

STATISTICS WORKSHEET-7

1. A die is thrown 1402 times. The frequencies for the outcomes 1, 2, 3, 4, 5 and 6 are given in the following table:

| | | | | | | |
|-----------|-----|-----|-----|-----|-----|-----|
| Outcome | 1 | 2 | 3 | 4 | 5 | 6 |
| Frequency | 400 | 300 | 157 | 180 | 175 | 190 |

Find the probability of getting 6 as outcome:

- a) 0.34
b) 0.135
c) 0.45
d) 0.78

Ans: The total number of throws = $400 + 300 + 157 + 180 + 175 + 190 = 1402$

The probability of getting 6 as outcome = frequency of 6 / total number of throws = $190/1402 \approx 0.135$

Therefore, the answer is **(b) 0.135**.

2. A telephone directory page has 400 telephone numbers. The frequency distribution of their unit place digit (for example, in the number 25827689, the unit place digit is 9 is given in table below:
First row refers to the digits
Second row to their frequencies.

| | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|----|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 44 | 52 | 44 | 44 | 40 | 20 | 28 | 56 | 32 | 40 |

What will be the probability of getting a digit with unit place digit odd number that is 1, 3, 5, 7, 9?

- a) 0.67
b) 0.60
c) 0.45
d) 0.53

Ans: The total frequency of unit place digits that are odd = frequency of digit 1 + frequency of digit 3 + frequency of digit 5 + frequency of digit 7 + frequency of digit 9 = $52 + 44 + 20 + 56 + 40 = 212$

Therefore, the probability of getting a digit with unit place digit odd number = frequency of odd digits / total frequency = $212/400 = 0.53$

Hence, the answer is option **(d) 0.53**.

3. A tyre manufacturing company which keeps a record of the distance covered before a tyre needed to be replaced. The table below shows the results of 1100 cases.

| | | | | |
|------------------|-------|-----------|------------|--------|
| Distance (miles) | <4000 | 4000-9000 | 9001-14000 | >14000 |
|------------------|-------|-----------|------------|--------|

| | | | | |
|-----------|----|-----|-----|-----|
| Frequency | 20 | 260 | 375 | 445 |
|-----------|----|-----|-----|-----|

If we buy a new tyre of this company, what is the probability that the tyre will last more than 9000 miles?

- a) 0.67
- b) 0.459
- c) 0.745
- d) 0.73

Ans:

The total frequency of tyres that last more than 9000 miles is:

$$375 + 445 = 820$$

The total frequency of all the cases is:

$$20 + 260 + 375 + 445 = 1100$$

Therefore, the probability of a new tyre from this company lasting more than 9000 miles is:

$$820/1100 = 0.745$$

So, the answer is **(c) 0.745**.

4. Please refer to the case and table given in the question No. 3 and determine what is the probability that if we buy a new tyre then it will last in the interval [4000-14000] miles?

a) 0.56
b) 0.577
c) 0.745
d) 0.73

Ans: The probability that the tyre will last in the interval [4000-14000] miles is the sum of the probabilities of the distance falling in this range:

$$P(4000 \leq \text{distance} \leq 14000) = P(4000-9000) + P(9001-14000)$$

$$= 260/1100 + 375/1100$$

$$= 0.236 + 0.341$$

$$= 0.577$$

Therefore, the answer is **(b) 0.577**.

5. We have a box containing cards numbered from 0 to 9. We draw a card randomly from the box. If it is told to you that the card drawn is greater than 4 what is the probability that the card is odd?

a) 0.5
b) 0.8
c) 0.6
d) 0.7

Ans: If we draw a card from the box, there are 10 possible outcomes, and each outcome has an equal probability of $1/10$ or 0.1. If we know that the card drawn is greater than 4, then there are 5 possible outcomes: 5, 6, 7, 8, and 9. The probability that the card drawn is odd given that it is greater than 4 is the ratio of the number of outcomes that are both odd and greater than 4 to the total number of outcomes that are greater than 4.

The odd numbers greater than 4 are 5 and 7, so the number of outcomes that are both odd and greater than 4 is 2. The total number of outcomes that are greater than 4 is 5. Therefore, the probability that the card drawn is odd given that it is greater than 4 is $2/5$ or **0.4**.

So the correct answer is not given among the options provided.

6. We have a box containing cards numbered from 1 to 8. We draw a card randomly from the box. If it is told to you that the card drawn is less than 4 what is the probability that the card is even?

a) 0.33
b) 0.40
c) 0.56
d) 0.89

Ans: If we draw a card from the box, there are 8 possible outcomes, and each outcome has an equal probability of $1/8$ or 0.125. If we know that the card drawn is less than 4, then there are 3 possible outcomes: 1, 2, and 3. The probability that the card drawn is even given that it is less than 4 is the ratio of the number of outcomes that are both even and less than 4 to the total number of outcomes that are less than 4.

The even numbers less than 4 are 2, so the number of outcomes that are both even and less than 4 is 1. The total number of outcomes that are less than 4 is 3. Therefore, the probability that the card drawn is even given that it is less than 4 is $1/3$ or 0.33.

So the correct answer is (a) 0.33.

7. A die is thrown twice and the sum of the numbers appearing is observed to be 7. What is the conditional probability that the number 6 has appeared at least on one of the die?

a) 0.45

b) 0.37

c) 0.33

d) 0.89

Ans: When a die is thrown twice, there are $6 \times 6 = 36$ possible outcomes. Let A be the event that the number 6 has appeared at least once, and B be the event that the sum of the numbers appearing is 7. We want to find the conditional probability $P(A|B)$, which is the probability that the number 6 has appeared at least once given that the sum of the numbers appearing is 7.

To find $P(A|B)$, we can use Bayes' theorem:

$$P(A|B) = P(B|A) * P(A) / P(B)$$

where $P(B|A)$ is the probability of getting a sum of 7 given that the number 6 has appeared at least once, $P(A)$ is the probability of the number 6 appearing at least once, and $P(B)$ is the probability of getting a sum of 7.

The probability of getting a sum of 7 given that the number 6 has appeared at least once can be found by counting the number of outcomes in which the sum is 7 and at least one die shows a 6. There are four such outcomes: (1,6), (2,5), (3,4), and (4,3). Therefore, $P(B|A) = 4/11$.

The probability of the number 6 appearing at least once can be found by counting the number of outcomes in which at least one die shows a 6. There are 11 such outcomes: (1,6), (2,6), (3,6), (4,6), (5,6), and (6,1), (6,2), (6,3), (6,4), (6,5), and (6,6). Therefore, $P(A) = 11/36$.

The probability of getting a sum of 7 can be found by counting the number of outcomes in which the sum is 7. There are six such outcomes: (1,6), (2,5), (3,4), (4,3), (5,2), and (6,1). Therefore, $P(B) = 6/36 = 1/6$.

Substituting these values into Bayes' theorem, we get:

$$P(A|B) = (4/11) * (11/36) / (1/6) = 4/9 = .444$$

So the correct answer is not given among the options provided.

8. Consider the experiment of tossing a coin. If the coin shows tail, toss it again but if it shows head, then throw a die. Find the conditional probability of the event that 'the die shows a number greater than 4' given that 'there is at least one Head'.

a) 0.1

b) 0.22

c) 0.38

d) 0.45

Ans: We need to find the conditional probability of the event that 'the die shows a number greater than 4' given that 'there is at least one head'. Let A be the event that the die shows a number greater than 4, and let B be the event that there is at least one head.

We can apply Bayes' theorem to find $P(A|B)$:

$$P(A|B) = P(B|A) * P(A) / P(B)$$

where $P(B|A)$ is the probability of getting at least one head given that the die shows a number greater than 4, $P(A)$ is the probability that the die shows a number greater than 4, and $P(B)$ is the probability of getting at least one

head.

First, let's find $P(B)$. There are two ways that we can get at least one head: either the first coin toss shows a head and we throw a die, or the first coin toss shows a tail, the second coin toss shows a head, and we throw a die. The probability of the first case is $1/2 * 1/6 = 1/12$, and the probability of the second case is $1/2 * 1/2 * 1/6 = 1/24$. So, the probability of getting at least one head is $1/12 + 1/24 = 1/8$.

Next, let's find $P(A)$. Since we throw a die only when the first coin toss shows a tail, the probability that the die shows a number greater than 4 is $2/6 = 1/3$.

Finally, let's find $P(B|A)$. If the die shows a number greater than 4, we know that the first coin toss showed a head. So, the probability of getting at least one head given that the die shows a number greater than 4 is 1.

Substituting these values into Bayes' theorem, we get:

$$\begin{aligned} P(A|B) &= P(B|A) * P(A) / P(B) \\ &= 1 * (1/3) / (1/8) \\ &= 8/3 \end{aligned}$$

So the correct answer is not given among the options provided.

9. There are three persons Evan, Ross and Michelle. These people lined up randomly for a picture. What is the probability of Ross being at one of the ends of the line?

- a) 0.66
- b) 0.45
- c) 0.23
- d) 0.56

Ans: There are three people, and each of them has an equal probability of being at any position in the line.

The total number of possible ways they can line up is $3! = 6$.

If Ross is at one of the ends, there are two possible positions for Ross (either at the beginning or at the end of the line). Once Ross is fixed, there are two remaining positions for the other two people. Thus, there are $2 * 2! = 4$ possible ways for Ross to be at one of the ends of the line.

Therefore, the probability of Ross being at one of the ends of the line is $4/6 = 2/3$.

So, the correct answer is **(a) 0.66**.

10. Let us make an assumption that each born child is equally likely to be a boy or a girl. Now suppose, if a family has two children, what is the conditional probability that both are girls given that at least one of them is a girl?

- a) 0.33
- b) 0.45
- c) 0.56
- d) 0.26

Ans: We can use Bayes' theorem to find the conditional probability of the event that both children are girls given that at least one of them is a girl. Let A be the event that both children are girls, and let B be the event that at least one of them is a girl.

First, let's find the probability of A, which is the probability that both children are girls. Since each child can be either a boy or a girl with equal probability, there are four equally likely outcomes: BB, BG, GB, and GG, where B represents a boy and G represents a girl. Out of these four outcomes, only one corresponds to both children being girls. Therefore,

$$P(A) = 1/4.$$

Next, let's find the probability of B, which is the probability that at least one of the children is a girl. There are three possible outcomes where at least one child is a girl: BG, GB, and GG. Out of these three outcomes, two correspond to the event A. Therefore, $P(B) = 2/4 = 1/2$.

Now we can use Bayes' theorem to find the conditional probability of A given B:

$$P(A|B) = P(B|A) * P(A) / P(B)$$

where $P(B|A)$ is the probability of at least one child being a girl given that both children are girls. Since both children are girls, the probability that at least one of them is a girl is 1. Therefore, $P(B|A) = 1$.

Substituting the values we have found, we get:

$$P(A|B) = P(B|A) * P(A) / P(B) = 1 * 1/4 / 1/2 = 1/2$$

Therefore, the conditional probability that both children are girls given that at least one of them is a girl is $1/2$ or 0.5 .

So, the correct answer is not given among the options provided.

11. Consider the same case as in the question no. 10. It is given that elder child is a boy. What is the conditional probability that both children are boys?

- a) 0.33
- b) 0.23
- c) 0.5
- d) 0.76

Ans: Let A be the event that both children are boys, and let B be the event that the elder child is a boy.

We are given that the elder child is a boy, which means that one child is a boy. So, the possible outcomes are BB, BG, and GB, with equal probability.

Out of these outcomes, only one corresponds to both children being boys. Therefore, the probability of A given B is:

$$P(A|B) = P(A \text{ and } B) / P(B)$$

The probability of A and B is the probability that both children are boys and the elder child is a boy, which is equal to 1/4.

The probability of B is the probability that the elder child is a boy, which is equal to 1/2 (since the gender of each child is independent and equally likely to be a boy or a girl).

Therefore, the conditional probability of both children being boys given that the elder child is a boy is:

$$P(A|B) = (1/4) / (1/2) = 1/2$$

So, the correct answer is (c) 0.5.

12. We toss a coin. If we get head, we toss a coin again and if we get tail we throw a die. What is the probability of getting a number greater than 4 on die?

- a) 0.166
- b) 0.34
- c) 0.78
- d) 0.25

Ans: The probability of getting a tail on the first toss of the coin is 1/2. If we get a tail, we throw a die, and the probability of getting a number greater than 4 is 2/6, or 1/3. So, the probability of getting a tail and a number greater than 4 on the die is:

$$P(\text{Tail and Die} > 4) = P(\text{Tail}) * P(\text{Die} > 4 | \text{Tail}) = (1/2) * (1/3) = 1/6$$

If we get a head on the first toss of the coin, we toss another coin. The probability of getting a head on this second toss is 1/2, and if we get a head, we have to start over and toss the first coin again. If we get a tail on the second toss, we throw a die, and the probability of getting a number greater than 4 is again 1/3. So, the probability of getting a head on the second toss, followed by a tail and a number greater than 4 on the die is:

$$P(\text{Head-Tail-Die} > 4) = P(\text{Head}) * P(\text{Tail}) * P(\text{Die} > 4 | \text{Head and Tail}) = (1/2) * (1/2) * (1/3) = 1/12$$

The total probability of getting a number greater than 4 on the die is the sum of these two probabilities:

$$P(\text{Die} > 4) = P(\text{Tail and Die} > 4) + P(\text{Head-Tail-Die} > 4) = 1/6 + 1/12 = 1/4$$

So, the correct answer is not one of the choices given. The correct answer is:

(d) 0.25.

13. We toss a coin. If we get head, we toss a coin again and if we get tail we throw a die. What is the probability of getting an odd number on die?

- a) 0.345
- b) 0.79
- c) 0.2
- d) 0.25

Ans: The probability of getting a tail on the first toss of the coin is $1/2$. If we get a tail, we throw a die, and the probability of getting an odd number is $3/6$ or $1/2$. So, the probability of getting a tail and an odd number on the die is:

$$P(\text{Tail and Odd}) = P(\text{Tail}) * P(\text{Odd} | \text{Tail}) = (1/2) * (1/2) = 1/4$$

If we get a head on the first toss of the coin, we toss another coin. The probability of getting a head on this second toss is $1/2$, and if we get a head, we have to start over and toss the first coin again. If we get a tail on the second toss, we throw a die, and the probability of getting an odd number is still $1/2$. So, the probability of getting a head on the second toss, followed by a tail and an odd number on the die is:

$$P(\text{Head-Tail-Odd}) = P(\text{Head}) * P(\text{Tail}) * P(\text{Odd} | \text{Head and Tail}) = (1/2) * (1/2) * (1/2) = 1/4$$

The total probability of getting an odd number on the die is the sum of these two probabilities:

$$P(\text{Odd}) = P(\text{Tail and Odd}) + P(\text{Head-Tail-Odd}) = 1/4 + 1/4 = 1/2$$

So, the correct answer is not one of the choices given. The correct answer is: **(d) 0.25.**

14. Suppose we throw two dice together. What is the conditional probability of getting sum of two numbers found on the two die after throwing is less than 4, provided that the two numbers found on the two die are different?

- a) 0.3
- b) 0.56
- c) 0.24
- d) 0.06

Ans: There are 36 possible outcomes when two dice are thrown together, since each die has 6 possible outcomes. Out of these, there are only 12 outcomes where the numbers on the two dice are different and their sum is less than 4: (1,2), (2,1), (1,3), (3,1), (1,2), (2,1).

Therefore, the conditional probability of getting a sum of two numbers less than 4 given that the two numbers on the two dice are different is:

$$P(\text{sum} < 4 | \text{different numbers}) = 6/12 = 0.5$$

So the correct answer is not listed in the options given.

15. A box contains three coins: two regular coins and one fake two-headed coin, you pick a coin at random and toss it. What is the probability that it lands heads up?

- a) $1/3$
- b) $2/3$
- c) $1/2$
- d) $3/4$

Ans: The probability of picking each coin is $1/3$. If a regular coin is picked, the probability of it landing heads up is $1/2$. If the two-headed coin is picked, the probability of it landing heads up is 1. Therefore, the overall probability of the picked coin landing heads up is:

$$(2/3) * (1/2) + (1/3) * 1 = 2/6 + 3/6 = 5/6$$

So the correct answer is not listed in the options given. The probability is actually higher than any of the given options.