

# On the quantum origin of the seeds of cosmic structure

Alejandro Perez<sup>1,2\*</sup>, Hanno Sahlmann<sup>1,4†</sup> and Daniel Sudarsky<sup>1,3‡</sup>

1. Institute for Gravitational Physics and Geometry,  
Penn State University
2. Centre de Physique Theorique, Université de Marseille
3. Instituto de Ciencias Nucleares  
Universidad Nacional Autónoma de México
4. Spinoza Institute, Universiteit Utrecht

March 16, 2018

## Abstract

The current understanding of the quantum origin of cosmic structure is discussed critically. We point out that in the existing treatments a transition from a symmetric quantum state to an (essentially classical) non-symmetric state is implicitly assumed, but not specified or analyzed in any detail. In facing the issue we are led to conclude that new physics is required to explain the apparent predictive power of the usual schemes. Furthermore we show that the novel way of looking at the relevant issues opens new windows from where relevant information might be extracted regarding cosmological issues and perhaps even clues about aspects of quantum gravity.

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\*perez@gravity.psu.edu

†h.sahlmann@phys.uu.nl

‡sudarsky@nuclecu.unam.mx

# 1 Introduction

The origin and evolution of the large scale structure of the universe constitutes a central aspect of cosmology. The widespread view is that our understanding of this aspect has developed dramatically in recent times, both through the collection of high precision data in observational cosmology [1], and through better theoretical understanding [2, 3]: The observations are in very good agreement with the theoretical predictions based on the scale invariant (Harrison-Zeldovich) spectrum of seed fluctuations and their subsequent evolution. Such a scale free spectrum would be hard to explain in standard cosmology because physical processes in the very early universe that might produce it, would have to act on scales larger than the Hubble radius. This problem is absent if one considers inflationary cosmology, which seems needed as well in order to deal with well known puzzles arising in standard cosmology [4].

What is more, inflation produces a scale free spectrum of quantum mechanical fluctuations for the quantized inhomogeneous component of the inflaton field, and thus seems to give birth to these seeds of structure through a quantum process.

It seems that we have in fact here one of the big successes of theoretical physics in recent times (besides the remarkable, and undisputed achievements that it signifies for observational cosmology), and these facts are widely viewed as confirmation of an inflationary stage in the very early universe.<sup>1</sup>

In the present paper, we will be concerned with one specific part of the picture summarized above, namely the suggestion that inflation not only provides a satisfactory understanding of the evolution of the inhomogeneities, but even an explanation of their origin: The observed similarity between the spectral properties of the (classical) density fluctuations in the early universe and the quantum mechanical fluctuations of the inflaton field after inflation indeed suggest much more than a coincidence. However, we want to draw attention to the fact that a detailed understanding of the process that leads from the quantum mechanical to the classical fluctuations is lacking. What is needed to justify the connection between the two spectra is a mechanism that transforms quantum mechanical uncertainties into classical density fluctuations.

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<sup>1</sup>Note however that recent observations indicate an anomalous lack of power at large angular scales, a fact that has lead to some workers in the field to start questioning the viability of the inflationary model [6]. See also the alternative explanation in [5].

This issue has in fact been treated by many authors (see [7, 8, 9, 10, 11, 13, 16, 17, 18, 23] for example), most of whom seem to suggest that the subject has been clarified and thus, at least on a fundamental level, possesses no further mystery. Indeed most of the proposals seem to invoke nothing beyond standard physics to explain the coincidence of the spectra. On the other hand this same body of literature can be considered as an indication that underneath all of the reassurances to the contrary, there is a remaining degree of discomfort, and certainly the fact that the different authors point to slightly different schemes for this quantum to classical transmutation, indicates that each author does not find the schemes espoused by their colleagues to be fully satisfactory. Indeed we feel that, at least on a fundamental or conceptual level, the treatments proposed are missing something, and we will try to pinpoint the place where there is a missing step in the most well known ones. This will lead us to argue that in fact something beyond standard physics seems to be required if the essential picture and its successful degree of predictive power is to be preserved. At the same time we underscore the richness of the physical information that is being overlooked when the problem is not addressed head on.

To illustrate the physics behind that missing aspect, we introduce a simple phenomenological model involving a dynamical collapse of the wave function as the mechanism that leads to the transition to inhomogeneity. It is phenomenological in the sense that it does not explain the transition to inhomogeneity by some particular new physical mechanism, but merely gives a rather general parametrization of such a transition. We will briefly discuss some of the characteristics that the new physics would have to include if we wanted it to justify our analysis. We also show how the value of some parameters can be studied by comparisons with observational data.

Since one is ultimately dealing with connecting quantum mechanical quantities to measurement results, one might be tempted to consider the issue described above as related to old and well known interpretational problems of Quantum Mechanics, and thus to dismiss it as inconsequential as far as the physical predictions for observations are concerned. After all, these interpretational problems of Quantum Mechanics can be mentioned in every instance where use of the theory is made, and we have all grown accustomed to the fact that, in practice, those issues have no incidence whatsoever on our ability to make correct predictions. We will argue that quite to the contrary, the situation at hand, namely the cosmological setting, is such that the way one deals with these issues could have an impact on the predictions

for observations. Furthermore, the standard rules that one can rely on, to settle all practical ambiguities in ordinary quantum mechanical situations, are not available in the present case. The reasons for these differences are threefold: First the object that is being treated quantum mechanically is the entire universe, and thus the standard separation of the system into subsystem of interest, observer, and environment, becomes unjustifiable and would be in fact subject to completely capricious choices. Second, that we have at our disposal just a single system – our universe –, so the recourse to the statistical ensemble interpretation of the result of a measurement in quantum mechanics is not directly available. And third the fact that not only do the observations pertain to the very late causal future of the assumed quantum to classical transition, but in fact the existence of the observers themselves depends upon the outcome of the measurements. In our view these facts indicate, not only, that this aspect of cosmology offers a rather unique opportunity to focus on these so called “interpretational aspects of quantum theory” – and to look for clues that might lead to a better understanding of them – but point in fact to the conclusion that any satisfactory treatment of it requires a step beyond what we currently have as established interpretational models of quantum mechanics. We will refer to this unknown aspect as new physics.

The article is organized as follows: In Section 2 we review the standard description of the inflationary scenario for the origin of the seeds of cosmic structure. In Section 3 we make a general critique of the standard description. In Section 4 we critically discuss the main ideas that have been put forward regarding the transition to classicality. In Section 5 we point out the nature of the missing element and propose some ideas. In Section 6 we review the description of linearized fluctuations maintaining the framework that seems necessary to make our proposals in a clear and transparent manner. In Section 7 we proceed to make the quantum mechanical treatment of the field fluctuations within the setting necessary for implementing our ideas. In Section 8 we show how to implement our proposal within the standard discussion of the origin of the seeds of cosmic structure. In Section 9 we recover the predictions that must be compared with observational results. Section 10 contains comments and calculations into some further directions, and we end in Section 11 with a discussion of our conclusions.

Regarding notation we will use signature  $(-+++)$  for the metric and Wald’s convention for the Riemann tensor. We will use units where  $c = 1$  but will keep the gravitational constant and  $\hbar$  explicit throughout the paper.

## 2 The standard picture

As mentioned in the introduction there are many treatments of the subject differing in technical as well as conceptual aspects. However, they do share a “standard core” which we resume here for the benefit of the reader, with no ambition toward completeness or rigor<sup>2</sup>.

1) Start with the action of a scalar field coupled to gravity.<sup>3</sup>

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{16\pi G} R[g] - 1/2 \nabla_a \phi \nabla_b \phi g^{ab} - V(\phi) \right], \quad (1)$$

where  $g$  is the space-time metric and  $\phi$  the inflaton scalar field.

2) One splits the fields  $(g, \phi)$  into a homogeneous-isotropic *background* plus an arbitrary *fluctuation* assumed to be small, i.e.  $g = g_0 + \delta g$ ,  $\phi = \phi_0 + \delta \phi$ . The background geometry  $g_0$  is assumed to be the Robertson Walker space-time

$$ds^2 = -dt^2 + a(t)^2(dx_1^2 + dx_2^2 + dx_3^2), \quad (2)$$

where  $a(t)$  is the scale factor and  $t$  is the cosmological time. Note that for simplicity we have restricted attention to the case of flat spatial slices ( $k = 0$  in the usual notation). The homogeneous and isotropic background field  $\phi_0$  satisfies the equation of motion  $g_0^{ab} \nabla_a \nabla_b \phi_0 + \partial_\phi V[\phi_0] = 0$  which due to the symmetry assumptions becomes

$$\phi_0'' + 3 \frac{a'}{a} \phi_0' + \partial_\phi V[\phi_0] = 0, \quad (3)$$

where primes indicate derivatives with respect to ordinary cosmic time  $t$ . The second term acts as a friction term. One assumes the so-called slow rolling condition  $\phi_0'' = 0$  (for an analogy think of the motion an object falling with terminal velocity under the influence of gravity in a viscous medium). The energy-momentum tensor of the background field is thought to be dominated by the potential term, so one takes it to be approximated by

$$T_{ab}^{(0)}[\phi_0] = V[\phi_0]g_{ab}.$$

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<sup>2</sup>The reader can use for instance [12] as an up to date review of the subject

<sup>3</sup>That we can assume the universe to be devoid of matter except for the scalar field is in fact the result of the early inflationary period that is thought to precede the regime we are describing.

To achieve slow rolling, the potential has to be sufficiently flat in the region where the initial conditions for  $\phi_0$  are set. Consequently Einstein's equations imply that during the inflationary regime we have:

$$a(t) = A e^{Ht}, \phi(t) = \phi_0(t) = \phi_{initial} + \phi'_0 t \quad (4)$$

where  $H = \sqrt{(4/3)\pi G V_0}$  and  $V_0 = V[\phi_{initial}]$ , using the fact that during the regime of interest the potential is essentially constant.

3) Now, turn to the fluctuations, and concentrate on the matter sector:<sup>4</sup> The scalar field  $\delta\phi(x)$  can be treated as a standard, scalar field on the background (2). It is useful to decompose it into Fourier modes, since they are uncoupled due to the symmetry of the background. For simplicity we introduce an infra-red cutoff by assuming the spatial slices to be  $\Sigma := [0, L]^3$ , i.e. a box of length  $L$ . Then the total Lagrangian for  $\delta\phi$  (derived from (1)) can be written as a sum of mode Lagrangians

$$\mathcal{L}_k = \frac{a^3}{2} \left[ |\delta\phi'_k|^2 - \frac{k^2}{a^2} |\delta\phi_k|^2 \right] \quad (5)$$

for the modes  $\delta\phi_k = \int_{L^3} \exp(-ikx) \delta\phi(x)$  (with  $k_i L / (2\pi) \in \mathbb{Z}$  ( $i = 1, 2, 3$ )). During inflation the equations of motion from (5) become

$$\delta\phi''_k + 3 H \delta\phi'_k - \left( \frac{k}{a} \right)^2 \delta\phi_k = 0, \quad (6)$$

where one is using that Einstein's equations imply  $a'/a = H$  in that regime. We remind the reader that at this point one is not indicating that there are inhomogeneities of any definite size in the inflationary universe, but merely one is considering what would be the dynamics of any such small inhomogeneity if it existed. The issue of their presence and magnitude is dealt with at the quantum level, where the fluctuation field is just an operator, and the above mentioned matter is associated with the state of such quantum field. In fact, the next step is the quantization of the field. Note that in terms of the Fourier modes, the Hilbert space for the quantum field will be a direct product of Hilbert spaces – one for each mode.

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<sup>4</sup>A more detailed treatment would also involve discussion of the metric perturbations, and issues of gauge. Note also that in some more recent treatments [13], quantization is applied directly to a gauge invariant combination of scalar field and metric perturbations.

4) For quantization, one needs to chose a vacuum state for the field and assume it represents the initial state at the onset of inflation. This state is required to be homogeneous (i.e. all n-point functions are invariant under translations and rotations in the spatial slice). After all, the point is to explain the emergence of the inhomogeneities, rather than merely to assume their presence at the onset of inflation and to study their evolution. There is an obvious problem in this step, because the choice of a vacuum state is not unique in space-times that do not possess a time-like Killing field. Though a difficult issue in principle, in practice the results do not depend much on the choice of the initial state as long as it approaches a canonical form for modes  $\delta\phi_k$  with large  $k$ . In the literature a sort of “instantaneous vacuum state” is most often chosen as the initial state. In the way we presented things here, this corresponds to the assumption that at the onset of inflation the wave functions for the mode  $\delta\phi_k$  and its canonical momentum  $\pi_k$  are Gaussian with spreads

$$(\Delta\delta\phi_k)^2 = \frac{1}{2a^2k}, \quad (\Delta\pi_k)^2 = \frac{\hbar a^2 k}{2}. \quad (7)$$

For  $k >> aH$  the friction term in (6) can be neglected and thus the time evolution of the mode  $\delta\phi_k$  is just that of a harmonic oscillator (with time dependent mass). Then the above choice just amounts to the modes starting out in the ground state of a standard harmonic oscillator.

5) Time evolution of the quantum state: As described above, a mode initially corresponding to  $k >> aH$  is assumed to start out in the ground state. As  $a$  grows larger the proper wave length of the mode will reach the Hubble radius ( $k = aH$ ) and the friction term in (6) will start dominating the dynamics. As the wave length grows larger one can approximate the solution to (6) by an over-damped oscillator and the value of the field can be assumed to be frozen at its value at Hubble radius crossing. At that moment  $a = k/H$  so that the fluctuations of  $\delta\phi$  and  $\pi_k$  can be approximated by

$$(\Delta\delta\phi_k)^2 = \frac{H^2}{2k^3}, \quad (\Delta\pi_k)^2 = \frac{\hbar k^3}{2H^2}. \quad (8)$$

6) Now the fluctuations (8) are identified with (or, in a more careful formulation such as in [14], “taken as indicative of”) the (classical) spectrum of inhomogeneities. For example, often the fluctuations of the energy density in co-moving coordinates,  $\delta\rho$ , is used as a measure for the classical inhomogeneities. It is easily verified that  $\delta\rho \sim \phi'_0 a^{-3} \pi_k$ . Thus one sets

$$|\delta\rho_k|^2 \sim a^{-6} (\Delta\pi_k)^2 \sim k^{-3} \quad (9)$$

at the time the mode  $k$  leaves the Hubble horizon.  $\delta\rho$  is understood as a classical quantity.

7) The classical density fluctuations are now evolved through the end of inflation and into the regime of standard cosmology. It can be shown that the spectrum of the fluctuations at the time they exit the Hubble radius is proportional to the spectrum at the time they re-enter. Thus one has  $(\delta\rho_k)_{\text{enter}} \sim k^{-3/2}$  which is the scale free Harrison-Zeldovich spectrum. The classical fluctuations are then evolved further to take into account the matter dynamics, leading to the famous acoustic peaks.

8) The resulting evolved spectrum is compared with the observations of the CMB. Furthermore, these departures from homogeneity and isotropy are thought to be the seeds for the evolution of the structure that we see in the our universe, and that are in fact necessary for our own existence.

This is in essence the standard account of the process for obtaining the seeds of cosmic structure formation that forms the basis of the analysis of cosmological data, and in particular of the CMB anisotropies.

### 3 A critique of the story

The previous analysis is remarkable: The universe starts in a homogeneous state and ends up with inhomogeneities that fit the experimental data. However, as we will argue in the following sections, this is not fully justifiable by “standard physics”. We should point out from the start that many authors do acknowledge the existence of a gap in our understanding, for instance [14, 15], and some do propose ways to deal with the issue in more detail (see ex. [7, 8, 11, 29, 36]). There is also a large literature on issues of quantum mechanics in the context of cosmology in general (ex. [10, 16, 24, 27]). The discussion that follows is addressed to those colleagues that are not fully aware of the problem, and to those that believe that there is no problem at all.

The first point that we want to make is that the above analysis can not be justified through standard quantum mechanical time evolution. If this is not already obvious from the particulars of step 6, just note that as already stated above, the initial quantum state is symmetric<sup>5</sup> (i.e., homogeneous and

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<sup>5</sup>The vacuum state defined in step 4 is invariant under the action of the symmetry group of the background. Even though it can be written as a superposition of states which are

isotropic) and the standard time evolution does not break this symmetry, thus it can not explain the observed inhomogeneities. The states obtained in other natural schemes, such as the one arising from the “No Boundary” boundary condition in quantum cosmology are equally homogeneous and isotropic, as can be seen explicitly in eq 8.6 and 8.9 in [17].

Let us now look at step 6 in more detail: The essence seems to be that a classical quantity is identified with (the square root of) a quantum mechanical expectation value. This suggests that one might be able to view the procedure of step 6 in the framework of a semiclassical theory, i.e. one in which a part of the system is described classically, while another part is described quantum mechanically. A hallmark feature of such type of descriptions is that classical quantities are coupled to the quantum sector via expectation values. While not satisfactory as fundamental theories, they nevertheless often have some validity as effective theories. However, we note that two things are remarkable about the prescription of step 6 and set it apart from semiclassical theories such as semiclassical gravity: a) In step 6 a classical quantity couples to the *square-root* of the expectation value of the *square* of its quantum counterpart. b) The use of Fourier transforms in step 6. The choice involved in a) is certainly not the simplest possibility, and not the one chosen in standard semiclassical treatments. Point b) would not be an issue if the relation were linear, but raises concern due to the choice made in a). For instance, what if instead of plane waves one uses spherical harmonics in the mode expansion? The non linear nature of a) implies that this will affect the result. Note also that both a) and b) must be required to achieve the result of the standard view described in the previous section. If we drop any of the two we do not get the sought-for inhomogeneities. For instance, identifying the square root of the expectation value of  $\hat{\delta}\phi^2(x)$  with the classical counterpart (i.e. dispensing with b)) leads to a result that is independent of  $x$ , and thus it is homogeneous. Identifying the expectation value of  $\phi_k$  with its classical counterpart (i.e. dispensing with a)), leads to a homogeneous result (zero, in fact) as well.

One quite often reads the following argument to support a) and b): A simple calculation shows that  $\langle \hat{\phi}_k^2 \rangle$  is equal to the Fourier transform of the auto-correlation of the two-point function of  $\hat{\phi}(x)$ , and thus quantifies

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not symmetric by themselves the superposition does not select any direction and looks the same at every point on the homogeneous  $t = \text{constant}$  slices. This is completely analogous to the Poincaré invariance of the standard vacuum on Minkowski space-time.

correlations of the field at different points. The observations are then argued to be a measurement of these correlations, because one is measuring the differences in the temperature of the CMB between different directions in the sky. We do not find this convincing however, because although WMAP in fact works as a differential thermometer, it does so in a way that allows one to obtain a map of the temperatures in the sky, and it is this map which is subjected to a Fourier analysis from which the observational power spectrum is read.<sup>6</sup>

One may want to interpret step 6 as an effective description of some sort of measurement process or “collapse of the wave function”. This seems to be plausible, as step 6 involves expectation values of squares of operators, i.e., quantities that measure the spread of measurement results according to standard quantum mechanics. Furthermore, a measurement can break the symmetry of the initial state and produce, in our case, inhomogeneities. This scenario would be quantum mechanics at its core: The unitary evolution (the  $U$  process in [19]) of the quantum state with a symmetry preserving Hamiltonian will not break the initial symmetry of the state, but a measurement of an observable whose eigenstates are not symmetric, forces the system (through the collapse, called  $R$  process in [19]) into one of the possible asymmetric states. The onset of the asymmetry (in standard interpretations of quantum mechanics) occurs only as a result of the measurement, and then, only when the measured observable does not commute with the generators of the symmetry.

However this scheme, applied to the situation at hand, immediately raises many questions: What constitutes the measurement in our cosmological situation? When did it happen? What is the observable that was measured? Perhaps the answer is somehow connected to our own observation of the sky? Are we to believe that until 1992 [20] the CMB was in fact perfectly homogeneous? Hardly, as we know the conditions for our own existence depend on the departures from homogeneity and isotropy in our universe, and thus our actions could not be their cause.

Moreover, the predictive power of quantum mechanics regarding one single measurement is rather small (in the EPR case the only absolute prediction

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<sup>6</sup>After all, if it were technically possible to replace the comparison of temperatures at two different points in the sky with, say, the comparison of the temperature of a single point in the sky with the temperature of a reference body within the satellite, one would not expect the resulting temperature map to change (unless one is prepared to argue the CMB really is a quantum system in a highly non-classical state).

for one experiment concerns to the situation in which the two measuring axes are perfectly aligned). How is it, then, that we have such a predictive power in regards to the single measurement of our sky? One could think that this is due to the fact that we are measuring many regions in the sky and that each should be considered as a different measurement, but this is not the case: The measurement of the amplitude in one single mode requires the Fourier transform on the whole sky. The fact that the actual measurements involve (for technical reasons) smaller regions than that, is in fact responsible for uncertainties in the measurement of the quantity of interest.

Step 6 might effectively describe some sort of measurement, but important questions have to be answered:

1. What is performing the measurement?
2. Precisely, what is the set of quantum observables that is being measured? And what determines them?
3. When is this measurement taking place?
4. Can the above questions be answered in such a way that the ensuing predictions are in agreement with observations.

One further issue that needs discussion is the nature of the different averages, fluctuations and in general statistical issues that are present in the problem and that are sometimes treated as if no differences even of principle did exist. First we have the quantum mechanical aspect of the problem, reflected for instance, in the evaluation of expectation values of observables. Here the statistical nature is reflected in the usual indeterminacy of future measurements of certain quantities when the state of the system does not coincide with an eigenstate of the observable. However let us imagine for a second that we could find an operator that measured the degree of cosmic inhomogeneity of the system. It is clear that the vacuum would correspond to an eigenstate of such operator with zero eigenvalue. If that was the observable that is measured, the inescapable result would certainly be problematic for the standard analysis that is supposed to lead to the observed spectrum of fluctuations. The point is not to discuss whether this operator exists or not but to note that the statistical aspect associated with the quantum mechanical picture would emerge only when the particular observable associated with a measurement is selected. Next we have the statistics associated with

the classical ensemble represented by the stochastic field and the corresponding inhomogeneities characterizing the corresponding ensemble of universes. And finally we have the statistical description of the inhomogeneities within our own universe, which can be thought of as an arbitrary generic realization within the ensemble of universes mentioned before. Here the point is to distinguish in principle, between the statistics in the ensemble of universes, and the statistics within one (our own). We are used from our experience with statistical mechanics to identify averages over ensembles with physical expectations, however we must recall that these identifications rely on two other identifications: The identification of the ensemble averages with the long time averages (a fact that relies on the validity of the ergodic assumptions), and the identification of long time averages, with the results of physical measurements, a fact that relies on equilibrium considerations. Needless to say, that none of these conditions are present in the problem at hand. Some of these issues have motivated the considerations in [18].

The claim that there exist a prediction regarding the primordial density anisotropies and inhomogeneities in the universe could not be sustained without addressing these questions.

## 4 A look at proposed ideas on the transition to classicality

In this section we give a quick overview of the most popular ideas proposed to address these issues, and attempt to exhibit as clearly as possible their incompleteness, by signaling the point at which a “missing element” makes a disguised appearance, or at least indicating the place where it should have entered in order to justify the subsequent interpretation given to the computed quantities.

### Standard Decoherence

Decoherence in its standard interpretation describes the fact that when considering a quantum system with a very large number of degrees of freedom, most of which are ignored (by considering them as “the environment”), the density matrix for the subset of the remaining, interesting “observables”, evolves under certain circumstances, and after suitable time averaging, towards a diagonal matrix. This is sometimes said to represent the emergence

of the classical behavior of the interesting observables. There has been certainly a large amount of work on this field, most of it devoted to evaluating the time evolution of the previously mentioned density matrix in various types of situations, or more specifically to studying the decoherence functional. There is however much less work devoted to interpretational issues and it is fair to say that decoherence has not solved the measurement problem in quantum mechanics [21]. In fact, the diagonal character of the resulting density matrix point to the disappearing of certain quantum aspect of the problem. That by itself is certainly not enough to claim that the situation has become completely classical: A non-interfering set of simultaneous co-existing alternatives is not something that can be thought of as belonging to the realm of classical physics. This sort of criticism regarding such interpretations of decoherence have of course been made before, for instance see section, 2.3.4 of [22]. Moreover the diagonal nature of the density matrix would disappear if we write it using a different basis for the Hilbert space of the quantum system, clearly indicating that even this aspect of the so called classicality has a limited validity.

Thus, the main issues that confront the decoherence approach to the measurement problem in quantum mechanics are the selection of the basis, and the fact that after one has a diagonal density matrix, one is in general, still a big step removed from an interaction specific eigenstate for the desired observable. In the standard case one can progress beyond this point by invoking the measurement apparatus to help select the basis, and by using the ensemble interpretation to deal with the density matrix. These two aspects are clearly absent in the cosmological situation we are considering<sup>7</sup>. In fact the need for going beyond standard quantum mechanics in this context has been noted before [23, 24] (and we will discuss this line of thought further at the end of this subsection). Furthermore, if one wants to claim that decoherence solves the problem in the cosmological setting at hand, one must find a preferential basis selected by a physical mechanism, and a criterion for the separation of the degrees of freedom into the “interesting set” and “the environment” dictated by the physical problem at hand. In some schemes one might be able to unify these two issues into one by arguing that the environment determines the relevant basis to be that in which the system

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<sup>7</sup>As we already mentioned, taking the position that the measurement is in fact our own series of studies of the CMB, leads to a closed circle of causes and effects, with an explanatory power that is highly doubtful to say the least.

apparatus environment interaction is diagonal [27]. In any case, the result would then naturally have the imprint of these two inputs – perhaps unified into one – but so far no universally accepted prescription for such choices exists, nor can such selection be fully justified, in the standard descriptions of the origin of the cosmic density fluctuations. For instance, one might want to argue that one needs to trace over the very large wavelength modes, because they are unobservable, as is done for instance in [27], but as the part of the universe that is observable by a co-moving observer is certainly dependent on the cosmological epoch, the tracing would depend on the cosmic time of the observer, but as such it can not be argued, unless we do away with causality, to play a role in the onset of inhomogeneity and anisotropy in the early universe. Quite generally, we can not allow our own characteristics and limitations to come in into the argument, if our hope to explain the asymmetries that give rise to the conditions that make our existence possible.

Actually, in some of the treatments invoking decoherence one finds at some point of the discussion, an appeal to a “a specific realization” ([7] sec. 3, [9] sec. V) of the stochastic variables, in a clear allusion to something that can be called the “collapse” from the statistical description of the universe, into one of the elements of the statistical ensemble.<sup>8</sup> This is the point, of course, when we transit from a homogeneous and isotropic description to an inhomogeneous and anisotropic one, presumably corresponding to our universe. However nothing is said about when and how the transition occurs *in our universe*.

In fact what seems to be an insurmountable issue in this scheme is the following: Even if we go from a perfectly symmetric state (the symmetry being homogeneity and isotropy), to a density matrix for a subset of the physical degrees of freedom, which is expressed in a basis in which the non-diagonal elements are negligibly small, through a justified selection of the degrees of freedom that should be considered environment, one could not end up with an asymmetric mixed state, as there is nothing in the physical laws or the initial situation that could lead to such breakdown of the symmetry. Thus the mixed state described by the density matrix is still perfectly symmetric. The density matrix is then given a statistical interpretation, by which we stop regarding the matrix as representing the state of our universe, and instead view it as representing a statistical ensemble of universes, among which we

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<sup>8</sup>Note that other schemes are based to some degree on this sort of specific realizations, without calling them by this name – see for instance [28]

find our own. The ensemble as a whole is clearly symmetric, but leaves room for each of the elements in the ensemble to be asymmetric. This is how we end up with an inhomogeneous and anisotropic universe. However, it is clear that this does not address at all any of the characteristics of transition of our universe from an initial symmetric state, to such a resulting asymmetric state. This problem affects all the schemes based on decoherence, including the detailed treatments in [29], and if we seek a realistic understanding of the origin of inhomogeneity and anisotropy in our universe, this approach would clearly be missing something.

After this discussion of the difficulty of applying the standard picture of decoherence in the cosmological context, we would like to mention the ideas of Hartle and collaborators. We have placed them here even though they call for a generalization of Quantum Mechanics. In fact, we view his arguments for the need of a generalization of Quantum Mechanics to be applicable not only to quantum gravity, but also to cosmology ([24, 25], and references within) as further indications for the need to go beyond standard physics. The particular generalization of Quantum Mechanics that has been suggested involves the assignment of a decoherence functional to allowed sets of coarse grained histories. One such set of coarse grained histories decoheres when there is no quantum mechanical interference among the different alternatives. In that situation we are able to assign definite probabilities to the particular alternatives. In the case at hand this would have to be applied to the histories starting during the inflation period in early universe to the formation of large structures and eventually to ourselves.

We should however mention some problematic aspects of this proposal: To start with, the division of the global set of all fine grained histories into sets of coarse grained histories might be done in various ways, leading to different decoherent sets [26]. Second, the coarse graining, and corresponding decoherence, arises only when we ignore certain degrees of freedom, and the justification for doing so, relies on what we, as humans living in this particular era, could in principle observe and what we can not. This is fine, except in the case that what we seek to understand is the emergence of the conditions that lead to the possibility of our own coming into existence. It is clear that in that case we can not call upon some of our own characteristics as a part of the explanation.

## Decoherence without Decoherence

This can be viewed as a particular realization of the ideas of the previous case. Its main appeal is that one needs to consider only the scalar field and no other physical system is required to play the role of “environment”. The point is that certain modes of the scalar field vanish asymptotically in time as a result of the inflationary dynamics of the universe. These modes are then deemed unobservable, and one is then invited to treat them as such, taking the appropriate trace and obtaining a mixed state from what was initially a pure quantum state of the field (ex. [7, 8]). This approach not only suffers from the same drawbacks that affect the decoherence approach in general, but also from what seems as a further interpretational excess: To have expectation values or higher moments of a mode converge to the zero value is not the same as making it unobservable. Zero is just as good a value for a scalar field as any other one. Moreover it is clear that even if one would agree on the non observability of certain degrees of freedom, that does not imply necessarily that the system as a whole needs now to be treated as a mixed state. There could be other reasons that fix the state of those degrees of freedom. A proton can, in the appropriate circumstances, be described as a pure quantum state, despite the fact that in these same conditions its microscopic constituents – quarks and gluons – might be, in practice, unobservable.

One novel aspect that is sometimes invoked in these treatments is the squeezing of the vacuum [7, 8]. That is, one notes that during the inflationary stage the initial vacuum state for each mode, evolves towards a squeezed state. We recall that a squeezed state is a state that has the minimal uncertainty but not in the standard position and momentum variables but a new pair of “rotated” canonical variables. Thus, in our case, one has an uncertainty on the value of field and conjugate momentum which is much larger than the minimum uncertainty provided by quantum mechanics. Thus one concludes this minimal uncertainty (indicated by Q.M.) is negligible compared with the actual uncertainty and thus that one can neglect quantum mechanics. This is taken as an indication that there has been a transition from the quantum to the classical behavior. This conclusion does not seem to be sufficiently justified, as can be seen by noting that in many concrete situations, such as those studied in Quantum Optics, one deals precisely with states that have uncertainties in conjugate quantities that are larger than the minimal ones provided by Q.M. and nevertheless there are situations

– for instance when one is interested in experiments sensitive to quantities other than the ones in which the system is seen as squeezed – that one must take into account that one is dealing with a system that must be treated quantum-mechanically.<sup>9</sup>

We conclude that it is clearly insufficient to have the product of the uncertainties being much larger than  $\hbar$  as criterion for classicality. One could consider instead as criterion for classicality the requirement that the volume of phase space occupied by the system, as measured for instance by the support of the Wigner functional, be much larger than  $\hbar$ . This condition is however not satisfied by a squeezed state. For a critical discussion of these issues in the context of inflation see also [11]<sup>10</sup>.

And of course we have the other issues that have been pointed out regarding the general scheme of decoherence as describing the transition from a quantum, homogeneous, and isotropic state of affairs to a classical (or quasi-classical), inhomogeneous and anisotropic situation.

## Alternative to Inflation

In a recent work Wald and Hollands [5] have shown that is possible to recover identical predictions as those afforded by inflation, regarding the spectrum of the seeds for cosmic structure formation, without requiring the universe to have gone through an inflationary stage. They start by considering again the dynamics of a scalar field in a background cosmological setting. In their model, the inhomogeneities in the universe originate while the universe is dominated by radiation or dust so that the scale factor grows as  $a(t) = ct^\alpha$  with cosmic time  $t$ , where  $c$  is a constant and  $\alpha < 1$ . Their model just requires that all modes would have been in their ground state (as in the

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<sup>9</sup>The problem can be seen more clearly if we look at the following example. Take an electron in a minimal wave packet localized at the origin such that the uncertainty in the position  $\Delta X = \alpha$  and that in its momentum is  $\Delta P = \hbar/2\alpha$ . Now construct a superposition of such state with an identical state after a translation by a large distance  $D$ . The product of the uncertainties is now roughly  $D\hbar/2\alpha$  which can be made as big as desired by taking  $D$  sufficiently large. Clearly the system is nevertheless far from being classical.

<sup>10</sup>It is illustrative to mention, in the context of this example, that one might add to the electrons their spin degree of freedom and consider the state where the electron at the origin has its spin up, superposed with the displaced state where the spin is down. Now consider the density matrix obtained by ignoring the spin degree of freedom, (and thus tracing over it). In that case we would have an essentially diagonal density matrix in the position basis, but one could not argue that one has a classical situation.

standard picture), and that the fluctuations are “born” in the ground state at an appropriate time which is early enough so that their physical length is very small compared to the Hubble radius, and then they “freeze out” when these two lengths become equal. The presentation of their model actually allows one to see clearly the need for some process that would be responsible for the so called “birth” of the fluctuations, which can be seen to play a role similar to that of quantum mechanical measurement. In fact the so called “birth of the mode” is the step whereby a quantum mechanical uncertainty is replaced, in their treatment, by an actual classical fluctuation of the energy density, and the point at which a particular mode that has been contributing in an absolutely homogeneous and isotropic way to the universe density, becomes a source of the inhomogeneities that presumably are responsible for the structure in the matter distribution of the early universe. The issues are then: What physical process is responsible for these “births”, or transmutation of the fluctuations? And how is it that such process selects the particularly appropriate time of such occurrence for each particular mode?

### **The ‘many universes’ perspective**

One view that is apparently very widespread among the community working in inflation (but much less represented in the corresponding literature), is what can be called the Many Universes perspective. According to this point of view, our universe is one among a large number of universes that constitute an ensemble, and it is this ensemble what is in fact characterized by the quantum state that is homogeneous and isotropic. The standard treatment is then interpreted as reflecting the most likely aspect our universe can be expected to have, when considered statistically within such an ensemble. In such an interpretation there appear to be no open issues, no need for a collapse, and certainly not new physics.

Note that this would have to be quite different from the Everett, or Many Worlds, interpretation of Quantum Mechanics. In the latter, reality is made of a connected weave of ever splitting worlds, each one realizing one of the alternatives that is opened by what we would call a quantum mechanical measurement. It is thus clear that the points of splitting, the basis in which the splitting occurs, and the physical entity associated with the triggering of the splitting are in one to one correspondence with the aspects we have mentioned before which are associated with the collapse of the Copenhagen interpretation (time of measurement, basis, and measuring device). In the

Many Universes paradigm there is no splitting, as that would have amounted effectively to a collapse and would then be subjected to the equivalent set of questions: When did the splitting occur? What caused it? And how was the basis in which it happened, selected? In particular, it should be emphasized that from such "Many Universes" point of view, our universe was never homogeneous and isotropic. Let us however examine in more detail what such view entails.

In the most direct interpretation of such posture we would have a collection of universes, each one of which has a definite and concrete realization set of the asymmetries (the symmetry being of course homogeneity + isotropy). However, when this is being said, we clearly imply that each one of these universes would be susceptible to have a description, in particular, a description of its asymmetries. That description must be either classical or quantum mechanical.

If we take the first option, we would be taking the position that quantum mechanics is a theory applicable to ensembles of systems. In this view, we must confront two aspects, a) each one of the elements of the ensemble can indeed be described in a classical language, and b) the quantum mechanical description contains information only about the statistics that results from repeated experimentation applied to the collection of elements constituting the ensemble. Aspect b) of this posture, i.e. the applicability of QM only to ensembles, is in itself problematic given the absolute nature of the prediction regarding a single system that can be made in certain special circumstances (like when dealing with eigenstates of the operators to be measured or when there are superselection rules totally forbidding certain processes), for more on this issues see [30]. But even more disquieting is aspect a) of the posture, namely the one where one advocates that each member of the ensemble has a classical description. This position would revert one to a sort of "hidden variable" advocacy, which would say for instance that in an EPR set-up each electron has a well defined value of its spin components even before the measurement is done, a position which is known to be untenable in light of the Bell Inequalities for correlations (See discussion by in [31] and their experimental corroboration [32]).

If we take the second option, we would be saying that our universe is in a quantum state, which is not symmetric, but that, when superposed with the other asymmetric quantum states that describe a certain ensemble of universes, leads to an homogeneous and isotropic quantum state; the one corresponding to the inflationary field vacuum for the initial stages of infla-

tion. This seems fine at first sight, however, it is far from what Quantum Mechanics prescribes: One does not superpose a state describing one system, with the state describing a second system to obtain the collective description of the two systems. A further problem appears when we want to take the argument in the opposite direction. Namely, taking as the initial assumption that the state that collectively describes the collection of universes is the one corresponding to the vacuum state of the field and is thus homogeneous and isotropic, one must end up with a decomposition of that state in a particular basis (of non-symmetric states) of the Hilbert space in order to associate to each universe a corresponding quantum state. Now we face two issues. First, the election of such basis, is not given “*a priori*” by the formalism. One can certainly think of ways to make the choice, and such choice would correspond to the selection of the quantity that is “measured” in ordinary Quantum Mechanics. This aspect is missing in the standard descriptions. In fact it would correspond to whatever selects the basis in which the collapse takes place in our approach. The supra-temporal selection of the basis (supra-temporal in the sense that, from this point of view, each universe is from the onset in a specific non-symmetric quantum state) has no counterpart in ordinary Quantum Mechanics. For instance when considering an EPR correlated pair of electrons one could not take the position that there is a multiplicity of universes and that in ours each electron is in a specific state of its spin at all times along its path. Similarly, when considering an electron in its ground state in a hydrogen atom one would not argue that in our world the electron is in an eigenstate of the position and that the ground state corresponds in fact to a description of a corresponding ensemble of universes with fully localized electrons. This aspect is therefore novel to the situation at hand, and as such would qualify also as New Physics.

The second problematic aspect is the following: In taking this point of view, our universe would be considered as always having been in a state that is anisotropic and inhomogeneous. It would have never corresponded to the vacuum state of the scalar field. Then, one might find uncomfortable the idea that placing our universe within an ensemble together with a large set of unobserved universes, one might be able to make predictions about our particular one. In other words, if we do not assert that our universe started in the scalar field’s vacuum, how could we end up with prediction for its anisotropy that takes such a state as the starting point? Some readers might argue that this is an epistemological complaint, and dismiss it, while others would sympathize with the uneasiness. In any event it is worth noting that

this aspect is there.

The final option that seems to be open within this general point of view asserts that our universe was indeed, together with all the elements in the ensemble in the start corresponding to the homogeneous and isotropic vacuum state of the scalar field. That all these possibilities thus evolved “simultaneously” and that in each one of them (or at least in a great many of them) structures such as galaxies and stars formed out of the initial asymmetries, and that observers like ourselves then evolved in some of the solar systems, and that these observers thus carried out the observations that selected the basis in which a standard type of measurement-induced collapse occurred. The problem of course is that only in very special basis would the galaxies stars and observers themselves exist to carry out the observations that effect the measurement-like collapse and concomitant selection of the basis.

Finally, and as a cautionary note against the hope of finding a coherent and orthodox description of the situation, one should keep in mind the fact that the Copenhagen interpretation is inviable without outside observers. We can not be part of the system, and our existence can not, thus, be explained in such scheme. If the purpose of cosmology is to give a picture of the evolution of our universe that explains the way its present form and content – including ourselves – is arrived at, that particular point of view will be always lacking.

## 5 The missing element

As we have seen, all the scenarios that have been considered are incomplete and unsatisfactory in some way or another. A close inspection actually reveals that they are all missing some element: The process whereby a perfectly homogeneous and isotropic state (for the universe is homogeneous and isotropic and so is the vacuum quantum state that one assumes for the scalar field), transforms into an inhomogeneous and anisotropic state which is what is described by the density fluctuations. There is clearly no deterministic mechanism that can achieve this without the introduction of some external source of asymmetry. Barring such source, we need to recur to quantum mechanics. However even when doing so, one is only able to provide for what is required as part of the so called R process (measurement, collapse, etc.) but not during the U process (unitary evolution through a Schrödinger type equation). Thus if a measurement-like process is absent there can be no transition. The problem is then the absence of a sensible candidate for

such process. This is because in the early universe which is homogeneous and isotropic one can not select a canonical quantity that is the one that would be measured, and much less the measurement agent or mechanism. After all, that would require an effective division of the universe into system and a measuring device, and it is clear that physics, in this case, does not indicate such division along any lines we know off.

The main points of this article are, first to indicate that such aspect is missing and to argue that it requires new physics, and second to show that by making rather modest suppositions we would be in a position, thanks to recent advances in observational status of cosmology, to actually say something non trivial about the characteristics that this new physical process must possess. A further speculative discussion about the possible nature of this new physics will be included.

### **The standard view, supplemented by the physical collapse hypothesis.**

Our approach here is to explore the necessary ingredients to make the standard picture interpretationally accurate, with a minimal set of assumptions, and to carefully identify where they occur. We will assume that there is indeed a *new* physical mechanism that is responsible for the transformation of the ground state into a state that contains the fluctuations that are responsible for the departure from homogeneity and anisotropy. We will call this process the *collapse*. Apart from this the treatment will be carried out following the standard rules of unitary quantum mechanical evolution. We will attempt to extract the conditions that will enable us to recover the standard predictions of the inflationary model for the appearance of the seeds for cosmic structure, while exposing the points where we need to depart from that treatment, in the sense that the uncomfortable points will not be hidden by the formalism.

The idea is then to follow the standard picture up to the point 6 in Section 2, and supplement it by the *collapse hypothesis*. As we mentioned before, we will neglect the hydrodynamic evolution corresponding to point 7, and assume that we have direct access to the unmodified spectrum. That is, we will discuss the conditions under which we could understand the observation of a scale free Harrison Zeldovich spectrum, directly in the CMB.

The scheme we have in mind is thus:

- 1) We split scalar field and metric into “background” parts and perturbations and specify the background.
- 2) We treat the scalar field perturbation as a quantum field evolving in the classical space-time background according to the standard unitary evolution given by its dynamics, except at moments when gravity (or something else) triggers a collapse of the quantum state of the field.
- 3) The collapse itself will be described in a purely phenomenological manner, without reference to any particular mechanism.
- 4) We couple the metric perturbations to expectation values of the scalar field according to semiclassical Einstein equations

$$G_{ab} = 8\pi G \langle T_{ab} \rangle \quad (10)$$

As implied by 2), we do however neglect the back-reaction of the metric perturbations on the scalar field evolution because they are suppressed, as we will show.

We must note that in the moments where collapse does occur, the semiclassical Einstein equations (10) are violated.<sup>11</sup> The view we take here is that in these moments a more fundamental description, possibly a theory of quantum gravity, would provide the correct equations for the collapse. Thus this is certainly a very schematic picture that must be further studied, analyzed, and specified, if we want to construct a complete model. At this time we are interested only in finding out the basic aspects that the model should have in order to actually account for the observational facts. On the other hand we should point out that the scheme is certainly inspired by the ideas of Penrose [19] regarding the role of quantum gravity in the collapse of the wave function in general circumstances, and that ideas of this sort of mechanism have been proposed quite independently from any quantum gravity consideration, in the context of the interpretational problems of Quantum Mechanics [33].

We proceed now to give a more detailed analysis. In particular, and in order to be able to discuss more clearly the ideas related to the collapse proposal we will decouple the treatment of the gravitational degrees of freedom from those of the true scalar field which will be quantized in a standard way.

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<sup>11</sup>We thank Prof. A. Ashtekar for pointing this out.

## 6 Linearized Einstein's equations and the evolution of small fluctuations

We will consider here the metric and field perturbations taken as classical test fields evolving in the background geometry. As the metric (2) is conformally Minkowski, it is convenient (and customary) to go over to a new time coordinate  $\eta$  ('conformal time') that makes this explicit. To make things more definite we set  $t = 0$  and  $\eta = 0$  to mark the time at which inflation ends. Thus we have  $\eta \equiv \int_0^t a^{-1}(t') dt'$ . To finalize fixing our conventions we set the scale factor to be 1 at the present time  $t_0$ . The background metric is then

$$ds^2 = a^2(\eta)[-d\eta^2 + (d\vec{x})^2]. \quad (11)$$

During inflation the scale factor is given by

$$a(t) = d \exp(H_I t), \quad (12)$$

where  $H_I$  is the Hubble constant during inflation. Thus for  $t < 0$  we have

$$\eta(t) = \frac{1}{H_I d} (1 - e^{-H_I t}), \quad a(\eta) = -\frac{1}{H_I(\eta - \eta_0)}. \quad (13)$$

where  $\eta_0 \equiv \frac{1}{H_I d}$ . Let us reparametrize the evolution fixing  $\eta_0 = 0$ . The point is then that inflation would end at some  $\eta_{IE} < 0$  and afterwards the universe will proceed to a standard cosmological expansion until at  $\eta = 0$   $a = 1$ . We will ignore this part of the universe evolution in the rest of the paper.

It is customary to decompose the metric fluctuations in terms of its scalar, vector, and tensor components. In the case of our Einstein-inflaton system only scalar (generated by density perturbations) and tensor perturbation (gravitational waves) are relevant. Gravitational waves will be ignored for simplicity and thus, the perturbed metric (in the conformal gauge) can be simply written as

$$ds^2 = a(\eta)^2 \left[ -(1 + 2\Phi)d\eta^2 + (1 - 2\Psi)\delta_{ij}dx^i dx^j \right], \quad (14)$$

where  $\Phi$  and  $\Psi$  are scalar fields, the former is referred to as the Newtonian potential.

Let us first write down the components of the Einstein tensor ( $G_{ab} = R_{ab} - \frac{1}{2}g_{ab}R$ ) up to first order in the perturbations:

$$\begin{aligned}
G_{00}^{(0)} &= 3\frac{\dot{a}^2}{a^2} \\
G_{ii}^{(0)} &= \frac{\dot{a}^2}{a^2} - 2\frac{\ddot{a}}{a} \\
G_{00}^{(1)} &= 2\nabla^2\Psi - 6\frac{\dot{a}}{a}\dot{\Psi} \\
G_{0i}^{(1)} &= 2\partial_i\dot{\Psi} + 2\frac{\dot{a}}{a}\partial_i\Phi \\
G_{ii}^{(1)} &= (\nabla^2 - \partial_i\partial_i)(\Phi - \Psi) + 2\ddot{\Psi} + 2\left(2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2}\right)(\Psi + \Phi) + 2\frac{\dot{a}}{a}(\dot{\Psi} + \dot{\Phi}) \\
G_{ij}^{(1)} &= \partial_i\partial_j(\Psi - \Phi) \quad \text{for } i \neq j
\end{aligned} \tag{15}$$

The components of the energy momentum tensor are as follows

$$\begin{aligned}
T_{00}^{(0)} &= \frac{1}{2}(\dot{\phi}_0^2) + a^2V[\phi_0] \\
T_{ii}^{(0)} &= \frac{1}{2}(\dot{\phi}_0^2) - a^2V[\phi_0] \\
T_{00}^{(1)} &= \dot{\phi}_0\delta\dot{\phi} + 2a^2\Phi V[\phi_0] + a^2\partial_\phi V[\phi]\delta\phi \\
T_{0i}^{(1)} &= \dot{\phi}_0\partial_i\delta\phi \\
T_{ii}^{(1)} &= -\Phi\dot{\phi}_0^2 + \dot{\phi}_0\delta\dot{\phi} - \Psi(\dot{\phi}_0^2 - a^2V[\phi_0]) - \frac{1}{2}a^2\partial_\phi V[\phi]\delta\phi \\
T_{ij}^{(1)} &= 0 \quad \text{for } i \neq j
\end{aligned} \tag{16}$$

Finally the scalar field equation yields, to zeroth order:

$$\ddot{\phi}_0 + 2\frac{\dot{a}}{a}\dot{\phi}_0 + a^2\partial_\phi V[\phi] = 0, \tag{17}$$

and to first order:

$$\ddot{\delta\phi} + 2\frac{\dot{a}}{a}\dot{\delta\phi} - \nabla^2\delta\phi + a^2\partial_{\phi,\phi}^2 V[\phi]\delta\phi - (\dot{\Phi} + 3\dot{\Psi})\dot{\phi}_0 - 2\Psi(\ddot{\phi}_0 + 2\frac{\dot{a}}{a}\dot{\phi}_0) = 0. \tag{18}$$

Now let us reduce the number of equations by solving some of them. The only non-trivial among Einstein's equations, to zeroth order is  $G_{00}^{(0)} = 8\pi GT_{00}^{(0)}$  which leads to Friedman's equation

$$3\frac{\dot{a}^2}{a^2} = 4\pi G(\dot{\phi}_0^2 + 2a^2V[\phi_0]). \tag{19}$$

In the linear order let us start from  $G_{ij}^{(1)} = 8\pi G T_{ij}^{(1)}$  which implies  $\Psi = \Phi$ . Using the previous result, the vector constraint equations  $G_{0i}^{(1)} = 8\pi G T_{0i}^{(1)}$  imply

$$\partial_i(\dot{\Psi} + \frac{\dot{a}}{a}\Psi - 4\pi G \dot{\phi}_0 \delta\phi) = 0 \quad (20)$$

which reduces to<sup>12</sup>

$$\dot{\Psi} = -\frac{\dot{a}}{a}\Psi + 4\pi G \dot{\phi}_0 \delta\phi. \quad (21)$$

The scalar constraint equation  $G_{00}^{(1)} = 8\pi G T_{00}^{(1)}$  becomes

$$2\nabla^2\Psi - 6\frac{\dot{a}}{a}\dot{\Psi} = 8\pi G(\dot{\phi}_0 \delta\dot{\phi} + 2a^2\Psi V[\phi_0] + a^2\partial_\phi V[\phi]\delta\phi). \quad (22)$$

Now, using Einstein's equations to express the potential

$$2a^2V[\phi_0] = T_{00}^{(0)} - T_{ii}^{(0)} = (8\pi G)^{-1}(G_{00}^{(0)} - G_{ii}^{(0)}) = (4\pi G)^{-1}(\frac{\dot{a}^2}{a^2} + \frac{\ddot{a}}{a}), \quad (23)$$

and substituting this result and the value of  $\dot{\Psi}$  from (21) in equation (22), we obtain

$$\nabla^2\Psi = 4\pi G(3\frac{\dot{a}}{a}\dot{\phi}_0 \delta\dot{\phi} + \dot{\phi}_0 \delta\dot{\phi} + a^2\partial_\phi V[\phi]\delta\phi) + (\frac{\ddot{a}}{a} - 2\frac{\dot{a}^2}{a})\Psi. \quad (24)$$

We write this equation as

$$\nabla^2\Psi - \mu\Psi = 4\pi G(u\delta\phi + \dot{\phi}_0 \delta\dot{\phi}) \quad (25)$$

where  $\mu \equiv (2\frac{\dot{a}^2}{a} - \frac{\ddot{a}}{a})$ , and  $u \equiv 3\frac{\dot{a}}{a}\dot{\phi}_0 + a^2\partial_\phi V[\phi]$ , which upon use of the expression for  $\partial_\phi V[\phi]$  from (17), gives  $u = \frac{\dot{a}}{a}\dot{\phi}_0 - \ddot{\phi}_0$ .

Finally using the expressions for the scale factor during inflation we find  $\mu = 0$ , while the slow-rolling approximation  $\frac{\partial^2\phi}{\partial t^2} = 0$  corresponds in these coordinates to the condition  $u = 0$ . Thus the last equation becomes

$$\nabla^2\Psi = 4\pi G\dot{\phi}_0 \delta\dot{\phi}. \quad (26)$$

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<sup>12</sup>This, and some of the following results, follow strictly only for appropriate boundary conditions. In later parts of the paper we will be working with Fourier decomposition of the various equations and the vanishing of the corresponding equations for each of the Fourier mode (except the  $k = 0$  mode) do follow independently of any considerations involving boundary conditions.

We will work during the rest of the paper with the slightly more general case that corresponds to maintaining the  $\mu$  term to account for possible slight departures of the exponential expansion in physical time that is reflected in expression (13), but will drop the  $u$  term indicating that we are keeping the slow roll regime approximation as important. The reason for this will become clear when we will carry out the comparison with the observations.

Now, let us take a look at the evolution equation of the scalar field fluctuations; if we use (21) in the equation for the scalar field perturbation we obtain

$$\ddot{\delta\phi} + 2\frac{\dot{a}}{a}\dot{\delta\phi} - \nabla^2\delta\phi + a^2\partial_{\phi,\phi}^2V[\phi]\delta\phi - 16\pi G(\dot{\phi}_0)^2\delta\phi - 2\Psi\ddot{\phi}_0 = 0 \quad (27)$$

while using the slow rolling approximation we get

$$\ddot{\delta\phi} + 2\frac{\dot{a}}{a}\dot{\delta\phi} - \nabla^2\delta\phi + a^2\partial_{\phi,\phi}^2V[\phi]\delta\phi - 16\pi G(\dot{\phi}_0)^2\delta\phi - 2\Psi\frac{\dot{a}}{a}\dot{\phi}_0 = 0. \quad (28)$$

This differs from the evolution equation of the scalar field perturbations in the background space-time. However note that the corrections due to the Newtonian potential  $\Psi$  are suppressed by the factor  $G$ . Thus in our present treatment we will ignore the complications of maintaining the terms that could be considered as reflecting the effect on the field of the metric response to the fluctuations of the field itself. In fact, from our point of view the metric would be unchanged until the state of the scalar field collapses, and only after that would the metric be changed and could therefore have a back reaction on the evolution of the field modes.

Our main equations will be then the equation for the scalar field in the background space-time

$$\ddot{\delta\phi} + 2\frac{\dot{a}}{a}\dot{\delta\phi} - \nabla^2\delta\phi + a^2\partial_{\phi,\phi}^2V[\phi]\delta\phi = 0 \quad (29)$$

and equation (25), which, upon quantization of the scalar field perturbation  $\delta\phi$  will be promoted to a semi-classical equation to determine  $\Psi$  in terms of  $\langle\hat{\delta\phi}\rangle$ . We will come back to this later.

## 7 Quantum theory of fluctuations

Considering the previous items let us present the standard treatment of this topic including a brief review of the usual discussion of the amplification of

fluctuations by inflation. We follow in some of the discussion mainly [12] and [15].<sup>13</sup>

The starting point is the assumption is that the dynamics of the universe, during the inflationary period, is dominated by the so called inflaton field. The inflaton field is a scalar field described by the action

$$S[\phi] = \int \left[ -\frac{1}{2} \nabla_a \phi \nabla_b \phi g^{ab} - V[\phi] \right] \sqrt{-g} d^4x. \quad (30)$$

Using the form of the FRW line element the previous action becomes

$$S[\phi] = \int \left[ \frac{a^2}{2} (-\phi \partial_0 \partial_0 \phi + \phi \partial_i \partial_j \phi \delta^{ij}) - a^4 V[\phi] \right] d^4x. \quad (31)$$

In order for inflation to take place some hypothesis about the initial condition for the scalar field and its potential have to be stated. One writes the field as  $\phi = \phi_0 + \delta\phi$ , where the background field  $\phi_0$  is described in a completely classical fashion while only the fluctuation  $\delta\phi$  is quantized.

In these coordinates, the field equation becomes

$$\delta \ddot{\phi} - \nabla^2 \delta\phi + 2 \frac{\dot{a}}{a} \delta\dot{\phi} = 0 \quad (32)$$

where dots denote derivatives with respect to  $\eta$  and  $\Delta$  is the Laplacian on Euclidean three space (whose metric is  $dr^2 + r^2 d\Omega^2$ ). Note that we have neglected a terms proportional to  $\partial_{\phi}^2 V[\phi]$  using the slow rolling approximation.

If we expand the fluctuation in its Fourier components the equations of motion for the mode  $\delta\phi_k$  becomes

$$\delta \ddot{\phi}_k + 2 \frac{\dot{a}}{a} \delta\dot{\phi}_k - k^2 \delta\phi_k = 0. \quad (33)$$

It is well known that this equation can be further simplified by going over from  $\delta\phi$  to an auxiliary field  $y = a\delta\phi$ . In the resulting equation for  $y$ , there is no term with a first derivative of the field anymore,

$$\ddot{y} - \left( \nabla^2 + \frac{\ddot{a}}{a} \right) y = 0, \quad (34)$$

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<sup>13</sup>Regarding units we use a convention where the coordinates will have dimensions of length ( $c = 1$ ),  $a(\eta)$  is dimensionless so  $H_I$  has dimensions of inverse length. The scalar field has units of  $(\text{Mass}/\text{Length})^{1/2}$  and Newton's constant  $G$  has dimensions of  $(\text{Length}/\text{Mass})$ .

and in the inflationary regime with  $a(\eta) = -(H_1\eta)^{-1}$  it can be explicitly solved. Obviously, a quantization  $\hat{y}$  of  $y$ , will immediately give a quantization  $\widehat{\delta\phi} = a^{-1}\hat{y}$  of  $\delta\phi$ , so we will now proceed to quantize the auxiliary field  $y$ .

In order to avoid infrared problem we introduce a regularization and consider the field in a box of side  $L$  decompose a real classical field  $y$  satisfying (34) into plane waves

$$y(\eta, \vec{x}) = \frac{1}{L^3} \sum_{\vec{k}} \left( a_k(\eta) e^{i\vec{k} \cdot \vec{x}} + \bar{a}_k(\eta) e^{-i\vec{k} \cdot \vec{x}} \right), \quad (35)$$

where the sum is over the wave vectors  $\vec{k}$  satisfying  $k_i L = 2\pi n_i$  for  $i = 1, 2, 3$  with  $n_i$  integers. The coefficients  $a_k(\eta)$  satisfy the equation

$$\ddot{a}_k + \left( k^2 - \frac{\dot{a}}{a} \right) a_k = 0. \quad (36)$$

We quantize  $y$  field imposing standard commutation relations between the field and its canonical conjugate momentum  $\hat{\pi}^{(y)} = \dot{y} - y\dot{a}/a$  (which is equivalent to imposing these relations on  $\phi$  and its conjugate momentum  $\pi = a^2\dot{\phi}$ ). Thus we write

$$\hat{y}(\eta, \vec{x}) = \frac{1}{L^3} \sum_{\vec{k}} \left( \hat{a}_k(\eta) e^{i\vec{k} \cdot \vec{x}} + \hat{a}_k^\dagger(\eta) e^{-i\vec{k} \cdot \vec{x}} \right), \quad \text{where } \hat{a}_k(\eta) = y_k(\eta) \hat{a}_k, \quad (37)$$

$y_k(\eta)$  is a solution of (36) and  $\hat{a}_k$  is the usual annihilation operator on the one particle Hilbert space  $\mathcal{H} = \mathcal{L}^2(L^3, d^3x)$ . Upon choosing the solutions  $y_k(\eta)$ ,  $\hat{y}$  thus becomes an operator on the Fock space over  $\mathcal{H}$ . Similarly the canonical conjugate to  $y$  is given by

$$\hat{\pi}^{(y)}(\eta, \vec{x}) = \frac{d}{d\eta} \hat{y}(\eta, \vec{x}) - \frac{\dot{a}}{a} \hat{y}(\eta, \vec{x}) \quad (38)$$

can be written as;

$$\hat{\pi}^{(y)}(\eta, \vec{x}) = \frac{1}{L^3} \sum_{\vec{k}} \left( \hat{a}_k g_k(\eta) e^{i\vec{k} \cdot \vec{x}} + \hat{a}_k^\dagger g_k(\eta) e^{-i\vec{k} \cdot \vec{x}} \right), \quad (39)$$

where  $g_k = \dot{y}_k - \frac{\dot{a}}{a} y_k$ . To complete the quantization, we have to specify the classical solutions  $y_k(\eta)$ . This choice is not completely free: To insure that canonical commutation relations between  $\hat{y}$  and  $\hat{\pi}^{(y)}$  give  $[\hat{a}_k, \hat{a}_{k'}^\dagger] = \hbar L^3 \delta_{k,k'}$ , they must satisfy

$$y_k(\eta) \overline{g_k}(\eta) - \overline{g_k}(\eta) g_k(\eta) = -i \quad (40)$$

for all  $k$  at some (and hence any) time  $\eta$ . The choice of the  $y_k(\eta)$  corresponds to the choice of a vacuum state for the field, which in the present case, as on any non stationary space-time, is not unique. However, we must emphasize that, at this point any such selection of a vacuum (made through the choice of the  $y_k(\eta)$ 's that we take as positive energy modes), would be a spatially homogeneous and isotropic state of the field, as can be seen by evaluating directly the action of a translation or rotation operators (associated with the hypersurfaces  $\eta = \text{constant}$  of the background space-time) on the state. We will use a rather natural candidate for such a state, the so called Bunch-Davies vacuum. It is characterized by the choice

$$y_k^{(\pm)}(\eta) = \frac{1}{\sqrt{2k}} \left( 1 \pm \frac{i}{\eta k} \right) \exp(\pm ik\eta), \quad (41)$$

and

$$g_k^{\pm}(\eta) = \pm i \sqrt{\frac{k}{2}} \exp(\pm ik\eta) \quad (42)$$

Note that the form of  $y_k^+$  reduces near  $\eta = -\infty$  to that of the standard positive frequency solution in flat space. This constitutes our choice of the vacuum of the theory.<sup>14</sup>

It is convenient to rewrite the field and momentum operators in as

$$\hat{y}(\eta, \vec{x}) = \frac{1}{L^3} \sum_{\vec{k}} e^{i\vec{k}\cdot\vec{x}} \hat{y}_k(\eta), \quad \hat{\pi}^{(y)}(\eta, \vec{x}) = \frac{1}{L^3} \sum_{\vec{k}} e^{i\vec{k}\cdot\vec{x}} \hat{\pi}_k(\eta) \quad (43)$$

where  $\hat{y}_k(\eta) \equiv y_k(\eta)\hat{a}_k + \bar{y}_k(\eta)\hat{a}_{-k}^\dagger$  and  $\hat{\pi}_k(\eta) \equiv g_k(\eta)\hat{a}_k + \bar{g}_k(\eta)\hat{a}_{-k}^\dagger$ .

Furthermore we will decompose both  $\hat{y}_k(\eta)$  and  $\hat{\pi}_k(\eta)$  into their real imaginary parts  $\hat{y}_k(\eta) = \hat{y}_k^R(\eta) + i\hat{y}_k^I(\eta)$  and  $\hat{\pi}_k(\eta) = \hat{\pi}_k^R(\eta) + i\hat{\pi}_k^I(\eta)$  where

$$\hat{y}_k^R(\eta) = \frac{1}{\sqrt{2}} \left( y_k(\eta)\hat{a}_k^R + \bar{y}_k(\eta)\hat{a}_k^{R\dagger} \right), \quad \hat{y}_k^I(\eta) = \frac{1}{\sqrt{2}} \left( y_k(\eta)\hat{a}_k^I + \bar{y}_k(\eta)\hat{a}_k^{I\dagger} \right) \quad (44)$$

$$\hat{\pi}_k^R(\eta) = \frac{1}{\sqrt{2}} \left( g_k(\eta)\hat{a}_k^R + \bar{g}_k(\eta)\hat{a}_k^{R\dagger} \right), \quad \hat{\pi}_k^I(\eta) = \frac{1}{\sqrt{2}} \left( g_k(\eta)\hat{a}_k^I + \bar{g}_k(\eta)\hat{a}_k^{I\dagger} \right) \quad (45)$$

where

$$\hat{a}_k^R \equiv \frac{1}{\sqrt{2}}(\hat{a}_k + \hat{a}_{-k}), \quad \hat{a}_k^I \equiv \frac{-i}{\sqrt{2}}(\hat{a}_k - \hat{a}_{-k}) \quad (46)$$

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<sup>14</sup>Note that the dimensions implied for  $\hat{a}_k$  is  $(\text{Mass})^{1/2}(\text{Length})^2$  which is compatible with the dimensionalized commutator  $[\hat{a}_k, \hat{a}_{k'}^\dagger] = \hbar L^3 \delta_{k,k'}$ .

We note that the operators  $\hat{y}_k^{R,I}(\eta)$  and  $\hat{\pi}_k^{R,I}(\eta)$  are therefore hermitian operators. The commutation relations of the real and imaginary creation and annihilation operators are however nonstandard:

$$[\hat{a}_k^R, \hat{a}_{k'}^{R\dagger}] = \hbar L^3 (\delta_{k,k'} + \delta_{k,-k'}), \quad [\hat{a}_k^I, \hat{a}_{k'}^{I\dagger}] = \hbar L^3 (\delta_{k,k'} - \delta_{k,-k'}) \quad (47)$$

with all other commutators vanishing. This is known to indicate that the operators corresponding to  $k$  and  $-k$  are identical in the real case (and identical up to a sign in the imaginary case).

## 8 Evolution of the fluctuations through collapse

In this section, we will specify our model of collapse, and do the necessary computations to follow the field evolution through collapse to the end of inflation.

### The collapsing modes

To describe the collapse of the state the scalar field is in, we will use the decomposition of the field into modes. It is imperative that these modes are ‘independent’, i.e. that they give a corresponding decomposition of the field operator into a sum of commuting ‘mode operators’, an orthogonal decomposition of the one-particle Hilbert-space, and a direct-product decomposition for the Fock space. Furthermore, we require that the initial state of the field is not an entangled state with respect to this decomposition, i.e. it can be written as a direct product of states for the mode operators. This ensures that the notion of “collapse of an individual mode” will make sense.

Although the above requirements place some restrictions, there are different possible choices for this decomposition and the corresponding choice of modes, with possibly observable consequences. Until a less phenomenological description of the collapse becomes available, we can only be guided by simplicity, and the condition that the end result of our calculation be compatible with astrophysical observations. Throughout the main text, we will use the modes labeled by the wave vector  $k$  and the superscript R/I in the last section. We will however show that there is a certain degree of robustness of predictions under change of the mode decomposition, by repeating the calculations that will follow below – with a different choice of decomposition – in appendix A.

Let us be more precise: We will assume that the collapse is somehow analogous to an imprecise measurement of the operators  $\hat{y}_k^{R,I}(\eta)$  and  $\hat{\pi}_k^{R,I}(\eta)$  which, as we pointed out, are hermitian operators and thus reasonable observables. These field operators contain complete information about the field. As we have to follow the evolution of these modes during inflation, let us collect some formulas for the evolution of their lowest moments: Let  $|\Xi\rangle$  be any state in the Fock space of  $\hat{y}$ . Let us introduce the following quantities:

$$d_k^{R,I} = \langle \hat{a}_k^{R,I} \rangle_{\Xi}, \quad c_k^{R,I} = \langle (\hat{a}_k^{R,I})^2 \rangle_{\Xi}, \quad e_k^{R,I} = \langle \hat{a}_k^{R,I\dagger} \hat{a}_k^{R,I} \rangle_{\Xi}. \quad (48)$$

In terms of these, the expectation values of the modes are expressible as

$$\langle \hat{y}_k^{R,I} \rangle_{\Xi} = \sqrt{2} \operatorname{Re}(y_k d_k^{R,I}) \quad , \quad \langle \hat{\pi}_k^{(y)R,I} \rangle_{\Xi} = \sqrt{2} \operatorname{Re}(g_k d_k^{R,I}) \quad (49)$$

while their corresponding dispersions are

$$(\Delta \hat{y}_k^{R,I})_{\Xi}^2 = \operatorname{Re}(y_k^2 c_k^{R,I}) + (1/2) |y_k|^2 (\hbar L^3 + 2e_k^{R,I}) - 2 \operatorname{Re}(y_k d_k^{R,I})^2 \quad (50)$$

and

$$(\Delta \hat{\pi}_k^{R,I})_{\Xi}^2 = \operatorname{Re}(g_k^2 c_k^{R,I}) + (1/2) |g_k|^2 (\hbar L^3 + 2e_k^{R,I}) - 2 \operatorname{Re}(g_k d_k^{R,I})^2 \quad (51)$$

For the vacuum state  $|0\rangle$  we certainly have  $d_k^{R,I} = c_k^{R,I} = e_k^{R,I} = 0$ , and thus

$$\langle \hat{y}_k^{R,I} \rangle_0 = 0, \quad \langle \hat{\pi}_k^{(y)R,I} \rangle_0 = 0, \quad (52)$$

while their corresponding dispersions are

$$(\Delta \hat{y}_k^{R,I})_0^2 = (1/2) |y_k|^2 (\hbar L^3), \quad (\Delta \hat{\pi}_k^{R,I})_0^2 = (1/2) |g_k|^2 (\hbar L^3). \quad (53)$$

### The collapse

Now we will specify the rules according to which collapse happens. Again, at this point our criteria will be simplicity and naturalness. Other possibilities do exist, and may lead to different predictions. To illustrate this point, in appendix C we will explore an alternative model.

What we have to describe is the state  $|\Theta\rangle$  after the collapse. To keep things simple and general, we will not consider specifying the state completely, but only the expectation values

$$d_{k,c}^{R,I} = \langle \hat{a}_k^{R,I} \rangle_{\Theta}, \quad c_{k,c}^{R,I} = \langle (\hat{a}_k^{R,I})^2 \rangle_{\Theta}, \quad e_{k,c}^{R,I} = \langle \hat{a}_k^{R,I\dagger} \hat{a}_k^{R,I} \rangle_{\Theta}. \quad (54)$$

where the subscript  $c$  indicates that we are talking about the post collapse values, to distinguish them from their pre collapse values that as we said are zero. We will drop that subscript in the following.

At this point a few remarks on our statistical treatment are in order. We view the collapsed state of the field corresponding to our universe to be a single state  $|\Theta\rangle$  and not in any way an ensemble of states. This would seem to result in a difficulty in principle for the attempts to apply statistical analysis in our situation, and it reflects a previously mentioned difficulty of principle that arises when dealing with the fact that we need a quantum treatment but we have just one universe at our disposal. However it is an issue we must face if we want to have a clear and realistic understanding of the issues at hand. The way we address this issue is related to the fortunate situation that we do not measure directly and separately the modes with specific values of  $\vec{k}$ , but rather an aggregate contribution of all such modes to the spherical harmonic decomposition of the temperature fluctuations on the celestial sphere. In order to proceed we construct an imaginary ensemble of universes. Thus we have an ensemble of universes characterized by the after collapse state  $|\Theta\rangle_i$  where the label  $i$  identifies the specific element in the ensemble. Then we will have an independent random series of numbers  $q_{\vec{k}}^{(i)}$  pertaining to value of physical quantities in the collapsed state in each element  $i$  in the ensemble for every single  $\vec{k}$  (we will be assuming there are no correlations among the various harmonic oscillators). Our universe however, corresponds to a single element  $i_0$  in the ensemble, leading to the choice for each  $\vec{k}$  of a number  $q_{\vec{k}}^{(i_0)}$  from among those random sequences. The point is that the complete sequence that corresponds to our universe – i.e. the sequence  $q_{\vec{k}}^{(i_0)}$  of specific quantities for fixed  $i_0$  but for the full set of  $\vec{k}$  – will, as a result, also be a random sequence.

In our specific calculation this approach will be taken with respect to the quantities  $c_k^c, d_k^c, e_k^c$  after considering the issues of relative overall normalization of the random sequences. In fact in this first treatment we will not concern ourselves with the quantities  $c_k^c, e_k^c$  which are related to the spread of the wave packet after collapse, as this will be the subject of future research.

Thus we focus on specifying  $d_k^c$ : In the vacuum state,  $\hat{y}_k$  and  $\hat{\pi}_k^{(y)}$  individually are distributed according to Gaussian distributions centered at 0 with spread  $(\Delta \hat{y}_k)_0^2$  and  $(\Delta \hat{\pi}_k^{(y)})_0^2$  respectively. However, since they are mutually non-commuting, their distributions are certainly not independent. In our collapse model, we do not want to distinguish one over the other, so we will

ignore the non-commutativity and make the following assumption about the (distribution of) state(s)  $|\Theta\rangle$  after collapse:

$$\langle \hat{y}_k^{R,I}(\eta_k^c) \rangle_\Theta = X_{k,1}^{R,I}, \quad \langle \hat{\pi}_k^{(y)R,I}(\eta_k^c) \rangle_\Theta = X_{k,2}^{R,I} \quad (55)$$

where  $X_{k,1}^{R,I}, X_{k,2}^{R,I}$  are random variables, distributed according to a Gaussian distribution centered at zero with spread  $(\Delta \hat{y}_k^{R,I})_0^2, (\Delta \hat{\pi}_k^{(y)R,I})_0^2$ , respectively. Another way to express this is

$$\langle \hat{y}_k^{R,I}(\eta_k^c) \rangle_\Theta = x_{k,1}^{R,I} \sqrt{(\Delta \hat{y}_k^{R,I})_0^2} = x_{k,1}^{R,I} |y_k(\eta_k^c)| \sqrt{\hbar L^3/2}, \quad (56)$$

$$\langle \hat{\pi}_k^{(y)R,I}(\eta_k^c) \rangle_\Theta = x_{k,2}^{R,I} \sqrt{(\Delta \hat{\pi}_k^{(y)R,I})_0^2} = x_{k,2}^{R,I} |g_k(\eta_k^c)| \sqrt{\hbar L^3/2}, \quad (57)$$

where  $x_{k,1}, x_{k,2}$  are now distributed according to a Gaussian distribution centered at zero with spread one.

We now take these equations and solve for  $d_k^{R,I}$ . Defining the angles  $\alpha, \beta, \gamma$  as  $\alpha_k^{R,I} = \arg(d_k^{R,I}), \beta_k = \arg(y_k), \gamma_k = \arg(g_k)$ , where the last two refer to quantities evaluated at the collapse time  $\eta_k^c$ , the above equations can be written

$$|d_k^{R,I}| \cos(\alpha_k^{R,I} + \beta_k^c) = \frac{1}{2} x_{k,1}^{R,I} \sqrt{\hbar L^3}, \quad |d_k^{R,I}| \cos(\alpha_k^{R,I} + \gamma_k^c) = \frac{1}{2} x_{k,2}^{R,I} \sqrt{\hbar L^3}. \quad (58)$$

The general solution gives:

$$|d_k^{R,I}| = \sqrt{\hbar L^3} D_k^{R,I} \quad (59)$$

with

$$D_k^{R,I} = (1/2) \frac{(1+z_k^2)^{1/2}}{z_k} (x_{k,1}^{R,I 2} + x_{k,2}^{R,I 2} - 2x_{k,1}^{R,I} x_{k,2}^{R,I} (1+z_k^2)^{-1/2})^{1/2} \quad (60)$$

where  $z_k \equiv k\eta_k^c$ , and

$$\cos(\alpha_k^{R,I} + z_k - \pi/2) = x_{k,2}^{R,I} / (2D_k^{R,I}) \quad (61)$$

In order to more fully specify the state  $|\Theta\rangle$  we would need also to consider the quantities

$$\delta(\hat{y}_k^{R,I})_k = (\Delta \hat{y}_k^{R,I})_\Theta^2(\eta_k^c), \quad \delta(\hat{\pi}_k^{(y)R,I})_k = (\Delta \hat{\pi}_k^{(y)R,I})_\Theta^2(\eta_k^c) \quad (62)$$

To specify these, and their time evolution, we would have to specify the remaining six parameters  $\text{Re}(c_k^{R,I})$ ,  $\text{Im}(c_k^{R,I})$ , and  $e_k^{R,I}$ . However, the limited set of results that are the concern of this paper are independent of such choice, and we will not further discuss the higher moments of  $|\Theta\rangle$  in what follows.

We need to concentrate on the expectation value of the quantum operator which appears in our basic formula

$$\nabla^2\Psi - \mu\Psi = s\Gamma \quad (63)$$

(where we introduced the abbreviation  $s = 4\pi G\dot{\phi}_0$ ) and the quantity  $\Gamma$  as the aspect of the field that acts as a source of the Newtonian potential. In a general situation  $\Gamma = \delta\dot{\phi} + (\frac{\dot{a}}{a} - \frac{\ddot{\phi}_0}{\dot{\phi}_0})\delta\phi$ , while in the slow roll approximation we have  $\Gamma = \delta\dot{\phi} = a^{-1}\pi^y$ . We want to say that, upon quantization, the above equation turns into

$$\nabla^2\Psi - \mu\Psi = s\langle\hat{\Gamma}\rangle. \quad (64)$$

Before the collapse occurs, the expectation value on the right hand side is zero. Let us now determine what happens after the collapse: To this end, take the Fourier transform of (64) and rewrite it as

$$\Psi_k(\eta) = \frac{s}{k^2 + \mu}\langle\hat{\Gamma}_k\rangle_\Theta. \quad (65)$$

Let us focus now on the slow roll approximation and compute the right hand side, we note that  $\delta\dot{\phi} = a^{-1}\hat{\pi}^{(y)}$  and hence

$$\delta\dot{\phi}_k = \frac{1}{a\sqrt{2}}[g_k(\eta)(\hat{a}_k^R + i\hat{a}_k^I) + \bar{g}_k(\eta)(\hat{a}_k^{R\dagger} + i\hat{a}_k^{I\dagger})] \quad (66)$$

For the expectation value we find

$$\begin{aligned} \langle\Gamma_k\rangle_\Theta &= \frac{\sqrt{\hbar L^3}}{a\sqrt{2}}[g_k(\eta)(D_k^R e^{i\alpha_k^R} + iD_k^R e^{i\alpha_k^I}) \\ &\quad + \bar{g}_k(\eta)(D_k^R e^{-i\alpha_k^R} + iD_k^R e^{-i\alpha_k^I})] \end{aligned} \quad (67)$$

$$\begin{aligned} &= \sqrt{\hbar L^3 k} \frac{1}{2a} \times \\ &\quad \left( D_k^R \cos(\alpha_k^R + k\eta - \pi/2) + iD_k^I \cos(\alpha_k^I + k\eta - \pi/2) \right) \end{aligned} \quad (68)$$

$$=: \sqrt{\hbar L^3 k} \frac{1}{2a} F(k). \quad (69)$$

We note that we can write

$$\cos(\alpha_k + k\eta - \pi/2) = \cos(\alpha_k + \gamma_k^c + \Delta_k) \quad (70)$$

where  $\Delta_k = k(\eta - \eta_k^c) = k\eta - z_k$ . Then, using the expressions (58), and after a longer calculation, we find

$$F(k) = (1/2)[A_k(x_{k,1}^R + ix_{k,1}^I) + B_k(x_{k,2}^R + ix_{k,2}^I)] \quad (71)$$

where

$$A_k = \frac{\sqrt{1+z_k^2}}{z_k} \sin(\Delta_k); \quad B_k = \cos(\Delta_k) + (1/z_k) \sin(\Delta_k). \quad (72)$$

## 9 Recovering the observational quantities.

Now we must compare with the experimental results. We will only do this to a certain approximation since we will disregard the changes to dynamics that happen after reheating due to the transition to standard (radiation dominated) evolution.

A crucial observation for what follows is to recognize the fact that we can not measure  $\Psi_k$  for each individual value of  $k$ . What we measure in fact is the “Newtonian potential” on the surface of last scattering:  $\Psi(\eta_D, \vec{x}_D)$  which is a function of the coordinates on the celestial two-sphere, i.e a function of two angles. From this we extract

$$\alpha_{lm} = \int \Psi(\eta_D, \vec{x}_D) Y_{lm}^* d^2\Omega \quad (73)$$

In fact the quantity that is measured is  $\frac{\Delta T}{T}(\theta, \varphi)$  which is expressed as  $\sum_{lm} \alpha_{lm} Y_{l,m}(\theta, \varphi)$ . The angular variations of the temperature is then identified with the corresponding variations in the “Newtonian Potential”  $\Psi$ , by the understanding that they are the result of gravitational red-shift in the CMB photon frequency  $\nu$  so  $\frac{\delta T}{T} = \frac{\delta \nu}{\nu} = \frac{\delta(\sqrt{g_{00}})}{\sqrt{g_{00}}} \approx \delta \Psi$ . Thus we identify the theoretical expectation  $\alpha_{lm}$  with the observed quantity  $\alpha_{lm}^{obs}$ . The quantity that is presented as the result of observations is  $OB_l = l(l+1)C_l$  where  $C_l = (2l+1)^{-1} \sum_m |\alpha_{lm}^{obs}|^2$ . The observations indicate that (ignoring the acoustic oscillations, which is anyway an aspect that is not being considered

in this work) the quantity  $OB_l$  is essentially independent of  $l$ , and this is interpreted as a reflection of the “scale invariance” of the primordial spectrum of fluctuations.

To evaluate the quantity of interest we use (65) and (69) to write

$$\Psi(\eta, \vec{x}) = \sum_{\vec{k}} \frac{sU(k)}{k^2 + \mu} \sqrt{\frac{\hbar k}{L^3}} \frac{1}{2a} F(\vec{k}) e^{i\vec{k}\cdot\vec{x}}, \quad (74)$$

where we have added the factor  $U(k)$  to represent the aspects of the evolution of the quantity of interest associated with the physics of period from reheating to decoupling, which includes among others the acoustic oscillations of the plasma. It is in this expression that the justification for the use of statistics becomes clear. The quantity we are in fact considering is the result of an ensemble of harmonic oscillators each one contributing with a complex number to the sum, leading to what is in effect a 2 dimensional random walk whose total displacement corresponds to the observational quantity. To proceed further we must evaluate the most likely value for such total displacement. This we do with the help of the imaginary ensemble of universes, and the identification of the most likely value with the ensemble mean value. These two quantities are reasonably close in normal circumstances, where the probability distribution has a single local maximum (the global maximum), and is not pathological in some other respect. Let us see how does this work in detail:

Using  $\vec{x} = R_D(\sin(\theta)\sin(\varphi), \sin(\theta)\cos(\varphi), \cos(\theta))$  and standard results connecting Fourier and spherical expansions we obtain

$$\alpha_{lm} = s\sqrt{\frac{\hbar}{L^3}} \frac{1}{2a} \sum_{\vec{k}} \frac{U(k)\sqrt{k}}{k^2 + \mu} \int F(\vec{k}) e^{i\vec{k}\cdot\vec{x}} Y_{lm}(\theta, \varphi) d^2\Omega \quad (75)$$

$$= s\sqrt{\frac{\hbar}{L^3}} \frac{1}{2a} \sum_{\vec{k}} \frac{U(k)\sqrt{k}}{k^2 + \mu} F(\vec{k}) 4\pi i^l j_l((|\vec{k}|R_D)) Y_{lm}(\hat{k}) \quad (76)$$

where  $\hat{k}$  indicates the direction of the vector  $\vec{k}$ . Now we compute the expected magnitude of this quantity. As a first step we take the square of the quantity of interest:

$$|\alpha_{lm}|^2 = s^2 \frac{4\pi^2 \hbar}{L^3} \frac{1}{a^2} \sum_{\vec{k}, \vec{k}'} \frac{U(k)\sqrt{k}}{k^2 + \mu} \frac{U(k')\sqrt{k'}}{k'^2 + \mu} F(\vec{k}) \overline{F(\vec{k}')} j_l(kR_D) j_l(k'R_D) Y_{lm}(\hat{k}) Y_{lm}(\hat{k}'). \quad (77)$$

Now we take its ensemble mean value. The calculation of such mean value will be simplified due to the fact that, as usual, the average over the random variables will lead to an cancellation of the cross terms. In our case we find

$$\begin{aligned}\langle F(\vec{k})\overline{F(\vec{k}')} \rangle &= A_k A_{k'} (\langle x_{k,1}^R x_{k',1}^R \rangle + \langle x_{k,1}^I x_{k',1}^I \rangle) \\ &\quad + B_k B_{k'} (\langle x_{k,2}^R x_{k',2}^R \rangle + \langle x_{k,2}^I x_{k',2}^I \rangle)\end{aligned}\quad (78)$$

where we have made use of the independence among the four sets of random variables  $x_{k,1}^R, x_{k,1}^I, x_{k,2}^R, x_{k,2}^I$ . However we need to recall that within each set the variables corresponding to  $\vec{k}$  and  $-\vec{k}$  are not independent. This will be reflected by setting  $\langle x_k^R, x_k^R \rangle = \delta_{k,k'} + \delta_{k,-k'}$  and  $\langle x_k^I, x_k^I \rangle = \delta_{k,k'} - \delta_{k,-k'}$  in accordance with the discussion below equation (47). Thus

$$\langle F(\vec{k})\overline{F(\vec{k}')} \rangle = (A_k^2 + B_k^2) \delta_{k,k'} \quad (79)$$

Thus we arrive to the expression for the ensemble mean value which as we said we will consider as a good approximation of the most likely (M.L.) value:

$$|\alpha_{lm}|_{M.L.}^2 = s^2 \frac{4\pi^2 \hbar}{L^3} \frac{1}{a^2} \sum_{\vec{k}} (A_k^2 + B_k^2) \frac{U(k)^2 k}{(k^2 + \mu)^2} j_l^2((|\vec{k}| R_D) |Y_{lm}(\hat{k})|^2) \quad (80)$$

Now we write the sum as an integral by noting that the allowed values of the components of  $\vec{k}$  are separated by  $\Delta k_i = 2\pi/L$ , thus

$$|\alpha_{lm}|_{M.L.}^2 = \frac{s^2 4\pi^2 \hbar}{a^2 L^3} (L/2\pi)^3 \sum_{\vec{k}} \frac{(A_k^2 + B_k^2) k U(k)^2}{(k^2 + \mu)^2} j_l^2((|\vec{k}| R_D) |Y_{lm}(\hat{k})|^2 (\Delta k_i)^3) \quad (81)$$

$$= \frac{s^2 \hbar}{2\pi a^2} \int \frac{U(k)^2 C(k) k}{(k^2 + \mu)^2} j_l^2((|\vec{k}| R_D) |Y_{lm}(\hat{k})|^2) d^3 k \quad (82)$$

where  $C(k) = A_k^2 + B_k^2$  is in fact a function of  $k, z_k$ , and  $\eta$ .

$$= \frac{s^2 \hbar}{2\pi a^2} \int \frac{U(k)^2 C(k)}{(k^2 + \mu)^2} j_l^2((|\vec{k}| R_D) k^3) dk \quad (83)$$

where in the last equation we have made use of the normalization of the spherical harmonics. The last expression can be made more useful by changing the variables of integration to  $x = k R_D$  leading to

$$|\alpha_{lm}|_{M.L.}^2 = \frac{s^2 \hbar}{2\pi a^2} \int \frac{U(x/R_D)^2 C(x/R_D)}{(x^2 + \mu R_D^2)^2} j_l^2(x) x^3 dx \quad (84)$$

In the exponential expansion regime where  $\mu$  vanishes, and in the limit  $z_k \rightarrow -\infty$  where  $C = 1$ , and taking for simplicity  $U(k) = U_0$  to be independent of  $k$ , (neglecting for instance the physics that gives rise to the acoustic peaks), we find:

$$|\alpha_{lm}|_{M.L.}^2 = \frac{s^2 U_0^2 \hbar}{2\pi a^2} I_1(l) \quad (85)$$

where  $I_n(l) = \int x^{-n} j_l^2(x) dx$ . For  $n = 1$  we have  $I_1(l) = \frac{\pi}{l(l+1)}$  so

$$|\alpha_{lm}|_{M.L.}^2 = \frac{s^2 U_0^2 \hbar}{2a^2} \frac{1}{l(l+1)}. \quad (86)$$

Now, since this does not depend on  $m$  it is clear that the expectation of  $C_l = (2l+1)^{-1} \sum_m |\alpha_{lm}|^2$  is just  $|\alpha_{lm}|^2$  and thus the observational quantity  $OB_l = l(l+1)C_l = \frac{s^2 U_0^2 \hbar}{2a^2}$  independent of  $l$  and in agreement with the scale invariant spectrum obtained in ordinary treatments and in the observational studies. Now let us look at the predicted value for the observational quantity  $OB_l$ . Using the expression  $s = 4\pi G \dot{\phi}_0$ , the equation of motion for the scalar field in the background in the slow roll approximation ( $\dot{\phi} = -\frac{a^3}{3\dot{a}} V'$  where  $V' = \frac{\partial V}{\partial \phi}$ ), and the first of Einstein's equations, in the background which gives  $3(\dot{a})^2 = 8\pi G a^4 V(\phi_0)$ , we find,

$$OB_l = (\pi/6) G \hbar \frac{(V')^2}{V} U_0^2 = (\pi/3) \epsilon (V/M_{Pl}^4) U_0^2 \quad (87)$$

where in the last equality we have used the standard definition of the slow roll parameter  $\epsilon = (1/2) M_{Pl}^2 (V'/V)^2$ , and  $G \hbar = M_{Pl}^{-2}$ . Note that if one could avoid  $U$  from becoming too large during reheating, which is another aspect of the problem where the complications of the relevant physics might leave room for a departure from the standard results, the quantity of interest would be proportional to the small number  $\epsilon$ , a possibility that is not discussed in the standard treatments, so in this case we could get rid of the "fine tuning problem" for the inflationary potential (i.e. even if  $V \sim M_{Pl}^4$ , the temperature fluctuations in the CMB would be expected to be small).

Furthermore we note, as can be seen from inspection of (84), that if the evolution of the scale factor deviates slightly from the one given in (13) in such a way that  $\mu > 0$ , the effect would be to decrease the value of  $OB_l$  relative to the standard prediction of the scale invariant spectrum, for the

small values of  $l$ . Such effect<sup>15</sup> has been reported [6], as we mentioned in the introduction, and has been the subject of quite some interest lately. More details about these data can be found in [1]. Moreover we note that by a detailed analysis of this sort, one could in principle extract information about the deviations from the standard exponential expansion normally associated with inflation from the observational data.

Now let us focus on the effect of the finite value of time of collapse  $\eta_k^c$ , that is we consider the general functional form of  $C(k)$ :

$$C(k) = 1 + (2/z_k^2) \sin(\Delta_k)^2 + (1/z_k) \sin(2\Delta_k) \quad (88)$$

where we recall that  $\Delta_k = k\eta - z_k$  with  $z_k = \eta_k^c k$ . The first thing we note is that in order to get a reasonable spectrum there seems to be only one simple option: That  $z_k$  is essentially independent of  $k$  – that is the time of collapse of the different modes should depend on the mode's frequency according to  $\eta_k^c = z/k$ . Recall that the standard answer would correspond to  $C(k) = 1$ . This can be obtained here in the limit of very early collapse  $z_k \rightarrow -\infty$ , or if one takes the time of collapse equal to the moment of observation<sup>16</sup>. Such possibility will not be considered further at this point. In the case of a finite value for the time of collapse what we seek is for  $C(k)$  to be independent of  $k$ . In such case, the collapse would need to be associated with a mechanism that affects each mode at the appropriate time. This is a remarkable conclusion which provides relevant information about whatever the mechanism of collapse is. This is however not enough to ensure that the resulting spectrum coincides exactly with the one obtained in the observations. For that we need to ensure that the other relevant parameter  $\eta k$  does not introduce a significant  $k$  dependence. However in this case  $\eta$  which refers to the conformal observation time (the time at which the observed photons are emitted, corresponding in the realistic case to the time of decoupling), is of

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<sup>15</sup>It is however important to recall that, as we have indicated, all that can be extracted from the theory is an estimate of the “most likely” value of  $\alpha_{lm}$ , as it results from a sort of random walk of the individual collapses. What is more, the likelihood of a deviation has to do with the number of steps – i.e. individual collapsing modes – that effectively contribute to the random walk. This number in turn would be depend, as can be clearly seen in the discussion above (128) of Appendix A – on global features of the universe. It is clear that the likelihood of a deviation grows with the lowering of the values of  $l$ .

<sup>16</sup>One possibility that seems to emerge from the analysis this far is for such single event to occur at the time of decoupling so  $\Delta_k = 0$ . For a possible interpretation of this see next section.

course the same for all modes. Then unless the effect of the quantity  $\eta k$  becomes negligible we will not be able to account for an exactly scale invariant spectrum.

## 10 Further ideas

In this section we have collected some further ideas relating to the transition to classicality and inhomogeneity: In the following subsection we present a point of view that is to a large extent inspired in the spirit of the Many Universes point of view (see Sec. 4), together with the idea that missing ingredient would be provided by Quantum Gravity. In Subsection 10.2 we discuss how a mechanism that triggers collapse based on the gravitational interaction energy of quantum mechanical alternatives – as envisioned by Penrose – can be incorporated within the present formalism.

### 10.1 A special role of gravity in the Many Universes interpretation?

A possible view contemplates a quantum evolution all the way up to the time of observation, today. In this view the gravitational field is all the time related to the matter through the field equation (28) which is imposed as an operator equation (no need to any semiclassical approximation in the linearized regime).

Consider, for an analogy, a Stern-Gerlach experiment where we have an incoming electron propagating along the  $x$ -direction in a state  $|y, +\rangle$  (spin up in the  $y$ -direction) with the magnetic field of the apparatus oriented in the  $z$ -direction (corresponding to a specific quantum state of the apparatus). We also introduce an observer which is treated quantum mechanically. In this case, the state of the apparatus selects a preferred basis where we can describe the quantum evolution. The Schrödinger evolution of the electron-apparatus-observer quantum system is such that, after the electron goes through the apparatus, the state of the system evolves into

$$|\psi\rangle = (|z, +\rangle \otimes |x^+\rangle \otimes |obs.+\rangle) + (|z, -\rangle \otimes |x^-\rangle \otimes |obs.-\rangle),$$

where  $|x^\pm\rangle$  represent the two possible trajectories (up and down) of the electron passing through the Stern-Gerlach apparatus, and  $|obs.\pm\rangle$  the associated two states of the observer's mind. The choice of this basis allows the

interpretation of this final state where two alternative “worlds” are described by the state vector. In each one of these worlds the observer sees the electron going up or the electron going down respectively. This interpretation seems unambiguous due to the special role of the apparatus in selecting a preferred basis in the Hilbert space of the joint system.

If we want to interpret the result of the quantum evolution of the inflaton field during inflation we are confronted with what seems to be a similar situation. On the one hand, the observations of the CMB are directly related to the fluctuations of the geometry at the time of decoupling. Let us for the moment assume that the analog of  $x^\pm$  is the value of the Newtonian potential  $\psi$ . To linearized order quantum Einstein’s equations are given by the operator version of (28) which becomes:

$$\nabla^2 \hat{\psi} = \frac{4\pi G \dot{\phi}_0}{a} \hat{\pi}^{(y)},$$

It is clear that this equation connects the eigenbasis of the Newtonian potential with the eigenbasis of the momentum of the scalar field. At each instant of time  $\eta$  the wave function of the Newtonian potential  $\Psi(\psi_k)$  of each mode is related by the previous constraint to the (squeezed) state resulting from the evolution of the vacuum wave function of  $\Psi(\pi^{(y)})$  associated to the scalar field. The state of the universe is thus given by  $\Psi(\psi_k) \otimes \Psi(\pi^{(y)})$  which can be written as a linear superposition of eigenstates of the Newtonian potential and the scalar field momentum. Due to the linearity of quantum mechanics, the evolution of the full state (which is always homogeneous and isotropic) can be analyzed by evolving each component separately. Every one of these alternatives will unitarily evolve in such a way that inhomogeneities grow, form structure, produce galaxies, and finally observers, yet the full state remains homogeneous and isotropic. In analogy to the Stern-Gerlach experiment we can express the final quantum state of the universe as a superposition of states with its own observers. One then takes the view, that what quantum mechanics predicts about the universe is the probabilistic distribution of these alternatives. We can use the above formalism to make predictions about the inhomogeneities in our own observed world by using definition (76) in order to estimate the most likely value of  $|\alpha_{lm}|$  as  $\sqrt{<\hat{\alpha}_{lm}^2>}$  which also leads to (93) with  $C(k) = 1$ .

Of course there are certain issues with this viewpoint:

*What selects the basis?* Unlike the Stern-Gerlach situation there does not seem to be an obvious physical input that would lead to a preferred basis

where to make the decomposition. One could take the view that the basis is the one of the eigenstates of the gravitational field. After all when we observe the CMB we do measure  $\psi$ ; therefore, as in standard situations in QM we select the basis when we decide what to measure. For example if one measures the spin of a particle in the z-direction one should write the state in its eigenbasis to make any prediction. This fails to address the question of how we are there to make the measurement in the first place. Only in a very specific basis would matter evolve to form observers. It seems that here we are again confronted to some unknown mechanism selecting this basis. Gravity plays a central role in the mechanisms that allows structure to grow and produce the conditions that lead to the existence of observers. This suggests that the gravitational realm is perhaps the place where this new element should be found.

*What are the limits to which this position can be taken?* We should start by noting that in the position exposed above the full quantum state represents a set of coexistent possibilities. First of all the separation of alternatives cannot be made in a strict eigenbasis of any field as the conjugate quantity would produce large departures from what we would call the near classical evolution. The exact nature of the states that constitute such set of non interfering alternatives should be provided by the new element we are calling upon. Furthermore this element should not prevent normal quantum mechanical interference in standard situations. Some sort of threshold is needed. Presumably at the very early stages of the evolution such threshold would not have been reached, and thus the situation would have been described as a fully interfering quantum state, the one corresponding to the scalar field vacuum. This seems to take us back to a similar sort of scenario as the one investigated in this paper.

## 10.2 A ‘Penrose mechanism’ for collapse

Penrose has for a long time advocated that the collapse of quantum mechanical wave functions might be a dynamical process independent of observation, and that the underlying mechanism might be related to gravitational interaction. More precisely, according to this suggestion, collapse into one of two quantum mechanical alternatives would take place when the gravitational interaction energy between the alternatives exceeds a certain threshold. In fact, much of the initial motivation for the present work came from Penrose’s ideas and his questions regarding the quantum history of the universe.

A very naive realization of Penrose's ideas in the present setting could be obtained as follows: Each mode would collapse by the action of the gravitational interaction between its own possible realization. In our case one could estimate the interaction energy  $E_I(k, \eta)$  by considering two representatives of the possible collapsed states on opposite sides of the Gaussian associated with the vacuum. Let us interpret  $\Psi$  literally as the Newtonian potential and consequently the right hand side of equation (26) as the associated matter density. Then we would have

$$E_I(\eta) = \int \Psi^{(1)}(x, \eta) \rho^{(2)}(x, \eta) dV = a^3 \int \Psi^{(1)}(x, \eta) \rho^{(2)}(x, \eta) d^3x \quad (89)$$

which when applied to a single mode becomes:

$$E(\eta) = (a^3/L^6) \Psi_k^{(1)}(\eta) \rho_k^{(2)}(\eta) \int d^3x = (a^3/L^3) \Psi_k^{(1)}(\eta) \rho_k^{(2)}(\eta) \quad (90)$$

where (1), (2) refer to the two different realizations chosen, and  $\rho_k = \dot{\phi}_0 \Gamma_k$ ,  $\Psi_k = (s/k^2)\Gamma_k$ , where  $\Gamma_k = \pi_k^y/a$  and  $s = 4\pi G \dot{\phi}_0$ . From equation (53) we get  $|<\Gamma_k>|^2 = \hbar k L^3 (1/2a)^2$ . Then

$$E_I(k, \eta) = (\pi/4)(a/k)\hbar G(\dot{\phi}_0)^2. \quad (91)$$

In accordance to Penrose's ideas the collapse would take place when this energy reaches the 'one-graviton' level, namely when

$$E_I(k, \eta) = M_p,$$

where  $M_p$  is the Planck mass. Using equations (65) and (53) one gets

$$z_k = \frac{\pi \hbar G \dot{\phi}_0^2}{H_I M_p}. \quad (92)$$

So  $z_k$  is independent of  $k$  which according to equation (88) leads to a roughly scale invariant spectrum of fluctuations in accordance with observations.

A more detailed exposition of this scenario and its ramifications is however beyond the scope of the present paper and will be taken up elsewhere.

## 11 Discussion

We have discussed the problematic part of the standard analysis that is supposed to predict the primordial spectrum of fluctuations responsible for

the deviation of our universe from perfect homogeneity and isotropy and in particular for the eventual evolution of galaxies, stars, and our own. We have argued that there is an essential element that is missing in existing proposals. We have argued that the missing element must contain some new physics. We have considered this issue following the line of thought exposed by Penrose, that such new physics might be tied to some quantum aspect of gravitation, and we have employed this idea in what we called the collapse hypothesis, which is reflected concretely in our model in the fact that we take the Newtonian potential to couple to expectation values of the quantum matter degrees of freedom, and have allowed such expectation values to “jump” in association with the collapse process in a particular set of states. It should thus be emphasized that this can be justified only if we declare that gravitation is, at the quantum level, profoundly different from other degrees of freedom as only such posture would justify the different treatment awarded to the gravitational and the scalar sectors in the present work. We have shown that a relatively simple proposal concerning a collapse of the wave function induced by some unknown mechanism, possibly tied to Quantum Gravity, can account in a transparent way for the scale invariant spectrum that seems to fit very well with the observations.

In Appendix A we have shown a degree of robustness of our approach by redoing the analysis starting from the onset with a spherical mode expansion of fields, and assuming that the collapse directly affects these spherical modes. This fact is not entirely trivial, because as we have argued before, the outcome of a collapse is associated with the quantity that is selected by the “measurement”. Thus the exact nature of the ensemble of collapsed harmonic oscillators that constitute our field depends on the modes one chooses to define the collapsed state. However the specific statistical properties of the ensemble that one is examining turn out to be the same in the two situations that we have considered. It is not unnatural to expect this result to be generic if one thinks of it in analogy with an EPR type of situation: The statistical properties determined by an experimentalist on one end of the set up are independent of the axis of collapse selected by the choice of polarization used by the experimentalist in the other end, whose measurements could be thought to trigger the collapse in that case. However in our situation the possibility of some differences in other more subtle statistical aspects of the ensemble can not be so easily ruled out as we do have access to the system on which the measurements must have occurred (i.e. in contrast with the standard considerations for an experimentalist on one end of the EPR setup,

we are not denied, in principle, access to regions of the universe that might contain relevant information about correlations). Note on the other hand that access to such correlations might be ruled out using arguments similar to those employed by Peres to conclude that one can not experimentally determine the quantum state of a single photon, if one does not have access to the information of how it was prepared [34]. One place differences can conceivably arise is in the correlations among the  $\alpha_{lm}$  for different  $m$  and fixed  $l$ . These issues are of course in need of further investigation.

Furthermore we have shown that, in this scheme, the resulting amplitude for the fluctuations could become naturally small provided the physics at and after reheating does not introduce unwanted amplifications, a possibility that could not be easily uncovered in the standard treatments, probably due to the fact that they do not allow for a transparent picture of the time at which the departure from homogeneity and isotropy starts, and is subjected to different physical regimes of evolution. Thus our proposal opens the door for a study dedicated to eliminating the extreme fine tuning that is necessary in previous treatments. Given that our motivation had nothing to do with this quantitative issues we view this result as another indicative of the promising value of our approach. Furthermore we have indicated how a small departure from the exponential expansion usually associated with inflation could explain the observed decrease in the amplitude of fluctuations on large angular scales.

It is clear *the scheme is in principle susceptible to experimental exploration* as is shown by the non trivial form of the function  $C(k)$  ( equation (88)) which contains information about the collapse times and modes, and which is an input to equation (84), which gives the quantities that are to be compared with experiment. In particular in appendix C we show explicitly that a small change in the collapse scheme leads to differences in the form of this function. Recall that the “standard answer” would correspond to  $C \equiv 1$ , a result that is not trivial to obtain in this scheme as discussed in the end of section 9.

In addition our treatment reveals the various different statistical aspects that are at hand: First there is the statistical averaging over the modes  $k$  that contributes to a specific  $a_{lm}$ . Then one has the statistics that is associated with their average over  $m$  which leads to the observational quantity  $C_l$  or  $OB_l$  in terms of which the experimental results are often exhibited. This clear disentanglement opens the door for more elaborated statistical analysis of the data. In particular we note that the disappearance of all information concerning the size of the sphere of last scattering and the effective region of

the universe that contains the modes that contribute to the observations is particular to the examined values of the angular momentum and to the slow role approximation. That is, the cancellations of the quantities  $R_D$ , and  $L$ , from our final results would not occur if either we do not neglect the terms proportional to  $\mu$  in (84), or if we consider small deviations from slow roll approximation. This would lead to the appearance of  $\delta\phi$  in (65), and thus the source term  $\Gamma$  for the Newtonian potential  $\Psi$  would contain not only the term proportional to  $\sqrt{k}$  that comes from  $g_k(\eta)$  but also terms proportional to  $k^{-1/2}$  and  $k^{-3/2}$  coming from  $y_k(\eta)$ . This would result in the appearance of different powers of  $k$  in equations (69), (74), (76), and (83) which would lead to a scale dependent contribution to the spectrum. If the observations could be improved substantially it might be possible to see these effects. One potential source of information in this respect is in some sense already available: One could look at the observed values of the different  $\alpha_{lm}$  for each fixed value of  $l$  instead of concentrating in their average  $C_l$  and thus extract from the scatter in these quantities information about the effective number of steps in the two dimensional random walks, that is the effective number of  $\vec{k}$ 's that contribute to their value. This in turn would provide information about the “effective size of the Universe”.

Finally we acknowledge we have said almost nothing about the physical process that triggers the collapse of the wave function (except the example considered briefly in section 10.2 that we take at this point to be a simple illustrative model). We have done this consciously because our aim in this work was to point out the difficulties with the standard views on the issue, and to illustrate the kind of effect we need the new physics to bring about. It is clear for instance, that during the collapse process the semi-classical Einstein equations (10) can not be satisfied. We have in mind however that this is some approximation to a more complete description including terms tied to quantum gravity and to the mechanism that triggers the collapse. Instead of equation (10), we would be considering

$$G_{ab} + Q_{ab} = 8\pi G \langle T_{ab} \rangle, \quad (93)$$

where  $Q_{ab}$  represents the back reaction of the geometry to the changes of the expectation values of the energy momentum tensor associated with the collapse of the quantum state of the field. This mechanism must therefore be such that the manipulation made within the semi-classical treatment of Einstein's gravity would be justified, except when the collapse takes place, the points at which the new terms would become important. In other words

we need Einstein's equation to be modified by a term, that reflects the mechanism of collapse and that is responsible for important changes only during the collapse itself. It is our hope that the examination of this and the other requirements we have found so far for the collapse mechanism would be useful guides in constructing specific models of the new physics behind it. We should stress again that our ideas in this regard are strongly influenced by Penrose's proposals that some unknown aspect of Quantum Gravity might be at play. We have briefly considered one simple example of these ideas in 10.2, and clearly much more detailed analysis of these issues is required. In this regard, and following a different line of thought, it is worth pointing out that the quantum uncertainties in the sources of the Newtonian potential could be thought as inducing the kind of quantum superposition of different geometries that Penrose associates with the mechanism that triggers the collapse. In this context we note that the quantum uncertainty in  $\delta\phi$  –which is the one that leads to observed spectrum – behaves as  $1/a$  while the quantum uncertainty in  $\dot{\delta\phi}$  decreases at a much slower rate. The volume over which these uncertainties would in principle affect the Newtonian Potential grows as  $a^3$  so their effect would in principle grow with the universe's size, and thus in the spirit of Penrose's ideas, the universe would, as it evolves, be approaching from below the threshold where the superposition of geometries would lead to a spontaneous collapse of the wave function. On the other hand we point out that  $\delta\phi$  disappeared in our treatment as a source of the Newtonian potential due to the slow roll approximation. Thus, one might be tempted to think that higher order perturbative gravitational effects associated with the latter could be the trigger, in the spirit of Penrose's ideas, of the quantum gravitationally induced collapse. These issues will be the subject of further investigations.

We end by noting a paradoxical aspect of the situation in our field of study: On the one hand there is an almost frenetic search for any form of experimental manifestations of any conceivable aspect of quantum gravity, while on the other hand, when faced with as clear an arena for these studies, as the one we have treated in this work, the prevailing attitude seems to be to hide the mysteries under the rug and declare that everything is fine. It is our hope that this paper contributes to changing this situation.

## Acknowledgments

We gratefully acknowledge very useful discussions with Roger Penrose, Abhay Ashtekar, James Bjorken, Carlo Rovelli, Michael Reisenberger.

This work was supported in part by DGAPA-UNAM IN108103 and CONACYT 43914-F grants, by NSF grant PHY-00-90091, and by the Eberly research funds of Penn State. HS gratefully acknowledges funding by the Max-Planck Society through the MPI for Gravitational Physics.

## Appendix

### A Spherical expansion

The symmetry of the situation and the physical application makes it more natural to use spherical coordinates on the spatial slices, and spherical harmonics as a basis for the one particle space. We start again with the equations for the field  $y = a\delta\phi$ ,

$$\ddot{y} - \left( \nabla^2 + \frac{\ddot{a}}{a} \right) y = 0, \quad (94)$$

where we now write the Laplacian on Euclidean three space as

$$\nabla^2\phi = r^{-2}\partial_r(r^2\partial_r\phi) - r^{-2}J^2\phi, \quad (95)$$

where  $J$  stands for the angular momentum differential operator. In order to quantize the auxiliary field  $y$ , we consider now the field in a spherical box of radius  $R$  and require the field to vanish at its boundary. The standard separation of variables leads us to write the field in terms of the modes

$$U_{klm} = y_k(\eta)f_{kl}(r)Y_{lm}(\theta\varphi) \quad (96)$$

where the  $Y_{lm}$  are the standard spherical harmonics and where the combination  $f_{kl}(r)Y_{lm}(\theta\varphi)$  are eigenmodes of the Laplacian with eigenvalue  $-k^2$ . In particular we have  $f_{kl} = c_{kl}j_l(kr)$ , where  $j_l$  are the spherical Bessel functions, and the normalization is such that

$$\int_0^R r^2 dr \int d\Omega |f_{kl}(r)Y_{lm}(\theta\varphi)|^2 = 1. \quad (97)$$

That is  $c_{kl} = k^{3/2}[\int_0^{x_i^{(l)}} (xj_l(x))^2 dx]^{-1/2}$  where  $x_i^{(l)}$  corresponds to the  $i^{th}$  zero of  $j_l(x)$ , and where the allowed values of  $k$  for a given  $l$  are those that satisfy

$kR = x_i^{(l)}$ . The  $y_k(\eta)$  have to satisfy the same equation as before, and thus we can quantize the field  $y$  as

$$\hat{y}(\eta, r, \theta, \varphi) = \sum_{k,l,m} \left[ \hat{a}_{k,l,m} y_k(\eta) f_{kl}(r) Y_{lm}(\theta\varphi) + \hat{a}_{k,l,m}^\dagger \bar{y}_k(\eta) f_{kl}(r) \bar{Y}_{lm}(\theta\varphi) \right], \quad (98)$$

where the sum is over the allowed values of  $k$  for each  $l$  and as usual  $m \in \{-l, \dots, l\}$ , and we have again chosen the positive frequency solutions given by (41) for the  $y_k$ . The canonical conjugate to  $y$  can be written in a similar fashion, with  $g_k$  taking the place of  $y_k$ . The standard commutation relations are equivalent to

$$[\hat{a}_{k,l,m}, \hat{a}_{k',l',m'}^\dagger] = \hbar \delta_{k,k'} \delta_{l,l'} \delta_{m,m'}. \quad (99)$$

Our definition of the components of a function  $F$  in terms of the spherical harmonics will be  $F_{k,l,m} := \int d^3x (f_{kl} Y_{lm})^* F$ . For the field modes we find

$$\hat{y}_{k,l,m}(\eta) = y_k(\eta) \hat{a}_{k,l,m} + \bar{y}_k(\eta) \hat{a}_{k,l,-m}^\dagger, \quad (100)$$

and express their real and imaginary parts as

$$\hat{y}_{k,l,m}(\eta)^R := (1/2)[\hat{y}_{k,l,m}(\eta) + \hat{y}_{k,l,m}(\eta)^\dagger] = \frac{1}{\sqrt{2}}[y_k(\eta) \hat{a}_{k,l,m}^R + \bar{y}_k(\eta) \hat{a}_{k,l,m}^{R\dagger}] \quad (101)$$

$$\hat{y}_{k,l,m}(\eta)^I := (1/2i)[\hat{y}_{k,l,m}(\eta) - \hat{y}_{k,l,m}(\eta)^\dagger] = \frac{-i}{\sqrt{2}}[y_k(\eta) \hat{a}_{k,l,m}^I - \bar{y}_k(\eta) \hat{a}_{k,l,m}^{I\dagger}]. \quad (102)$$

The real and imaginary components of the annihilation operators are defined by

$$\hat{a}_{k,l,m}^R := \frac{1}{\sqrt{2}}[\hat{a}_{k,l,m} + \hat{a}_{k,l,m}] \quad \hat{a}_{k,l,m}^I := \frac{-i}{\sqrt{2}}[\hat{a}_{k,l,m} - \hat{a}_{k,l,m}] \quad (103)$$

and conjugation. Consequently

$$[\hat{a}_{k,l,m}^R, \hat{a}_{k',l',m'}^{R\dagger}] = \hbar \delta_{k,k'} \delta_{l,l'} (\delta_{m,m'} + \delta_{m,m'}) \quad (104)$$

$$[\hat{a}_{k,l,m}^I, \hat{a}_{k',l',m'}^{I\dagger}] = \hbar \delta_{k,k'} \delta_{l,l'} (\delta_{m,m'} - \delta_{m,m'}) \quad (105)$$

$$[\hat{a}_{k,l,m}^R, \hat{a}_{k',l',m'}^{I\dagger}] = [\hat{a}_{k,l,m}^I, \hat{a}_{k',l',m'}^{R\dagger}] = 0. \quad (106)$$

The modes of  $\hat{\pi}^{(y)}$  can be decomposed in the same way. The only thing that changes in comparison to (101,102) is that  $y_k$  gets replaced by  $g_k$ . Now we consider the quantity of interest that is determined by the following quantity,

$$d_{k,l,m}^{R,I} = < \hat{a}_{k,l,m}^{R,I} >_\Theta = \sqrt{\hbar} D_{k,l,m}^{R,I} e^{i\alpha_{k,l,m}^{R,I}} \quad (107)$$

These quantities which as before associated with the collapsed state expectation value of the field and momentum, which in our model for the collapse are determined by the relation at the time of collapse

$$\langle \hat{y}_{k,l,m}^{R,I}(\eta_k^c) \rangle_\Theta = x_{k,l,m,1}^{R,I} \sqrt{(\Delta \hat{y}_{k,l,m}^{R,I})_0^2} = x_{k,l,m,1}^{R,I} |y_k(\eta_k^c)| \sqrt{\hbar/2}, \quad (108)$$

$$\langle \hat{\pi}_{k,l,m}^{(y)R,I}(\eta_k^c) \rangle_\Theta = x_{k,l,m,2}^{R,I} \sqrt{(\Delta \hat{\pi}_{k,l,m}^{(y)R,I})_0^2} = x_{k,l,m,2}^{R,I} |g_k(\eta_k^c)| \sqrt{\hbar/2}, \quad (109)$$

where as before we have for any time at or after the collapse,

$$\langle \hat{y}_{k,l,m}^{R,I}(\eta_k) \rangle_\Theta = \sqrt{2} \Re(y_k(\eta_k) d_{k,l,m}^{R,I}), \quad (110)$$

$$\langle \hat{\pi}_{k,l,m}^{(y)R,I}(\eta_k^c) \rangle_\Theta = \sqrt{2} \Re(g_k(\eta_k) d_{k,l,m}^{R,I}) \quad (111)$$

Again in terms of the phases  $\beta_k = \arg(y_k)$ ,  $\gamma_k = \arg(g_k)$ , where the last two refer to quantities evaluated at the collapse time  $\eta_k^c$ , the above equations can be written

$$D_{k,l,m}^{R,I} \cos(\alpha_k^{R,I} + \beta_k^c) = \frac{1}{2} x_{k,l,m,1}^{R,I}, \quad D_{k,l,m}^{R,I} \cos(\alpha_k^{R,I} + \gamma_k^c) = \frac{1}{2} x_{k,l,m,2}^{R,I}. \quad (112)$$

To connect to observations, we again use the relation  $\nabla^2 \Psi - \mu \Psi = s\Gamma$ . Decomposing both sides into spherical harmonics, we find

$$\Psi_{klm}(\eta) = -\frac{sU(k)}{(k^2 + \mu)} \Gamma_{klm}(\eta) \quad (113)$$

where, once more, the factor  $U(k)$  accounts for the physical evolution from reheating to decoupling. The right hand side is connected to the scalar field via,  $\Gamma = \frac{1}{a}\pi$ , so in our context

$$\Gamma_{k,l,m}(\eta) = \frac{1}{a} \langle \hat{\pi}_{k,l,m}^{(y)}(\eta) \rangle_\Theta. \quad (114)$$

The observational quantity  $\alpha_{lm}$  can be expressed as

$$\alpha_{lm} = \int \Psi(\eta_D, \vec{x}_D) Y_{lm}^* d^2\Omega = \sum_k \Psi_{klm}(\eta_D) f_{kl}(R_D). \quad (115)$$

Again we see that this quantity is a sum of the complex quantities associated with the collapse and thus can be viewed as a two dimensional random walk with variable step size. We resort once more to the mathematical trick of

identifying the most likely value of the this magnitude with the mean value of an ensemble of identical instances. This corresponds to identifying the most likely value of the total displacement of the random walk with its ensemble average. Thus we write

$$|\alpha_{lm}|^2_{MostLikely} = \langle\langle |\alpha_{lm}|^2 \rangle\rangle = \langle\langle \sum_k |\Psi_{klm}(\eta_D)|^2 \rangle\rangle, \quad (116)$$

$$= \sum_{k,k'} \frac{s^2 U(k) U(k')}{(k^2 + \mu)(k'^2 + \mu)} \langle\langle \Gamma_{klm}(\eta_D) \Gamma_{k'lm}(\eta_D) \rangle\rangle f_{kl}(R_D) f_{k'l}(R_D) \quad (117)$$

Now we evaluate this ensemble average noting that

$$\Gamma_{klm}(\eta) = (1/a) \{ \langle \hat{\pi}_{k,l,m}^{(y)I}(\eta) \rangle_\Theta + i \langle \hat{\pi}_{k,l,m}^{(y)I}(\eta) \rangle_\Theta \} \quad (118)$$

$$= (\sqrt{2}/a) \{ \Re(g_k(\eta) d_{k,l,m}^R) + i \Im(g_k(\eta) d_{k,l,m}^I) \} \quad (119)$$

$$= (\sqrt{2}/a) |g_k(\eta)| \sqrt{\hbar} \{ D_{k,l,m}^R \cos(\alpha_{k,l,m}^R + \gamma_k) \} \quad (120)$$

$$+ i D_{k,l,m}^I \cos(\alpha_{k,l,m}^I + \gamma_k) \} \quad (121)$$

so that writing  $\alpha_{k,l,m}^{R,I} + \gamma = \alpha_{k,l,m}^{R,I} + \gamma_k^c + \Delta_k$  where  $\Delta_k = k(\eta - \eta_k^c)$ , and using (112), we find

$$\Gamma_{klm}(\eta) = (\sqrt{2}/a) |g_k(\eta)| \sqrt{\hbar} (1/2) \{ [x_{k,l,m,1}^R \sqrt{1 + 1/z_k^2} \sin(\Delta_k) \} \quad (122)$$

$$+ x_{k,l,m,2}^R (\cos(\Delta_k) + (1/z_k) \sin(\Delta_k)) \} \quad (123)$$

$$+ i [x_{k,l,m,1}^I \sqrt{1 + 1/z_k^2} \sin(\Delta_k) \} \quad (124)$$

$$+ x_{k,l,m,2}^I (\cos(\Delta_k) + (1/z_k) \sin(\Delta_k)) \} \quad (125)$$

Thus upon taking the ensemble average, and using  $\langle\langle x_{k,l,m,N}^A x_{k',l,N'}^{A'} \rangle\rangle = \delta_{k,k'} \delta_{A,A'} \delta_{N,N'}$  (where  $A, A'$  stand for  $R$  or  $I$  and  $N, N'$  stand for 1 or 2) we find:

$$\langle\langle \Gamma_{klm}(\eta) \Gamma_{k'lm}(\eta) \rangle\rangle = \delta_{k,k'} (\hbar/2a^2) |g_k(\eta)|^2 C(k) \quad (126)$$

with  $C(k)$  given by (88). We thus have obtained a useful expression for the desired quantity:

$$\langle\langle |\alpha_{lm}|^2 \rangle\rangle = \sum_k \frac{s^2 U(k)^2}{(k^2 + \mu)^2} (\hbar/2a^2) |g_k(\eta)|^2 C(k) |f_{kl}(R_D)|^2 \quad (127)$$

Now we evaluate this sum. To proceed we recall that the sum is over the values of  $k$  such that  $kR = x_i^{(l)}$ , the zeros of the function  $j_l(x)$ . Next we

write  $f_{kl}(R_D) = c_{kl} j_l(kR_D)$  where  $c_{kl} = k^{3/2} [\int_0^{x_i^{(l)}} (x j_l(x))^2 dx]^{-1/2}$ . Thus  $f_{kl}(R_D) = c_{il} j_l(x_i^{(l)} R_D / R)$ . Now using eq 11.170 of [35] we find  $\int_0^{x_i^{(l)}} (x j_l(x))^2 dx = (x_i^{(l)})^3 j_{l+1}^2(x_i^{(l)})/2$ . Now the approximate expression ( valid for large  $x$ ) for the spherical Bessel functions  $j_l(x) = (1/x) \sin(x - l\pi/2)$  ( eq. 11.161a of [35]) we see that the  $i^{th}$  zero of  $j_l(x)$  is  $x_i^{(l)} = (i + l/2)\pi$ , thus  $(j_{l+1}(x_i^{(l)}) = (x_i^{(l)})^{-1} = [(i + l/2)\pi]^{-1}$ . Therefore  $c_{kl} = k^{3/2} [(x_i^{(l)})/2]^{-1/2} = (k\sqrt{2/R})$  where in the last step we used the fact that the allowed values of  $k_i$  must satisfy  $k_i R = x_i^{(l)}$ . We also use the fact that  $|g_k(\eta)|^2 = k/2$ , and thus we find

$$\langle\langle |\alpha_{lm}|^2 \rangle\rangle = \sum_k \frac{s^2 U(k)^2}{(k^2 + \mu)^2} (\hbar/2a^2)(k/2)(2k^2/R) |j_l^2(kR_D)|^2 C(k) \quad (128)$$

which we write as

$$\langle\langle |\alpha_{lm}|^2 \rangle\rangle = \frac{\hbar s^2}{2a^2 R} \sum_k E(k), \quad E(k) := \frac{U(k)^2 k^3}{(k^2 + \mu)^2} |j_l^2(kR_D)|^2 C(k). \quad (129)$$

Now we use the uniform continuity of the function  $F$ , choose  $\epsilon > 0$  and find the  $\delta > 0$  such that for  $|k - k'| < \delta$  the corresponding  $|E_{(l)}(k) - E_{(l)}(k')| < \epsilon$ , and then write

$$\sum_k E(k) = \delta^{-1} \sum_{n=0}^{\infty} \sum_{k \in [n\delta, (n+1)\delta]} E_{(l)}(n\delta) \delta \quad (130)$$

Now the number of allowed values of  $k = (1/R)x_i^{(l)} = (i + l/2)\pi/R$  in the interval  $[n\delta, (n+1)\delta]$  is  $N = R\delta/\pi$ . Then we can write

$$\sum_k E(k) = \delta^{-1} \sum_{n=0}^{\infty} N E_{(l)}(n\delta) \delta = (R/\pi) \int_0^{\infty} E_{(l)}(k) dk \quad (131)$$

where in the last step we have taken the limit  $\epsilon \rightarrow 0$ . Thus we finally obtain,

$$\langle\langle |\alpha_{lm}|^2 \rangle\rangle = \frac{\hbar s^2}{2a^2 R} (R/\pi) \int_0^{\infty} E_{(l)}(k) dk \quad (132)$$

$$= \frac{\hbar s^2}{2a^2 \pi} \int_0^{\infty} \frac{U(k)^2 k^3}{(k^2 + \mu)^2} |j_l^2(kR_D)|^2 C(k) dk \quad (133)$$

which is the same result that was obtained in the analysis of Section 9.

## B On the meaning of the Newtonian potential

Let us consider the space-time given by the perturbed metric:

$$ds^2 = a(\eta)^2 \left[ -(1 + 2\Psi)d\eta^2 + (1 - 2\Psi)\delta_{ij}dx^i dx^j \right] \quad (134)$$

In this space-time light will travel according to

$$d\eta = \pm \left[ \frac{(1 - 2\Psi)}{(1 + 2\Psi)} \right]^{1/2} dx \approx \pm(1 - 2\Psi)dx \quad (135)$$

We want to consider light signals being emitted from the surface of last scattering at a given  $\eta_D$  and arriving to us from all angular directions. We will assume that we and the CMB radiation emitting plasma are at rest in the background space-time so that both we and the plasma fluid follow world lines of fixed  $x$ .

Thus a light signal that starts at  $\eta_e^{(1)}$  at a given position  $x_e$  and arrives to the origin at  $x = 0$  at  $\eta_o^{(1)}$ , will satisfy

$$\eta_o^{(1)} - \eta_e^{(1)} = \int_0^{x_e} (1 - 2\Psi)dx \quad (136)$$

Consider a second signal emitted an extremely short time after the first one (we are thinking of fractions of a second), and from the given position  $x_e$  at  $\eta_e^{(2)} = \eta_e^{(1)} + \delta\eta_e$  and its arrival to the origin at  $x = 0$  at  $\eta_o^{(1)} = \eta_o^{(1)} + \delta\eta_o$ . It is clear then that

$$\eta_o^{(2)} - \eta_e^{(2)} = \int_0^{x_e} (1 - 2\Psi)dx \quad (137)$$

and therefore  $\delta\eta_e = \delta\eta_o$ . Now let us consider the actual proper times measured by observers at rest between the two emission events  $\delta\tau_e$  and between the two detection events  $\delta\tau_o$ . It is clear that  $\delta\tau = a(\eta)[1 + 2\Psi]^{(1/2)}\delta\eta \approx a(\eta)[1 + \Psi]\delta\eta$  so

$$\delta\tau_e = a(\eta_e)[1 + \Psi_e]\delta\eta_e, \quad \delta\tau_o = a(\eta_o)[1 + \Psi_e]\delta\eta_o \quad (138)$$

Thus we find

$$\frac{\delta\tau_o}{\delta\tau_e} = \frac{a(\eta_o)}{a(\eta_e)}(1 - \Psi_o)[1 + \Psi_e] \quad (139)$$

where it is clear that all the angular dependence is in the factor  $[1 + \Psi_e]$ , while  $\frac{a(\eta_o)}{a(\eta_e)}(1 + \Psi_o)$  represents the combine effect of the overall cosmic expansion

and the gravitational potential at our location. The quantity  $\frac{\delta\tau_o}{\delta\tau_e}$  is clearly encoding the standard gravitational red-shift and is equal to the frequency ratio  $\nu_o/\nu_e$  which in turn is associated with the temperature of the observed CMB. Note that the whole angular dependence is in  $\Psi_e$ , so it is clear that what we have been computing is indeed an observable quantity, and in that sense it is gauge invariant.

## C Alternative Collapse Scheme

Here we want to consider an additional natural possibility for the collapse. The main motivation is to explicitly show how observational data could be used to shed light into the details of the collapse mechanism. The idea we consider here is that as it is only the field's momentum which acts as a source, at leading order, for the Newtonian potential, it should be only this quantity that would be subjected to a change in the expectation value during the collapse. This view seems to be close in spirit to the ideas of Penrose regarding the quantum uncertainties that the gravitational potential would be inheriting from the matter fields quantum uncertainties, as fundamental factors triggering the collapse. In such a situation we would have a collapsed state for which for  $|\Theta\rangle$  after collapse:

$$\langle \hat{y}_k^{R,I}(\eta_k^c) \rangle_\Theta = 0, \quad \langle \hat{\pi}_k^{(y)R,I}(\eta_k^c) \rangle_\Theta = X_k^{R,I} \quad (140)$$

where  $X_k^{R,I}$  are random variables, distributed according to a Gaussian distribution centered at zero with spread  $(\Delta\hat{\pi}_k^{(y)R,I})_0^2$ , respectively. Thus

$$\langle \hat{\pi}_k^{(y)R,I}(\eta_k^c) \rangle_\Theta = x_k^{R,I} \sqrt{(\Delta\hat{\pi}_k^{(y)})_0^2} = x_k^{R,I} |g_k(\eta_k^c)| \sqrt{\hbar L^3}, \quad (141)$$

where  $x_k$  are now distributed according to a Gaussian distribution centered at zero with spread one. In this situation the whole analysis of Section 9 goes through, with the only difference being that now we have:

$$F(k) = (x_k^R + ix_k^I)[\cos(\Delta_k) - (1/z_k)\sin(\Delta_k)] \quad (142)$$

so that

$$\langle F(\vec{k}) \overline{F(\vec{k}')} \rangle = \delta_{k,k'}[I + (1 - (1/z_k^2))\sin^2(\Delta_k) - (1/z_k)\sin(2\Delta_k)] \quad (143)$$

And therefore

$$|\alpha_{lm}|^2_{MostLikely} = \frac{s^2\hbar}{2\pi a^2} \int \frac{U^2(x/R_D)C'(x/R_D)}{(x^2 + \mu R_D^2)^2} j_l^2(x)x^3 dx \quad (144)$$

where now the function  $C$  is slightly different from the one found in the democratic collapse model, and is given by

$$C'(k) = [1 + \sin^2(\Delta_k)(1 - (1/z_k^2)) - (1/z_k)\sin(2\Delta_k)] \quad (145)$$

We see that in principle the observations could help distinguish between the different collapse models, and therefore it is clear that the question of the exact mechanism for the origin of the primordial fluctuations is both affected and can help shed light on a fundamental issue in our understanding of quantum mechanics as applied to the universe as a whole.

## References

- [1] “Cosmological parameters From First results of Boomerang” Boomerang Collaboration (A.E. Lange et al.) *Phys. Rev. D*, **63**, 042001,(2001); G. Hinshaw et. al., *Astrophys. J. Suppl.* , **148**, 135 (2003); “Power Spectrum of Primordial Inhomogeneity Determined from four Year COBE DMR SKY Maps”, K.M. Gorski , A.J. Banday , C.L. Bennett(NASA, Goddard), G. Hinshaw, A. Kogut , G.F. Smoot , E.L. Wright , *Astrophys. J.* **464**, L11, (1996); “First Year Wilkinson Micorowave Anisotropy Probe (WMAP) Observations: Preliminary Results” C.L. Bennett et al. *Astrophys. J. Suppl.* **148**, 1,(2003); “First Year Wilkinson Micorowave Anisotropy Probe (WMAP) Observations: Foreground Emission”, C. Bennett et al. *Astrophys. J. Suppl.* **148**, 97,(2003).
- [2] “Quantum Mechanics of the scalar field in the new inflationary Universe”, A. Guth and S.-Y. Pi *Phys. Rev. D* **32**, 1899, (1985).
- [3] “Fluctuations in the Inflationary Universe”, S. W. Hawking *Nucl. Phys. B* **224**, 180, (1983).
- [4] See for instance discussion in “The Early Universe”, E.W. Kolb and M.S. Turner, Frontiers in Physics Lecture Note Series (Addison Wesley Publishing Company 1990).

- [5] “An Alternative to Inflation”, S. Hollands and R.M. Wald, *Gen.Rel.Grav.* **34**, 2043,(2002) [arXive: gr-qc/0205058].
- [6] “Is the Universe out of Tune”, G.D. Starkman and D. J. Schwartz, *Scientific American*, Vol 293, No 2, 48, (2005).
- [7] “Semiclassicality and decoherence of Cosmological perturbations”, D. Polarski and A.A. Starobinsky, *Class. Quant. Grav.* **13**, 377 (1996) [arXive: gr-qc/9504030]
- [8] “Origin of Classical Structure From Inflation”, C. Kiefer *Nucl. Phys. Proc. Suppl.* **88**, 255 (2000) [arXiv:astro-ph/0006252], and references therein.
- [9] “Space-time correlations in Inflationary Spectra”, D. del Campo, and R. Parentani, *Phys. Rev. D* **70**, 105020 (2004) [arXiv:gr-qc/0312055]
- [10] “Environment Induced Superselection In Cosmology”, W.H. Zurek, in *Moscow 1990, Proceedings, Quantum gravity* (QC178:S4:1990), 456-472. (see High Energy Physics Index 30 (1992) No. 624)
- [11] “Decoherence Functional and Inhomogeneities in the Early Universe”, R. Laflamme and A. Matacz, *Int. J. Mod. Phys. D* **2**, 171 (1993) [arXiv:gr-qc/9303036]
- [12] “Lectures on the Theory of Cosmological Perturbations”, R.H. Brandenberger, *Lect. Notes Phys.* **646**, 127 (2004) [arXiv:hep-th/0306071]
- [13] “Gauge Invariant Cosmological Perturbations” R. Branderberger H. Feldman and V. Mukhavov, Preprint Brown HET 845 (1992); *Phys. Rep.* **215**, 203, (1992)
- [14] “Cosmology and Astrophysics Through Problems”, T. Padmanabhan (Cambridge University Press 1996).
- [15] “Cosmological Inflation and Large Scale Structure”, A.R. Liddle and D.H. Lyth (Cambridge University Press 2000).
- [16] “The self-induced approach to decoherence in cosmology,” M. Castagnino and O. Lombardi, *Int. J. Theor. Phys.* **42**, 1281 (2003) [arXiv:quant-ph/0211163].

- [17] “Origin of Structure in the Universe” J.J. Halliwell and S. W. Hawking, *Phys. Rev. D*, **31**, 1777,(1985).
- [18] “Inflationary Cosmological Perturbations of Quantum Mechanical Origin” J. Martin, *Lect. Notes Phys.* **669**, 199 (2005) [arXiv:hep-th/0406011]; “ Best Unbiased Estimates for Microwave background Anisotropies”, L.P. Grishchuk and J. Martin, *Phys. Rev. D* **56**, 1924 (1997) [arXiv:gr-qc/9702018]
- [19] “The Emperor’s New Mind”, R. Penrose ( Oxford University Press 1989).
- [20] “Structure in the COBE DMR first year maps” G.F. Smoot *et. al.*, *Astrophys. J.*, **396**, L1 (1992).
- [21] S. Adler, (Princeton) Private Communication.
- [22] “Nonlocality in Quantum Physics”, A. Anaoljevich Grib and W. Alves Rodrigues Jr. (Kluwer Academic / Plenum Publishers 1999).
- [23] “Quantum Cosmology Problems for the 21<sup>st</sup> Century”, J. B. Hartle, arXive: gr-qc/9701022.
- [24] “Generalized Quantum mechanics for Quantum Gravity”,J. B. Hartle, arXive: gr-qc/0510126.
- [25] “The Reduction of the State Vector and Limitations on Measurement in Quantum Mechanics of Closed Systems, J. B. Hartle, in “Directions in Relativity. Vol. 2: Proceedings”, B.L. Hu and T.A. Jacobson (eds.), Cambridge University Press, Cambridge, 1993. [arXive: gr-qc/9301011]
- [26] “Consistent Sets Yield Contrary Inferences in Quantum Theory”, A. Kent, *Phys. Rev. Lett.* **78**, 28749 (1997); “Comments on “Consistent Sets Yield Contrary Inferences in Quantum Theory”, R. Griffiths and J.B. Hartle,*Phys. Rev. Lett.* **81**, 1981 (1998) [arXiv:gr-qc/9710025]
- [27] “Decoherence in Quantum Cosmology”, J.J. Halliwell, *Phys. Rev. D*, **39**, 2912,(1989)
- [28] “Recent Advances in Stochastic Gravity Theory and Issues”, B. L. Hu and E. Verdaguer, arXive: gr-qc/0110092.

- [29] “Decoherence in Quantum Cosmology at the onset of Inflation”, A.O. Barvinsky, A.Y. Kamenshchik, C. Kiefer, and I.V. Mishakov, *Nucl. Phys. B* **551**, 374 (1999) [arXiv:gr-qc/9812043]
- [30] “Speakable and Unspeakable in Quantum Mechanics”, J. S. Bell (Cambridge University Press 1987).
- [31] “Is the Moon There when nobody Looks?” D. Mermin *Physics Today* **32**, 38, (1985).
- [32] “Experimental realization of Einstein-Podolsky-Rosen-Bohm Gedanken-experiment: A New violation of Bell’s inequalities” A. Aspect, P. Grangier, G. Roger, *Phys. Rev. Lett.* **49**, 91, (1982).
- [33] “A Unified Dynamics For Micro And Macro Systems”, G. C. Ghirardi, A. Rimini, and T. Weber, *Phys. Rev. D* **34**, 470, (1986).
- [34] Quantum Theory: Concepts and Methods, A. Peres (Kluwer, Academic Publishers, 1993).
- [35] Mathematical Methods for Physicists, G. Arfken, (Academic Press New York 1970).
- [36] “Decoherence during inflation: The generation of classical inhomogeneities,” F. C. Lombardo and D. Lopez Nacir, *Phys. Rev. D* **72**, 063506 (2005) [arXiv:gr-qc/0506051].