Transformações de Lorentz

$$\mathbb{V} = \frac{d\mathbb{r}}{dt} = c\,\mathbb{n}$$

$$dr^2 - c^2 dt^2 = 0$$

* Equações de Maxwell preservam a velocidade da luz.

$$(\mathbb{r}, t) \longrightarrow (\mathbb{r}', t')$$

$$c = c'$$

- Transformação afim

$$\mathbb{X} = \begin{pmatrix} ct \\ \mathbb{r} \end{pmatrix}$$

$$\mathbb{X}' = M\mathbb{X}$$

. .

$$\begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \alpha & \beta & \gamma & \delta \\ \alpha_1 & \beta_1 & \gamma_1 & \delta_1 \\ \alpha_2 & \beta_2 & \gamma_2 & \delta_2 \\ \alpha_3 & \beta_3 & \gamma_3 & \delta_3 \end{bmatrix} \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix}$$

Simplificando em uma direção

$$y' = y \quad e \quad z' = z$$

$$t' = \alpha t + \frac{\beta}{c} x \quad e \quad x' = \alpha_1 ct + \beta_1 x$$

$$c^2 dt'^2 = \left(\alpha dt + \frac{\beta}{c} dx\right)^2 = c^2\alpha^2 dt^2 + 2c^2 \frac{\alpha\beta}{c} dt\, dx + \beta^2 dx^2$$

$$dx'^2 = c^2\alpha_1^2 dt^2 + 2\alpha_1 \beta_1 c\, dt\, dx + \beta_1^2 dx^2$$

$$-c^2\alpha^2 dt^2 - 2c\alpha\beta dt\, dx - \beta^2 dx^2 + c^2\alpha_1^2 dt^2 + 2\alpha_1\beta_1 c\, dt\, dx + \beta_1^2 dx^2 = -c^2 dt^2 + dx^2$$

$$\begin{cases} \alpha_1^2 - \alpha^2 = -1 \\ \alpha\beta = \alpha_1\beta_1 \\ -\beta^2 + \beta_1^2 = 1 \end{cases}$$

Se $c \to \infty$

$$x' = x + vt$$

$$x' = \alpha_1 c t + \beta_1 x \quad ; \quad x' = \beta_1 x + \left(\frac{\alpha_1 c}{v}\right) vt$$

$$[\alpha_1] = [\beta_1] \Rightarrow \frac{c\alpha_1}{v} = \beta_1 = \gamma$$

$$\begin{cases} \left(\frac{\gamma v}{c}\right)^2 - \alpha^2 = -1 \\ \alpha\beta = \gamma^2 v / c \\ -\beta^2 + \gamma^2 = 1 \end{cases}$$

$$\gamma^2 \frac{v^2}{c^2} - \left(\frac{\gamma^2 v}{\beta c}\right)^2 = -1$$

$$\beta^2 = -1 + \gamma^2$$

$$\frac{\gamma^2 v^2}{c^2}\left[1 + \frac{\gamma^2}{(1-\gamma^2)}\right] = -1 \quad ; \quad \gamma^2 \frac{v^2}{c^2}\left(\frac{1}{1-\gamma^2}\right) = -1$$

$$\gamma^2 \frac{v^2}{c^2} + 1 - \gamma^2 = 0 \Rightarrow \boxed{\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}}$$

$$\beta^2 = -1 + \frac{1}{(1 - v^2/c^2)} \quad ; \quad \boxed{\beta = \frac{v}{c}\gamma}$$

$$\boxed{\alpha = \gamma}$$

$$\begin{cases} t' = \gamma\left(t + \frac{vx}{c^2}\right) \\ x' = \gamma(x + vt) \\ y' = y \quad e \quad z' = z \end{cases} \Longrightarrow \text{Transformação de Lorentz.}$$

$$\begin{array}{l} \Delta x' = \gamma(\Delta x + v\Delta t) \\ \Delta t' = \gamma\left(\Delta t + \frac{v\Delta x}{c^2}\right) \end{array} \Longrightarrow \boxed{v' = \frac{u + v}{(1 + vu/c^2)}}$$

– Contração espacial

$$\Delta t = 0 \qquad \boxed{L = \gamma L_0}$$

– Dilatação temporal : $\Delta x = 0$ $\boxed{t = \frac{t_0}{\gamma}}$