1. From fourier transform we know, the tent function is a convolution of 2 equal box function

$$f(t) = g(t) * g(t)$$

Also we know the FT of a box function is a Sinc function and using the fourier transform we get:

$$F[f(t)] = F[g(t) * g(t)]$$

$$= Sinc(u) . Sinc(u)$$

$$= Sinc^2(u)$$

2. a) $\delta(t) * \delta(t-t_0)$

Using fourier transform we get:

$$= \int_{-\infty}^{\infty} \delta(\tau)\delta(t-t_0-\tau)\,d\tau$$

we know that the FT of impulse $\delta(t)$ is 1 if $t=0$:
therefore, $t-t_0-\tau = 0$ giving us $\tau = t-t_0$

Substituting the value of τ in the 1st equation we get:

$$= \int_{-\infty}^{\infty} \delta(t-t_0)\,\delta(t-t_0-t-t_0)\,d\tau$$

$$= \delta(t-t_0) \int_{-\infty}^{\infty} \delta(0)\,d\tau$$

Giving us $\delta(t-t_0)$

2. b) $\delta(t-t_0)$ and $\delta(t+t_0)$

Using Fourier we get,

$$\delta(t-t_0) * \delta(t+t_0) = \int_{-\infty}^{\infty} \delta(\tau - t_0)\,\delta(t+t_0-\tau)\, d\tau$$

The fourier transform $\delta(t)$ of impulse is 1 if $t=0$.

$\therefore$, $t+t_0-\tau=0$ giving us $\tau = t+t_0$

Substituting the value of τ in the 1st equation:

$$= \int_{-\infty}^{\infty} \delta(t+t_0-t_0)\,\delta(t+t_0-t-t_0)\, d\tau$$

$$= \int_{-\infty}^{\infty} \delta(t)\,\delta(0)\, d\tau$$

$$= \delta(t) \int_{-\infty}^{\infty} \delta(0)\, d\tau$$

$$= \delta(t)$$

6.1) $\pi_{Model, price}(\sigma_{Name='Aston Martin'}(Car \bowtie Manufacturer))$

2) $\pi_{VIN}(\sigma_{Name='James Bond'}(Car \bowtie Customer))$

3) $\pi_{model}(\sigma_{Price<100000 \text{ AND } year \geq 2016 \text{ AND } Name='Ford'}(Car \bowtie Manufacturer))$

7.1) $\pi_{Username}(\pi_{SellerUsername}(Auction) \cup \pi_{BuyerUsername}(Sold))$

2) $R = Auction \bowtie Sold$

$S = \pi_{SellerUsername}(\sigma_{Auction.AuctionID = Sold.AuctionID}(R))$

$T = \pi_{SellerUsername}(Auction) - S$

$Result = \pi_{Username}(T)$

3) $R = Auction \bowtie (\pi_{AuctionID}(Sold))$

$Result = \pi_{Auction.AuctionID}(\sigma_{Auction.EndTime < CurrentTime \text{ AND } Auction.auctionID \text{ NOT IN } (\pi_{Sold.auctionID}(Sold))}(R))$

4) $R = Auction \bowtie Item$

$S = R - ((R \bowtie (\rho_{MaxPrice}(\pi_{Price}(Auction))))$
$- (\pi_{ItemID, SellerUsername, Price}(R)))$

$Result = \pi_{Item.itemName, Auction.SellerUsername}(S)$

OMIT

5) Butterworth filter can be defined as,

$$H(u,v) = \frac{1}{1+\left[\frac{D_0}{D(u,v)}\right]^{2n}}$$

Using this we can construct the homomorphic filter as:

$$H(u,v) = (\gamma_H - \gamma_L)\left[\frac{1}{1+\left[\frac{D_0}{\left[1+[D(u,v)]^{2n}\right]}\right]} + \gamma_L\right]$$

~~BONUS~~

BON

a) $\sum_{x=0}^{M-1}$

b) $\frac{1}{MN}$

c) $\sum_{x=0}^{a-1}$

d) $\frac{1}{AB}$

horic filter a:

$+ Y_2$

BONUS 1

a) $$\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} \delta(x-y)\, e^{-j2\pi\left(\frac{xu}{M}+\frac{yv}{N}\right)}$$

$e^0 = 1$

$\delta(0,0) = 1$

$x = y = 0$

b) $$\frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} MN\,\delta(u,v)\, e^{j2\pi\left(\frac{xu}{M}+\frac{yv}{N}\right)}$$

We know $u = v = 0$

$= \underline{\underline{1}}$

c) $$\sum_{x=0}^{a-1}\sum_{y=0}^{b-1} \delta(x-x_0,\ y-y_0)\, e^{-j2\pi\left(\frac{xu}{a}+\frac{yv}{b}\right)}$$

$x - x_0 = 0$; $x = x_1$

$y - y_0 = 0$ $\quad y = y_0$

$\Rightarrow e^{-j2\pi\left(\frac{x_0u}{a}+\frac{y_0v}{b}\right)} \Rightarrow e^{-j2\pi\left(\frac{X_0u}{M}+\frac{Y_0v}{N}\right)}$

d) $$\frac{1}{\cancel{AB}}\sum_{x=0}^{A-1}\sum_{y=0}^{B-1} \cancel{AB}\,\delta(u-u_0,\ v-v_0)\, e^{j2\pi\left(\frac{xu}{A}+\frac{yv}{B}\right)}$$

$u - u_0 = 0$; $u = u_0$

$v - v_0 = 0$; $v = v_0$

$= e^{j2\pi\left(\frac{xu_0}{A}+\frac{yv_0}{B}\right)}$

$\Rightarrow e^{j2\pi\left(\frac{xu_0}{M}+\frac{yv_0}{N}\right)}$

e) $$\frac{1}{AB}\sum_{x=0}^{A-1}\sum_{y=0}^{B-1}\frac{AB}{2}\left[\delta(u+u_0, v+v_0)+\delta(u-u_0, v-v_0)\right] e^{-j2\pi\left(\frac{xu}{A}+\frac{yv}{B}\right)}$$

$$= \frac{1}{2}\sum_{x=0}^{A-1}\sum_{y=0}^{B-1}\left[\delta(u+u_0, v+v_0)\, e^{-j2\pi\left(\frac{xu}{A}+\frac{yv}{B}\right)}\right.$$

$u+u_0=0 ; u=-u_0$

$v+v_0=0 ; v=-v_0$

$$\left. + \underbrace{\delta(u-u_0, v-v_0)\, e^{-j2\pi\left(\frac{x_0u}{A}+\frac{y_0v}{B}\right)}}\right]$$

$u-u_0=0 \;;\; u=u_0$

$v-v_0=0 \;;\; v=v_0$

$$\Rightarrow \frac{1}{2}\left[e^{-j2\pi\left(-\frac{xu_0}{A}-\frac{v_0y}{B}\right)} + e^{-j2\pi\left(\frac{xu_0}{A}+\frac{v_0y}{B}\right)}\right]$$

$$= \frac{1}{2}\left[e^{j2\pi\left(\frac{xu_0}{A}+\frac{yv_0}{B}\right)} + e^{-j2\pi\left[\frac{xu_0}{A}+\frac{yv_0}{B}\right]}\right]$$

$$= \cos\left(\frac{2\pi x_0}{A}+\frac{2\pi y_0}{B}\right)$$

$$= \cos\left(\frac{2\pi x_0}{M}+\cancel{u_0}\frac{2\pi y v_0}{N}\right)$$

BONUS

2. During the process of obtaing the right side image we are taking the conjugate and performing the inverse DFT ($\mathfrak{J}^{-1}$) on it. The complex conjugate basically transforming a variable in its negative form, for example, a to -a in the inverse transform.

Now, if $f(x,y)$ is the orginial image with $M \times N$ size and $F(u,v)$ is the DFT then in general the image of the right side can be given as:

$$\mathfrak{J}^{-1}\left[F^*(u,v)\right] = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} F(u,v)\, e^{-j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

$$\mathfrak{J}^{-1}\left[F^*(u,v)\right] = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} F(u,v)\, e^{j2\pi\left(\frac{u(-x)}{M}+\frac{v(-y)}{N}\right)}$$

$$\mathfrak{J}^{-1}\left[F^*(u,v)\right] = f(-x,-y)$$

As, $f(x,y)$ is the original image then $f(-x,-y)$ is basically mirroring the orginal image with respect to the origin which is resulting the image on the right side.