and V_D are, respectively, the total energy and the volume of the interacting GDE. Here T is the temperature of the horizon (20) and p_D is the pressure of the interacting GDE. In addition, S_D is the entropy of the interacting GDE which consists two parts,

$$S_D = S_D^{(0)} + S_D^{(1)}, \tag{36}$$

where $S_D^{(1)} = -\frac{1}{2}\ln\left(CT^2\right)$ is a logarithmic correction to the thermodynamic entropy which is due to the fluctuations around equilibrium, and is valid in all thermodynamic systems [81]. C is the heat capacity defined by

$$C = T\frac{\partial S_D^{(0)}}{\partial T} = T\frac{\partial S_D^{(0)}}{\partial \tilde{r}_A^0}\frac{\partial \tilde{r}_A^0}{\partial T} = T\frac{\partial S_D^{(0)}}{\partial \tilde{r}_A^0}\left(\frac{-1}{2\pi T^2}\right). \tag{37}$$

It is a matter of calculation to show that

$$C = -8\pi^2\alpha\left(\tilde{r}_A^0\right)^3\left[\frac{2}{3}-\frac{1}{2-\Omega_D^0}\right] \tag{38}$$

and therefore,

$$S_D^{(1)} = -\frac{1}{2}\ln\left[-2\alpha\tilde{r}_A^0\left(\frac{2}{3}-\frac{1}{2-\Omega_D^0}\right)\right]. \tag{39}$$

Using Eq. (35) and following the recipe of previous section, we get

$$dS_D = 8\pi^2\alpha\left(\tilde{r}_A\right)^2\left(\frac{2}{3}+\omega_D\right)d\tilde{r}_A, \tag{40}$$

as the changes of the total entropy. Combining this equation with (34) and using Eq. (36), we obtain

$$Q = \frac{\alpha(2-\Omega_D)}{\left(\tilde{r}_A\right)^2} - \frac{3\alpha}{2\left(\tilde{r}_A\right)^2} - \frac{3\left(2-\Omega_D\right)}{16\pi^2\left(\tilde{r}_A\right)^4}\left[\frac{dS_D^{(0)}}{d\tilde{r}_A}+\frac{dS_D^{(1)}}{d\tilde{r}_A}\right]. \tag{41}$$

Since

$$\frac{dS_D^{(0)}}{d\tilde{r}_A} = \frac{\partial S_D^{(0)}}{\partial \tilde{r}_A^{(0)}}\frac{d\tilde{r}_A^{(0)}}{d\tilde{r}_A} = 8\pi^2\alpha\left(\tilde{r}_A^0\right)^2\left(\frac{2}{3}-\frac{1}{2-\Omega_D^0}\right)\frac{d\tilde{r}_A^{(0)}}{d\tilde{r}_A}, \tag{42}$$

and

$$\frac{dS_D^{(1)}}{d\tilde{r}_A} = \frac{\partial S_D^{(1)}}{\partial \tilde{r}_A^{(0)}} \frac{d\tilde{r}_A^{(0)}}{d\tilde{r}_A} = -\frac{1}{2\tilde{r}_A^{(0)}} \frac{d\tilde{r}_A^{(0)}}{d\tilde{r}_A}, \tag{43}$$

we reach at

$$Q = \frac{\alpha(2 - \Omega_D)}{(\tilde{r}_A)^2} - \frac{3\alpha}{2(\tilde{r}_A)^2} + \frac{d\tilde{r}_A^{(0)}}{d\tilde{r}_A} \left[\frac{3\alpha \left(\tilde{r}_A^0\right)^2 (2 - \Omega_D)}{2(\tilde{r}_A)^4 (2 - \Omega_D^0)} - \frac{\alpha \left(\tilde{r}_A^0\right)^2 (2 - \Omega_D)}{(\tilde{r}_A)^4} + \frac{3(2 - \Omega_D)}{32\pi^2 (\tilde{r}_A)^4 \tilde{r}_A^0} \right]. \tag{44}$$

Finally, inserting $\tilde{r}_A = 1/H$ and $\tilde{r}_A^0 = 1/H_0$ into this equation, we get

$$Q = \alpha H^2 (2 - \Omega_D) - \frac{3\alpha H^2}{2} + \frac{d\tilde{r}_A^{(0)}}{d\tilde{r}_A} \left[\frac{3\alpha H^4 (2 - \Omega_D)}{2H_0^2 (2 - \Omega_D^0)} - \frac{\alpha H^4 (2 - \Omega_D)}{H_0^2} + \frac{3H^4 H_0 (2 - \Omega_D)}{32\pi^2} \right], \tag{45}$$

which is an expression for the interaction between the GDE and DM components of the Universe. In order to solve the coincidence problem, DE should decay into the DM sector meaning that $Q > 0$ [82]. Therefore, the permissible thermal fluctuations are those leading to $Q > 0$ and vice versa. The latter means that, when the mutual interaction between the dark sectors meets the $Q > 0$ condition, then it leaves a physically acceptable fluctuation into the system. Since we have considered interactions in the $Q = 3b^2 H(\rho_D + \rho_m)$ form, the $Q > 0$ condition leads to $b^2 > 0$. In order to investigate the validity of this criterion, we use Eq. (33)

$$\frac{2b^2}{\Omega_D} + 1 = (\Omega_D - 2)\omega_D. \tag{46}$$

Because the LHS of this equation is positive and $0 < \Omega_D < 1$, ω_D should meet the $\omega_D < 0$ condition. Loosely speaking, when the state parameter of DE meets the $\omega_D < 0$ condition, its mutual interaction with DM may lead to solve the coincidence problem and induces thermal fluctuations into the system in accordance with Eq. (45). In this way we provide a relation between the dark components interaction and the thermal fluctuations around the equilibrium state.

IV. THERMODYNAMICAL DESCRIPTION OF THE GGDE

Here, we consider the flat FRW universe filled by the GGDE and the DM whereas the dark sectors do not interact with each other. Therefore, for the energy density of GGDE, we have

$$\rho_D = \alpha H + \beta H^2, \tag{47}$$

where β is a free parameter and can be adjusted for better agreement with observations [91]. In addition, for the dimensionless density parameter of the GGDE we obtain

$$\Omega_D = \frac{\rho_D}{\rho_{cr}} = \frac{(\alpha + \beta H)}{3m_p^2 H}, \tag{48}$$

where the density parameter of the DM obeys Eq. (5), and therefore $\Omega_m + \Omega_D = 1$. Since the dark components do not interact with each other, the energy-momentum conservation implies

$$\dot{\rho}_m + 3H\rho_m = 0, \tag{49}$$

$$\dot{\rho}_D + 3H\rho_D \left(1 + \omega_D^0\right) = 0. \tag{50}$$

Combining these equations with the time derivative of Eq.(3), one obtains

$$\dot{H} = -\frac{\rho_D \left(1 + u + \omega_D^0\right)}{2m_p^2}. \tag{51}$$

Now, using the time derivative of Eq. (47), we arrive at

$$\dot{\rho}_D = -\frac{\rho_D}{2m_p^2} \left(1 + u + \omega_D^0\right) (\alpha + 2\beta H), \tag{52}$$

which yields

$$1 + \omega_D^0 = \frac{\frac{1}{6m_p^2 H}\alpha u + \xi u}{1 - \frac{1}{6m_p^2 H}\alpha - \xi}, \tag{53}$$

and we have defined $\xi = \beta/(3m_p^2)$. If one inserts Eq. (47) into the Eq. (3), after using Eq. (12), then one reaches

$$\frac{\alpha}{6m_p^2 H} = \frac{1 - \xi(u+1)}{2(u+1)}. \tag{54}$$

Using this equation, one can write Eq. (53) as

$$1 + \omega_D^0 = \frac{\Omega_D - \Omega_D^2 + \xi - \xi\Omega_D}{\Omega_D(2 - \Omega_D - \xi)}, \tag{55}$$

leading to

$$\omega_D^0 = \frac{\xi - \Omega_D^0}{\Omega_D^0(2 - \Omega_D^0 - \xi)}, \tag{56}$$

which is an expression for the state parameter of the non-interacting GGDE. The result of the GDE is available in the $\xi \to 0$ limit. In order to find the entropy changes of the GGDE, we assume that the first law of thermodynamics is available on the apparent horizon, where the pressure of the GGDE is

$$p_D = \rho_D \omega_D^0 = \left(\alpha H_0 + \beta H_0^2\right)\omega_D^0 = \left(\frac{\alpha}{\tilde{r}_A^0} + \frac{\beta}{(\tilde{r}_A^0)^2}\right)\omega_D^0. \tag{57}$$

and following the recipe of section II, we reach

$$dS_D^{(0)} = 8\pi^2(\tilde{r}_A^0)^2 d\tilde{r}_A^0 \left[\frac{2}{3} + \frac{\xi - \Omega_D^0}{\Omega_D^0(2 - \Omega_D^0 - \xi)}\right]\alpha + 8\pi^2 \tilde{r}_A^0 d\tilde{r}_A^0 \left[\frac{1}{3} + \frac{\xi - \Omega_D^0}{\Omega_D^0(2 - \Omega_D^0 - \xi)}\right]\beta. \tag{58}$$

Again, the subscript/superscript (0) indicates that the GGDE and the DM do not interact with each other. As a desired result, one can reach to Eq. (24) by taking appropriate limit ,$\beta \to 0$, from Eq. (58).

V.   THERMODYNAMICAL DESCRIPTION OF THE INTERACTING GGDE

Here, based on the thermodynamic fluctuations, we give an expression for the entropy changes of the interacting GGDE. In addition, we find a relation between the dark components interaction and the thermodynamic fluctuations. Since the GGDE and the DM interact with each other, the energy-momentum conservation implies

$$\dot{\rho}_m + 3H\rho_m = Q, \tag{59}$$
$$\dot{\rho}_D + 3H\rho_D(1 + \omega_D) = -Q. \tag{60}$$

Again, $Q = 3b^2 H(\rho_m + \rho_D) = 3b^2 H\rho_D(1 + u)$ is the interaction term, where b^2 is a coupling constant [79]. Combining Eqs. (59) and (60) with the time derivative of Friedmann equation (9), and using the time derivative of Eq. (47) leads to

$$\omega_D = -\frac{1}{2 - \Omega_D - \xi}\left(1 + \frac{2b^2}{\Omega_D} - \frac{\xi}{\Omega_D}\right). \tag{61}$$

Now, since $Q = 3b^2 H\rho_D(1 + u)$, we get

$$b^2 = \frac{Q\Omega_D}{3\alpha H^2 + 3\beta H^3}, \tag{62}$$

where we have used Eq. (47). Combining this equation with Eq. (61), one obtains

$$\omega_D = -\frac{1}{2 - \Omega_D - \xi}\left(1 + \frac{2Q}{\left(\frac{3\alpha}{\tilde{r}_A^2} + \frac{3\beta}{\tilde{r}_A^3}\right)} - \frac{\xi}{\Omega_D}\right). \tag{63}$$

as the relation for the state parameter of the interacting GGDE. It is obvious that, the results of sections IV and III are available in the $Q \to 0$ and $\beta \to 0$ limits respectively. In addition, the result of section II is obtainable if one takes the appropriate limit ($Q \to 0$ and $\beta \to 0$) from this equation. Since the mutual interaction between the GGDE and the DM induces the thermodynamic fluctuations around the equilibrium, the first law of thermodynamics on the apparent horizon is available (19). For the heat capacity we have

$$C = T \frac{\partial S_D^{(0)}}{\partial T} = T \frac{\partial S_D^{(0)}}{\partial \tilde{r}_A^0} \frac{\partial \tilde{r}_A^0}{\partial T} = T \frac{\partial S_D^{(0)}}{\partial \tilde{r}_A^0} \left(\frac{-1}{2\pi T^2} \right) = -\tilde{r}_A^0 \frac{\partial S_D^{(0)}}{\partial \tilde{r}_A^0}, \tag{64}$$

and since

$$\frac{\partial S_D^{(0)}}{\partial \tilde{r}_A^0} = 8\pi^2 (\tilde{r}_A^0)^2 \left[\frac{2}{3} + \frac{\xi - \Omega_D^0}{\Omega_D^0 (2 - \Omega_D^0 - \xi)} \right] \alpha + 8\pi^2 \tilde{r}_A^0 \left[\frac{1}{3} + \frac{\xi - \Omega_D^0}{\Omega_D^0 (2 - \Omega_D^0 - \xi)} \right] \beta, \tag{65}$$

we get

$$C = -8\pi^2 (\tilde{r}_A^0)^3 \left[\frac{2}{3} + \frac{\xi - \Omega_D^0}{\Omega_D^0 (2 - \Omega_D^0 - \xi)} \right] \alpha - 8\pi^2 (\tilde{r}_A^0)^2 \left[\frac{1}{3} + \frac{\xi - \Omega_D^0}{\Omega_D^0 (2 - \Omega_D^0 - \xi)} \right] \beta. \tag{66}$$

Therefore, for the logarithmic correction of entropy due to the thermal fluctuation we reach

$$S_D^{(1)} = -\frac{1}{2} \ln \left(CT^2 \right) = -\frac{1}{2} \ln \left[-2\tilde{r}_A^0 \left[\frac{2}{3} + \frac{\xi - \Omega_D^0}{\Omega_D^0 (2 - \Omega_D^0 - \xi)} \right] \alpha - 2 \left[\frac{1}{3} + \frac{\xi - \Omega_D^0}{\Omega_D^0 (2 - \Omega_D^0 - \xi)} \right] \beta \right]. \tag{67}$$

Considering the first law of thermodynamics and following the recipe of section IV, we obtain

$$dS_D = 8\pi^2 \tilde{r}_A^2 d\tilde{r}_A \left[\frac{2}{3} + \omega_D \right] \alpha + 8\pi^2 \tilde{r}_A d\tilde{r}_A \left[\frac{1}{3} + \omega_D \right] \beta, \tag{68}$$

where $\tilde{r}_A$ is the apparent horizon radii (2) of the universe filled with interacting GGDE and differs from those of the universe filled by the non-interacting GGDE unless we have $H = H_0$ which is the non-interaction limit. Now, using Eq. (63), we get

$$\frac{dS_D}{d\tilde{r}_A} = 8\pi^2 \tilde{r}_A^2 \left[\frac{2}{3} - \frac{1}{2 - \Omega_D - \xi} \left(1 + \frac{2Q}{\frac{3\alpha}{\tilde{r}_A^2} + \frac{3\beta}{\tilde{r}_A^3}} - \frac{\xi}{\Omega_D} \right) \right] \alpha + 8\pi^2 \tilde{r}_A \left[\frac{1}{3} - \frac{\left(1 + \frac{2Q}{\frac{3\alpha}{\tilde{r}_A^2} + \frac{3\beta}{\tilde{r}_A^3}} - \frac{\xi}{\Omega_D} \right)}{2 - \Omega_D - \xi} \right] \beta. \tag{69}$$

Since $\frac{dS_D}{d\tilde{r}_A} = \frac{dS_D^{(0)}}{d\tilde{r}_A} + \frac{dS_D^{(1)}}{d\tilde{r}_A}$, we need to evaluate $\frac{dS_D^{(0)}}{d\tilde{r}_A}$ and $\frac{dS_D^{(1)}}{d\tilde{r}_A}$. Calculations lead to

$$\frac{dS_D^{(0)}}{d\tilde{r}_A} = \frac{\partial S_D^{(0)}}{\partial \tilde{r}_A^{(0)}} \frac{d\tilde{r}_A^{(0)}}{d\tilde{r}_A} = \left[8\pi^2 (\tilde{r}_A^0)^2 \left[\frac{2}{3} + \frac{\xi - \Omega_D^0}{\Omega_D^0 (2 - \Omega_D^0 - \xi)} \right] \alpha + 8\pi^2 \tilde{r}_A^0 \left[\frac{1}{3} + \frac{\xi - \Omega_D^0}{\Omega_D^0 (2 - \Omega_D^0 - \xi)} \right] \beta \right] \frac{d\tilde{r}_A^{(0)}}{d\tilde{r}_A} \tag{70}$$

and

$$\frac{dS_D^{(1)}}{d\tilde{r}_A} = \frac{\partial S_D^{(1)}}{\partial \tilde{r}_A^{(0)}} \frac{d\tilde{r}_A^{(0)}}{d\tilde{r}_A} = \frac{-1}{2\tilde{r}_A^{(0)}} \frac{d\tilde{r}_A^{(0)}}{d\tilde{r}_A}. \tag{71}$$

Finally, we get

$$\frac{dS_D^{(0)}}{d\tilde{r}_A} + \frac{dS_D^{(1)}}{d\tilde{r}_A} = \left[8\pi^2 (\tilde{r}_A^0)^2 \left[\frac{2}{3} + \frac{\xi - \Omega_D^0}{\Omega_D^0 (2 - \Omega_D^0 - \xi)} \right] \alpha + 8\pi^2 \tilde{r}_A^0 \left[\frac{1}{3} + \frac{\xi - \Omega_D^0}{\Omega_D^0 (2 - \Omega_D^0 - \xi)} \right] \beta - \frac{1}{2\tilde{r}_A^{(0)}} \right] \frac{d\tilde{r}_A^{(0)}}{d\tilde{r}_A}. \tag{72}$$

Combining this equation with Eq. (69), we reach

$$\begin{aligned} Q &= \frac{3}{2\tilde{r}_A^4} [\alpha ([\frac{2}{3} + \frac{\xi - \Omega}{\Omega (2 - \Omega - \xi)}] \tilde{r}_A^2 - (\tilde{r}_A^{(0)})^2 [\frac{2}{3} + \frac{\xi - \Omega^0}{\Omega^0 (2 - \Omega^0 - \xi)}] \frac{d\tilde{r}_A^{(0)}}{d\tilde{r}_A}) \\ &+ \beta ([\frac{1}{3} + \frac{\xi - \Omega}{\Omega (2 - \Omega - \xi)}] \tilde{r}_A - \tilde{r}_A^{(0)} [\frac{1}{3} + \frac{\xi - \Omega^0}{\Omega^0 (2 - \Omega^0 - \xi)}] \frac{d\tilde{r}_A^{(0)}}{d\tilde{r}_A})] + \frac{3}{4\tilde{r}_A^{(0)} \tilde{r}_A^4} \frac{d\tilde{r}_A^{(0)}}{d\tilde{r}_A}. \end{aligned} \tag{73}$$

Again, $\tilde{r}_A^{(0)}$ is the radii of the apparent horizon when the GGDE does not interact with the DM and therefore, there is no thermal fluctuations around the equilibrium state of the universe. In addition, $\tilde{r}_A$ is the radii of the universe accelerated by an interacting GGDE. In order to alleviate the coincidence problem we should have $b^2 > 0$. From Eq. (30) we get

$$-\left(\frac{2b^2}{\Omega_D} + 1\right) = \omega_D(2 - \Omega_D - \xi) - \frac{\xi}{\Omega_D}. \tag{74}$$

Since the LHS of this equation is negative, we should have $\omega_D(2 - \Omega_D - \xi) - \frac{\xi}{\Omega_D} < 0$ leading to the $\omega_D < \frac{1}{\Omega_D(\frac{2}{\xi} - \frac{\Omega_D}{\xi} - 1)}$ condition for ω_D. Therefore, when the equation of state parameter of DE meets the $\omega_D < \frac{1}{\Omega_D(\frac{2}{\xi} - \frac{\Omega_D}{\xi} - 1)}$ condition, DE decays into DM in agreement with the solving of the coincidence problem leading to leave thermal fluctuations in the system. Finally, we should note that the Eq. (73) is indeed, a relation between the interaction and the thermodynamic fluctuations.

VI.   SUMMARY AND DISCUSSION

We have investigated thermodynamics of the GDE enclosed by the apparent horizon in the flat FRW Universe, and found expressions for its entropy as well as the equation of state parameter. Since some observation data points to the mutual interaction between the dark components of the Universe, we have studied thermodynamics of the interacting GDE. Indeed, such interactions may lead to perturbations in the Hubble parameter yielding different radii and temperature for the apparent horizon. Therefore, it seems that one can consider such interactions as the cause of the fluctuations around the equilibrium state which is the state of the Universe when the dark sectors do not interact with each other. In addition, we obtained an expression for the equation of state parameter of the interacting GDE, as well as an expression for its entropy changes. Bearing the logarithmic correction to the entropy in mind, we got a relation between the mutual interaction of the dark components of the Universe and the thermal fluctuations around the equilibrium. In order to study a more realistic model, we pointed to the GGDE model where its energy density profile consists two parts including the term proportional to H and subleading term which is proportional to H^2. This subleading term may play the key role in the early Universe [91]. We investigated thermodynamics of GGDE, and found expressions for its entropy changes and the equation of state parameter. In continue, we got an expression for the equation of state parameter of the interacting GGDE as well as a relation between the thermal fluctuations and the mutual interaction of the GGDE and the pressureless DM. Finally, we showed that the coincidence problem makes a limitation on the possible mutual interaction between the dark sectors and thus the state parameter of the DE candidates. The corresponding limitations on the state parameter were derived for both of the GDE and GGDE models. Briefly, we think our survey shows that the interaction between the dark sectors of the Universe can be considered as the cause of the thermal fluctuations and thus, the logarithmic correction to the entropy of the GDE and GGDE models.

Acknowledgments

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