

Time: 2H.

Marks:  $(4 \times 10)$

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**Note:** All the computations must be up to four decimal point precision.

1. Suppose  $f : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  defined as:

$$f_1(x, y) = x^2 - 2x + y^2$$

$$f_2(x, y) = x^2 + y^2 - 1$$

Use Newton's method for the approximation of  $x$  and  $y$  when  $X^{(0)} = [0, -1]^T$ . Perform two iterations.

2. Produce a tabulated data set for the function,  $f(x) = \log_{10}(x)$ , from  $x = 1$  to  $x = 10$  such that error in the linear interpolation is less than or equal to  $10^{-4}$ . Find the bound for the step-size (for independent variable) for this table. What value of step-size will guarantee that  $x = 10$  is included in the data set?
3. Find the value of  $\omega_{opt}$  which will produce the fastest convergence for SOR method for the following linear system:

$$3x_1 - x_2 + x_3 = 1$$

$$3x_1 + 6x_2 + 2x_3 = 0$$

$$3x_1 + 3x_2 + 7x_3 = 4$$

Perform two iterations of SOR with  $X^0 = [0, 0, 0]^T$ .

4. Following data set defines concentration of dissolved oxygen ( $o$ ) in a river, as a function of temperature ( $T$ ):

$T (^{\circ}C) :$	0	10	20	30
$o (mg/L) :$	14.6211	11.8431	9.8701	8.4181

Determine the cubic splines fit for this data set. Find the value of  $o(19)$ .

\*\*\*End of Part QP\*\*\*