Métrica

$$g_{\mu\nu} = \mathbb{e}_\mu \cdot \mathbb{e}_\nu$$

$$\mathbb{V} = V^\mu \mathbb{e}_\mu \quad ; \quad V^2 = V^\mu V^\nu \, \mathbb{e}_\mu \cdot \mathbb{e}_\nu$$

$$V^2 = g_{\mu\nu} V^\mu V^\nu = V^\mu V_\mu$$

$$\boxed{V_\mu = g_{\mu\nu} V^\nu}$$

$$d\mathbb{X} = dX^\mu \mathbb{e}_\mu$$

$$dS^2 = d\mathbb{X} \cdot d\mathbb{X}$$

$$dS^2 = g_{\mu\nu} dX^\mu dX^\nu$$

$$g^{\mu\nu} = (g_{\mu\nu})^{-1} \quad ; \quad g_{\mu\nu} g^{\mu\lambda} = \delta^\lambda_\nu$$

$$\begin{cases} x' = \gamma(x - vt) \\ y' = y \\ z' = z \\ t' = \gamma\left(t - \frac{vx}{c^2}\right) \end{cases} \qquad (x^\mu \to x^a) \; ; \quad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$e^a{}_\mu = \frac{\partial x^a}{\partial x^\mu} = \begin{bmatrix} \gamma & -\gamma \frac{v}{c} & 0 & 0 \\ -\gamma \frac{v}{c} & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\frac{\partial x'^0}{\partial x} = \frac{c\, \partial t'}{\partial x} = c\left(-\frac{v}{c^2}\right)\gamma = -\gamma \frac{v}{c}$$

$$ds^2 = g_{ab}\, dx^a dx^b = g_{\mu\nu}\, dx^\mu dx^\nu$$

$$= g_{ab}\, e^a{}_\mu dx^\mu\, e^b{}_\nu dx^\nu = g_{\mu\nu}\, dx^\mu dx^\nu$$

$$\boxed{g_{\mu\nu} = e^a{}_\mu e^b{}_\nu\, g_{ab}}$$

$$g_{00} = e^a{}_0\, e^b{}_0\, g_{ab} = \gamma\, e^b{}_0\, g_{0b} + \left(-\frac{\gamma v}{c}\right) e^b{}_0\, g_{1b}$$

$$= \gamma^2 g_{00} - \frac{\gamma^2 v}{c} g_{01} - \frac{\gamma^2 v}{c} g_{10} + \left(\frac{\gamma v}{c}\right)^2 g_{11}$$

$$g_{10} = e^a{}_0\, e^b{}_1\, g_{ab} = \gamma\, e^b{}_1\, g_{0b} + \left(-\frac{\gamma v}{c}\right) e^b{}_1\, g_{1b}$$

$$= -\frac{\gamma^2 v}{c} g_{00} + \gamma^2 g_{01} + \left(\frac{\gamma v}{c}\right)^2 g_{01} - \frac{\gamma^2 v}{c} g_{11}$$

$$g_{11} = e^a{}_1\, e^b{}_1\, g_{ab} = -\frac{\gamma v}{c} e^b{}_1\, g_{0b} + \gamma\, e^b{}_1\, g_{1b}$$

$$= \left(\frac{\gamma v}{c}\right)^2 g_{00} - \frac{\gamma^2 v}{c} g_{01} - \frac{\gamma^2 v}{c} g_{10} + \gamma^2 g_{11}$$

$$\begin{cases} (1-\gamma^2)\, g_{00} = -2\frac{\gamma^2 v}{c} g_{01} + \frac{\gamma^2 v^2}{c^2} g_{11} \\ \left(1-\gamma^2-\frac{\gamma^2 v^2}{c^2}\right) g_{01} = -\frac{\gamma^2 v}{c} g_{00} - \frac{\gamma^2 v}{c} g_{11} \\ (1-\gamma^2)\, g_{11} = -2\frac{\gamma^2 v}{c} g_{01} + \frac{\gamma^2 v^2}{c^2} g_{00} \end{cases}$$

$$g_{00} = -g_{11} \quad ; \quad g_{01} = 0 \quad ; \quad g_{11} = a$$

$$g_{\mu\nu} = \begin{bmatrix} \gamma & -\frac{\gamma v}{c} & 0 & 0 \\ -\frac{\gamma v}{c} & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -a & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \gamma & -\frac{\gamma v}{c} & 0 & 0 \\ -\frac{\gamma v}{c} & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$g_{\mu\nu} = \begin{bmatrix} \gamma & -\frac{\gamma v}{c} & 0 & 0 \\ -\frac{\gamma v}{c} & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -a\gamma & \frac{\gamma a v}{c} & 0 & 0 \\ -\frac{a\gamma v}{c} & \gamma a & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -a\gamma^2 + a\gamma^2\frac{v^2}{c^2} & 0 & 0 & 0 \\ 0 & -\gamma^2\frac{v^2}{c^2}a + a\gamma^2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -a & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$ds^2 = -a\,c^2dt^2 + a\,dx^2 + dy^2 + dz^2 \quad ; \quad a = 1$$

$$ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2$$

$$d\tau^2 = -\frac{ds^2}{c^2}$$

$$d\tau^2 = dt^2\left(1 - \frac{v^2}{c^2}\right) \; ; \quad \boxed{\tau = \int dt \sqrt{1 - v^2/c^2}}$$

$$\eta_{ab} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \; ;$$ Minkowski