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A historical review of how the cosmological constant has fared in general relativity and cosmology

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Abstract

This paper reviews the ups and downs of the cosmological constant since its introduction into general relativity theory by Einstein in 1917. Although it has never really been loved, and is a major headache to particle physicists, it keeps on coming back in various guises or with minor modifications. At present it plays a manor role in cosmology both in terms of the 'false vacuum' of inflationary theory in the very early universe, and as a large-scale repulsive force in the late universe, as evidenced by observations of distant supernovae. Its origin in terms of fundamental physics remains as obscure as ever.
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1. Introduction

The cosmological constant, usually referred to as either Λ or λ, plays a recurring role in the history of general relativity theory and cosmology. It is a long-range repulsive force that acts on everything, but has an appreciable effect only on very large scales, indeed a significant fraction of the Hubble radius. It was introduced reluctantly, and dropped as soon as it seemed to be unecessary, but keeps coming back. It played a major role in the first two quantitative cosmological models (the Einstein static and de Sitter universes), and has made a strong comeback in recent times: a term acting like a cosmological constant is now believed to be important both in the very early universe, when inflation took place, and at the present time, when it causes an accelerating expansion. However its role as the vacuum energy of quantum field theory remains enigmatic and is still not well understood-this remains one of the major unsolved problems of theoretical physics. Before turning to a historical survey of its role, I first summarise how it occurs in the field equations in general, and in the equations governing the evolution of the standard world models of cosmology in particular [1].

1.1. The Einstein field equations and the cosmological constant

The total stress–energy–momentum tensor T_{ab} of all matter and fields present in a space-time governed by the Einstein field equations (EFE) of general relativity necessarily obeys the energy–momentum conservation equations

$$T^{ab}_{;b} = 0. \tag{1}$$

The EFE were orginally formed by setting the stress tensor proportional to the unique combination of curvature tensor and metric tensor terms (the *Einstein tensor*) that forms a symmetric divergence-free tensor with two indices:

$$G_{ab} := (R_{ab} - \tfrac{1}{2}Rg_{ab}) = \kappa T_{ab}. \tag{2}$$

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This can be shown to be the only such form of the field equations that guarantees validity of the conservation equation (1)—a crucial feature resulting in consistency of the constraint and propagation equations when a $1+3$ time-space splitting is employed. Einstein then determined one could add a 'constant' term proportional to the metric tensor to obtain a generalised form of the EFE:

$$G_{\mathrm{ab}} + \Lambda T_{\mathrm{ab}} = \kappa T_{\mathrm{ab}}, \tag{3}$$

while still maintaining this crucial feature: that is

$$\text{Eq. (3)} \Rightarrow \text{Eq.(1)} \ \{\text{iff } \Lambda_{,a} = 0\} \iff \{\Lambda = \text{const.}\}. \tag{4}$$

It is for this reason that talk of a 'time varying cosmological constant' does not make sense, in the context of classical general relativity theory. Any such term that varies in either space or time will be of such a form that (4) is not true and the EFE are no longer generally consistent. Hence any additional term that is broadly like a cosmological constant but varies in space or time should be regarded not as a 'cosmological constant', but as a *time-varying field*, and hence as a contribution to the matter tensor T_{ab} (i.e. it belongs on the right hand side of the EFE rather than the left). This will then be conserved by itself (i.e. will by itself satisfy (1)) only if it does not interact with any other energy components.

1.2. The cosmological constant and Friedmann–Lemaître world models

For a Friedmann–Lemaître (FL) world model, local isotropy everywhere implies the stress tensor necessarily necessarily has a perfect fluid form:

$$T_{\mathrm{ab}} = (\mu + p)u_{\mathrm{a}}u_{\mathrm{b}} + pg_{\mathrm{ab}}, \ u^a u_{\mathrm{a}} = -1, \tag{5}$$

where $\mu(t)$ and $p(t)$ are respectively the total energy density and pressure contributions from all sources, and in standard coordinates, $u^a = \delta^a_0$. The non-trivial part of the energy conservation Eq. (1) then takes the form

$$\dot{\mu} + 3(\mu + p)\frac{\dot{S}}{S} = 0, \tag{6}$$

where $S(t)$ is the FL scale factor. The dynamics of the universe model are governed by the remaining non-trivial EFE, namely the Friedmann equation

$$\frac{\dot{S}^2}{S^2} = \frac{\kappa\mu}{3} + \frac{\Lambda}{3} - \frac{k}{S^2} \tag{7}$$

and the Raychaudhuri equation

$$3\frac{\ddot{S}}{S} = -\frac{\kappa}{2}(\mu + 3p) + \Lambda \tag{8}$$

relating the scale factor $S(t)$, energy density $\mu(t)$, pressure $p(t)$, curvature constant k (normalized to ± 1 if it is non-zero), and cosmological constant Λ. The dynamical effect of the cosmological constant arises through these two equations. Eq. (7) is a first integral of (8) and (6) whenever $\dot{S} \neq 0$ and Λ is indeed constant (mirroring the property (4)).

2. Twenties

Einstein introduced the cosmological constant into the field equations [9] in order to obtain a static universe with closed spatial sections, where boundary condition problems are solved (see North [32]:81–87 and Pais [33]:284–288). He was also motivated in its introduction by its effects in Newtonian theory, where it solves problems of infinities in a cosmological context (it was proposed in 1896 by Neumann [31], see also Seeliger [39,40]). It follows immediately from the Raychaudhuri equation (8) that a static universe demands a positive cosmological constant, provided the energy condition $\mu + 3p > 0$ is satisfied:

$$\ddot{S} = 0 \Rightarrow \frac{\kappa}{2}(\mu + 3p) = \Lambda = \text{const} > 0 \tag{9}$$

and the Friedmann equation (7) then shows that the spatial curvature must be positive if $\mu > 0$:

$$\dot{S} = 0 \Rightarrow \frac{k}{S^2} = \frac{\kappa\mu}{3} + \frac{\Lambda}{3} = \text{const} > 0. \tag{10}$$

In the case of pressure-free matter considered by Einstein, this gives

$$\mu_E = \frac{2}{\kappa}\Lambda,\ k = 1,\ S_E = \sqrt{\frac{2}{\kappa\mu_E}} = \sqrt{\Lambda}.$$

Einstein believed this model showed that Mach's principle was successfully incorporated in general relativity with a cosmological constant. However de Sitter [4,6] showed one could also have an empty ('vacuum') static universe [1] with these field equations—a space-time of constant curvature, showing Einstein's hope that the cosmological constant would solve Mach's principle is wrong (see North [32] and Pais [33] for a discussion). Then the stationary form of the de Sitter universe was discovered [26]: a FL universe with

$$\{\mu = p = 0,\ k = 0,\ \Lambda > 0\} \Rightarrow \dot{S}/S = \sqrt{\frac{\Lambda}{3}} = \text{const} \equiv H \iff S = \exp Ht. \tag{11}$$

Thus this solution of (8) and (7) also demands a positive cosmological constant. Finally first Friedmann [17,18] and then Lemaître [27,28] showed non-static solutions with a cosmological constant existed. However the static paradigm held sway until 1930, even though Lemaître demonstrated that his model could explain the systematic redshifts of galaxies observed by Slipher and Hubble.

3. Thirties

A key step was the demonstration by Eddington in 1930 of the instability of the Λ—dominated Einstein static universe [7]; this follows immediately from (8) and (6) when $\mu + p > 0$ (to see this, perturb the solution away from the static values $\mu = \mu_E$, $S = S_E$ given by (9) and (10)). Then expanding universes with a cosmological constant were adopted by Eddington and Lemaître, e.g. [7], allowing a coasting period which could last indefinitely long (the universe starts of asymptotically in an Einstein static state in the past). However it was then realised that there were good expanding universe models without a cosmological constant, see Einstein [12] and Einstein and de Sitter [13], leading to Einstein now expressing a preference for $\Lambda = 0$ models, because the unaesthetic cosmological constant no longer served any useful purpose [11,8]. This then became the general position, see e.g. Tolman and Ward [41], with Einstein famously declaring his introduction of the term to be "the greatest blunder of my life" [19]. However Eddington and others disagreed, because the argument leading from (2) to (3) remains irrefutable: there can consistently be a cosmological constant and the main desirable features of the field equations remain, so there is no good reason to exclude such a term, which is in effect a constant of integration in the field equations (see e.g. North [32] for this argument). Additionally Eddington strongly desired a fixed scale that could be used in surveying the world, and this was provided by a non-zero cosmological constant, and so he defended it with extreme statements (see Chandrasekhar [5]). Robertson's beautiful survey paper [37] is typical of the moderate view that included it for generality, without taking a strong position either way, surveying all the possibilities. Models without a cosmological constant start from a singular state and recollapse or expand forever according to whether $k > 0$ or $k \leqslant 0$. Negative cosmological constants necessary lead to a universe that simply expands and then recollapses. Positive cosmological constants lead to a wide variety of behaviour when the curvature constant is positive ($k > 0$): the Einstein static universe, models that are asymptotic to the Einstein static universe in the past or the future, models collapsing from infinity and then re-expanding, and models that start from a singularity and then either expand forever, perhaps with a long period of 'hesitation' in an almost stationary state before expanding exponentially, or that recollapse. Models with a positive cosmological constant and $k \leqslant 0$ on the other hand have much simpler behaviour: thay can only expand forever from a singular state.

What was clear in particular was that if one considered a universe containing with positive density and non-negative pressure, then a non-singular model (one that 'bounces' after collapsing from infinity, or asymptotically starts at the Einstein static state in the past) requires both a positive cosmological constant and a positive spatial curvature:

$$\text{non-singular with } (p \geqslant 0,\ \mu > 0) \Rightarrow \{\Lambda > 0,\ k > 0\}, \tag{12}$$

the second condition being required to obtain an extremum by (7), and the first being required to obtain a bounce there by (8).

[1] Note that the static form of this universe does not take the form of a Robertson–Walker metric, as it is based on timelike Killing vectors.

4. Forties and fifties

The cosmological constant effectively was introduced in a new guise through the inrtroduction of the steady state universes by Bondi and Gold and Hoyle, see Bondi [2] for a discussion. This is just the de Sitter space-time in the steady-state frame, justified by the "Perfect Cosmological Principle" (everything is invariant in both space and time), but envisaged as due to altered field equations and conservation equations (made explicit by Hoyle [23]) rather than as a vacuum solution of the EFE with cosmological constant, as in the original de Sitter universe. However McCrea [30] then put a new twist on this possibility: he pointed out the equivalence to a solution of the standard EFE with a fluid source with the degenerate equation of state $(\mu + p) = 0$. In this case, general relativity theory allows expansion of the universe without the density dying away: by (6):

$$\{(\mu + p) = 0\} \Rightarrow \{\dot{\mu} = 0\},\ (\mu + 3p) = -2\mu = \mathrm{const} < 0, \tag{13}$$

the latter condition leading to exponential expansion via (8). Thus this kind of fluid (with non-zero matter density) is equivalent to a cosmological constant (with zero matter density). This gives an alternative interpretation of the de Sitter universe:

$$\{\mu = -p > 0,\ k = 0,\ \Lambda = 0\} \Rightarrow \dot{S}/S = \sqrt{\frac{\kappa\mu}{3}} = \mathrm{const} \equiv H \Longleftrightarrow S = \exp Ht. \tag{14}$$

However it should be noted that this equation of state in fact means that the inertial mass density of this 'fluid' is zero, and consequently the associated four-velocity is undefined; equivalently all timelike vectors are eigenvectors of the fluid stress tensor because it now takes the form $T_{\mathrm{ab}} = pg_{\mathrm{ab}}$. Hence we may expect velocity ambiguities or instabilities to be associated with this model (and this is reflected in the multiple frames possible in the de Sitter universe, see [38] for details).

A further point of interest is that the confusion over horizons in de Sitter space-time was resolved by Rindler [36], who inter alia showed that an event horizon occurs in this universe because the positive cosmological constant leads to divergence of the integral $u_{\mathrm{eh}}(t) = \int_t^\infty \mathrm{d}t/a(t)$. This was later important in terms of the link it sets up to black hole properties and Hawking radiation, and hence to creation of perturbations in inflation.

5. Sixties and seventies

Perhaps the best motivation for keeping the cosmological constant in standard cosmology was that it could solve the age problem that had plagued cosmology since the initial measurements of the Hubble constant in 1929 (this seems to have been the main reason why Hubble himself never really adopted the expanding universe picture [24]). Thus for example in his authoritative book, Bondi wrote in 1952 that such Lemaître models, with cosmological constant-dominated 'hesitation' period interposing between the decelerating hot early period when elements were formed and a late epoch of accelerating expansion driven by Λ, were "the best relativistic cosmology can offer" ([2], p. 121). He comments that such theories predicted a density parameter close to the critical value and an end to galaxy formation when the cosmological constant started to dominate, so no galaxies should in this case be younger than about 2/3 the Hubble constant. However by 1960 the Thackeray–Baade revision of distance estimates and hence of the Hubble constant resolved the age problem, and there was no longer any need for the cosmological constant. Furthermore the steady state model, with its effective cosmological constant, was dropped in the early 1960s firstly because of a conflict with radio source number counts and secondly because of the discovery of the 3 K cosmic background radiation (CBR), which strongly confirmed the Gamow–Teller–Alphe–Hermann theory of element formation in an early hot big bang epoch in an evolving universe.

There was a brief revival of interest in a Lemaître model with a 'hesitation' period in the mid 1960s because of an apparent excess of quasar-stellar objects at a redshift of 1.9, which could be explained by such "loitering universe" models ([35], and see [43], p. 616 for further references), but this effect went away with further observations. Thus by the 1970s, Λ had been effectively dropped from standard cosmology, see e.g. Weinberg [43], where models with a cosmological constant are relegated to 'Cosmology: Other Models', and the review by Gott, Gunn, Schramm and Tinsley [20], which proposed as best fit to the data, a low-density model without cosmological constant. This viewpoint may have been strengthened by two further points. First, it became clear that although a cosmological constant could in principle avoid the initial singularity as mentioned above, this could not work in practice because a (constant!) cosmological constant could not possibly have dominated either the Hot Big Bang epoch or earlier times as would be required for it to cause a turn-around at earlier times (for then it would have conflicted with the by now well established theory of nucleosynthesis [43]). Indeed observation of galaxies at higher and higher redshifts showed the universe had expanded through at least a factor 4, and already by then matter must have dominated any cosmological constant there may be (because if indeed present, it does not dominate matter by much at the present time, cf. [16]). Secondly, one could argue that any true cosmological constant violated Newton's fundamental law that action and reaction must be equal and opposite, for it amounted to a universal presence that affected everything but was affected by nothing [15]. Einstein had shown that even space-time was not fixed and immutable but rather was subject to this fundamental principle, but it would not apply to any genuinely constant ''cosmological constant''. Thus in this sense such a term was philosophically objectionable.

6. Eighties and nineties

The foundation for the next step had been laid by Zeldovich already in 1967 by his investigation of the stress–energy tensor of the dynamic vacuum of quantum field theory [46,47]. The key point is that this stress–energy tensor takes the form of an effective cosmological constant, and can arise as an asymptotic form of the energy density of a scalar field. In 1980, this idea became the core of the inflationary paradigm introduced by Guth [21]. In classical terms, the effective energy density of a scalar field in a Robertson–Walker geometry will take the form of a perfect fluid with

$$\mu = \tfrac{1}{2}\dot{\phi}^2 + V(\phi), \ p = \tfrac{1}{2}\dot{\phi}^2 - V(\phi),$$

where the field obeys the Klein Gordon equation

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0$$

which guarantees that both its energy and momentum are conserved. In the slow-rolling approximation, this field behaves like a cosmological constant:

$$\dot{\phi}^2 \ll V(\phi) \Rightarrow \mu + p = \tfrac{1}{2}\dot{\phi}^2 \approx 0, \ \mu + 3p = \dot{\phi}^2 - 2V(\phi) < 0$$

and can generate an exponential expansion for as long as the slow rolling lasts. This situation can arise in various ways: through broken symmetries leading to a meta-stable potential-dominated state, or due to particular initial conditions arising from a previous quantum gravity era, in either case leading to an effective cosmological constant driving an exponential expansion through many e-foldings. During this time the universe expands enormously but the energy density remains constant because the anomalous vacuum equation of state $(\mu + p) = 0$ is closely approximated by the dominant scalar field (the ̀inflaton'), while any pre-existing matter or radiation gets rapidly diluted away. This leads to a de Sitter like state; in effect, the steady state universe is resurrected in a new guise in the very early universe. But if this continued unchanged, there would remain only an empty and featureless universe. Hence the need for a ''dynamic lambda'' that, unlike a true cosmological constant, decays away to create matter and radiation and so allow subsequent formation of structure (and hence is not subject to the philosophical objection raised in the previous section). Indeed in many cases slow rolling will eventually no longer apply and the effective cosmological constant decays away (thus it is a 'false vacuum'), the resultant radiation-dominated expansion being the start of the hot big bang era in the early universe.

This scenario explains the flatness, horizon, and homogeneity problems through the accelerated expansion caused by the effective cosmological constant [21,25]. Furthermore the fluctuations associated with particles created by Hawking radiation arising from the event horizon in de Sitter space-time, again caused by the cosmological constant, become the seed inhomogeneities that eventually lead to galaxy formation, providing a major potential link between particle physics on the one hand and the large scale structure of the universe on the other. A particular popular version is chaotic inflation, based on a quadratic potential and where there are different values of various physical parameters, including the cosmological constant, in different regions. On the largest scales the universe has a fractal-like structure with interleaved exponentially expanding domains and 'pocket universes' (including the one in which we live) where the inflationary field has decayed to matter and radiation, which can then pursue a standard thermal history if the 'constant of nature' there are not too different from those in the observable universe [29,22].

This is a very attractive and popular scenario, even thought the details have not been tied down (indeed there are something like 60 competing versions on offer). It received a great boost in the mid to late nineties when its predictions for the cosmic background radiation (namely, that there should be a series of peaks in the angular power spectrum) were confirmed by balloon measurments. However it has some problems, the principle one being that the inflaton has not been identified, and indeed on a simple analysis based on quantum field theory the effective energy density should be quite different than what we observe. This is the puzzle of the quantum vacuum [44]: simple field theory gives an answer for the energy density of Λ that is wrong by 120 orders of magnitude. There is as yet no agreed resolution of this problem. One possibility is that it can be solved by invoking the anthropic principle (we have to live in a universe that admits life, so in an ensemble of universes, or of expanding universe regions such as may occur in chaotic inflation, the one we inhabit must have a low value for Λ for otherwise galaxies and planets will not come into existence [45]). Many people however are not convinced by this argument, particularly because its scientific status is unclear (how do you verify that it is true when all the other universes in the ensemble are completely unobservable, indeed they will in general have no causal contact of any kind with us?) Note also that if random values of the cosmological constant were laid down in this way, half the universes resulting would have negative cosmological constants and so would necessarily be fated to recollapse, rather than expanding forever as is possible (but not inevitable) with a positive cosmological constant.

7. Late nineties and early two thousands

In the late nineties, the cosmological constant made a spectacular come-back when it was discovered to be the major dynamical effect in the late expansion of the universe. This was determined through the Hubble diagram for supernovae in distant galaxies [42,34], the key point being that type Ia supernovae can serve as good standard candles because their maximum luminosity is correlated with their decay rate. The resulting data show an accelerating universe, and can be explained by the effect of either a constant positive Λ, or the energy density of a variable field (as in the case of inflation in the early universe), dubbed "quintessence". The former case is simpler but the latter may make structure formation easier; they can in principle be observationally distinguished by accurate enough measurements of the supernova Hubble diagram. Interpreting the data in this way as evidence for a large-scale repulsive force requires a thorough investigation of the alternatives: a major inhomogeneity in the form of a low density region surrounding the local group, or effects of local lumpiness on observations, or supernovae evolution effects, or absorbtion effects.

In principle the cosmological constant can also be detected by number counts and lensing data, but astrophysical modelling is necessary to interpret the data in each case and introduces extra parameters that also have to be determined. The value proposed must also be compatible with the data from CBR anisotropies. The current best fit models to all the observations have values of approximately

$$\Omega_m \simeq 0.3, \ \Omega_\Lambda \simeq 0.7 \Rightarrow \Omega_k \simeq 0 \tag{15}$$

for the effective densities of all the matter, the cosmological constant, and the curvature respectively. This solves the age problem and the flatness problem, and so is compatible with inflationary predictions for spatial curvature, but unlike standard inflationary models has a low density of matter, as evidenced in many other ways [3]. This data strongly suggests the universe will expand forever, for this will certainly be the case if Λ continues to dominate, as will be true if it remains constant. It is possible it is not constant (if it is 'quintessence' rather than an exactly constant quantity) and might still decay away in the future, however; one cannot easily tell if this will be the case or not. If this does happen, it could conceivably go negative—a case not envisaged in any of the standard scenarios—causing a recollapse, but presumably it is more likely that it will go to zero. Then conversion of the quintessence field into radiation might or might not lead to a low-density ever-expanding model at very late times, because while the present-day matter density is way below that needed to cause a turn-around, the additional energy density resulting from the decay might lead to a late-time value of Ω_k greater than zero. Indeed if we could measure the first two quantities in (15) accurately enough we could determine the sign of the third one, and that will determine the far future of the universe if the effective cosmological constant decays to zero. But until we identify more closely the nature of this presently dominant constant or field, we are unable to tell whether this decay will in fact happen or not.

8. Conclusion

Thus in this historical perspective, we are faced with the triumph of an unwanted guest: it seems defeated time after time, yet keeps coming back in new guises. It now seems that an effective cosmological constant dominates the universe both at very early times and at late times. In early times it serves a very useful purpose by first flattening out the universe and then indirectly generating perturbations about this smooth background; however at late times it seems to serve no useful purpose, indeed it has the rather negative effect of causing large scale structure formation to cease and creating event horizons that limit the domain in the universe we shall ever be able to connect with. We do not understand the physics behind either epoch, although we have numerous speculations about the possibilities in each case. Clarifying the nature of the underlying physics of the cosmological constant remains an important challenge to physicists and cosmologists.

References

[1] Bond JR et al. In: Proceedings of the IAU Symposium 201 (PASP), 2000;
Wang X et al. astro-ph/0105091, 2001;
Douspis M et al. Astronomy and Astrophysics 2001;379:1;
Efstathiou G et al. astro-ph/0109152, 2001;
Lahav O. Review talk. In: Lemos J et al., editor. X Encontro Nacional de Astronomia e Astrofisica, Lisbon, 2000;
Abroe ME et al. astro-ph/0111010, 2001.
[2] Bondi H. Cosmology. Cambridge: Cambridge University Press; 1960.
[3] Coles P, Ellis GFR. The density of matter in the universe. Cambridge: Cambridge University Press; 1997.
[4] de Sitter W. On the Relativity of Inertia. Remarks concerning Einstein's Latest Hypothesis'. In: Proceedings of the Akad Wetensch Amsterdam, 19. 1917. p. 1217–25.
[5] Chandrasekhar S. In: Eddington: the most distinguished astrophysicist of his time. Cambridge: Cambridge University Press; 1983. p. 36–9.
[6] de Sitter W. On Einstein's theory of gravitation and its astronomical consequences. III' Mon Not Roy Ast Soc 1917;78:3–28.
[7] Eddington A. On the instability of Einstein's spherical world'. Mon Not Roy Ast Soc 1930;90:668–78.
[8] Einstein A. PAW. 1913, p. 235 (reference from Pais (1982)).
[9] Einstein A. Kosmologische betrachtungen zur allgemeinen Relativitatstheorie. S-B Preuss Akad Wiss 1917:142–52.
[11] Einstein A. Letter to H Weyl May 23, 1923 (quoted in Pais (1982)).
[12] Einstein A. Zum kosmologischen problem der Allgemeinen elativitatstheorie'. S_B Preuss Akad Wiss 1931:235–7.
[13] Einstein A, de Sitter W. On the relation between the expansion and mean density of the universe'. Proc Acad Nat Sci 1932;18: 213–4.
[15] Ellis GFR. In: Sachs RK, editor. General relativity and cosmology (Varenna lectures). New York: Academic Press; 1971.
[16] Ellis GFR, van Elst HJ. In: Lachieze-Ray M, editor. Theoretical and observational cosmology (Cargese lectures). Kluwer; 1999.
[17] Friedmann A. Uber die Krummung des Raumes. Zs f Phys 1922;10:377–86.
[18] Friedmann A. Uber die Moglichkeit einer Welt mit konstanter negativer krummung. Zs f Phys 1924;21:326–32.
[19] Gamow G. In: My world line. Viking Press; 1970. p. 44.
[20] Gott JR, Gunn JE, Schramm DN, Tinsley BM. An unbound universe? Astrophys J 1974;194:543.
[21] Guth A. Phys Rev D 1981;23:347.
[22] Guth A. astro-ph/0101507, 2001.
[23] Hoyle F. Mon Not Roy Ast Soc 1948;108:372.
[24] Hubble E. The law of redshifts. Mon Not Roy Ast Soc 1953;113:658–66.
[25] Kolb EW, Turner MS. The early universe. Wiley; 1990.
[26] Lemaître G. Note on de Sitter's universe. J Math Phys (MIT) 1925;4:188–92.
[27] Lemaître G. Un univers Homogene de Masse Constante et de Rayon Croissant. Ann Soc Sci Brux 1927;47A:49–59.
[28] Lemaître G. Mon Not Roy Ast Soc 1931;91:483.
[29] Linde A. Particle physics and inflationary cosmology. Chur: Harwood Academic; 1990.
[30] McCrea WH. Relativity theory and the creation of matter. Proc Roy Soc Lond A 1951;206:562.
[31] Neumann C. Allgemeine Untersuchungen uber das Newtonsche Prinzip der Fernwerkungen (Leipzig), 1896.
[32] North JD. The measure of the universe: a history of modern cosmology. Oxford: Clarendon Press; 1965.
[33] Pais A. Subtle is the Lord. Oxford: Oxford University Press; 1982.
[34] Perlmutter S et al. Astrophys J 1999;517:565.
[35] Petrosian V, Salpeter EE, Szekeres P. Astrophys J 1967;147:1222.
[36] Rindler W. Visual horizons in world models. Mon Not Roy Ast Soc 1956;116:662.
[37] Robertson HP. Relativistic cosmology. Rev Mod Phys 1933;5:62–90.
[38] Schroedinger E. Expanding universes. Cambridge: Cambridge University Press; 1957.
[39] Seeliger H. Astron Nach 1895;137:129.
[40] Seeliger H. Munch Ber Math Phys Kl 1896:373.
[41] Tolman RC, Ward M. On the behaviour of non-static universes when the cosmological term is omitted. Phys Rev 1932;39:835–43.
[42] Schmidt BP et al. Astrophys J 1998;507:46.
[43] Weinberg SW. Gravitation and cosmology. New York: Wiley; 1972.

[44] Weinberg SW. Rev Mod Phys 1989;61:1.
[45] Weinberg SW. The Cosmological Constant Problems astro-ph/0005265, 2000. See also astro-ph/0104482.
[46] Zeldovich YaB. Cosmological constant and elementary particles. Sov Phys JETP Lett 1967;6:316–7.
[47] Zeldovich YaB. The cosmological constant and the theory of elementary particles. Sov Phys Uspekhi 1968;11:381–93.