

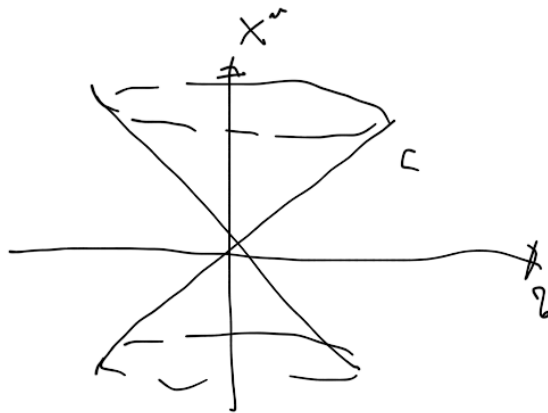
## Energia - Momento

$$U^\mu = \frac{dx^\mu}{d\tau} \quad ; \quad U = U^\mu e_\mu$$

$$U^2 = U \cdot U \quad ; \quad ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

$$d\tau^2 = -\frac{ds^2}{c^2} \quad \Rightarrow \quad -c^2 d\tau^2 = g_{\mu\nu} dx^\mu dx^\nu$$

$$U^2 = U^\mu U_\mu = -c^2$$



Cono de luz  
do referencial comovido.

$$p^\mu = m U^\mu \quad ; \quad p^\mu p_\mu = m^2 U_\mu U^\mu$$

$$p^\mu p_\mu = -m^2 c^2$$

$$p^\mu = \left( \frac{E}{c}, p^i \right)$$

$$p^i = 0 \quad \Rightarrow \quad \frac{-E^2}{c^2} = -m c^2$$

$$\boxed{E = m c^2}$$

$$ds^2 = -c^2 d\tau^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2$$

$$\frac{d\tau}{dt} = \left( 1 - \frac{v^2}{c^2} \right)^{1/2}$$

$$u^\mu = \frac{dx^\mu}{d\tau} = \frac{dx^\mu}{dt} \frac{dt}{d\tau} = \gamma v^\mu$$

$$p^\mu = \gamma m v^\mu \quad ; \quad v^0 = c$$

$$p^0 = \frac{E}{c} \quad ;$$

$$E = \gamma m c^2$$

$$|P = p^\mu e_\mu \quad ; \quad \frac{dP}{d\tau} = U^\nu \nabla_\nu p^\mu e_\mu$$

$$\nabla_\nu p^\mu = \frac{\partial p^\mu}{\partial x^\nu} + \Gamma_{\alpha\nu}^\mu p^\alpha$$

$$|P = U e$$

$$U^\nu \nabla_\nu p^\mu = \frac{d}{d\tau} (\gamma U^\mu) + \Gamma_{\alpha\nu}^\mu \gamma U^\alpha U^\nu$$

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma_{\alpha\nu}^\mu \frac{dx^\alpha}{d\tau} \frac{dx^\nu}{d\tau} = 0$$

$$\pi_{\alpha\nu}^{\mu} = 0$$

$$\frac{d^2 X^{\mu}}{d\tau^2} = 0 \quad \Rightarrow \quad \text{Eq. de movimento.}$$

$$\vec{F} = \frac{d\vec{p}}{dt} = F^{\mu} e_{\mu}$$

$$F^{\mu} = U^{\nu} \nabla_{\nu} p^{\mu}$$

- Recuperando a notação 3D

$$\vec{p} = p^i e_i \quad ; \quad p^{\mu} = \left( \frac{E}{c}, p^i \right)$$

$$p^{\mu} p_{\mu} = -m^2 c^2$$

$$p^0 p_0 + p^i p_i = -m^2 c^2 \quad ; \quad p_0 = -p^0$$

$$-\frac{E^2}{c^2} + \vec{p} \cdot \vec{p} = -m^2 c^2$$

$$\vec{p} \cdot \vec{p} = p^2$$

$$-\frac{\vec{E}^2}{c^2} + p^2 = -m^2 c^2$$

$$E = c (m^2 c^2 + p^2)^{1/2}$$