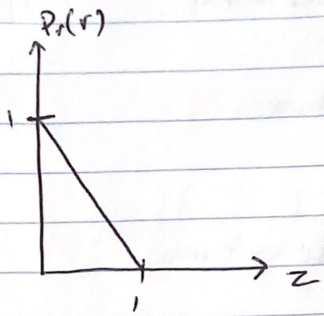
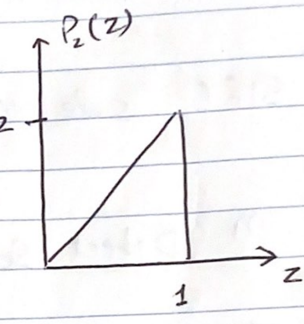
1.





Expression of $P_r(r)$ in terms of r

$$P_r(r) = 2(1-r)$$

Doing histogram equalization transformation

$$S = T(r)$$
$$= \int_0^r P_r(w)\,dw$$
$$= 2\left[r - \frac{r^2}{2}\right]$$
$$S = 2r - r^2$$

Exp. for $P_z(z)$ in terms of z

$$P_z(z) = 2z$$
$$V = T(z)$$
$$= \int_0^z P_z(x)\,dx$$

Equate V & S to get the f^n in terms of r and z

$$z^2 = 2r - r^2 \Rightarrow \boxed{z = \sqrt{2r - r^2}}$$

4.a) $V = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ $W^T = \begin{bmatrix} 2 & 1 & 1 & 3 \end{bmatrix}$

$$VW^T = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 1 & 3 \end{bmatrix}$$

$$VW^T = \begin{bmatrix} 2 & 1 & 1 & 3 \\ 4 & 2 & 2 & 6 \\ 2 & 1 & 1 & 3 \end{bmatrix}$$

A kernel is separable if it can be expressed as the outer product of two vectors. As VW^T can be expressed as the element wise product of V and W^T it is **separable.**

b) $W = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 6 & 2 \end{bmatrix}$

STEP 1: Choose a non zero element

$E \rightarrow 6$

STEP 2: Column & row for that specific element

$\therefore \quad C = \begin{bmatrix} 3 \\ 6 \end{bmatrix} \qquad R = \begin{bmatrix} 2 & 6 & 2 \end{bmatrix}$

STEP 3: Let $V = C$ and $W^T = R/E$

$V = \begin{bmatrix} 3 \\ 6 \end{bmatrix} \qquad W^T = \left[1/3, 1, 1/3 \right]$

$\therefore W_1 = V$ & $W_2 = W^T$

c) STEP 1: $E = 2$

STEP 2: $C = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ $R = \begin{bmatrix} 4 & 2 & 2 & 6 \end{bmatrix}$

STEP 3: $V = C = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ $W^T = \frac{R}{E} = \begin{bmatrix} 2 & 1 & 1 & 3 \end{bmatrix}$

We get,

$W_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ $W_2 = \begin{bmatrix} 2 & 1 & 1 & 3 \end{bmatrix}$

7. a) The Laplacion kernels are not separable as their rank is not 1

a = [[0,1,0] [1,-4,1] [0,1,0]]

rank(a) = 2

b = [[1,1,1] [1,-8,1] [1,1,1]]

rank(b) = 2

b) The Robert cross gradient kernels are not sepearable as their rank is not 1.

a = [[-1,0] [0,-1]]

rank(a) = 2

c = [[0,-1] [-1,0]]

rank(c) = 2

c) The sobel kernels are separable as their ranks are 1.