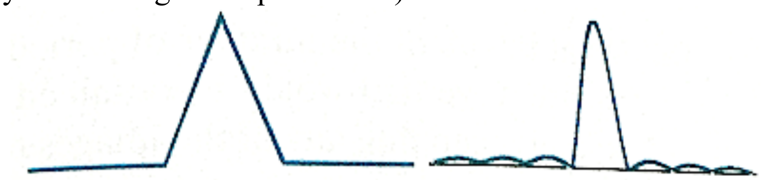
Assignment 3
CSC 8260 – Advanced Image Processing
Due Date: February 24th Friday, 2023, 11:59 pm

Q1. As the figure below shows the Fourier transform of a tent function(on the left) is a squared sinc function(on the right). Advance an argument that shows that the Fourier transform of a tent function can be obtained from the Fourier transformation of a box function. (Hint: The tent itself can be generated by convolving two equal boxes.)



Q2. What is the convolution of two 1-D impulses:

a. $\delta(t)$ and $\delta(t-t_0)$
b. $\delta(t-t_0)$ and $\delta(t+t_0)$

Q3. Show that $\Im\{e^{j2\pi t_0 t}\} = \delta(\mu-t_0)$, where t0 is a constant. (Hint: Study Example 4.2 to be followed from textbook, pasting sample below for reference)

EXAMPLE 4.2: Fourier transform of an impulse and an impulse train.

The Fourier transform of a unit impulse located at the origin follows from Eq. (4-20):

$$\Im\{\delta(t)\} = F(\mu) = \int_{-\infty}^{\infty} \delta(t) e^{-j2\pi\mu t} dt = \int_{-\infty}^{\infty} e^{-j2\pi\mu t} \delta(t) dt = e^{-j2\pi\mu}$$

where we used the sifting property from Eq. (4-12). Thus, we see that the Fourier transform of an impulse located at the origin of the spatial domain is a constant in the frequency domain (we discussed this briefly in Section 3.4 in connection with Fig. 3.30).

Similarly, the Fourier transform of an impulse located at $t = t_0$ is

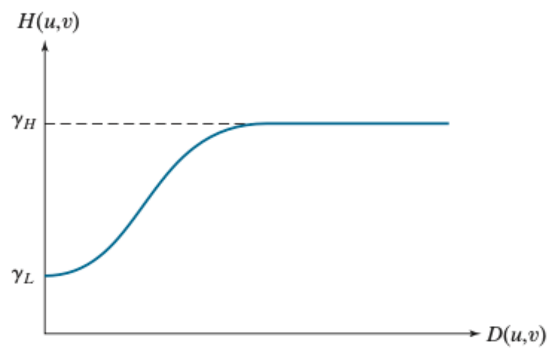
$$\Im\{\delta(t-t_0)\} = F(\mu) = \int_{-\infty}^{\infty} \delta(t-t_0) e^{-j2\pi\mu t} dt = \int_{-\infty}^{\infty} e^{-j2\pi\mu t} \delta(t-t_0) dt = e^{-j2\pi\mu t_0}$$

Q4. Show that the following expressions are true.(Hint: Make use of the solution to Problem Q3):

a. $\Im\{\cos(2\pi\mu_0 t)\} = \frac{1}{2}[\delta(\mu-\mu_0) + \delta(\mu+\mu_0)]$
b. $\Im\{\sin(2\pi\mu_0 t)\} = \frac{1}{2j}[\delta(\mu-\mu_0) - \delta(\mu+\mu_0)]$

Q5. Use a Butterworth highpass filter to construct a homomorphic filter transfer function that has the same general shape as the function in Fig. 4.59.

FIGURE 4.59 Radial cross section of a homomorphic filter transfer function..



Q6. Highpass fiter transfer functions

a. Write a function H = hpFilterTF4e(type, P, Q, param) to generate a P X Q high pass filter transfer function, H, with the following properties, If type = ‘ideal’, param should be a scalar equal to the cut-off frequency D_0 in Eq. (4-119). If type = ‘gaussian’, param should be a scalar equal to the standard deviation D_0, in Eq(4-120). If type = ‘butterworth’, param should be a 1x2 array (vector) the cutoff frequents and filter order, [D_0, n], in Eq (4-121)

b. Generate an ideal highpass filter transfer function of size 512 × 512 with D_0 = 96. Display your result as an image.

c. Generate a highpass Gaussian filter transfer function of size 512 × 512 with D_0 = 96. Display your result as an image.

d. Generate a highpass Butterworth filter transfer function of size 512 × 512. Choose D_0 = 96 and n = 2.

IDEAL, GAUSSIAN, AND BUTTERWORTH HIGHPASS FILTERS FROM LOWPASS FILTERS

As was the case with kernels in the spatial domain (see Section 3.7), subtracting a lowpass filter transfer function from 1 yields the corresponding highpass filter transfer function in the frequency domain:

$$H_{HP}(u,v) = 1 - H_{LP}(u,v) \tag{4-118}$$

where $H_{LP}(u,v)$ is the transfer function of a lowpass filter. Thus, it follows from Eq. (4-111) that an ideal highpass filter (IHPF) transfer function is given by

$$H(u,v) = \begin{cases} 0 & \text{if } D(u,v) \le D_0 \\ 1 & \text{if } D(u,v) > D_0 \end{cases} \tag{4-119}$$

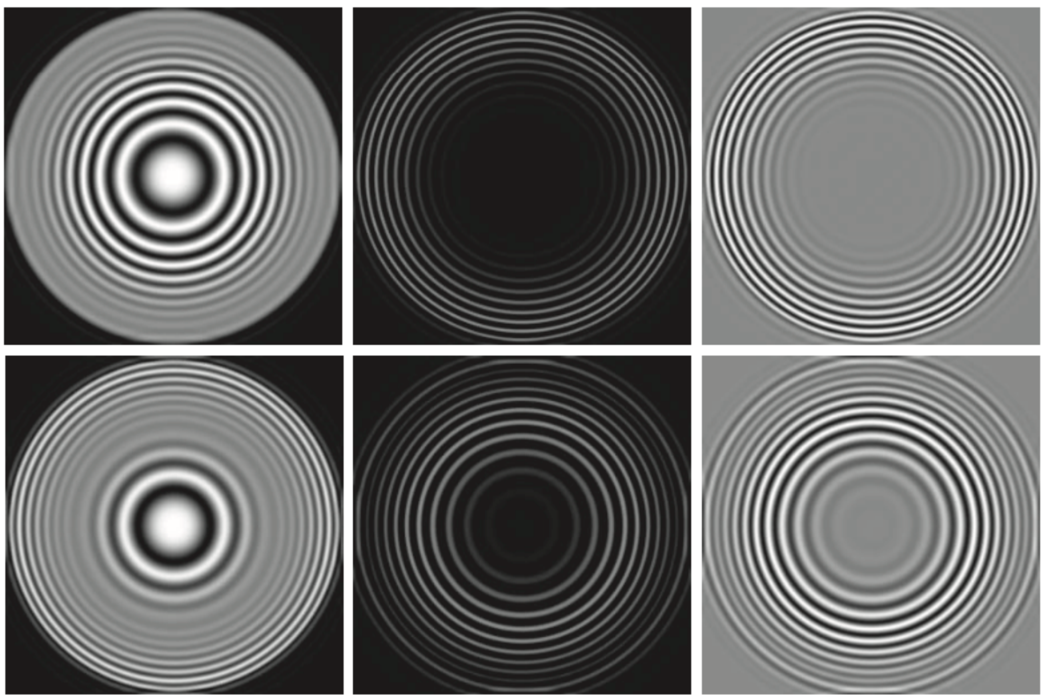
The above equation is referred in 4th edition. Use below image to find the equations in other editions of textbook.

Q7. Lowpass filtering in the frequency domain. Show the image resulting after each of the following:

a. Read the image testpattern512.tif and lowpass filter it using a Gaussian filter so that the large letter "a" is barely readable, and the other letters are not.

b. Read the image testpattern512.tif. Lowpass filter it using a Butterworth filter of your specification so that, when thresholded, the filtered image contains only part of the large square on the top, right. (Hint: It is more intuitive to work with the negative of the original image.)

Q8. The objective of this project is to duplicate the results in below using frequency domain filtering. The image used is zoneplate.png. For consistency scale the intensity range of this image to [0,1] using MATLAB function im2double or project function intScaling4e with the default setting. To simplify experimenting, do the filtering without padding. (Hint: To separate frequency bands requires filters with sharp cutoffs, so use Butterworth filter transfer functions with n = 3 throughout.)



Extra Credit

Q1. Show the validity of the following 2-D discrete Fourier transform pairs from Table 4.4 of textbook(check the below screenshot of the table from 4th edition and refer with the one in your textbook)

a. $\delta(x,y) \Leftrightarrow 1$
b. $1 \Leftrightarrow MN\delta(u,v)$
c. $\delta(x-x_0,y-y_0) \Leftrightarrow e^{-j2\pi(\frac{ux_0}{M}+\frac{vy_0}{N})}$
d. $e^{j2\pi(\frac{ux_0}{M}+\frac{vy_0}{N})} \Leftrightarrow MN\delta(u-u_0,v-v_0)$
e. $\cos\left(\frac{2\pi u_0 x}{M}+\frac{2\pi v_0 y}{N}\right) \Leftrightarrow (MN/2)\,[\delta(u+u_0,v+v_0)+\delta(u-u_0,v-v_0)]$

f. $\sin\left(\frac{2\pi u_0 x}{M} + \frac{2\pi v_0 y}{N}\right) \Leftrightarrow (jMN/2)\,[\delta(u+u_0,v+v_0) - \delta(u-u_0,v-v_0)]$

TABLE 4.4 Summmary of DFT pairs. The closed-form expressions in 12 and 13 are valid only for continuous variables. They can be used with discrete variables by sampling the continuous expressions.

	Name	DFT Pairs
1)	Symmetry properties	See Table 4.1
2)	Linearity	$af_1(x,y) + bf_2(x,y) \Leftrightarrow aF_1(u,v) + bF_2(u,v)$
3)	Translation (general)	$f(x,y)e^{j2\pi(u_0x/M + v_0y/N)} \Leftrightarrow F(u-u_0, v-v_0)$ $f(x-x_0, y-y_0) \Leftrightarrow F(u,v)e^{-j2\pi(ux_0/M + vy_0/N)}$
4)	Translation to center of the frequency rectangle, $(M/2, N/2)$	$f(x,y)(-1)^{x+y} \Leftrightarrow F(u-M/2, v-N/2)$ $f(x-M/2, y-N/2) \Leftrightarrow F(u,v)(-1)^{u+v}$
5)	Rotation	$f(r, \theta+\theta_0) \Leftrightarrow F(\omega, \varphi+\theta_0)$ $r = \sqrt{x^2+y^2}$ $\theta = \tan^{-1}(y/x)$ $\omega = \sqrt{u^2+v^2}$ $\varphi = \tan^{-1}(v/u)$
6)	Convolution theorem†	$f \star h)(x,y) \Leftrightarrow (F \bullet H)(u,v)$ $(f \bullet h)(x,y) \Leftrightarrow (1/MN)[(F \star H)(u,v)]$

Q2. Consider the images shown. The image on the right was obtained by: (a) multiplying the image on the left by $(-1)^{x+y}$; (b) computing the DFT;(c) taking the complex conjugate of the transform; (d) computing the inverse DFT; and (e) multiplying the real part of the result by $(-1)^{x+y}$. Explain (mathematically) why the image on the right appears as it does.