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FROM THE HUBBLE EXPANSION TO HIGHER DIMENSIONS A REVIEW OF MODERN COSMOLOGY

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Abstract

The marriage of particle physics and cosmology over the last decade or so has led to a great deal of progress in the understanding of the physics of the early universe. In this review we discuss a range of theories and phenomena which define the study of modern cosmology.

1. Introduction

The study of cosmology in its modern sense has relatively recent origins. Its beginning can be placed around 1929 when Hubble discovered that the universe was of a dynamic nature and was, in fact, expanding. The most amazing thing about this discovery was the universality of the expansion: *all* galaxies, which may be regarded as the basic "atoms" of the universe, are moving away from our galaxy. Therefore, the observed expansion is neither a local phenomenon nor a random statistical event but a property of the universe as a whole. All galaxies move away from each other with enormous velocities which, at large distances, approach the speed of light. This discovery verified some of the most daring predictions of Einstein's theory of general relativity. Until 1929 these ideas had been regarded as interesting but unappealing.

The first theorists to construct models of an expanding universe, using general relativity, were de-Sitter (1917), Friedmann (1922) and particularly Lemaitre (1927) who introduced the concept of a 'primordial atom'. These models showed that solutions could be found to Einstein's relatively new equations of gravitation in which the cosmology was evolving dynamically. These solutions could not therefore be excluded as possible descriptions of the universe. However, the majority of astronomers did not take these models seriously since the concept of a dynamic universe was contradictory to most beliefs at that time. Einstein himself 'marred' the beautiful simplicity of his equations by the inclusion of a cosmological constant in order to retain the notion of a static cosmos in which he believed. (The importance of the existence of an object akin to this cosmological constant will be discussed in later sections). The momentous discovery of Hubble in 1929 meant that for the first time the study of the universe as a whole became the object of serious physical research, subject now to observational constraints. The great advancement of cosmology that followed was due to systematic research in both observations *and* theory. Hubble, himself, initiated a large scale study of the universe, starting from the nearby galaxies. Modern astronomical techniques have taken the subject far beyond the local galactic neighbourhood to distant objects such as quasars and 'primeval' galaxies at distances some three orders of magnitude farther than Hubble's original sample.

The subject of cosmology is concerned mainly with this extragalactic world. It is a study of the large-scale nature of the universe extending to distances of giga-parsecs, a study of the overall dynamic and physical behaviour of a myriad of galaxies spread across vast distances and of the evolution of this enormous system over several billion years. In this review we will discuss a range of topics which have contributed much to our current understanding of the universe. In Section 2 we describe briefly the range of structure observed in the universe and discuss some of the theories put forward to explain the nature of this structure in Section 3. Section 4 discusses the physical properties of the early universe and the impact of the ideas from elementary particle physics on our understanding of the first seconds of the evolution of the universe. In Section 5 we take a look at more exotic cosmological theories, involving higher dimensional cosmologies, with concluding remarks in Section 6.

2. The Observable Universe

It became clear from the catalogue of the positions of bright galaxies, compiled by Shapley and Ames (1932), that galaxies were not spread uniformly across the sky but rather seemed to be segregated into compact clusters, many of which appeared spherically symmetric. Abell (1958) chose a homogeneous sample of such clusters and noticed in addition that the clusters of galaxies tended to be grouped into larger structures, now called *superclusters*, indicating some form of higher-order clustering. These superclusters are found to be of the order of 50Mpc in size which are extremely large when compared with the typical galactic scale of 30-50kpc. There can be as many as 10 rich clusters with a total mass of $10^{15} - 10^{17}\mathrm{M}_\odot$ in a large supercluster and the largest structures could have as many as 10^5 member galaxies. Most galaxies belong to such large dynamic structures and it is estimated that $\simeq$90% of all galaxies belong to some form of hierarchical structure. The clusters of galaxies are generally spherical in shape whereas almost all superclusters are flat. For this reason superclusters are sometimes called *pancakes*, after the work of Zel'dovich (1970).

At first glance the observations of clusters and superclusters indicate that their distribution is random. However, after detailed observation and data analysis it becomes clear that their distribution is not in the least uniform. There exist huge volumes in the universe which contain almost no galaxies, appropriately called "voids", de Lapparent et al. (1986). It is estimated that only 10% of space is occupied by superclusters while the rest does not contain any luminous matter. These voids may reach dimensions of up to the order of 50Mpc. More recent surveys have also demonstrated the existence of a significant amount of structure on very large scales; Maddox et al. 1990 (APM survey) and Rowan-Robinson et al. 1990 (QDOT survey). Recent theories of the structure of superclusters can, in fact, explain the creation of condensations of matter in spherical form and also the formation of large scale filamentary and flat structures, (cf. Peebles 1965, Zel'dovich 1970 and Saarinen et al. 1987). Numerical simulations of the distribution of matter on large scales give structures of comparable size, shape and distribution to those observed (White et al. 1987).

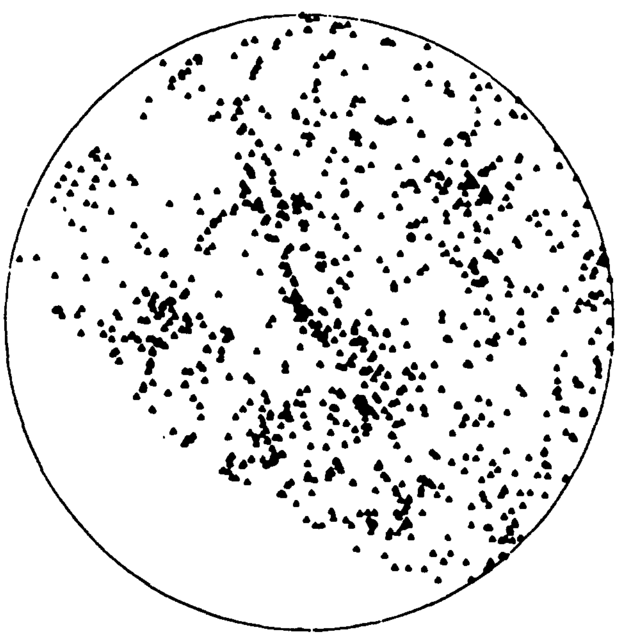

Figure 1: Equal area projections of the galaxy distribution in the northern sky from the CfA survey volume limited to $4000 \mathrm{kms}^{-1}$.

The universe as a whole appears to be isotropic and homogeneous on very large scales. Isotropy means that the universe looks the same in every direction, homogeneity means that the universe will appear the same to any observer, independently of their position in the universe. In other words, all observers would measure the same density and generally the same properties of the universe. This is termed the *cosmological principle*. Observationally it is found that on small scales, the distribution of galaxies is inhomogeneous but becomes increasingly more homogeneous as the scale increases, with the greatest degree of isotropy in the universe being exhibited by the microwave background radiation (MBR). This is observed to be isotropic to an extremely high degree (cf. Davies et al. 1987 and the COBE results: Smoot et al. 1991) and may in fact be too isotropic for the good of the standard hot big-bang cosmology (see Section 6).

The MBR was first discovered by Penzias and Wilson (1965) and became one of the major cosmological discoveries of all time. This radiation fills the whole of space and is thought to be the remnant of the radiation of the early universe. It is an extremely diffuse radiation which comes uniformly from all directions and corresponds to a black body spectrum of approximately 3°K. Such a spectrum cannot be due to discrete objects such as stars or galaxies but is consistent with what would be expected from an early concentration of matter in the universe with a temperature of about 3000°K. At this temperature hydrogen recombines and the mean free path of photons in the universe would become as large as the horizon. The photons would therefore, act as a background sea of radiation (the MBR), the temperature of which would cool due to the general expansion of the universe, giving it a temperature at the present epoch of order 3°K. Evidence that the origin of this radiation is primordial comes from the fact that its spectrum is almost exactly that of a black body. The recent remarkable results from the Cosmic Background Explorer satellite (COBE) have shown that the microwave background radiation displays a spectrum which has a black body form to a very good degree of accuracy, $\delta T/T \leq 10^{-5}$; Smoot et al. 1991. The implications of this to the theories of the formation of large scale structure in the universe are discussed below.

Thus, on very large scales, the universe is observed to be extremely isotropic and homogeneous and any viable cosmological model must contain this exactly or at least in the limit. On the scale of superclusters ($\simeq$50Mpc), however, observations show that the universe has some filamentary and bubble-like structure, cf. Figure 1, making it generally inhomogeneous. There are several theories which attempt to explain this large-scale structure.

3. Theories for Large-scale Structure in the Universe

The so-called "Hot Big Bang" model of the universe has been extremely successful. This model predicts that the universe should be filled with a low level of radiation left over from the initial 'explosion' and the observation of this radiation by Penzias and Wilson became the first major success of the big-bang model. This and subsequent successes, such as the prediction of the correct abundances of the light elements formed during the period of cosmological nucleosynthesis, has given us the confidence to trace the history of the universe back into the first second of its existence.

Questions that remain unanswered, however, are: why is the universe "lumpy" and how did it get that way? The big-bang model treats the universe as completely smooth and uniform and we described in the previous section that, on the very large scale, matter does indeed appear to be spread out evenly everywhere. However, on smaller scales a great deal of structure exists. Recently observations have revealed structures such as huge empty regions (voids), the largest $\simeq$60Mpc in diameter (Kirschner et al. 1983), giant filaments, i.e. roughly linear overdense regions in the distribution of galaxies about 100Mpc long and 5Mpc across (Giovanelli and Haynes 1982) and in more complete surveys most galaxies appear to lie on the surfaces of 'bubbles', $\simeq$50Mpc across (de Lapparent et al. 1986). For an excellent collection of articles on this subject see Cornell (1989).

Before we consider the formation of these complicated large-scale structures we must address the problem of how the basic matter condensations are formed. We shall concentrate on the mechanism of gravitational instability based on the work of Jeans (1902).

Weizsäcker (1951) proposed that the alternative mechanism of turbulence in the early universe could give rise to the formation of matter condensations. However, there have been some serious arguments raised against this theory and we will not consider it further (Peebles 1971).

The basic idea behind the Jeans instability analysis for the growth of density perturbations in an otherwise homogeneous medium is the competition between gravity, which causes a local density excess to grow as it attracts more and more matter and gas pressure, which tends to disperse any density enhancement and restore the system to homogeneity. Jeans (1902) found that for small scale perturbations the gas pressure dominates and the perturbations quickly disperse. On the other hand, for large scale perturbations the gravitational field is dominant and the density enhancement grows. This latter process is known as the *Jeans instability.* The scale at which the instability sets in is known as the *Jeans length*, λ_j, and the mass lying within this radius is called the *Jeans mass*, M_j. In a sphere of radius greater than λ_j gravity overcomes the pressure and forms a matter condensation. If the radius of the sphere is less than λ_j then gas pressure causes the initial perturbation to be damped and it disperses.

The mass concentrations formed via the Jeans instability may evolve to form a star, a galaxy or even larger scale structures. The amount of matter condensed depends upon the initial density of gas and the propagation speed of the perturbation (local sound speed). In the early universe, prior to the recombination of hydrogen, matter and radiation were strongly coupled and this led to the sound speed in the universe being very large. After recombination the interaction length of the photons became comparable with the Hubble radius and they ceased contributing to the pressure causing a sharp drop in the sound speed. The Jeans mass dropped correspondingly from $\simeq 10^{17} M_\odot$ before recombination to $\simeq 10^5 M_\odot$ after.

We will discuss two extremes of matter condensations which form via gravitational instability: isothermal and adiabatic (cf. Contopoulos and Kotsakis 1987). In the former case, the temperature inside the perturbation is kept the same as the surrounding cosmic temperature by the free streaming of photons which remain uniformly distributed while the matter condenses. In the latter case, the temperature increases along with the density with the photon to baryon ratio the same inside and outside the perturbation. Two different theories have been proposed for the formation of galaxies and clusters of galaxies based on each of these types of perturbation.

In the theory of isothermal perturbations (Peebles 1965) any density enhancements in the initial matter distribution of the universe remain unchanged until gravity begins to dominate which happens around the time of recombination. At this time any perturbation greater than $10^5 \mathrm{M}_\odot$ starts to accrete the surrounding matter and begins to build up larger and larger concentrations of mass (clusters, superclusters, etc.). Thus, the larger the structure, the longer its formation time.

There are two basic arguments in favour of this theory. The first is that a study of observational data on the galaxy distributions shows that there are no distinctive scales for groups of galaxies up to supercluster size, cf. Peebles (1980). The second argument is based upon the clustering obtained in numerical models investigating the growth of density perturbations in an expanding universe. Aarseth et al. (1979) found that an initially uniform distribution of points (each point representing a galaxy) condensed as the universe expanded. Together with these matter concentrations, the numerical simulations also produced large empty regions devoid of galaxies in between the clumps of matter. The results of these model calculations, therefore, seem to be compatible with many characteristics of the observed universe.

Zel'dovich (1970) proposed an alternative to the Peebles approach in which the initial perturbations were adiabatic. In this theory only large perturbations survive the dissipative effects of viscosity present in the early stages of the universe's evolution. Only matter concentrations greater than $10^{13} M_{\odot}$ remain intact until the recombination era when they collapse via the Jeans instability. Thus, in this theory, it is the larger scale structures (superclusters) which form first and fragment to form structure on smaller scales (clusters etc.). The initial concentrations formed are very flat in shape and this has led to this theory being called the ***pancake theory***. These "pancakes" eventually fragment to form galaxies. (The Peebles picture is often referred to as the ***bottom-up*** scenario whereas the Zel'dovich picture is called the ***top-down*** scenario).

The top-down theory has several attractive characteristics. For example, observations of the distribution of galaxies (Figure 1) show galaxies and clusters distributed in several long filamentary structures, reminiscent of Zel'dovich's pancakes. The numerical simulations are also found to be consistent with this picture if we allow the points to represent particles, rather than galaxies. Recently Uson et al. (1991) have discovered a pancake shaped cloud of neutral hydrogen at a large redshift of 3·4. These authors have suggested that because this corresponds to a very early epoch it may be an embryonic cluster of galaxies. This is strong evidence for the top-down theory of structure formation in the universe. It is still too soon, however, to say which of the two theories better describes the formation of large-scale structure and until more observations are made both should be considered equally possible.

One current field of study, which incorporates the above theories in order to produce the large-scale structure, is that of the theories of dark matter in the universe. Dark matter is the 'unseen' matter which most astronomers believe surrounds the luminous stars and galaxies and makes up the vast bulk of the mass of the universe. Dark matter betrays itself by the gravitational effect it has on the matter we can see. Observational evidence shows that dark matter is present on all distance scales, from its effect on the motion of the sun, the rotation of galaxies, in the dynamics of clusters and superclusters and also in the expansion of the universe (Kormendy and Knapp 1987).

At present, there is good evidence that the dark matter is in a non-gaseous, effectively collisionless form, and therefore the evolutionary phases of the structure in the universe can be studied, quite easily, by N-body numerical methods (eg. White et al. 1984). There are essentially two forms that this dark matter can take; it may be composed of baryonic or non-baryonic matter.

Baryonic Dark Matter

The observed ratio of the abundances of deuterium and helium together with the standard model for primordial nucleosynthesis constrains the fraction of the density of the universe contributed by baryons (luminous and dark) to be

$$0.056h^{-2} \leq \Omega_{baryons} \leq 0.14h^{-2} \tag{1}$$

where the parameter h is chosen such that $h = H_0/50kms^{-1}Mpc^{-1}$ (observationally $1 \leq h \leq 2$) and $\Omega = \rho/\rho_c$, the ratio of the density of the universe to the critical (or closure) density. Since the dynamically measured value of Ω is $\simeq 0\cdot 1 - 0\cdot 2$, then it is possible that all of the dark matter could be baryonic in the form of distributions of objects such as brown dwarfs or black holes within galaxies and clusters. Such constituents of dark matter may eventually be detected. Recent work has shown, however, that the abundances of the light elements may be very different from the primordial abundances. Dimopoulos et al. (1988) invoke a late decaying particle, the products of which interact with the ambient protons and^{4}He causing hadronic showers and ^{4}He destruction. In this model the primordial abundances of all the other light elements (D,^{3}He,^{6}Li,^{7}Li) are found to be completely independent of the physics before the keV era whose only important role is to provide at least the observed abundance of ^{4}He. Delbourgo-Salvador et al. (1985) discuss a model of galactic evolution which changes significantly the light element primordial abundances, principally through the destruction of deuterium. Applegate et al. (1987) consider the effects of baryon diffusion through QCD fluctuations and photodiffusion from decaying massive neutrinos/gravitinos respectively, on the primordial abundances leading to very different interpretations of the observed abundances. These processes, can allow the density parameter for baryons, $\Omega_{baryons}$, to be as high as $\simeq 0\cdot 3$.

Non-Baryonic Dark Matter

Belief in an inflationary universe, discussed below, strongly biases most cosmologists and there is almost universal agreement that our universe should be flat with $\Omega = 1$. Due to the constraints imposed on baryonic matter by nucleosynthesis this seemingly suggests that most of the matter in the universe should be in some non-baryonic form. Also the existence of galaxies and clusters today requires that perturbations in the density must become non-linear before the present epoch. In a baryonic universe, for adiabatic perturbations at recombination, this implies that present-day fluctuations in the microwave background radiation should be much larger than present observational limits, Kodama et al. (1987).

One of the currently fashionable possibilities is that dark matter consists of relic WIMPs; Weakly Interacting Massive Particles (there is also some talk nowadays of a baryonic distribution of objects with the more inspiring name of MACHOs or Massive Accreting Compact Halo Objects). It is postulated that these WIMPs are left over from the very hot, early epoch of the universe. The early universe and modern particle theories working together have provided a very generous list of candidates for the dark matter (cf. Turner 1987). For the purpose of illustration we shall consider only two.

The standard model of particle physics [a gauge theory which undergoes spontaneous symmetry breaking at a temperature $T \simeq 300GeV$: $SU(3) \times SU(2) \times U(1) \longrightarrow SU(3) \times U(1); 1GeV = 10^{13}K$] supplies no candidates, other than the rather exotic quark nuggets, for dark matter beyond the ordinary baryons in some non-luminous guise. Virtually all extensions of the standard model, however, provide us with a generous supply of dark matter candidates. The two we shall consider are massive neutrinos and axions. [Table 1 provides a summary of the conversion scales between temperature, energy, size of the universe and time after the big bang for a hot big bang model].

Temperature	Energy	Size of universe	time after big bang (s)	Remarks
3K	3.10^{-4}eV	1	$\simeq 10^{18}$	Present epoch
3000K	0.3eV	10^{-3}	10^{13}	Recombination of Hydrogen
10^9K	0.1MeV	10^{-9}	100	Big-bang Nucleosynthesis
10^{11}K	10MeV	10^{-11}	0.01	
10^{13}K	1GeV	10^{-13}	10^{-6}	Quark/Hadron transition
10^{15}K	100GeV	10^{-15}	10^{-10}	End of electro-weak unification
10^{27}K	10^{14}GeV	10^{-27}	10^{-34}	End of grand unification
$> 10^{31}$K	$> 10^{19}$GeV	$< 10^{-31}$	$< 10^{-43}$	Planck era - Quantum gravity

Table 1: Conversion factors between temperature and energy for significant times in the history of the very early universe.

Massive neutrinos are a product of the standard model extension known as the Majoron model, cf. Gelmini and Roncadelli (1981). (The symmetry broken in this theory is the lepton number). We know neutrinos exist and if they have a mass then they would be ideal dark matter candidates. If neutrinos are massive then it can be shown that the density of neutrinos relative to the critical density is given by

$$\Omega_\nu = 0.12\frac{n_\nu}{n_\gamma}h^{-2}\sum(m_\nu) \tag{2}$$

where n_γ is the photon number density in the microwave background, masses are in eV and we are summing over the number of neutrino species. In the standard model $n_\nu/n_\gamma \simeq 3/11$. Consequently, one species of mass $\simeq 25h^2eV$ suffices to give $\Omega_\nu = 1$.

Another popular, though more exotic, dark matter candidate is the axion. Peccei and Quinn (1977) proposed extending the standard model by adding one additional Higgs doublet. (We can add as many scalar, or Higgs, fields to the theory as we like by relating them to the free energy of the system). This extension introduces another symmetry (the PQ symmetry) which is also spontaneously broken. The existence of such a broken symmetry leads to a new light pseudoscalar boson called the axion, Weinberg (1978).

The scale at which the PQ symmetry is broken, f_{PQ}, determines all of the properties of the axion: its mass, lifetime and coupling to ordinary matter, viz.,

$$\begin{aligned} m_a &= 10^{-5} eV(10^{12} GeV/f_{PQ}) \\ \tau(a \rightarrow 2\gamma) &= 10^{41} yrs.(f_{PQ}/10^{12} GeV)^5 \\ g &= m_e/f_{PQ} \end{aligned} \tag{3}$$

where g is the coupling of the axion to the electron, cf. Turner (1987). From helium burning constraints in various stars it is found that f_{PQ} is required to be $>10^8$GeV and it can be shown that if the energy density is to be of the order unity, $\Omega_a \simeq 1$, then we require a PQ breaking scale of $\simeq 10^{12}$ GeV, corresponding to an axion mass of 10^{-5} eV. Thus, for the allowed values of f_{PQ} we have

$$10^{-5} eV \leq m_a \leq 10^{-1} eV \tag{4}$$

What then are the implications of the existence of these WIMPs for the formation of structure in the universe?

Density perturbations in a self-gravitating fluid, in which the mean free path of the fluid constituents is finite, will undergo Landau damping (cf. Bond and Szalay 1983) and the damping scale determines the characteristic size of the initial structures. For instance, the damping scale, λ_{FS}, for a massive neutrino is of the order of 40Mpc(m/30eV) whereas for an axion it is $<10^{-5}$Mpc. Physically, the damping scale λ_{FS} is the comoving distance that a WIMP could have travelled since the big-bang. The separation of damping scales allows the WIMPs to be split into three categories:

Cold	$\lambda_{FS} \ll$1Mpc eg. axions	galactic size perturbations survive free-streaming
Warm	λ_{FS}=1Mpc	
Hot	$\lambda_{FS} \geq$1Mpc eg. massive ν's	only perturbations on scales much larger than galactic scales survive free-streaming

This is more clearly illustrated in Figure 2, where we can see the power spectra at late times in a universe now dominated by WIMPs. The quantity $k^3. \mid \delta_k \mid^2$ is the local power in plane wave perturbations of scale $\lambda = 2\pi/k$ and δ_k is the amplitude of the relative density fluctuations in some particle or radiation field. Objects of this size will condense out of the general expansion when $k^3. \mid \delta_k \mid^2 \approx 1$. Until non-linear effects are important the spectra shown evolve by increasing their amplitude while maintaining their shape.

We see from the figure that the characteristic scale for hot dark matter, such as massive neutrinos, is

$$\lambda \simeq \frac{17}{\Omega h^2} Mpc \simeq 17(100eV/m_x) Mpc \tag{5}$$

which is of the order of supercluster size (m_x is the mass of the particle). Thus, structure in a neutrino-dominated universe will grow according to a variant of Zel'dovich's "pancake" scenario. On the other hand, an axion-dominated model will cluster hierarchically in the manner discussed by Peebles (1965).

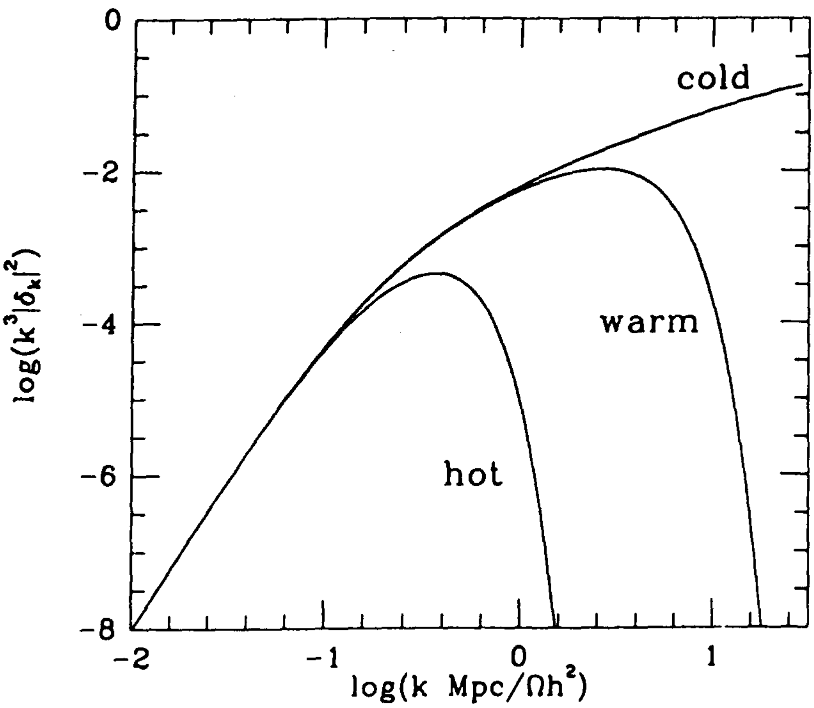

Figure 2: Linear power spectra as a function of spatial frequency at late times in a universe dominated by collisionless particles. The three cases are differentiated by the random velocities of the particles involved (White 1987).

White et al. (1987) have shown that cosmologies dominated by cold dark matter produce mass distributions which fit the observed galaxy distribution, (i) if Ω=0·1-0·2 and galaxies trace the mass distribution or (ii) Ω=1, $H_0 \leq 50\mathrm{kms}^{-1}\mathrm{Mpc}^{-1}$ and galaxies form preferentially in high density regions (cf. biased galaxy formation models; eg. Dekel and Silk 1986). These cold dark matter model catalogues differ from the real data in that their clusters are somewhat tighter and the associated velocities somewhat higher. Cold dark matter models can, therefore, reproduce the observed galaxy-galaxy correlation function of Peebles (1980) but not the cluster-cluster correlation. If Ω is indeed unity, galaxies cannot trace the mass. Rather they must be over-represented by a factor of about five in the dense regions from which dynamical estimates are obtained (Kaiser 1985). However, the recent APM (Maddox et al. 1990) and QDOT (Rowan-Robinson et al. 1990) surveys have indicated that there is more structure observed on large scales than can be accounted for in the standard cold dark matter models of White et al. (1987).

A major opponent of the dark matter models for the large-scale structure of the universe is the theory of galaxy formation based on cosmic strings. The cosmic string model does not preclude the existence of dark matter but the mechanism which generates the structure is somewhat different.

In the spontaneously broken gauge theories of elementary particle physics there are, in addition to the fundamental particles of the theory, topological entities, which form as defects in the process of breaking the symmetry. (These objects correspond to classical configurations of the gauge and Higgs fields). A class of these Grand Unified Theories (GUTs) leads to the prediction of topological entities which are line singularities and are referred to as *cosmic strings* (Vilenkin 1985).

GUTs all begin with the assumption that at the very high energies of the first moments after the big-bang, there was no distinction between three of the four fundamental forces of nature (electromagnetism, weak and strong interactions). Soon after the big-bang, the symmetry broke and energy settled into fundamental particles, such as quarks and leptons. However, it was postulated that when this occurred, at about 10^{-35}s, frozen bits of unified field got trapped in long *cosmic strings* (Kibble 1976). These defects contained remnants of the high energy that existed just after the big-bang. The existence of cosmic strings is highly speculative. Nevertheless, many unified theories do predict that the universe would fill up with a network of such strings, as defects of the time of the symmetry breaking, which is inherent to these theories. These cosmic strings would be very heavy (typically 10^4kgcm^{-1}) and very thin ($\simeq 10^{-7}$cm) and they would have a very strong gravitational field. (It can be shown that the production of cosmic strings in the very early universe leads to isothermal perturbations in the matter distribution of a definite spectrum and amplitude (Vilenkin 1985) which allows for a theory of structure in the late universe).

Cosmic strings are found to occur either in the form of closed loops or as infinitely long strings (Turok 1987). Most ($\approx$80%) of the strings are actually in long 'infinite' strings as large as the universe horizon size. The remainder are in the form of a scale invariant distribution of closed loops. The infinite strings that form are not straight but meander about in the form of Brownian random walks and the whole collection of strings forms a network that permeates all of space. The mean velocity of a piece of string is of the order of 10^{-1}c (Albrecht and Turok 1985). Thus the individual cosmic strings intersect frequently.

The evolution of a network of cosmic strings depends crucially on what happens when two strings intersect. For instance, if cosmic strings were to pass right through each other, then the physical length in string would expand as fast as the scale factor of the universe, R(t), and hence the energy density in strings would only decay as $R^{-2}(t)$, compared to the energy density in radiation which falls off as $R^{-4}(t)$. Thus the energy in strings would rapidly become the predominant form of matter-energy in the universe. A universe dominated by cosmic strings would look very different from the one that we observe today. Cosmic strings would be plainly visible all around us and the additional energy of the cosmic strings would cause the universe to expand much faster than the observed rate, eg. in a radiation-dominated Friedmann-Robertson-Walker period, the energy density in the strings would cause the universe to expand as $\simeq$t. If on the other hand, strings, as they cross, could also break and reconnect the other way, long strings would form loops (Figure 3) and this would avoid the scenario of a universe dominated by strings.

(a)

(b)

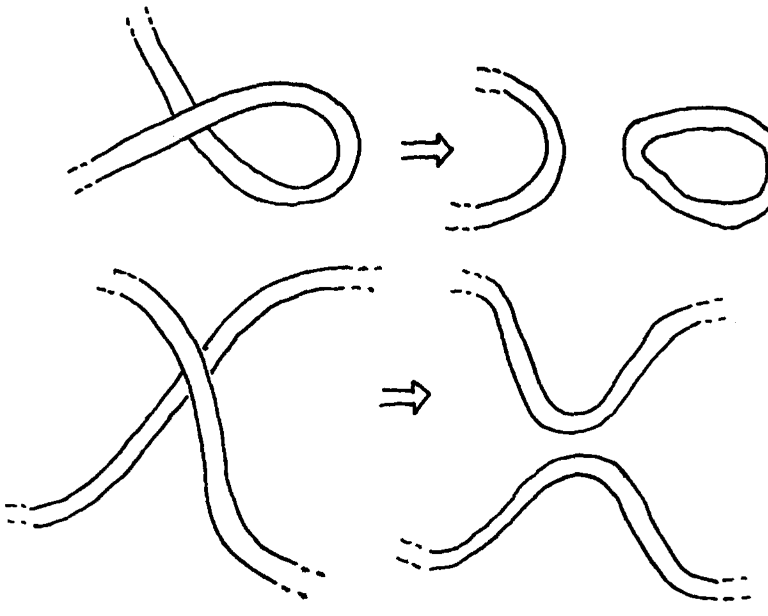

Figure 3: Intercommutation of cosmic strings: (a) a single self-intersecting string (b) two infinite strings intersect.

As a string moves around it loses energy by radiating gravitational waves. This effect will eventually cause a loop (unlike an infinite string) steadily to decrease in size until nothing remains but radiation. It is this conversion of the energy of the string to radiation which prevents strings from dominating the dynamics of the universe. Shellard (1987) discovered that provided their relative speed is less than $0{\cdot}9c$, two intersecting cosmic strings always break and reconnect to produce smaller loops.

The evolution of strings is strongly influenced by the expansion of the universe. Because the infinite strings are not straight but meander with a curvature about the Hubble length in size, they cross themselves and produce loops, also about a Hubble length in size. At later epochs, as the Hubble length increases, the loops which form are correspondingly larger. Once formed, the tension in the loops causes them to oscillate. Oscillating mass gives rise to gravitational radiation and so the loops decay by radiating gravitational waves.

It can be shown (Brandenberger 1987) that the loops decay completely into radiation after about 10^6 oscillations. Thus, at any time there is a 'debris' of loops left behind by the network, ranging in size from the Hubble length down to zero.

Zel'dovich (1980) and Vilenkin (1981) suggested that strings could produce density fluctuations sufficient to explain the galaxy formation of the universe. The gravitational fields of the loops accrete matter leading to the build up of the structures which exist in the universe. A loop accretes a mass proportional to its own mass, so smaller loops accrete a smaller amount of matter. Large loops not only accrete more matter but also the smaller loops around them. Thus, in this scenario, smaller loops formed galaxies and larger loops formed clusters of galaxies. The evolution of the network determines the number of loops of different sizes. The size of the loop also determines its mass and therefore how much matter it will accrete. The mass of a loop is its length times the mass per unit length, μ, of the cosmic string, a quantity that is not uniquely predicted by the underlying field theory. The mass per unit length, μ, depends on the value of the (string-generating) scalar field for which the potential energy is at a minimum and so should be the same for all loops.

The mass per unit length, μ, can be determined by counting the number of galaxies and then using the theory to predict which size of loop appears in the same quantity. We can then choose the value of μ which gives loops of the right size to accrete a galaxy. Remarkably these two independent determinations of μ give the same value, Turok and Brandenberger (1985). The value obtained also lies within the range most preferred by the underlying field theory.

A model based on cosmic strings should predict more than just the total number of galaxies or clusters. The distribution of these objects should also be a reflection of the distribution of loops of cosmic string in space. Clusters offer a clearer test because they are too far apart for gravity to have moved them around very much since their formation. A simple way to measure the degree of clustering in the distribution of objects is to use the two-point correlation function (Peebles 1980). For the observed clusters of galaxies, this is found to be identical with that of the corresponding calculations for loops of cosmic strings. Cosmic strings are, therefore, more likely to explain the existence of voids, filaments and sheets in the universe. However, they do have a problem when it comes to predicting the observed masses of galaxies. If cosmic strings are combined with a cold dark matter universe the galaxy masses obtained are too high (Turok 1987). In a universe dominated by hot dark matter with strings, however, galaxies look very different than with cold dark matter. If the dark matter was in the form of massive neutrinos then it can be shown (Tremaine and Gunn 1979) that 30eV neutrinos cannot cluster on scales smaller than about 10kpc. Consequently the inner regions are almost entirely baryonic while the holes are comprised of neutrinos. Unfortunately, the masses of galaxies in this scenario tend to be underestimated. For a more detailed discussion of cosmic strings with hot dark matter see Brandenberger et al. (1987). We conclude then by stating that cosmic strings do offer an intriguing alternative to the scenario of a WIMP-dominated universe as a viable model for structure formation. A more recent alternative to cosmic strings are the higher dimensional topological defects known as *textures*, cf. Turok (1989).

Cosmic strings and dark matter candidates were found to originate in the very early universe. Therefore, it would seem that in order to fully comprehend the evolution of the universe we must consider the contribution made by phenomena occurring in the early stages of this evolution.

4. The Early Universe

The models of Friedmann (1922) and Lemaitre (1927) and the discovery of extragalactic recession by Hubble (1929) established securely the concept of an expanding universe. A simple extrapolation back in time leads directly to an initial big-bang state of high density. The idea of a hot and dense early universe was put on a firm foundation by the discovery of the 3^oK background radiation by Penzias and Wilson (1965) and its identification by Dicke et al. (1965). However, prior to this discovery the idea had played an active and prominent role in the work of Gamow and colleagues, with excellent reviews in Gamow (1953) and Alpher et al. (1953). Advocates of cold big bang theories, in which the microwave background does not have a primordial origin, face the problems of providing a mechanism which generates the observed thermal background and producing the observed cosmic helium abundance. With the assumption of a hot big bang, the early universe becomes an extremely fascinating and physically intricate subject for study.

The thermal history of the standard big bang from a temperature of 10^{12}K is illustrated in Figure 4. The very early universe (T> 10^{12}K) consists of a dense sea of hadrons, which interact via the strong nuclear force, and leptons together with their antiparticles. As the temperature falls most of the hadrons annihilate, leaving behind the few surviving nucleons which now make up the galaxies and structure present in the universe today. At this time the universe consists mainly of leptons, antileptons and photons and enters what is known as the *lepton era*. The dynamics of the universe, which is determined mainly by its density, is dominated by leptons.

As the temperature drops below $\simeq 8\times 10^{11}$K, muon pairs begin to annihilate while the surviving electrons and positrons remain in a state of thermal equilibrium until the temperature approaches $\simeq 4\times 10^{9}$K. At this temperature, electron pairs annihilate significantly faster than they are created and we reach the *radiation era*. This era lasts until the very important stage when hydrogen recombination occurs at a temperature of $\simeq 3000^o$K. Neutral atoms can be formed at this temperature because the photons no longer have enough energy to ionise the matter completely. The photons therefore decouple from the matter and form a thermal background of radiation. During most of this relatively long period the universe is dominated by radiation and contains only a trace of matter. As time passes, matter becomes increasingly important and around the time of recombination the universe becomes matter-dominated. The radiation era, however, does not necessarily terminate at the instant the universe becomes matter-dominated since the important fact is that the universe remains dominated by radiation pressure until the latter stages of recombination. Helium is synthesised during the early stages of the radiation era and later, when the Hubble mass has increased, the various precursory inhomogeneities of galaxy formation begin to take effect. A more detailed discussion of the physical processes occurring in the different regimes of the early universe is given by Harrison (1973).

Thus, the standard model, whereby the scale factor of the universe follows the Friedmann equations, allows us to extrapolate backwards, from the present extremely isotropic universe, to times of the order of 0·01 seconds. However, to describe completely the evolution of the universe we need to know what happens during that first one hundredth of a second. The remarkable developments in elementary particle physics, in the search for a unified theory of the forces of nature, have allowed cosmologists to 'probe' the very early stages of the universe.

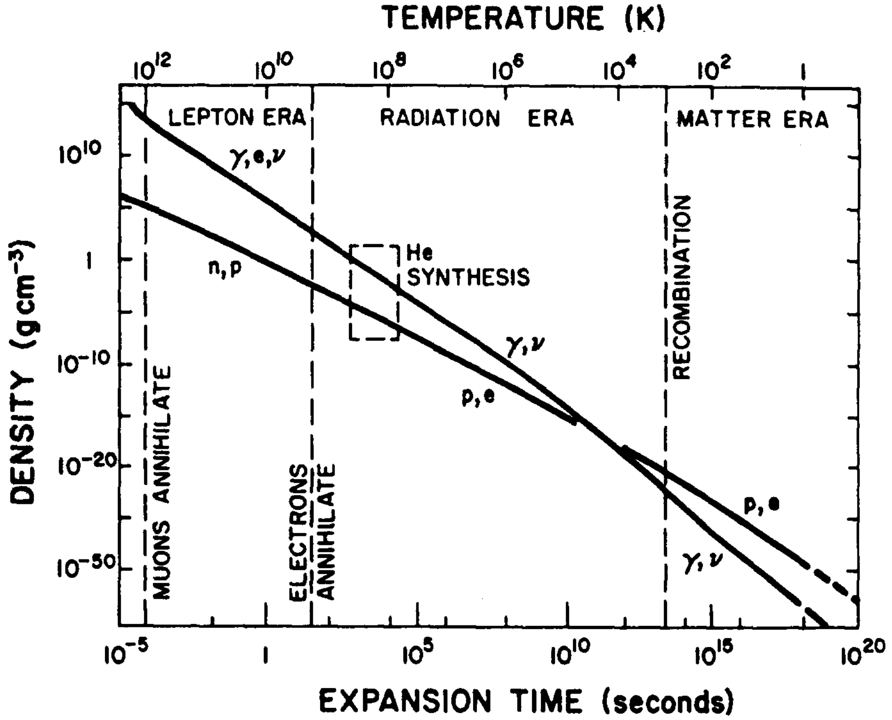

Figure 4: Thermal history of a standard big bang universe to a temperature of 10^{12}K. Note that recombination does not coincide with the universe becoming matter-dominated (after Harrison 1973).

The gauge theories of the particle physicists have been very successful, so far, in describing and predicting the behaviour of fundamental particle interactions, where the forces controlling the particles are described by the spontaneous breaking of symmetries imposed by the theories. The point of spontaneous symmetry breaking (SSB) is that although the laws of physics may be intrinsically symmetrical, that symmetry is not manifest below a certain temperature or energy level because the lowest energy state of the system (the vacuum) is a particular solution of the equations that does not possess their symmetry. In spontaneously broken gauge theories the properties of the vacuum state are of vital importance and are described by the vacuum expectation value of a scalar field - the Higgs field (Abers and Lee 1973). The appearance of a non-zero vacuum expectation value signals SSB.

The mathematical elegance and experimental vindication of the SU(3), strong force symmetric gauge, and SU(2)×U(1), electro-weak gauge theory, models have led to their incorporation within proposals for the grand unification of the strong and the electro-weak interactions. In such models a spontaneous breakdown of complete symmetry between the strength and properties of these three interactions occurs when the temperature falls to 10^{28}K, corresponding to an energy of about 10^{15}GeV. Such energies are never likely to be attained by terrestrial particle accelerators, which currently can only attain energies $\leq 10^{3}$GeV. However, the early universe provides an ideal 'theoretical laboratory' in which to test the ideas of the particle physics theories. According to the hot big bang model, temperatures corresponding to average particle energies as large as 10^{19}GeV should have been reached in the early universe.

This standard description of the evolution of the universe, while extremely successful, has a number of problems, which have no natural explanation within the big-bang theory. There are three specific problems we shall consider: the isotropy problem, the horizon problem and the flatness problem.

The observed isotropy of the microwave background radiation (Smoot et al. 1991) cannot be explained as a consequence of classical dissipative processes acting during the early stages of the universe (cf. Barrow and Matzner 1977; Collins and Hawking 1973). Isotropy is an unstable property of physically realistic cosmological initial conditions yet it is an observed property of the universe. This is the isotropy problem.

The horizon problem can be best described by considering the radiation-dominated epoch of the universe. The observed homogeneity and isotropy of the microwave background indicates that the universe is homogeneous and isotropic in a region corresponding to the size of the past light cone evaluated at the time (t_e) when radiation was last scattered. Using the equations of a radiation dominated Friedmann cosmology it can be shown that the homogeneity region is larger than the particle horizon at t_e), the distance light can travel between $t=0$ and $t=t_e$). Thus at t_e) the universe is homogeneous on scales larger than the size of any possible causally connected regions. In the standard big-bang cosmology, therefore, there is no causal explanation for the homogeneity and isotropy of the universe on the scales observed.

It is not clear whether or not the universe contains sufficient material to recollapse at a finite time in the future. This is because the universe is expanding at a rate very close to the critical density (Einstein-de Sitter) model, i.e. the universe is almost exactly flat. In order for the density parameter Ω to be so near to unity (current observations give $\Omega \simeq 0{\cdot}1 - 0{\cdot}2$) deviations from the critical density must have been smaller than one part in 10^{57} at the Planck time $[(\Omega - 1) \leq 10^{-57}]$. This is the flatness problem.

Guth (1981) proposed a new picture for the early stages of the hot big bang model which provided a 'natural' explanation for this collection of cosmological problems. The picture was dubbed the 'inflationary universe'. In particle physics theories which undergo SSB the Lorentz invariant energy density associated with the vacuum changes during a phase transition and creates an effective cosmological constant, $\rho_0 \approx O(T_c^4)$, where T_c is the critical temperature of the phase transition. As the universe cools below T_c bubbles of the low temperature phase (asymmetric vacuum) nucleate and grow and eventually the entire universe is in the asymmetric phase. However, Guth pointed out that if the nucleation rate was sufficiently small, then the universe would remain in the high temperature phase (symmetric vacuum) for a non-negligible period: the symmetric vacuum is metastable. During this interval the initial cosmological term, ρ_0, would soon begin to dominate the expansion dynamics and the universe would expand exponentially and 'supercool', erasing all knowledge of its previous history. (Exponential expansion was first discussed by de-Sitter in 1917). When the transition to the asymmetric vacuum state does occur an enormous latent heat (the energy difference between the two vacuum states) is released, reheating the universe so its subsequent evolution is that of the standard big bang model. As a result of the exponential expansion phase in its early evolution, the portion of the universe that is observable today should be extremely uniform and expanding at a critical rate (that is, Ω, should be equal to unity).

Without this phase of exponential expansion, the present uniformity of the universe and the proximity of its expansion rate to the critical value would remain unexplained. The only solution, other than some inflationary model, would be to appeal to very special initial conditions such as in the flatness problem described above. Guth's inflationary universe dispensed with the need for such special initial conditions. Furthermore, it also solved the problem of the over-production of magnetic monopoles expected during the symmetry-breaking stage of the Grand Unified Theories (GUTs) which describe how the electromagnetic and nuclear forces combine into the one theory. In the earlier, more conventional, models of the GUT phase transition monopoles were predicted to contribute a density more than 10^{12} times larger than the maximum allowed by the observed deceleration of the universe (Preskill 1979, Zel'dovich and Khlopov 1979). In the inflationary model the period of accelerating expansion dilutes the monopole density to a small and observationally permissible level. Thus, the inflationary universe seemed to avoid the problems associated with the standard big bang model. Unfortunately, in this model of inflation, once the universe reaches the symmetric phase, it remains trapped there giving rise to a sustained exponential expansion.

Linde (1982), Albrecht and Steinhardt (1982) and Hawking and Moss (1982) analysed a different class of GUTs (those which undergo SSB of a characteristic type first studied by Coleman and Weinberg 1973) and discovered that in these models a period of inflationary expansion was attained without the difficulty of being stuck in the symmetric vacuum of de-Sitter space. In their so-called 'new inflationary model' the universe takes a long but finite time to evolve from the symmetric to the asymmetric vacuum. During the evolution to the asymmetric state the universe expands exponentially due to the large energy density of the symmetric vacuum as was the case of the original inflationary model. However, in this new model the evolution to the true vacuum takes long enough for sufficient exponential expansion to occur to explain the uniformity, expansion rate and monopole-free composition of the present-day universe.

One problem with the new inflationary model is that the matter inhomogeneities, which are spontaneously generated by quantum fluctuations during the inflationary phase, are found to be far too large to lead to galaxies, rather everything would evolve to form superdense objects or black holes (Gibbons et al. 1983). Other versions of inflationary models have been proposed in order to produce perturbations of the required magnitude, cf. La and Steinhardt (1989).

All of the inflationary models predict that today the density parameter Ω should equal unity to a very high degree of precision. However, the best astronomical determinations of Ω all consistently suggest a much smaller value, less than 0·2. A possible way out of this conflict would be if the universe was dominated ($\Omega \simeq 1$) by non-baryonic dark matter, which coalesces on very large scales so that the observations to date would not have been sensitive to the presence of this matter, cf. previous section. Recently a deep survey of IRAS 60-μm sources by Rowan-Robinson et al. (1990) has implied that in order to explain the observed amplitude of the IRAS dipole, the value of the cosmological density parameter $\Omega = 0{\cdot}71$ (+0·29, -0·18). This is based on the assumptions that the IRAS galaxies satisfy a universal luminosity function at every location with the local density of galaxies being proportional to the total density of matter and that the IRAS galaxy distribution traces the total mass. Because of the far infrared nature of the IRAS survey, elliptical and early-type galaxies are absent from the surveys and hence they do not contain most of the galaxies in rich clusters like Coma. Thus, in order to reconstruct the galaxy density field a correction must be applied to take account of the missing galaxies, cf. Jones and van de Weygaert (1990).

The fact that inflationary models 'guarantee' isotropy does not preclude the existence of significant anisotropies and inhomogeneities before the onset of inflation. Indeed, several authors (Diósi et al. 1984, Waga et al. 1986) have raised the possibility that bulk viscosity in the early universe could be the driving force of an accelerated expansion akin to inflation. These authors have suggested that bulk viscosity arising around the time of a GUT phase transition could lead to a negative pressure thereby driving an inflationary expansion. In order for structure such as galaxies and clusters of galaxies to form, the universe must develop density inhomogeneities at some time during its evolution and it is unrealistic to assume that the universe could contain such inhomogeneities without being at least somewhat anisotropic, since density fluctuations tend to generate shearing motions via tidal stresses (see Liang 1974 and Barrow 1977). Quite apart from the anisotropy which is generated in this way it is feasible that the universe started off endowed with a lot of *primordial* anisotropy.

Relaxing the requirement of isotropy, i.e. permitting the cosmological flow to rotate and shear as it expands, allows more freedom in the choice of solutions. Various cosmological models of this kind have been formulated in which the rates of expansion in different directions are unequal (Taub 1951, Ellis and MacCallum 1969). A more physical interpretation of these models is that very long wavelength gravitational standing waves are present throughout the evolving matter distribution. Such a gravitational wave field determines preferred directions and orientations in space. If this picture is simultaneously true everywhere, the models are said to be spatially homogeneous (cf. Bianchi models; Bianchi 1897, Ryan and Shepley 1975).

The simplest of these anisotropic models is the Kasner model (Kasner 1921) in which the vorticity and acceleration of the flow lines are absent and in which shear and expansion are completely specified once the expansion rate in one direction is known. Such Kasner solutions can be easily extended to higher dimensional cosmologies and these will be discussed in the next section. Suffice it to say that within the very generous confines of 'conventional' cosmology the early universe has a great deal to offer to the understanding of the universe in its entirety. (A collection of papers, on the subject of the physics of the very early universe, is presented in Gibbons et al. 1983).

5. Exotic Cosmologies

In the preceding sections we have discussed the various 'conventional' methods of devising cosmological models in an attempt to explain the universe as it exists, within the framework of the general theory of relativity. There have been models which attempt to 'usurp' general relativity by providing an alternative theory (eg. Brans and Dicke 1961, Smalley 1974). None of these alternatives, however, has been able to match the phenomenal success of relativity theory in satisfying most of the available observational tests. Less radical, but more enlightening, have been the extensions of general relativity to higher dimensions (eg. Kaluza-Klein models, supersymmetry theories, superstring theories and extended Kasner models). All of these theories exist in an attempt to unify the fundamental forces of nature.

In general relativity, the force of gravity appears as a result of distortions in four-dimensional space. Kaluza (1921) extended the equations of general relativity by adding an extra spatial dimension. He found that the five-dimensional equivalent of the equations of general relativity automatically divided into two sets of equations in four dimensions. One set corresponded to the familiar gravitational equations while the other set exactly corresponded to the equations of electromagnetism. Five- dimensional "relativity" seemed to unify the two forces of nature known in 1921.

Kaluza proved his results only for the case where the fields were weak and the velocities small. However, Klein (1926a) showed that these two constraints were irrelevant; unification should not depend on the weak field, small velocity approximation. Klein employed the results obtained by the rapid developments of quantum physics in the early 1920's (Schrödinger 1926) and translated Kaluza's five- dimensional theory into quantum terms by writing down a version of Schrödinger's equation with five variables (each one effectively corresponding to a dimension) instead of four. He showed that this five-dimensional Schrödinger's equation had solutions which corresponded to gravitational and electromagnetic waves in four- dimensional space.

In these early studies no real attempt was made to justify the use of an extra dimension nor to explain why it did not manifest itself observationally. However, Klein (1926b) suggested that the extra dimension could be 'rolled up' or 'compactified' so that it was undetectable in the everyday world. (The usual analogy is a hosepipe, which when viewed from a long way away, looks like a one-dimensional line but turns out to be a two-dimensional object as one get nearer. Each point on the 'line' is a circle around the circumference of the pipe). Klein suggested that every point in the observed three-dimensional space might really be a circle looping around a fourth spatial dimension. Calculations suggest that each loop would be about 10^{-30} to 10^{-33} centimetres across, i.e. of the order of the Planck length, Schwarz (1985). This compactification argument has become standard in the study of higher dimensional cosmologies today, known collectively as Kaluza-Klein models.

The original Kaluza-Klein theory helped unify the two known fundamental forces at that time, gravity and electromagnetism. However, in the decades that followed, experiments in particle physics led to the discovery of two more fundamental forces of nature; the strong interaction and the weak interaction. Witten (1981) demonstrated that the Kaluza-Klein approach could be extended to unify the strong, weak and electromagnetic interactions, if a minimum of eleven dimensions were employed; ten spatial and one temporal. It is found that the resulting eleven-dimensional space can compactify in one of two ways; either four dimensions curl up leaving seven-dimensions or seven dimensions compactify leaving a four dimensional world. The 'odd' force out in these considerations is gravity. As yet there has been little success in the search for a consistent quantum theory of gravity. The idea of a supergravity has been developed, however, which like Einstein's theory is a geometrical theory of gravity. Supergravity goes beyond general relativity and attempts to unify gravity with the other forces of nature. Theories of supergravity can be made to work in different numbers of dimensions, but only up to a maximum of eleven (cf. Duff et al. 1986).

Today, the most favoured variation of the Kaluza-Klein approach is ten-dimensional superstring theory, which involves fundamental particles that are one-dimensional strings, not mathematical points, and also incorporates a version of supergravity. The justification for working in ten or eleven dimensions comes purely from theoretical particle physics and is not founded in any observations. Present accelerators have probed matter at distances as small as 10^{-16}cm without finding any evidence of extra dimensions, which is not too surprising as the extra dimensions are expected to have a size characteristic of the Planck length ($\simeq 10^{-33}$cm).

Supersymmetry (Duff et al. 1986) is a symmetry between fermions and bosons. In a supersymmetric theory there is a bosonic counterpart for every fermion and vice versa. Supersymmetric particles have not yet been detected and until they are the theory remains somewhat speculative. The motivation for supersymmetry is that mathematically it is very elegant and it is the ultimate symmetry we have available to impose. If supersymmetry is made a gauge symmetry the additional property of covariance is achieved, i.e. general relativity is automatically incorporated into the theory. A supersymmetric gauge theory is called supergravity. Such a theory offers the possibility of unifying gravity with the other forces of nature. Another advantageous effect of supersymmetry is that it solves a problem encountered by all GUTs, namely the discrepancy between the symmetry breaking scales of a GUT and the weak interaction. These scales may be set to very different energies. However, quantum corrections to the theory tend to raise the weak scale up to the GUT scale. Supersymmetry can be used to stabilise the weak scale (Turner 1987).

There is no evidence that the universe is supersymmetric at the present epoch. Therefore, supersymmetry must also be a broken symmetry. If the weak scale is to be stabilised then the supersymmetry breaking scale must occur at this scale. This means that the supersymmetric partners, or spartners, of all known particles must have masses of the order of the weak scale, between a few GeV and a TeV. Such supersymmetric particles include squarks and sleptons which are the scalar partners of quarks and leptons respectively. Supersymmetric partners of photons, gluons and gravitons also exist and are known as photinos, gluinos and gravitinos respectively.

Almost all supergravity models are supersymmetric GUTs with a unification scale higher than in normal GUTs; more like 10^{16}GeV compared to 10^{14}GeV. These theories are supposed to describe physics up to the Planck scale, 10^{19}GeV. Because of this, supersymmetry/supergravity models also predict all of the additional particles that GUTs do - magnetic monopoles, massive neutrinos, axions and even cosmic strings in some cases providing a large number of WIMPs for dark matter candidates (see previous section).

Superstring theories (Schwarz 1985) combine most of the ideas discussed above; supersymmetry, gauge symmetry, extra dimensions and one new one, strings (not to be confused with cosmic strings). In an analogue to Klein's idea of compactification the fundamental particles are not regarded as point-like but rather as one-dimensional string-like entities. The concept of superstrings was devised in an attempt to produce a unified theory in which all of the physical parameters were determined. At energies below the Planck scale the strings are regarded as essentially point-like objects which allows the theory at 'low energies' to resemble closely the very successful standard model of particle physics. The quantum fields present in the string theory determine the spectra, couplings and gauge structure of the 'low energy' model.

Only a small number of distinct string theories exist. However, there are a large number of distinct classical string vacua (Green et al. 1987). Since the low energy aspects of the string theory are sensitive to the structure of the vacuum, this ambiguity in the vacua causes difficulties in the calculation of the large number of free parameters present in the standard model. This is a problem with the present formulation of string theory and it is hoped that a truly quantum theory of string will remove this vacuum degeneracy problem. String theories have been formulated in both ten dimensional and four dimensional spacetimes (Greene 1990). For a discussion of string theory and cosmology see Tseytlin and Vafa (1991).

In principle, starting from the superstring (which describes physics at or above the Planck scale) we can calculate everything - the masses of all the fermions, the GUT etc. The so-called field theory limit of a superstring theory is supposed to be a supersymmetry/supergravity GUT. Thus, all the WIMP candidates for dark matter predicted by supersymmetric GUTs are also predicted by superstring theories. GUTs attempt to describe physics up to around 10^{14}GeV, supersymmetric GUTs up to 10^{19}GeV and superstring theories at energies above 10^{19}GeV.

All of the higher-dimensional cosmologies, discussed above, rely on the compactification of the additional dimensions. One simple way of understanding how extra dimensions can be incorporated into general relativistic cosmology is to consider the Kasner solutions discussed briefly above. The Kasner solutions (Kasner 1921) were the prototypes for cosmological models with great asymmetry in a few degrees of freedom. The four-dimensional Kasner metric is given by

$$ds^2 = dt^2 - t^{2\alpha}dx_1^2 - t^{2\beta}dx_2^2 - t^{2\gamma}dx_3^2 \tag{6}$$

where α, β and γ are constants satisfying the constraints,

$$\alpha + \beta + \gamma = \alpha^2 + \beta^2 + \gamma^2 = 1 \tag{7}$$

Thus, each t=constant hypersurface of this model is a flat three-dimensional space. This model represents an expanding universe, since the volume element is constantly increasing. However, it is an ***anisotropically*** expanding universe. Distances parallel to the x_1-axis expand at one rate $R_1 \propto t^{\alpha}$, while those along the x_2-axis can expand at a different rate, $R_2 \propto t^{\beta}$. More interesting is the fact that along one of the axes, distances contract rather than expand. This contraction shows up mathematically in the fact that equations (7) require one of α, β or γ, say γ, to be non-positive:

$$-\frac{1}{3} \leq \gamma \leq 0 \tag{8}$$

We can see immediately the extension to higher dimensions, eg. five (four spatial and one temporal). The metric would then take the form

$$ds^2 = dt^2 - t^{2\alpha}dx_1^2 - t^{2\beta}dx_2^2 - t^{2\gamma}dx_3^2 - t^{2\delta}dx_4^2 \tag{9}$$

where we have introduced the new 'scale factor', t^δ, for the additional dimension. The constraints on the expansion rates are then

$$\alpha + \beta + \gamma + \delta = \alpha^2 + \beta^2 + \gamma^2 + \delta^2 = 1 \tag{10}$$

and again we see that at least one of the expansion rates must be non-positive, δ say. Thus, compactification of the extra dimension follows quite naturally in such a model. As $t \to \infty, t^\delta \to 0$ and the dimension x_4 is 'lost' while the remaining dimensions grow large. This analysis can be continued for as many extra dimensions as we like, being only restricted by the fact that observationally all but three of the spatial dimensions must become vanishingly small. For a discussion of the compactification of extra dimensions in superstring theory see Tseytlin and Vafa (1991).

Finally, it is interesting to discuss a five-dimensional cosmology, which uses the mass of the universe as an extra dimension (Wesson 1984). This approach is equivalent to the creation of a fourth dimension by multiplying time by the speed of light to obtain a measure of distance; special and general relativity use the length ct as a coordinate. In a similar manner the constant of gravity, G, can be used to convert masses into distances. The parameter Gm/c^2 (where m is the rest mass of a particle) has units of length and so may be used as a coordinate, to give a five- dimensional version of general relativity.

Relativity in four dimensions implies that the strength of gravity is constant, a fact which has been verified by many experiments. However, because of the nature of the underlying physical laws, if the strength of gravity is proportional to time, the properties of astronomical systems are almost exactly the same as they are if the quantity Gm/c^2 remains constant. This parameter is therefore a more natural measure of the 'strength' of the gravitational force associated with a particle of mass m. In other words, it seems to make no difference whether the rate of change of this parameter is steady and finite or zero. Thus, we can allow for a steady rate of change of the rest mass, m, keeping G as strictly constant.

A similar argument, for a varying Gm/c^2 was given by Dirac (1938) in what he called the Large Numbers Hypothesis, where dimensionless ratios of the physical constants of nature are found to be typically of the order of 10^{40}. For example, if we compare the relative strength of the electrical and gravitational forces between the electron and the proton we find that a large dimensionless number is obtained given by

$$\frac{e^2}{Gm_pm_e} = 2{\cdot}3 \times 10^{39} \tag{11}$$

where e is the charge of the electron, G is the gravitational constant and m_p, m_e are the masses of the proton and electron, respectively. Similarly, if we compare the length scale associated with the universe, c/H_0, and the length scale associated with the electron, $e^2/m_e c^2$, we obtain the ratio

$$\frac{m_e c^3}{e^2 H_0} = 3{\cdot}7 \times 10^{40} h_0^{-1} \tag{12}$$

Dirac pointed out that (12) contained the Hubble constant, H_0, and therefore the magnitude computed in this formula varies with the epoch in the standard Friedmann model. If so, the near equality of (11) and (12) has to be a coincidence of the present epoch in the universe, unless the constant in (11) also varies in such a way as to maintain the state of near equality with (12) at all epochs. This would imply that at least one of the so-called constants involved in (11) (e, m_p, m_e and G) must vary with epoch. Because G has macroscopic significance, whereas the other constants are atomic quantities, Dirac postulated that the gravitational constant must vary with time in such a manner as to keep the ratios (11) and (12) of roughly the same magnitude. There is no observation which would be capable of distinguishing between the approaches of Dirac and Wesson.

In the description of Wesson (1984) the equations become the familiar equations of four-dimensional relativity in the limit where the rest masses of particles vary infinitely slowly. According to Wesson's theory the amount of mass in the universe today has built up at a steady rate. Such a cosmology would not contain a state of infinite density at the origin thereby avoiding one of the main difficulties associated with the standard hot big bang model.

The extensions of general relativity and particle physics theories to higher dimensions have proved extremely fruitful despite the lack of observational evidence. The ultimate aim of combining the four fundamental forces into a single unified theory has yet to be achieved. However, much of the current work in modern experimental particle physics concentrates on the determination of the existence of the high energy particles expected from the theories and it is hoped that these will be successful in the near future. Cosmology, in particular the study of the early universe, has provided much of the impetus for progress in these areas, and will continue to stimulate new ideas.

6. Conclusions

The standard 'hot big-bang' model of the universe together with the Grand Unified and supersymmetry theories of particle physics are seemingly able to trace the history of the universe back to the very first moments of its existence. Observational astrophysics and particle theories are probably most closely linked in the investigation of the nature of dark matter and its distribution in the universe. The belief that the universe exhibited a period of inflationary expansion and consequently that the universe must be flat, Ω=1, has led to the conclusion that most of the matter in the universe must be dominated by non-baryonic particles (WIMPs). A host of dark matter candidates have been provided by particle physics including massive neutrinos, axions and various supersymmetric particles.

Despite the apparent successes of these theories they are not without difficulties. The big-bang theory has been founded on only three major observations: the universal expansion, the microwave background radiation and the abundance of cosmological helium.

Recent observations, however, have posed some severe problems for this theory. The remarkable results of the COBE satellite have shown that the microwave background is isotropic to an extremely high degree while observations of very large scale structures and recently of a cloud of neutral hydrogen at a large redshift demonstrate that structure existed very early on in the universe. It is becoming increasingly more difficult to reconcile these two types of observation. If structure existed shortly after the big bang it should cause inhomogeneities at an observable level in the microwave background.

Present day particle accelerators are unable to attain the energies required to prove the existence of the high energy particles associated with Grand Unified and supersymmetry theories. Thus, at the present time, there is no experimental evidence to support these theories. One driving force for the existence of such particles is the belief that the universe is dominated by non-baryonic dark matter. Observationally, there is no convincing evidence for a density parameter in excess of about 0·2 (although see the recent IRAS results discussed in Section 4). Thus, other than a theoretical bias there is no reason to invoke a distribution of non-baryonic matter. Inflationary scenarios which were introduced to answer some of the difficulties of the big bang cosmology require a flat (Ω=1) universe. Thus the very successful standard model of cosmology may be in need of significant modification if theory is to remain consistent with observation.

The last decade or so of cosmological research has been extremely exciting and many interesting discoveries have been made and new issues raised. The next decade promises to be an exciting one.

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